GENERAL ASSEMBLY

INTRO to DATA SCIENCE

Lecture 16: networks & graphs

Last time:

- big data
- map reduce

questions?

I. networks
II. Network statics
III. Network dynamics

I. networks

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Given a large set of social network data, what types of questions do you think would be interesting to ask?

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These are questions about network *representation*.

Some natural questions arise when considering social network data, in particular:

- Who are the most central and/or influential actors in a network?
- Can the network be decomposed into coherent smaller groups?
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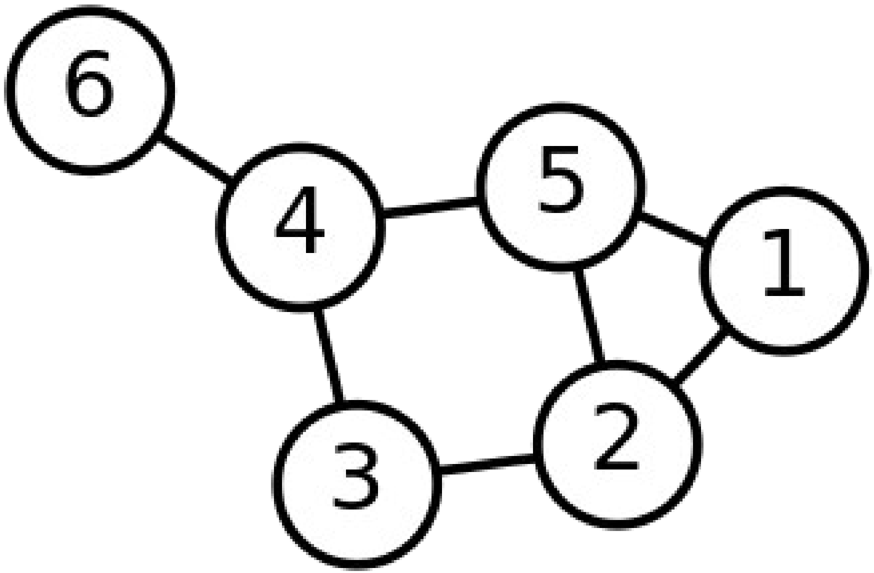
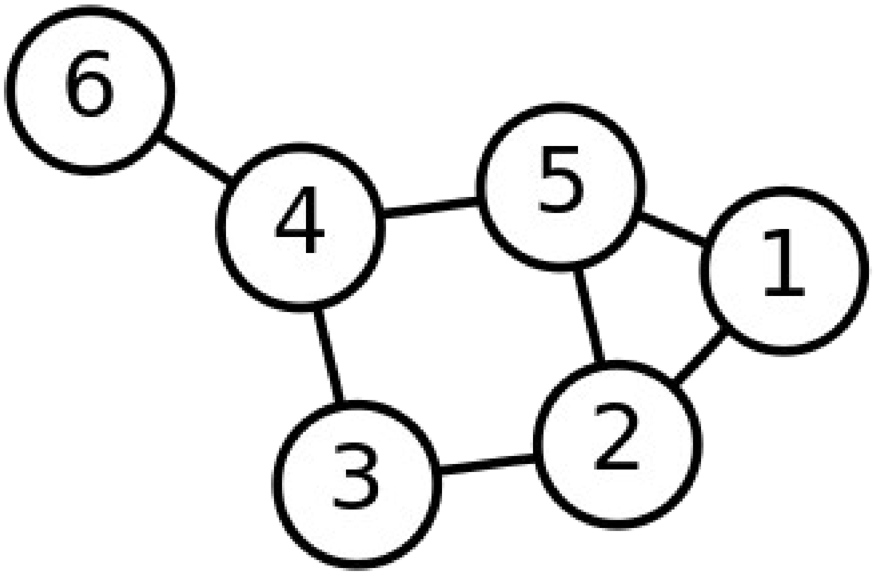
These are questions about network *behavior*.

The mathematical representation of a network is an object called a graph, which is a configuration of *nodes* connected by *edges*.

