Estimating Volatilities and Correlation

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Abstract

The purpose of this vignette is to demonstrate methods for estimating volatility and correlation as outlined in Chapter 10 of Foundations of Risk Management.

Contents

1 Estimating Volatility

We define σ_n as the volatility on day n, as estimated at the end of day $n-1$. This section describes the standard approach to estimating σ_n from historical data.

First we define the continuously compounded return between the end of day $i-1$ and the end of day i.

$$u_i = \ln \frac{S_i}{S_{i-1}} \tag{1}$$

where:

S_i is the value of the market variable at the end of day i (e.g. asset prices)

An unbiased estimate of the variance rate per day on day n, σ_n^2, using the m most recent observations is

$$\sigma_n^2 = \frac{1}{m-1} \sum_{i=1}^{m} (u_{n-1} - \bar{u})^2 \tag{2}$$

where $\bar{u}$ is the mean of u_i for $i = 1, 2, ..., m$

$$\bar{u} = \frac{1}{m} \sum_{i=1}^{m} u_{n-1} \tag{3}$$

A few changes can be made to simplify the equation for monitoring daily volatility.

1. define u_i as the percentage change in the market variable between the end of day $i-1$ and the end of day i.
$$u_i = \frac{S_i - S_{i-1}}{S_i} \tag{4}$$

2. Assume $\bar{u}$ to be zero

3. Replace $m-1$ with m

These changes simplify the formula for the variance rate to

$$\sigma_n^2 = \frac{1}{m}\sum_{i=1}^{m} u_{n-1}^2 \tag{5}$$

This equation gives equal weight to each of the previous m observations. A model that allows one to assign weights to the previous observations is

$$\sigma_n^2 = \sum_{i=1}^{m} \alpha_i u_{n-1}^2 \tag{6}$$

Where α_i is the weight given to the observation i days ago.
TODO: add information about the ARCH model
A special case of equation is the Exponentially Weighted Moving Average (EWMA) Model.

2 Exponentially Weighted Moving Average Model

The Exponentially Weighted Moving Average (EWMA) Model is a special case of a weighted moving average where the weights α_i decrease exponentially as we move backwards through time. Greater weights are given to more recent observations.

This weighting scheme leads to simple formula for updating volatility estimates. The predictive version of the variance rate of day n is given as

$$\hat{\sigma}_n^2 = \lambda\hat{\sigma}_{n-1}^2 + (1-\lambda)u_{n-1}^2 \tag{7}$$

where:

$\hat{\sigma}_{n-1}^2$ is the estimated variance rate of period $n-1$

u_{n-1} is the return of preiod $n-1$

λ is a constant between 0 and 1

The value for λ determines how responsive the volatility estimate is to the most recent percentage change, u_{n-1}. A lower (higher) value for λ leads to a greater (lesser) weight given to u_{n-1}. One way to think of this is that values of λ close to 1 produce volatility estimates that respond relatively slow to new information coming into the market provided by u_{n-1}.

Load the package and data. Unless noted otherwise, the weekly returns of Microsoft (MSFT) will be used as the asset return data.

```r
library(GARPFRM)

## Loading required package:  xts
## Loading required package:  zoo
##
## Attaching package:  'zoo'
##
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
##
## Loading required package:  PerformanceAnalytics
##
## Attaching package:  'PerformanceAnalytics'
##
## The following object is masked from 'package:graphics':
##
##     legend

data(crsp_weekly)

# Use the weekly MSFT returns
R <- largecap_weekly[, "MSFT"]
```

Here we calculate the volatility estimates of the MSFT weekly returns using the EWMA model. We choose `lambda=0.94`. The RiskMetrics database, originally created by J.P. Morgan and made publicly available in 1994, uses the EWMA model with $\lambda = 0.94$ for updating daily volatility in its RiskMetrics database. An `initialWindow=15` is specified to use the first 15 periods to calculate the initial conditions, u_0 and σ_0.

```r
lambda <- 0.94
initialWindow <- 15
volEst <- EWMA(R, lambda, initialWindow, type="volatility")
volEst

## EWMA Estimate
##
## Parameters
## lambda: 0.94
## initialWindow: 15
## type: volatility
##
## Final Period EWMA Estimate:
##               MSFT
## 2010-12-28 0.03364
```

