

CSCI 2824 - Discrete Structures

Homework 6

You MUST show your work. If you only present answers you will receive minimal credit. This homework is worth 100pts.

Due: Wednesday July 19

1. For the following question consider rolling two dice. One dice is red and one is blue, but they are otherwise normal dice.
 - (a) (4 points) How many outcomes result in a sum of 2? In a sum of 12?
 - (b) (4 points) How many outcomes have the blue die showing 2?
 - (c) (4 points) How many outcomes give an even sum?
2. For the following questions consider an eight-bit string.
 - (a) (2 points) How many eight-bit strings begin with 1100?
 - (b) (2 points) How many eight-bit strings have exactly one 1 in them?
 - (c) (5 points) How many eight-bit strings have exactly two 1's in them?
 - (d) (5 points) How many eight-bit strings have at least one 1 in them?
3. For the following questions determine how many permutations (orderings) can be formed from $\{A, B, C, D, E\}$ subject to the given constraints.
 - (a) (4 points) The ordering contains the substring ACE (in that order directly, as in $ACEDB$ is one, but $ADCBE$ is not)
 - (b) (6 points) Does not contain the substring AB nor CD (again directly, $ACEDB$ is acceptable).
 - (c) (10 points) A appears before C which appears before E (here there can be letters between).
4. (10 points) Prove that a number is divisible by 3 if and only if the sum of its digits is divisible by 3. (E.g. 27 is divisible by 3 since $2 + 7 = 9$ is divisible by 3).
5. (15 points) All numbers have some divisibility rule associated with them. There are a couple for 7. The one we focus on here is that a number n is divisible by 7 if and only if when you subtract 2 times the least significant digit from the number without its least significant digit the result is also divisible by 7. E.g. 14 is divisible by 7 since $1 - 4 \cdot 2 = -7$ is divisible by 7. 343 is divisible by 7 since $34 - 3 \cdot 2 = 28$ which is divisible by 7. Prove this.
6. For the following questions find the number of (unordered) five-card poker hands, selecting from an ordinary 52-card deck, having the properties indicated.
 - (a) (5 points) Containing four of a kind, that is, four cards of the same denomination.
 - (b) (5 points) Containing cards of exactly two suits
 - (c) (5 points) Containing two of one denomination, two of another denomination and one of a third denomination.
7. The following questions deal with selecting a committee from a club consisting of six distinct men and seven distinct women.
 - (a) (4 points) In how many ways can we select a committee of three men and four women?
 - (b) (4 points) In how many ways can we select a committee of four persons that has at least one man?
 - (c) (6 points) In how many ways can we select a committee of four persons that has at most one man?