Math6450 Assignment1: Linear Regression Analysis

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Part 1: Data Exploration and Preparation

BOSTON HOUSING DATASET ANALYSIS

1.1 DATASET DIMENSIONS
Number of observations (rows): 506
Number of variables (columns): 14
Dataset shape: (506, 14)

Column names: ['crim', 'zn', 'indus', 'chas', 'nox', 'rm', 'age', 'dis', 'rad', 'tax', 'ptratio', 'b', 'lstat', 'medv']

1.2 DESCRIPTIVE STATISTICS
Descriptive statistics for TARGET VARIABLE (medv):

```
count    506.000
mean      22.533
std        9.197
min        5.000
25%       17.025
50%       21.200
75%       25.000
max       50.000
Name: medv, dtype: float64
```

Descriptive statistics for PRIMARY FEATURE (lstat):

```
count    506.000
mean      12.653
std        7.141
min        1.730
25%        6.950
50%       11.360
75%       16.955
max       37.970
Name: lstat, dtype: float64
```

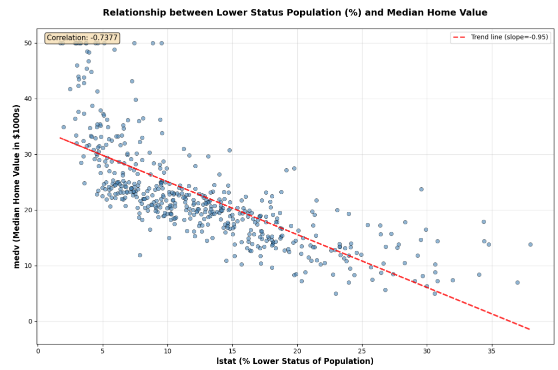
Additional statistics for medv:
Variance: 84.5867
Standard deviation: 9.1971
Skewness: 1.1081
Kurtosis: 1.4952

Additional statistics for lstat:
Variance: 50.9948
Standard deviation: 7.1411
Skewness: 0.9065
Kurtosis: 0.4932

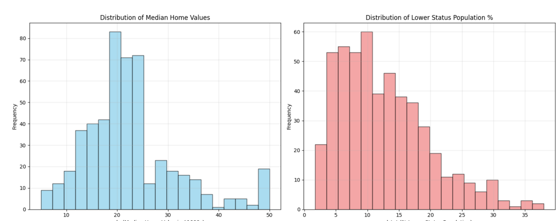
1.3 CORRELATION ANALYSIS
Correlation coefficient between medv and lstat: -0.7377

INTERPRETA1TION:
- The correlation coefficient of -0.7377 indicates a strong negative relationship
- This means that as lstat (% lower status population) increases, medv (median home value) tends to decrease
- The relationship explains approximately 54.4% of the variance (R^2 = 0.5441)
- Statistical significance: p-value = 5.08e-88
- The correlation is statistically significant at α = 0.05

1.4 SCATTER PLOT ANALYSIS



PATTERN OBSERVED IN SCATTER PLOT:
- The scatter plot reveals a clear negative relationship between lstat and medv
- As the percentage of lower status population increases, median home values tend to decrease
- The relationship appears to be non-linear, showing a curved pattern rather than a straight line
- There's more variability in home values at lower lstat percentages
- The relationship seems stronger (steeper decline) at lower lstat values and levels off at higher lstat values
- There are some potential outliers, particularly homes with high values despite higher lstat percentages
- The data points form a characteristic negative exponential or power-law pattern



SUMMARY:
- Dataset contains 506 observations and 14 variables
- Strong negative correlation (-0.7377) between lstat and medv
- Non-linear relationship visible in scatter plot
- Both variables show reasonable distributions for regression analysis

Part 2: Linear Regression Model Fitting

$$\text{medv} = \hat{\beta}_0 + \hat{\beta}_1 \times \text{lstat}$$

COEFFICIENTS:
Intercept (β_0): 34.5538
Slope (β_1): -0.9500

2.1 ESTIMATED REGRESSION EQUATION
medv = 34.5538 + (-0.9500) $\times$ lstat
medv = 34.5538 - 0.9500 $\times$ lstat

Alternative notation:
ŷ = 34.5538 + (-0.9500)x
where ŷ = predicted median home value and x = lstat

2.2 INTERPRETATION OF INTERCEPT (β_0)
Intercept value: 34.5538

INTERPRETATION:
- The intercept represents the predicted median home value when lstat = 0
- This means when 0% of the population has lower status, the predicted median home value is $34.55k
- In practical terms: $34554

PRACTICAL MEANING:
- Observed lstat range: 1.73% to 37.97%
- Since the minimum observed lstat is 1.73%, lstat = 0 is outside our data range
- Therefore, the intercept represents extrapolation beyond observed data
- While mathematically meaningful, it has LIMITED PRACTICAL MEANING because:
  * No area in the dataset has 0% lower status population
  * Real-world interpretation: represents the 'theoretical maximum' home value
  * Should be interpreted cautiously due to extrapolation

2.3 INTERPRETATION OF SLOPE (β_1)
Slope value: -0.9500

INTERPRETATION:
For each 1% increase in lstat (lower status population), the median home value decreases by $0.9500k on average, holding all other factors constant.

