Estimating Volatilities and Correlation

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Abstract

The purpose of this vignette is to demonstrate methods for estimating volatility and correlation as outlined in Chapter 10 of Foundations of Risk Management.

Contents

1 Estimating Volatility

We define σ_n as the volatility on day n, as estimated at the end of day $n-1$. This section describes the standard approach to estimating σ_n from historical data.

First we define the continuously compounded return between the end of day $i-1$ and the end of day i.

$$u_i = \ln \frac{S_i}{S_{i-1}} \tag{1}$$

where:

S_i is the value of the market variable at the end of day i (e.g. asset prices)

An unbiased estimate of the variance rate per day on day n, σ_n^2, using the m most recent observations is

$$\sigma_n^2 = \frac{1}{m-1} \sum_{i=1}^{m} (u_{n-1} - \bar{u})^2 \tag{2}$$

where $\bar{u}$ is the mean of u_i for $i = 1, 2, ..., m$

$$\bar{u} = \frac{1}{m} \sum_{i=1}^{m} u_{n-1} \tag{3}$$

A few changes can be made to simplify the equation for monitoring daily volatility.

1. define u_i as the percentage change in the market variable between the end of day $i-1$ and the end of day i.

$$u_i = \frac{S_i - S_{i-1}}{S_i} \tag{4}$$

2. Assume $\bar{u}$ to be zero

3. Replace $m-1$ with m

These changes simplify the formula for the variance rate to

$$\sigma_n^2 = \frac{1}{m}\sum_{i=1}^{m} u_{n-1}^2 \tag{5}$$

This equation gives equal weight to each of the previous m observations. A model that allows one to assign weights to the previous observations is

$$\sigma_n^2 = \sum_{i=1}^{m} \alpha_i u_{n-1}^2 \tag{6}$$

Where α_i is the weight given to the observation i days ago.
TODO: add information about the ARCH model
A special case of equation is the Exponentially Weighted Moving Average (EWMA) Model.

2 Exponentially Weighted Moving Average Model

The Exponentially Weighted Moving Average (EWMA) Model is a special case of a weighted moving average where the weights α_i decrease exponentially as we move backwards through time. Greater weights are given to more recent observations.

This weighting scheme leads to simple formula for updating volatility estimates. The predictive version of the variance rate of day n is given as

$$\hat{\sigma}_n^2 = \lambda\hat{\sigma}_{n-1}^2 + (1-\lambda)u_{n-1}^2 \tag{7}$$

where:

$\hat{\sigma}_{n-1}^2$ is the estimated variance rate of day $n-1$

u_{n-1} is the return of day $n-1$

λ is a constant between 0 and 1