4

c) $\log_5^n = \Theta(\log n)$

$c_1 \log n \leq \log_5^n \leq c_2 \log n \;;\quad \forall n \geq n_0$

$c_1 \log n \leq \dfrac{\log n}{\log 5} \leq c_2 \log n \qquad : \log n$

$c_1 \leq \dfrac{1}{\log 5} \leq c_2$

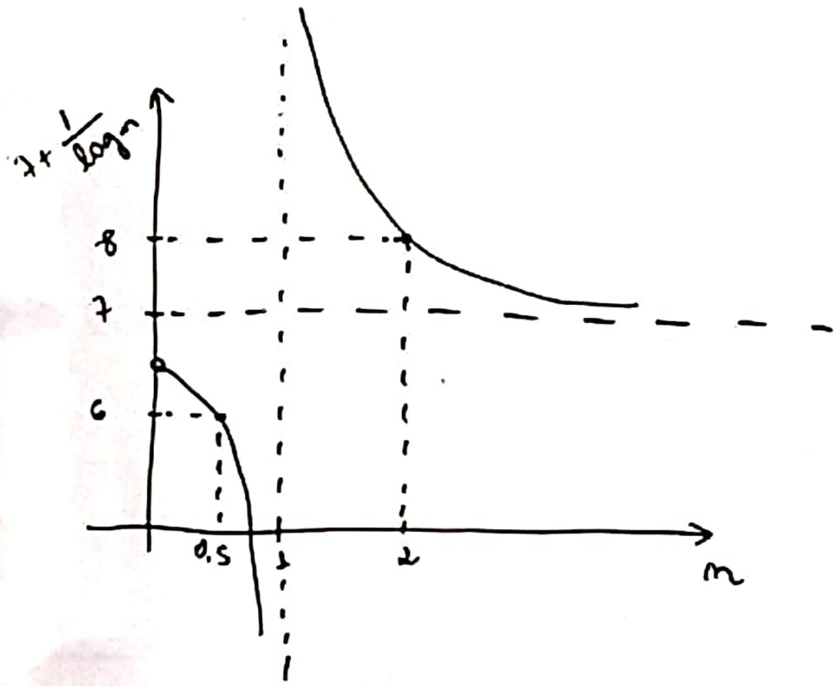
$c_1 \leq \log_5^2 \leq c_2$

$$\boxed{\begin{aligned} c_1 &= \log_5^2 \\ c_2 &= \log_5^2 \\ n_0 &= 1 \end{aligned}}$$

d) $7n\log n + n = \Theta(n\log n)$

$c_1 n\log n \leq 7n\log n + n \leq c_2 n\log n \; ; \forall n \geq n_0 : n\log n$

$c_1 \leq 7 + \dfrac{1}{\log n} \leq c_2$



$$\boxed{\begin{array}{l} C_1 = 7 \\ C_2 = 8 \\ n_0 = 2 \end{array}}$$

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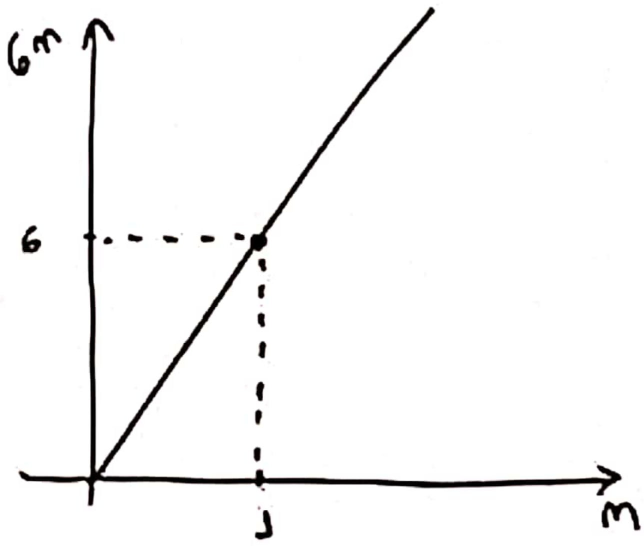
⑤ $6n^3 \neq \Theta(n^2)$

Vamos tentar provar que $6n^3 = \Theta(n^2)$. Para isso devemos achar constantes $c_1 > 0$, $c_2 > 0$ e $n_0 > 0$, tal que:

$$c_1 n^2 \leq 6n^3 \leq c_2 n^2 \; ; \; \forall n \geq n_0 \quad : n^2$$

$$c_1 \leq 6n \leq c_2$$

Não existe uma constante c_2 que seja maior ou igual a $6n$ para todo $n \geq n_0$.



$$\boxed{6n^3 \neq \Theta(n^2)}$$

$f(n) = O(g(n))$

$f(n) \le c\, g(n) \; ; \; \forall n \ge n_0$

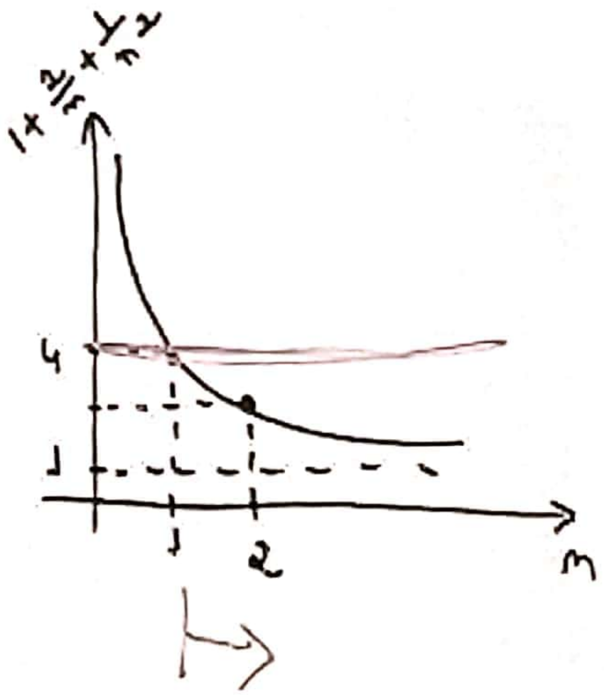
$(n+1)^2 = O(n^2)$

$n^2 + 2n + 1 \le c\, n^2 \; ; \; \forall n \ge n_0 \quad : n^2$

$1 + \frac{2}{n} + \frac{1}{n^2} \le c$

$c \ge 1 + \frac{2}{n} + \frac{1}{n^2}$

$\boxed{\begin{array}{l} c = 4 \\ n_0 = 1 \end{array}}$



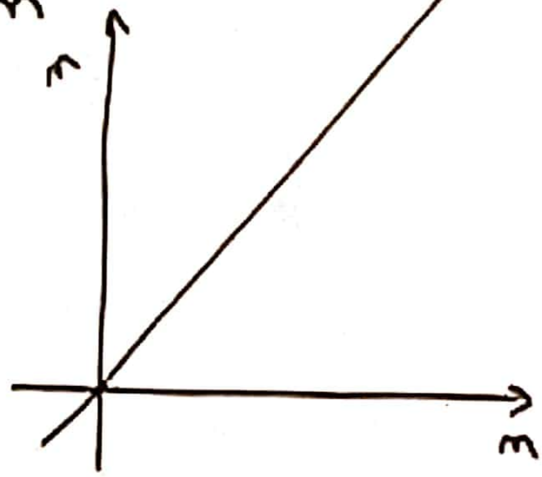
$n^2 \neq O(n)$

$n^2 \leq cn \; ; \forall n \geq n_0$

$:n$

$n \leq c$

$c \geq n$



$\nexists c$