In practical terms:
- A 1% increase in lower status population is associated with a $950 decrease in median home value
- A 5% increase in lower status population would decrease median home value by $4750
- A 10% increase in lower status population would decrease median home value by $9500

2.4 CONFIDENCE INTERVALS AND SIGNIFICANCE TESTING
95% CONFIDENCE INTERVALS:

	0	1
Intercept	33.448	35.659
lstat	-1.026	-0.874

DETAILED CONFIDENCE INTERVALS:
Intercept (β_0): [33.4485, 35.6592]
Slope (β_1): [-1.0261, -0.8740]

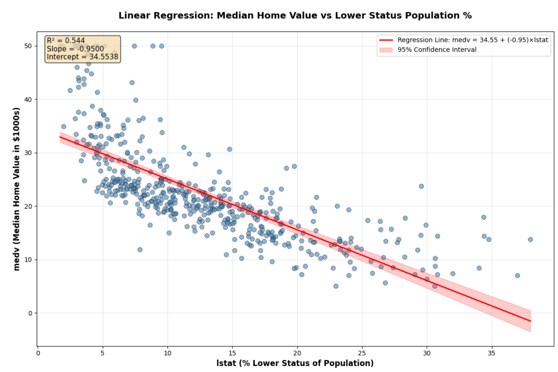
SIGNIFICANCE TESTING:
H_0: β = 0 (coefficient equals zero)
H_1: $\beta \neq 0$ (coefficient is significantly different from zero)

INTERCEPT (β_0) ANALYSIS:
- 95% CI: [33.4485, 35.6592]
- Contains zero? No
- Conclusion: The intercept IS significantly different from zero
- This means we can be 95% confident the true intercept is between 33.4485 and 35.6592

SLOPE (β_1) ANALYSIS:
- 95% CI: [-1.0261, -0.8740]
- Contains zero? No
- Conclusion: The slope IS significantly different from zero
- This means we can be 95% confident the true slope is between -1.0261 and -0.8740

P-VALUES (for additional confirmation):
Intercept p-value: 3.74e-236
Slope p-value: 5.08e-88
Both p-values < 0.05: True

MODEL SUMMARY STATISTICS:
R-squared: 0.5441
Adjusted R-squared: 0.5432
F-statistic: 601.62
F-statistic p-value: 5.08e-88
Standard Error: 6.2158



FINAL SUMMARY:
- Regression equation: medv = 34.5538 + (-0.9500) × lstat
- Both coefficients are statistically significant at α = 0.05
- The model explains 54.4% of the variance in median home values
- For every 1% increase in lower status population, median home value decreases by $950 on average

2.5 R-SQUARED ANALYSIS
R-squared value: 0.5441
R-squared as percentage: 54.41%

INTERPRETATION:
- R^2 = 0.5441 means that 54.41% of the variation in median home values is explained by the percentage of lower status population (lstat)
- The remaining 45.59% of variation is due to other factors not included in this model
- This indicates a moderate relationship
- In practical terms: knowing the lstat value allows us to predict about 54.4% of the variation in home values

2.6 ROOT MEAN SQUARE ERROR (RMSE)
Mean Squared Error (MSE): 38.6357
Root Mean Square Error (RMSE): 6.2158

INTERPRETATION:
- RMSE = 6.2158 thousands of dollars
- In actual dollars: $6216
- This means the typical prediction error is approximately $6216
- On average, our predictions are off by about ±$6216 from the actual median home value

CONTEXT:
- Mean home value: $22.53k ($22533)
- Standard deviation of home values: $9.20k
- Range of home values: $45.00k
- RMSE as % of mean: 27.6%
- RMSE as % of standard deviation: 67.6%

2.7 F-STATISTIC AND OVERALL MODEL SIGNIFICANCE
F-statistic: 601.6179
F-statistic p-value: 5.08e-88
Degrees of freedom: Model = 1.0, Residual = 504.0

HYPOTHESIS TEST:
H_0: The model has no explanatory power (β_1 = 0)
H_1: The model has explanatory power ($\beta_1 \neq 0$)