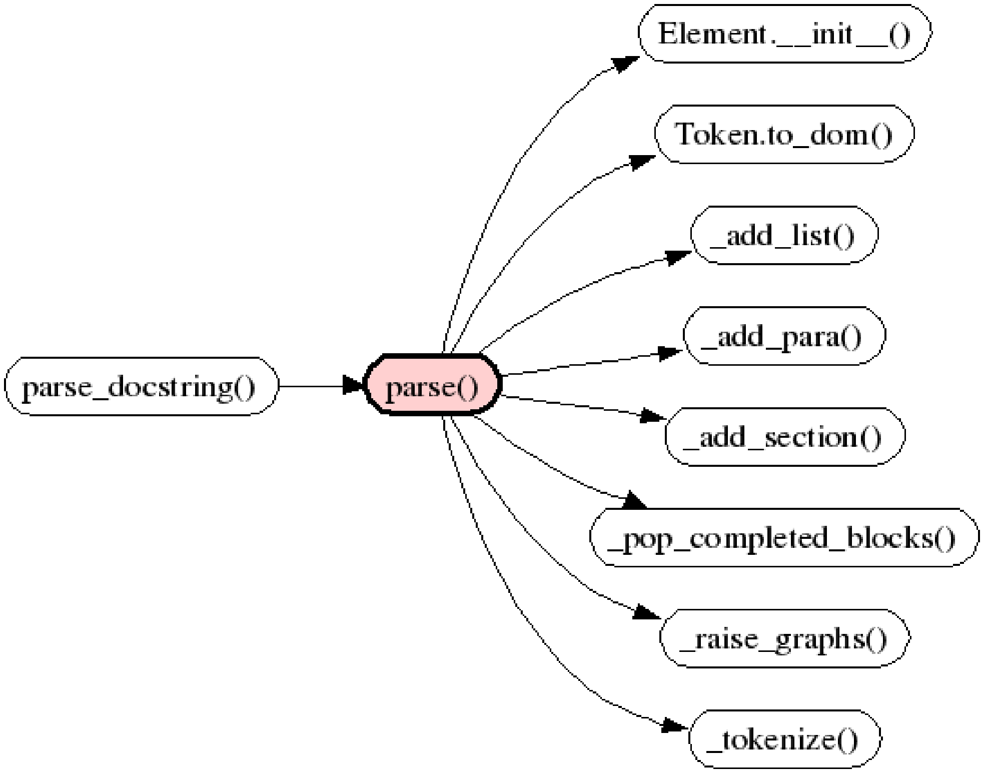


Network representation

Nodes represent *actors* in the graph, and edges represent the *relationships* between actors.



