CSCI 2824 - Discrete Structures
Homework 2

You MUST show your work. If you only present answers you will receive minimal credit. This homework is worth 100pts.

Due: Wednesday June 21

1. (2 points) Prove that the sum of two odd integers is an even integer.
2. (4 points) Prove that for all real numbers x and y, if $xy \leq 2$, then either $x \leq \sqrt{2}$ or $y \leq \sqrt{2}$.
3. (10 points) Prove that the real numbers have the Archimedean Property. That is given any positive real numbers x and y prove that there is an integer n such that $xn > y$.
4. (6 points) Suppose a, b and c are integers. If a^2 divides b and b^3 divides c, then a^6 divides c. [Divides, is a formal statement in math, x divides y means that y is a multiple of x. For example 5 divides 10, but 4 does not divide 2. This is often written $5|10$, $x|y$, $4 \not| 2$ etc. For this problem it may help to think in terms of multiples, $x|y$ means that $y = kx$ for some integer k.]
5. (7 points) Prove that for all *positive* integers m, n: $2m + 5n^2 = 20$ has no solution.
6. (5 points) Prove that that the difference between an irrational number x and a rational number y is irrational.
7. (4 points) Prove that for all integers n if $n^3 + 5$ is odd then n is even.
8. (8 points) Verify the following equation:

$$1^2 - 2^2 + 3^2 - \cdots + (-1)^{n+1}n^2 = \frac{(-1)^{n+1}n(n+1)}{2}$$

9. (6 points) Prove that $7^n - 1$ is divisible by 6 for all integers $n \geq 1$.
10. (8 points) Prove the following by cases:

    (a)

    $$\max\{x, y\} = \frac{x + y + |x - y|}{2}$$

    for all real numbers x and y.

    (b)

    $$\min\{x, y\} = \frac{x + y - |x - y|}{2}$$

    for all real numbers x and y.

11. (10 points) Verify the inequality:

$$2n + 1 \leq 2^n, \; n \geq 3$$

12. (10 points) Show that postage of 24 cents or more can be achieved by using only 5-cent and 7-cent stamps.
13. (20 points) Suppose we have two piles of cards containing n cards each. Two players play the following game. Each player, in turn, chooses one pile and then removes any number of cards from the chosen pile. The player who removes the last card on the table wins the game. Show the second player can always win the game. [Note: You need to PROVE the second player can always win. Coming up with a strategy is a necessary step, but then you'll need to prove that strategy will always work. Hint: Induction]