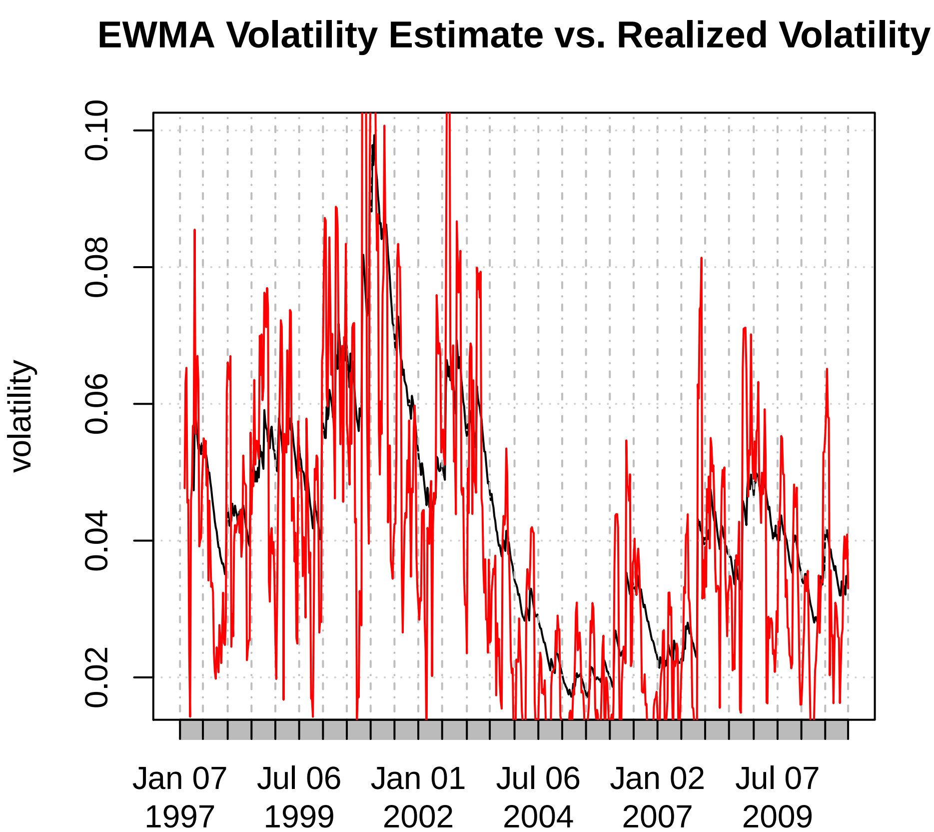
An important point to note is that we are using weekly returns to estimate weekly volatility while the lambda value used in the RiskMetrics database is for daily volatility. A data driven approach for selecting a value for λ is to determine the λ that minimizes the sum of squared errors between the realized volatility and the estimated volatility from the EWMA model.

Here we calculate the realized volatility defined as the equally weighted average of the standard deviation of the subsequent n periods.

```
vol <- realizedVol(R, n=5)
```

Now we plot the estimated volatility from the EWMA model and the realized volatility.

```
plot(volEst, main="EWMA Volatility Estimate vs. Realized Volatility")
lines(vol, col="red")
```



The `estimateLambda` function estimates the value for λ by minimizing the mean squared error between the realized volatility and the EWMA model estimated volatility.

```
# Estimate lambda
# Use initialWindow = 15 for the EWMA volatility estimate and
# n = 5 to calculate the realized volatility
lambda <- estimateLambdaVol(R, initialWindow, n=5)
lambda
```

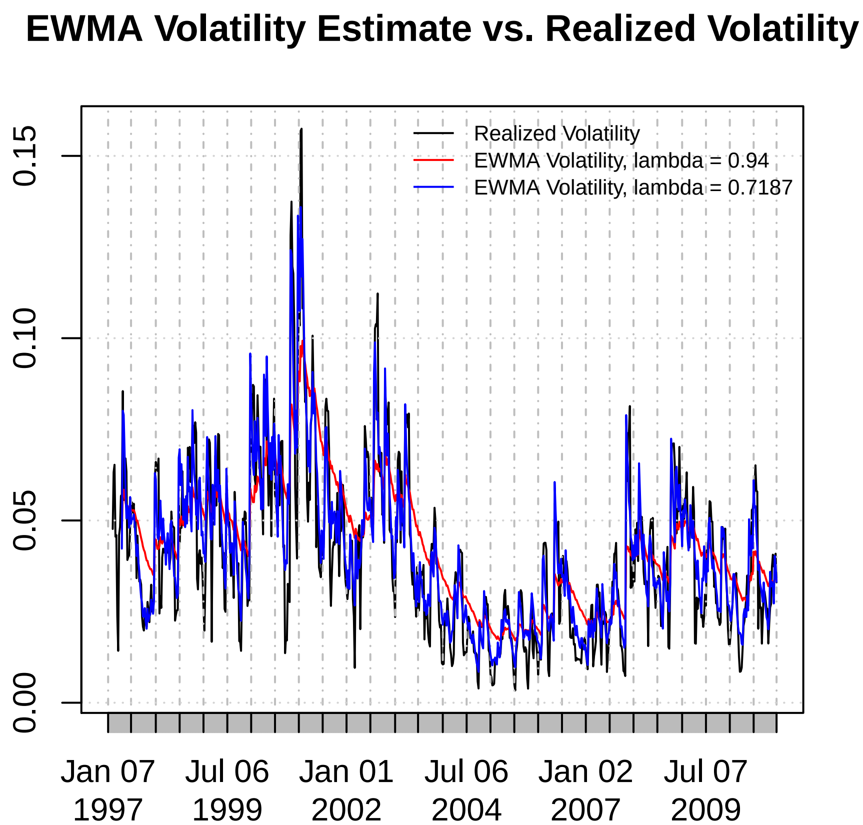
```
## [1] 0.7359
```

```
volEst2 <- EWMA(R, lambda, initialWindow, type="volatility")
volEst2

## EWMA Estimate
##
## Parameters
## lambda: 0.7359
## initialWindow: 15
## type: volatility
##
## Final Period EWMA Estimate:
##                MSFT
## 2010-12-28 0.03317
```