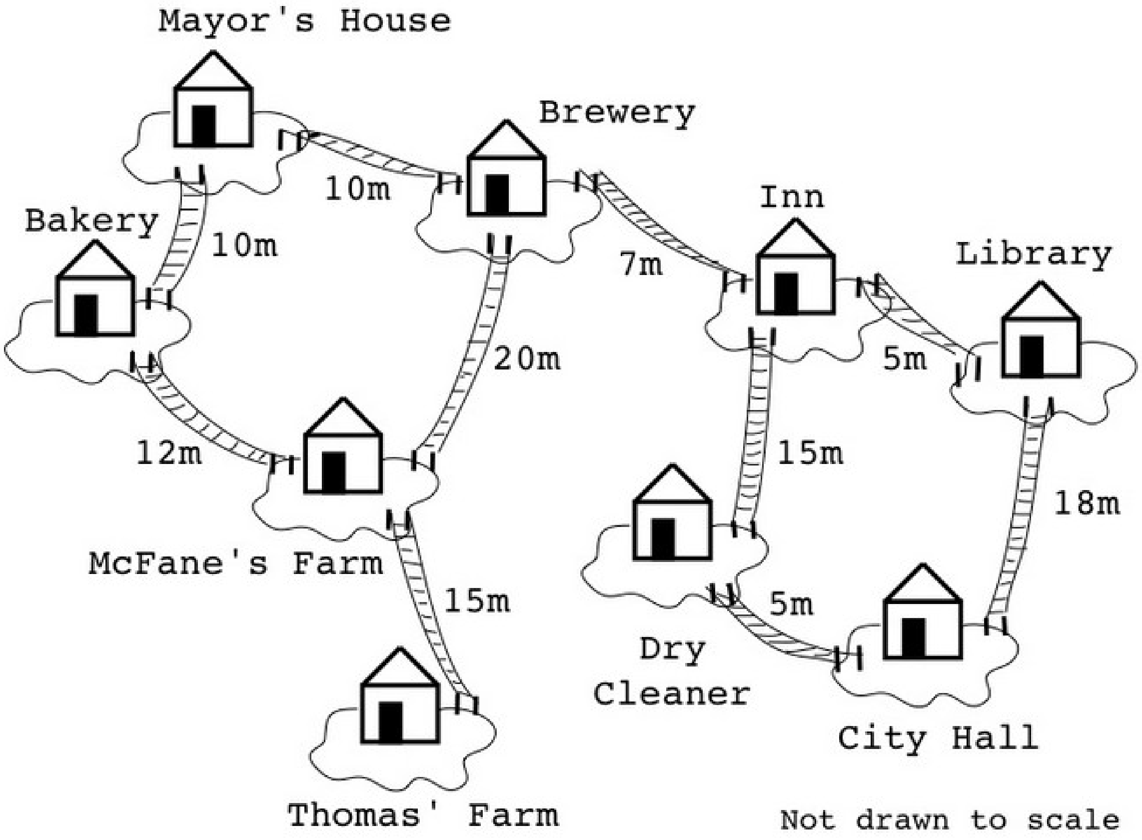
Network representation



NOTE

A *directed graph* has edges that point from one node to another.

