Infinite Sets

Recall we define |A| to be the Codinality of A, the number of elements in A.

Proposition: If f: A -> B is one-to-one then |A| & lB|.

If: f Beign one-to-one \Rightarrow each element of f(A) his exactly one presmape. $\Rightarrow |A| = |f(A)| \circ \& f(A) \leq B \Rightarrow |f(A)| \leq |B| \circ |A| \leq |B|$

Det: Aset has an infinite number of elements if it blocks not have a finite number of elements.

We now split infinity into two groups, Countable & un countable.

Def: A settlet is finite or has the same Cordinality of N is count-ble O
A settlet is not contrible is uncommoble.

Note: two Sets have the some Cardinality iff I a bijection between the sets.

Ex A: {1,2,3} B: {a,b,c} how the Same Conditable}

66 1130 is a bijection.

2136
3136

EC C: {4.5,6} D: {d.e} hon dilhours Condinalizations, no may can be injective.

Seems unnecessarily complicated, but in necessary for infinite sets.

Ex: Show the Set of odd positive intgers is cubintrolle (Some Conditionlingers N)

Pf: Consider the fan: f: N -> O = Seroto N posson intger f(n) = 2n+1

inj: if nfm then 2nt 2m => 2nt | \$2mt | Some ME I but mouth = 20
if m<0 => 2m <0 So 2mt <1 Some N.

This fis a bijection. So O is countably infinite.

Ex: Hilberts Hotel: Ingire a hotel with Countrbly in finite rooms, one for each number A & N. Now assume every room Doccupred.

Now a new person arrives & asks for a cosm. Con you a comodule them?

Yes! ask every one to move to ther room to. If your in room k

go to room ktl. new person gets room 0.

Now infinitely many ppl error, less label them -1, -2, -3, Con use accome dute them?

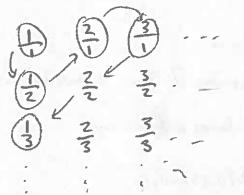
Yes! ask person in room k to move to room 2k & then the new group

gets room 2|m|-1 So -1->1 -2->3 etc.

This is the proof that |Z/= |N/, cazy huh? It gets correct!

Ex: The set of reports Q is country le.

Pf: This is a bit more hand wery. We are going to constant a good:



This will list every replaced number, & a few others. For this = # (D)
Only reduced numbers one.

To be countable we need a method to assign what the first element is, the Second etc. Call of the first then go do wn a row, \$\frac{1}{2} is second then diggoral of: \frac{7}{2} is the third then right + \frac{3}{2} is the flath then diggoral down, skip \frac{7}{2} be we've seen of so \frac{1}{3} is the fifth etc. This williand every number ingrad to hats no number more than once \Rightarrow bijectron

=) Possitive nationals ore countable. Then do the Same back as
Hilberts hatel to get negatives too!

Any countrible set is said to be well-ordered. A well ordered set is one which has a first element & given an element you know what the next element is.

leg. Nis well ordered. Ois first, given k kt I follows.

Any countrible set is well ordered ble I f abstraction f: N -> S

So f(0) is the first clement of S & given s& S : f(k) so f(k+1)

is the Next.

Ex In Pus. 1205. I is first than I is next. Given in we can find which clement comes next (a bush of work to this cose),

It may seem that all sets are countrable if even Q is. Why even have uncountable. There out IR is knowntable.

OEX: Ris uncomposte

Pf: By Contradiction, assume IRis Courtable. Then the Subset of reals between Oblis also countable, Put all numbers between oblin some order (we condition this ble Countable => well ordered). So we have

ri= 0. dudades . -

ra : O. dridudes -

13 = 0. dzi dze dzz

r = O.dnidnzdm ..

dije {0,1,43,4,5,6,7,8,9}

NOW Construct

r = 0. d, de, d3 ...

when di = { 4 if dif4 }

e.g. 0.23 754102

0.44590138

0.09118764

0.80553900

r= 4544 ...

then by construction r is not on our list but is between ORI.

Notice r disagrees u/ r; at spot ri; so r fr; \(\forall i \text{N}\)

Then (0,1) is not courted \(\epsilon\) \(\rightarrow\) R is not counterly.

Note by conversion IIH = 1P(N)|

This proof secretly relies on 1=0,999...

Because we need every real number to be distinct from other,

So less prove this LOTS of Work, Ready?!

Ex: 1= 0,999

lt Let X = 0.999 -..

9 2 0. 11111 ---

10x: 9,999 ... = 9 + 0,999 ... Or = 9 + X

1= 9 - 0.999999

1x = 9

x = 1

Thre are energie proofs as well that demon strate there is no number between 160,9619...

Matrices:

Def: Ametrix is a rectangular corresp of numbers, Ametrix with m rows In Columns is called an Mxn matrix. If M= n then the Matrix is square.

Ex: [34] is a 3x2 monterx.