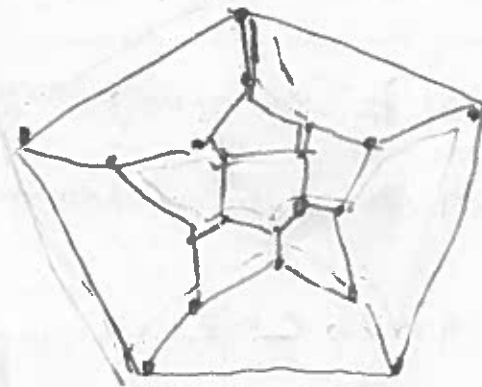## Hamilton Paths / circuits
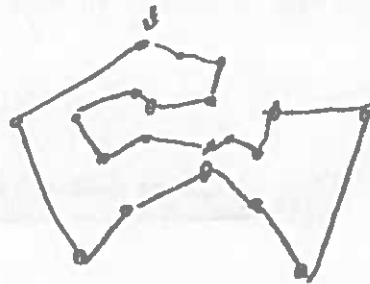
<u>Def</u>: A simple path in a graph G that passes through every vertex exactly once is called a <u>Hamilton path</u>. If the path is a circuit it is a <u>Hamilton Circuit</u>.

This comes from a 19th century puzzle, called the icosian puzzle with a dodechahedron (12 sides, all pentagons). Each vertex had a peg you needed to visit each peg with string exactly once.

EX:

Sol:

Unfortunately there is no nice necessary & sufficient condition for the existence of a Hamiltonian cycle. There are some sufficient conditions, and some necessary ones (like no node of degree one).
More over if a node has $\deg(v) = 2$ then both edges must be used.

<u>Theorem</u>: Every complete graph k_n for $n \geq 3$ has a Hamilton circuit.
All nodes are connected so you can go anywhere (in a cycle).

<u>Theorem</u> (Dirac's Theorem): If G is a simple graph with n vertices with $n \geq 3$ such that $\deg(v) \geq \frac{n}{2}$ then G has a Hamilton cycle.