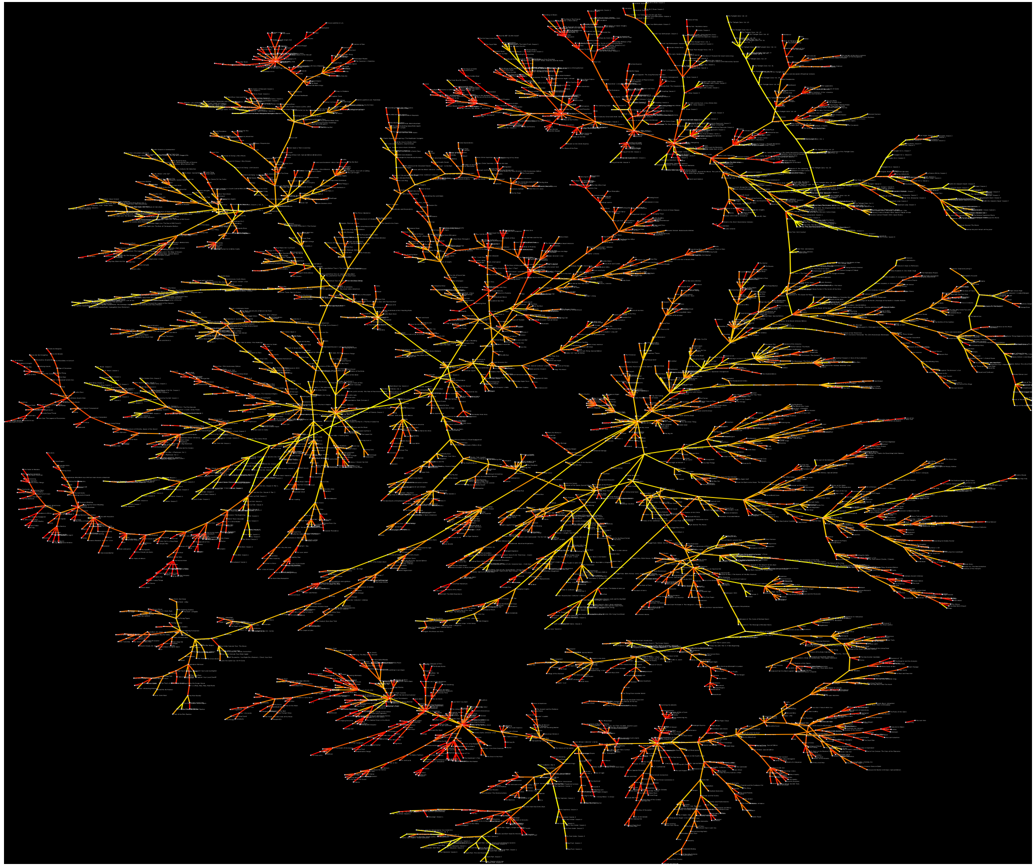
Network representation



NOTE

A *weighted graph* contains edges associated with real-valued numbers, eg to measure distance or importance.

Network representation



Source: netflix prize team The Ensemble
http://www.flickr.com/photos/chef_ele/3791293142/sizes/o/in/set-72157621825510293/

Network representation

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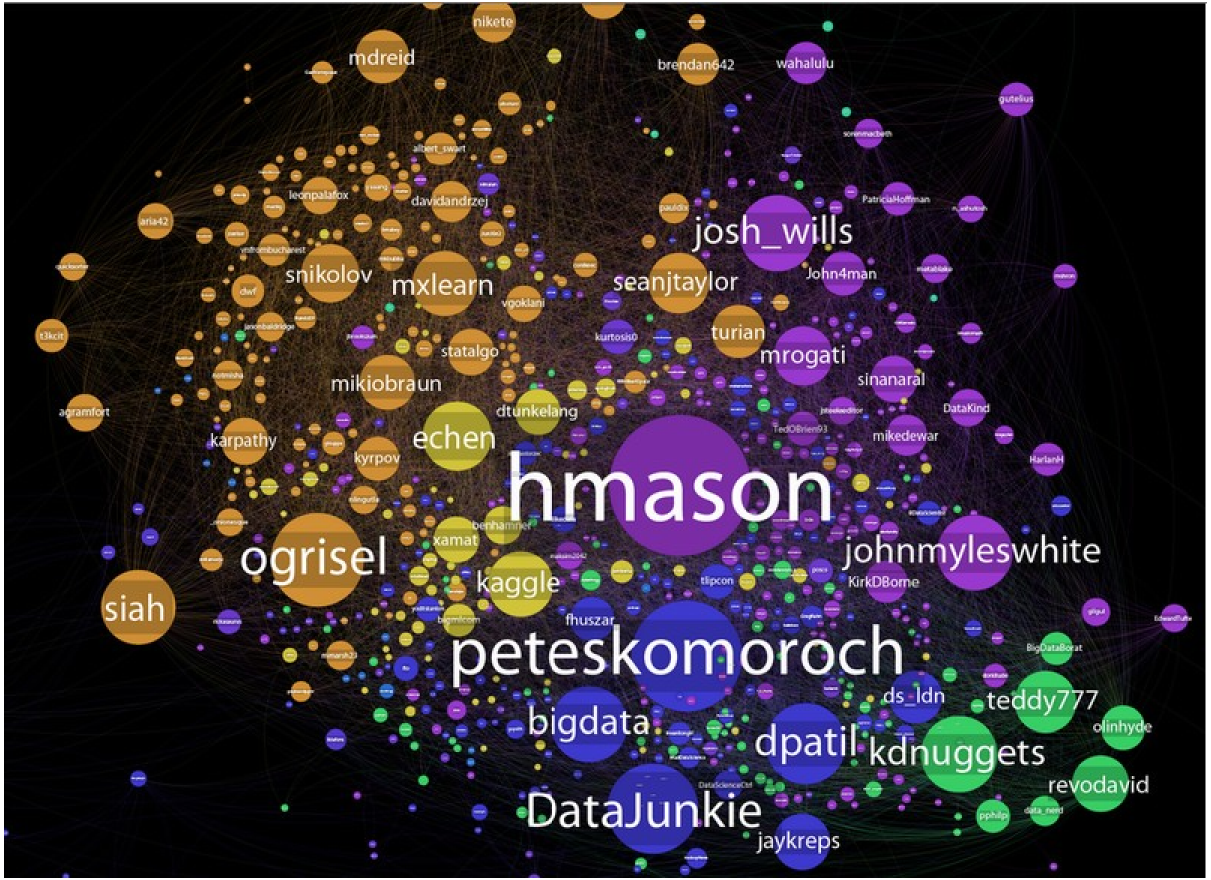

In practical terms, we need some data structures to represent and manipulate our network data.

