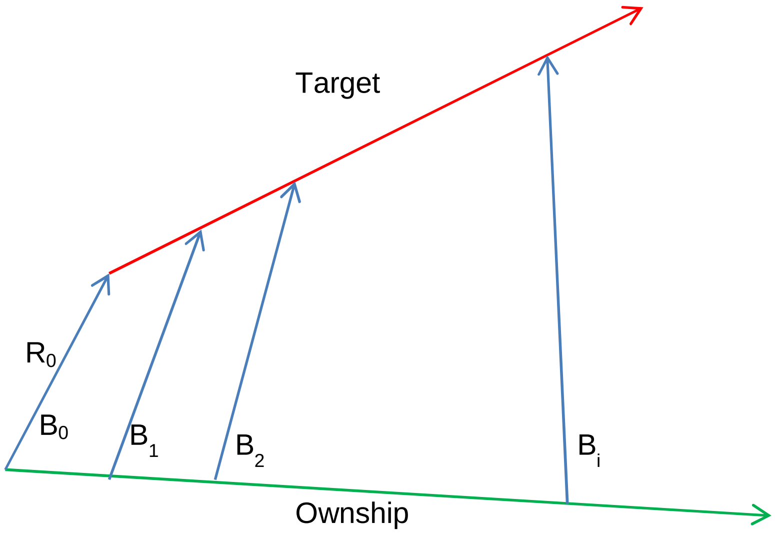
We have a set of n bearing cuts $\{(B_0^M, t_0), (B_1^M, t_1), (B_2^M, t_2), \ldots, (B_n^M, t_n)\}$

Ownship is on a constant course and speed and we assume that the target is as well i.e. we have something like this:



C_O – Ownship course, S_O – Ownship Speed

C_T – Target course, S_T – Target speed

We know ownship course and speed but in what follows, we don't actually have to use them because they get combined with the target course and speed and the range.

Range is R_0 at time t_0 and is unknown.

Simplest way to do this is to convert courses and speeds to X and Y components

Ownship $\dot{X}_O = S_O.\sin(C_O)$ $\dot{Y}_O = S_O.\cos(C_O)$

Target $\dot{X}_T = S_T.\sin(C_T)$ $\dot{Y}_T = S_T.\cos(C_T)$

Then bearing B_i at time t_i in the scenario above is given by

$$B_i = \arctan\left(\frac{R_0 \sin B_0 + (\dot{X}_T - \dot{X}_O).(t_i - t_0)}{R_0 \cos B_0 + (\dot{Y}_T - \dot{Y}_O).(t_i - t_0)}\right) = \arctan\left(\frac{\sin B_0 + \frac{(\dot{X}_T - \dot{X}_O)}{R_0}.(t_i - t_0)}{\cos B_0 + \frac{(\dot{Y}_T - \dot{Y}_O)}{R_0}.(t_i - t_0)}\right)$$
$$= \arctan\left(\frac{\sin B_0 + P.(t_i - t_0)}{\cos B_0 + Q.(t_i - t_0)}\right)$$

The sum of squared bearing errors is given by

$$\sum_{i=0}^{n} \left(B_i - B_i^M\right)^2$$

We then minimise the sum of squared bearing errors with respect to B_0, P and Q. We're not actually interested in B_0, P or Q. It's the sum of squared bearing errors we want. However it's probably worth checking that the values of B_0, P and Q make sense – B_0 should be close to the first bearing cut and P and Q should represent some sort of sensible combination of speed and range.

The main thing to be careful of here is the (normal) problem of bearings that go through 360 plus the problem of getting the right quadrant for the arctan. Can solve the arctan by using the arctan2 function if it's available.

To use this as a manoeuvre detection method, we assume first of all that the target hasn't manoeuvred between t_0 and t_n and solve as above.

We then hypothesis that the target manoeuvred some time between t_0 and t_n, say at t_{man}. We should take out the bearings around t_{man} to make sure we only have bearings on straight target legs. We then minimise the bearing errors exactly as before but separately for the bearings between t_0 and t_{man} and then again for the bearings between t_{man} and t_n. We do that for several different values of t_{man} between t_0 and t_n.

Because there are different numbers of cuts involved, we need to normalise the sum of squares by dividing by the number of cuts. We then compare the normalised sum of squares for the complete straight leg with the sum of the two normalised sums of squares on the two straight legs.

If the target has not manoeuvred we should find that the sum of the two straight legs is always less than the single leg but not by much. However, if there is a manoeuvre at time t_{man}, then the sum of the two straight legs either side of t_{man} should be significantly less that the single straight leg or for any other pair of straight legs where the manoeuvre time is different from t_{man}.