1. **Describe how one can calculate the advanced heuristic value for any state of the puzzle.**

First, calculate the Manhattan distance for the 2x2 goal piece from its current position to the goal position. Then, introduce a penalty into the heuristic based on the position of the 2x2 goal piece. Consider the following factors:

- add 1 to the heuristic function when the 2x2 goal piece is not in a corner or near an edge
- add 0 to the heuristic function if the 2x2 goal piece is in a favorable position (e.g., in a corner or near an edge)

Combine the Manhattan distances and additional factors into a single heuristic value. This can be achieved by summing the distances and applying appropriate weights (0 or 1) based on the position of the 2x2 goal piece

2. **Why is your advanced heuristic admissible?**

The advanced heuristic is admissible because it never overestimates the cost of reaching the goal. The Manhattan distance component alone is already admissible, as it provides a lower bound on the number of moves required to reach the goal state. By adding additional penalty, the advanced heuristic might increase the estimated cost, but it will never underestimate the actual cost since the penalty is at most 1.

3. **Why does your advanced heuristic dominate the Manhattan distance heuristic?**

The advanced heuristic dominates the Manhattan distance heuristic because it incorporates more information and captures additional aspects of the puzzle's complexity. The Manhattan distance heuristic only considers the geometric distance between the current and goal positions of the pieces. In contrast, the advanced heuristic considers factors like the position of the 2x2 goal piece. Hence the advanced heuristic must dominate the Manhattan distance heuristic as the advanced heuristic is either larger by 1 or the same as the Manhattan distance heuristic.