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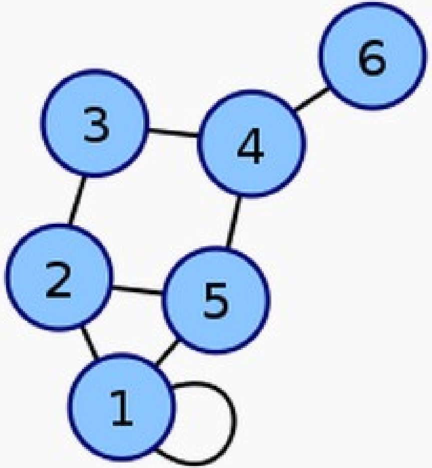
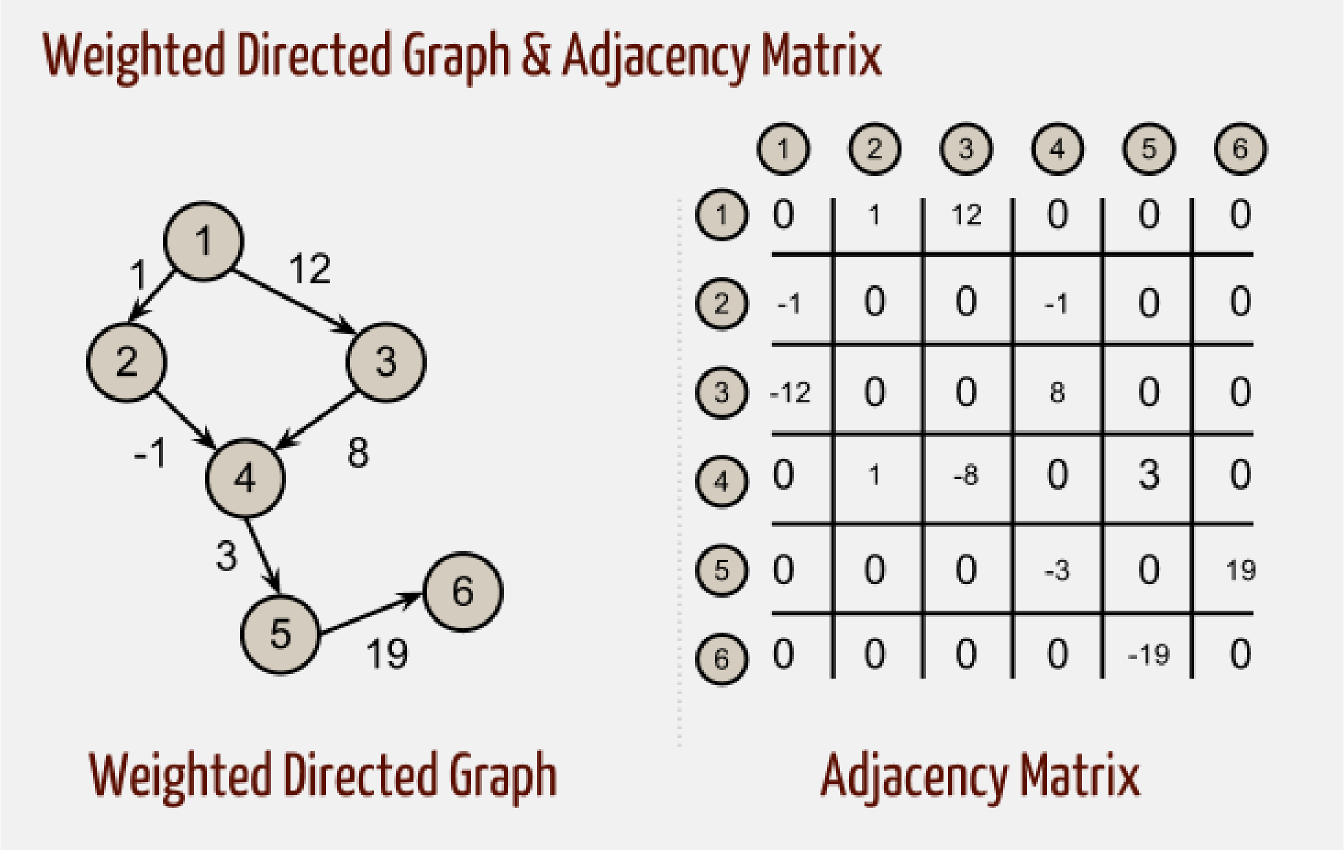
One common graph representation is the adjacency matrix. An *n*-node undirected graph can be represented by a symmetric $n \times n$ adjacency matrix A whose nonzero off-diagonal entries A_{ij} represent an edge between nodes i and j.

