

## CSCI 2824 - Discrete Structures

### Homework 6

You MUST show your work. If you only present answers you will receive minimal credit. This homework is worth 100pts.

**Due: Wednesday July 19**

1. For the following question consider rolling two dice. One dice is red and one is blue, but they are otherwise normal dice.
  - (a) (4 points) How many outcomes result in a sum of 2? In a sum of 12?
  - (b) (4 points) How many outcomes have the blue die showing 2?
  - (c) (4 points) How many outcomes give an even sum?
2. For the following questions consider an eight-bit string.
  - (a) (2 points) How many eight-bit strings begin with 1100?
  - (b) (2 points) How many eight-bit strings have exactly one 1 in them?
  - (c) (5 points) How many eight-bit strings have exactly two 1's in them?
  - (d) (5 points) How many eight-bit strings have at least one 1 in them?
3. For the following questions determine how many permutations (orderings) can be formed from  $\{A, B, C, D, E\}$  subject to the given constraints.
  - (a) (4 points) The ordering contains the substring  $ACE$  (in that order directly, as in  $ACEDB$  is one, but  $ADCBE$  is not)
  - (b) (6 points) Does not contain the substring  $AB$  nor  $CD$  (again directly,  $ACEDB$  is acceptable).
  - (c) (10 points)  $A$  appears before  $C$  which appears before  $E$  (here there can be letters between).
4. (10 points) Prove that a number is divisible by 3 if and only if the sum of its digits is divisible by 3. (E.g. 27 is divisible by 3 since  $2 + 7 = 9$  is divisible by 3).
5. (15 points) All numbers have some divisibility rule associated with them. There are a couple for 7. The one we focus on here is that a number  $n$  is divisible by 7 if and only if when you subtract 2 times the least significant digit from the number without its least significant digit the result is also divisible by 7. E.g. 14 is divisible by 7 since  $1 - 4 \cdot 2 = -7$  is divisible by 7. 343 is divisible by 7 since  $34 - 3 \cdot 2 = 28$  which is divisible by 7. Prove this.
6. For the following questions find the number of (unordered) five-card poker hands, selecting from an ordinary 52-card deck, having the properties indicated.
  - (a) (5 points) Containing four of a kind, that is, four cards of the same denomination.
  - (b) (5 points) Containing cards of exactly two suits
  - (c) (5 points) Containing two of one denomination, two of another denomination and one of a third denomination.
7. The following questions deal with selecting a committee from a club consisting of six distinct men and seven distinct women.
  - (a) (4 points) In how many ways can we select a committee of three men and four women?
  - (b) (4 points) In how many ways can we select a committee of four persons that has at least one man?
  - (c) (6 points) In how many ways can we select a committee of four persons that has at most one man?