Here we plot the realized volatility along with the EWMA estimated volatility with $\lambda = 0.94$ and $\lambda = 0.7359253$ to gain intuition through visualization of the EWMA volatility estimates.

```
# Realized volatility
plot(vol, main="EWMA Volatility Estimate vs. Realized Volatility")
# EWMA volatility estimate, lambda = 0.94
lines(volEst$estimate, col="red")
# EWMA volatility estimate, lambda = 0.7187
lines(volEst2$estimate, col="blue")
legend("topright", legend=c("Realized Volatility",
                            "EWMA Volatility, lambda = 0.94",
                            "EWMA Volatility, lambda = 0.7187"),
       col=c("black", "red", "blue"), lty=c(1, 1, 1), cex=0.7, bty="n")
```



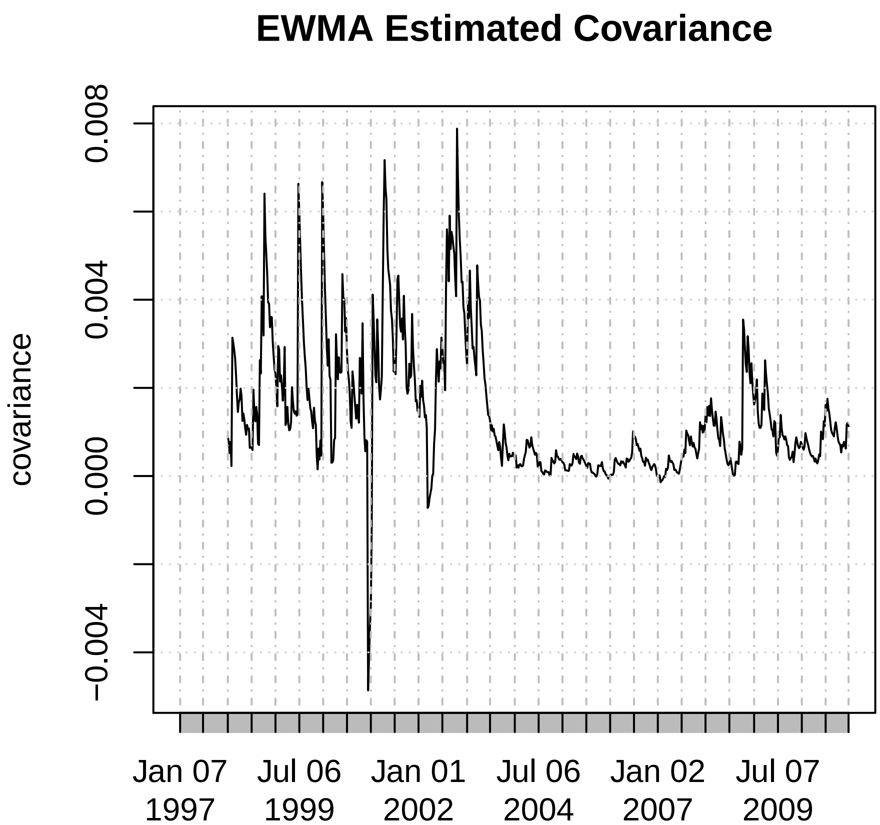
Now we move to using the EWMA model to calculate the covariance between the returns of two assets. Note that we set `lambda=NULL` in the `EWMA` function. If `lambda = NULL`, the optimal λ value is estimated by minimizing the mean squared error between the estimated covariance and realized covariance.

```
# Use the first 2 columns of the large
R <- largecap_weekly[,1:2]
initialWindow <- 52
covEst <- EWMA(R, lambda=NULL, initialWindow, n=10, "covariance")
covEst

## EWMA Estimate
##
## Parameters
## lambda: 0.8665
## initialWindow: 52
## type: covariance
##
## Final Period EWMA Estimate:
```

```
##              ORCL.MSFT
## 2010-12-28   0.00113

plot(covEst, main="EWMA Estimated Covariance")
```



In a similar fashion, we can also use the EWMA model to calculate the correlation between the returns of two assets.

```
corEst <- EWMA(R, lambda=NULL, initialWindow, n=10, "correlation")
corEst

## EWMA Estimate
##
## Parameters
## lambda: 0.8404
## initialWindow: 52
## type: correlation
##
```

```
## Final Period EWMA Estimate:
##              ORCL.MSFT
## 2010-12-28     0.831

plot(corEst, main="EWMA Estimated Correlation")
```