Network representation

Labeled graph	Adjacency matrix
	$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$ Coordinates are 1-6.

Network representation



NOTE

A directed graph has an *asymmetric* adjacency matrix.

Can you see why?

Another useful tool is the adjacency list (actually a dict!):

```
graph = {'A': ['B', 'C'],
         'B': ['C', 'D'],
         'C': ['D'],
         'D': ['C'],
         'E': ['F'],
         'F': ['C']}
```

Does this adjacency dict represent a directed or undirected graph? How could you generalize this to represent a weighted graph?

II. network statics

One key concept in the study of network structure is centrality. The centrality of a node is a measure of its importance in the network.

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The simplest centrality measure is the degree of a node, which is simply the number of edges connected to it. Using the adjacency matrix notation for an undirected graph, we can express the degree *ki* of node *i* as:

$$k_i = \sum_{j=1}^{n} A_{ij}.$$

A more sophisticated measure called eigenvector centrality allows important edges to give larger contributions to centrality:

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Here the eigenvector centrality x_i of node i is proportional to the average centrality of i's network neighbours.

Another useful centrality measure is based on the idea of shortest-distance (or *geodesic*) paths through the graph.

See http://oracleofbacon.org/movielinks.php

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If σ_{st} is the number of geodesic paths from node *s* to node *t*, and $\sigma_{st}(v)$ is the number of these paths that cross node *v*, then the betweenness centrality of node *v* is given by:

$$g(v) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

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NOTE

Betweenness centrality measures the proportion of geodesic paths passing through a node.

This gives an idea of the node's *influence* in the network.

