

Introduction to Counting

Now we're going to learn to count!

Specifically, we'll count the number of possible outcomes.

First we'll need some rules!

The product rule: If a procedure can be broken into two (or more) tasks & there are n_i ways to do the i th task there are $n_1 n_2 n_3 \dots n_k$ ways to do the procedure.

Ex: If a menu consists of the following main courses & drinks:

Hamburger	Tea
Cheeseburger	Milk
Fish Filet	Soda
	Coffee

How many ways are there to have a meal, which consists of a main course & optionally a drink?

3 main courses 5 drinks (one no-drink)

= 15 possible meals.

Ex: How many strings of length 4 can be formed using the letters A B C D E if repetitions are allowed? Not allowed? How many begin with B?

(a) $\underline{5} \quad \underline{4} \quad \underline{3} \quad \underline{2} = 5! = 120$ (b) $\underline{5} \quad \underline{5} \quad \underline{5} \quad \underline{5} = 5^4$

1st letter 2nd 3rd 4th

of choices for each letter

(c) $\underline{1} \quad \underline{4} \quad \underline{3} \quad \underline{2} = 4! = 24$

Ex How many possible Random Colorado license plates are there?

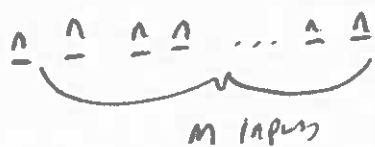
3 letters then 3 numbers or 3 numbers then 3 letters.

$$\underline{26} \underline{26} \underline{26} \quad \underline{10} \underline{10} \underline{10}$$

$$\underline{10} \underline{10} \underline{10} \quad \underline{26} \underline{26} \underline{26}$$

$$= 26^3 \cdot 10^3 \cdot 2$$

Ex: How many functions are there from a set of m elements to a set with n elements?



Ex: How many 1-to-1 fns?

if $m > n$ then 0 if $m \leq n$

$$\underbrace{\underline{n} \quad \underline{n-1} \quad \underline{n-2} \quad \underline{n-3} \quad \underline{n-4} \quad \dots \quad \underline{n-m+1}}_m = \frac{n!}{n!}$$

Sum Rule: If a task can be done in either n_1 ways or n_2 ways & ~~neither~~ of the n_1 or n_2 ways are the same then there are $n_1 + n_2$ ways to do the task.

Ex: In how many ways can we select two books from different subjects among 5 distinct CS books, 3 distinct math books, 2 distinct art books?

Prod rule Says we can select a CS & math book in 15 ways
a CS & art book in 10 ways
a math & art book in 6 ways

Sum rule says two books in different subjects in $15 + 10 + 6 = 31$ ways.

Ex: A six person Committee composed of Alize, Bob, Charlie, Dylan, Elle, & Frank is to select a chairperson, Secretary, treasury from themselves.

- (a) How many ways can this be done?
- (b) If either Alice or Bob must be chairperson?
- (c) If Elle must hold one position?
- (d) If Both Dylan & Frank must hold positions?

(a) $6 \cdot 5 \cdot 4 = 120$ product rule

(b) $2 \cdot 5 \cdot 4 = 20$ product rule

(c) $1 \cdot 5 \cdot 4 + 5 \cdot 1 \cdot 4 + 5 \cdot 4 \cdot 1 = 3 \cdot 20$ (sum & product rules)

(d) More complicated. we have many cases here

- 1. Find a role for Dylan
- 2. Find a role for Frank
- 3. Fill remaining position

1. 3 options

2. 2 options (1 taken by Dylan)

3. 4 options (other 4 fill remaining position)

$$= 3 \cdot 2 \cdot 4 = 24$$

Ex: In the programming language BASIC (older version) the name of variables needed to be ^{exactly} two characters (letter or number) uppercase was not distinguished from lowercase. Variables had to start with letters & could not be one of the 5 reserved 2 letter words. How many possible variable names were there?

e.g. $a5 \checkmark$ $A5 = a5$ if X $abcX$ $55X$ $a \checkmark$ $5X$

How many 1 character ones? 26

How many 2 characters? $\underline{26} \underline{36}$ but 5 are reserved
 $26 \cdot 36 - 5$

Altogether $26 + 26 \cdot 36 - 5 = 957$ options.

Ex passwords: How many possible passwords are there of length 8 allowing letters, upper & lower numbers & symbols

32

total number of options $32 + 10 + 26 + 26 = 94$

$$\text{So } \underline{94} \underline{94} \underline{94} \underline{94} \underline{94} \underline{94} \underline{94} \underline{94} = 94^8 = 6095689385410816$$

What if one symbol, one upper, one lower case, & one number are required?

Many tasks: 1. Place Symbol $\rightarrow 8$

2. Place upper $\rightarrow 7$

3. Place lower $\rightarrow 6$

4. Place number $\rightarrow 5$

5. fill remaining 4 spaces $\rightarrow 94^4$

$$8 \cdot 7 \cdot 6 \cdot 5 \cdot 94^4$$

$$= 131165825280$$

2×10^{-7} % of possible 8 length passwords!

Password restrictions might make dictionary searches more difficult but allow brute force searches! (We keepass!).

Ex: How many bit strings of length 8 either start with 1 or end with 00 but not both?

Need the Inclusion-exclusion principle - purposely overcount then count how much you overcount:

$$\text{start } 1 \quad \underline{1} \underline{} \underline{} \underline{} \underline{} \underline{} \underline{} \underline{} = 2^7$$

$$\text{end with } 00 \quad \underline{} \underline{} \underline{} \underline{} \underline{} \underline{} \underline{1} \underline{1} = 2^6$$

$$\text{Both } \underline{1} \underline{} \underline{} \underline{} \underline{} \underline{} \underline{1} \underline{1} = 2^5$$

$$= 2^7 + 2^6 - 2^5 = 160 \text{ ways.}$$