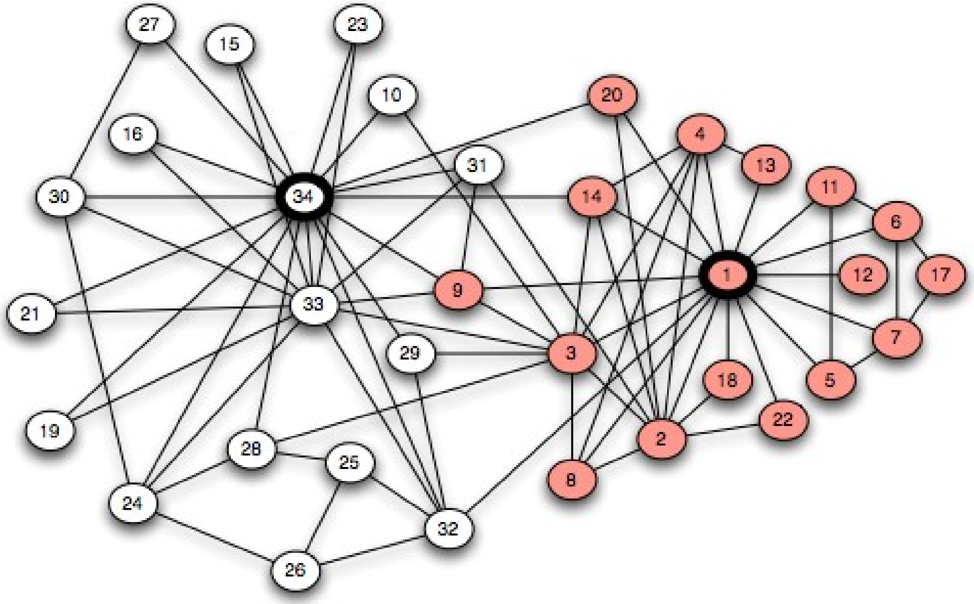

Figure 3.13: A karate club studied by Wayne Zachary [421] — a dispute during the course of the study caused it to split into two clubs. Could the boundaries of the two clubs be predicted from the network structure?

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NOTE

This is where the phrase “six degrees of separation” comes from.

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II. network dynamics

Suppose we're interested in the idea of how information (or behavior) spreads through a network:

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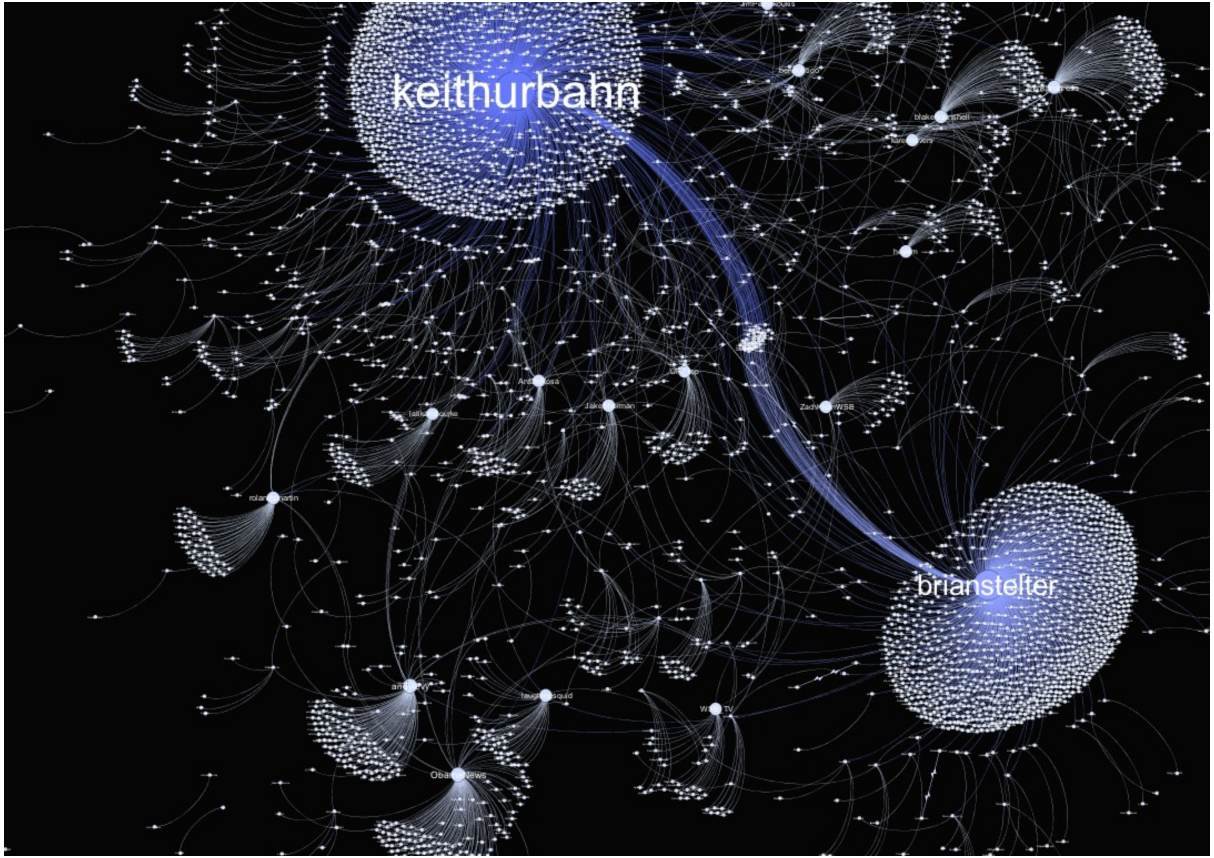
- How do members of a social network influence each other to adopt a new technology/product/behavior?
- How did information about the bin Laden raid spread over Twitter?
- What's the best way to use a social network to market your product?

Network dynamics



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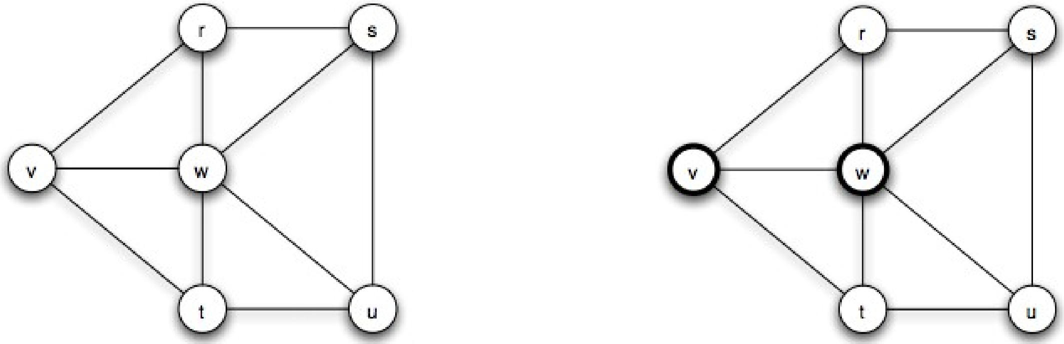
direct benefit effects – people may have incentives to use the same products/technology/etc as their network neighbors

Studies of informational effects have shown that while initial lack of information makes innovations risky to adopt, adopters ultimately benefit.

Furthermore, *early adopters* share certain common traits (eg higher socio-economic status, wider travel experience), and they influence their neighbors by providing indirect information about the innovation.

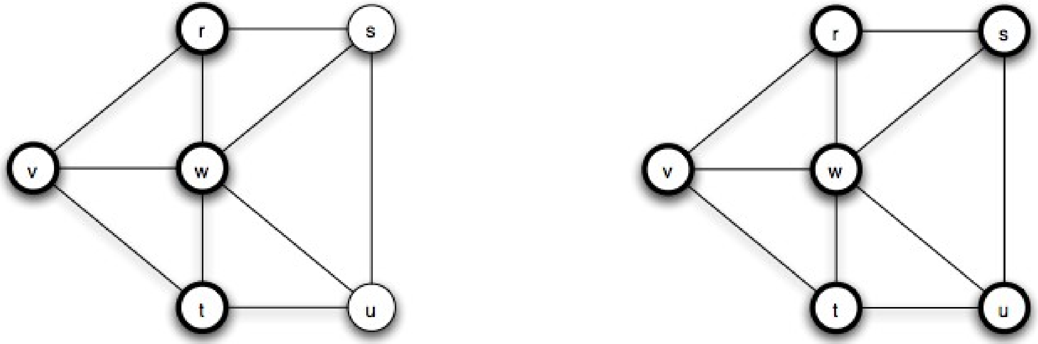
Adoption depends not only on the relative payoffs, but also on the structure of the network (eg, how many neighbors *a node* has, and which particular nodes these neighbors are).

Network dynamics



(a) *The underlying network*

(b) *Two nodes are the initial adopters*



(c) *After one step, two more nodes have adopted*

(d) *After a second step, everyone has adopted*

NOTE

Since all nodes have adopted, this is called a *complete cascade* (at threshold q).

Here's an interesting question: how can you identify which (non-adopting) nodes are most important to allowing the cascade to continue?

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Answering this question effectively is the idea behind *viral marketing*.

Very Good Source on graph Analysis

https://github.com/swinton/Web-Science-Summer-School-2011