INTERPRETATION:
- F-statistic = 601.6179 with p-value = 5.08e-88
- Since p-value < 0.05, we REJECT the null hypothesis
- Conclusion: The model IS statistically significant
- This means lstat DOES have significant explanatory power for predicting medv

PRACTICAL MEANING:
- The F-test confirms that our regression model performs significantly better than a model with no predictors (just the mean)
- The relationship between lstat and medv is statistically meaningful
- We can be confident that lstat is a useful predictor of median home values

2.8 ADJUSTED R-SQUARED COMPARISON
R-squared: 0.544146
Adjusted R-squared: 0.543242
Difference: 0.000904

WHY THERE MIGHT BE A DIFFERENCE:
- Regular R^2: 0.544146
- Adjusted R^2: 0.543242
- The difference of 0.000904 is very small

WHAT ADJUSTED R-SQUARED ACCOUNTS FOR:
- Number of predictors in the model: 1.0
- Sample size: 506 observations
- Degrees of freedom penalty for adding predictors

FORMULA EXPLANATION:
Adjusted R^2 = 1 - [(1 - R^2) × (n - 1) / (n - k - 1)]
where n = sample size (506) and k = number of predictors (1.0)
Manual calculation: 0.543242

INTERPRETATION:
- The very small difference suggests our model is not overfitting
- With only one predictor, the adjustment is minimal
- Both R^2 and adjusted R^2 tell essentially the same story

PRACTICAL IMPLICATIONS:
- For model comparison: Use adjusted R^2 when comparing models with different numbers of predictors
- For interpretation: Both values are nearly identical, indicating a robust single-predictor model
- The penalty for our one predictor is minimal given the sample size of 506 observations

FINAL SUMMARY:
- R^2 = 0.5441 (54.41% of variance explained)
- Adjusted R^2 = 0.5432 (54.32% of variance explained)
- RMSE = $6216 (typical prediction error)
- F-statistic = 601.6179, p < 0.05 (highly significant model)
- Model explains 54.4% of home value variation using just lstat
- Typical prediction accuracy: ±$6216 (27.6% of mean home value)

Part 3: Statistical Inference and Hypothesis Testing

3.1 HYPOTHESIS TESTING SETUP
TESTING THE SLOPE COEFFICIENT:
H_0: β_1 = 0 (The slope coefficient is zero)
→ lstat has no linear relationship with medv
→ There is no linear association between % lower status population and median home value

H_1: $\beta_1 \neq 0$ (The slope coefficient is not zero)
→ lstat has a significant linear relationship with medv
→ There is a significant linear association between % lower status population and median home value

Type of test: Two-tailed test
Significance level: α = 0.05

3.2 T-STATISTIC AND P-VALUE ANALYSIS
TEST STATISTICS:
t-statistic: -24.527900
p-value: 5.08e-88
Degrees of freedom: 504.0
Critical t-value (α = 0.05, two-tailed): ±1.9647

DECISION MAKING:
Decision rule: Reject H_0 if |t| > 1.9647 OR if p-value < 0.05
Observed: |t| = 24.5279, p-value = 5.08e-88

CONCLUSION AT 5% SIGNIFICANCE LEVEL:
REJECT H_0: The slope coefficient IS significantly different from zero
- |t| = 24.5279 > 1.9647
- p-value = 5.08e-88 < 0.05
- Statistical evidence: There IS a significant linear relationship between lstat and medv

PRACTICAL INTERPRETATION:
- We can be 95% confident that changes in % lower status population have a real, measurable effect on median home values
- The relationship observed in our sample is unlikely to be due to random chance
- The effect size: each 1% increase in lstat is associated with a $950 decrease in median home value

3.3 CONFIDENCE INTERVAL ANALYSIS
CONFIDENCE INTERVALS FOR SLOPE COEFFICIENT:
95% Confidence Interval: [-1.026148, -0.873951]
99% Confidence Interval: [-1.050199, -0.849899]

INTERVAL WIDTH COMPARISON:
95% CI width: 0.152198
99% CI width: 0.200300
Width increase: 0.048102
Percent increase in width: 31.6%

INTERPRETATION:
95% CONFIDENCE INTERVAL:
- We are 95% confident that the true slope coefficient lies between -1.026148 and -0.873951

- In practical terms: each 1% increase in lstat decreases median home value by between \$874 and \$1026

99% CONFIDENCE INTERVAL:
- We are 99% confident that the true slope coefficient lies between -1.050199 and -0.849899
- In practical terms: each 1% increase in lstat decreases median home value by between \$850 and \$1050

COMPARISON ANALYSIS:
- The 99% CI is wider than the 95% CI by 0.048102
- This represents a 31.6% increase in width
- WHY: Higher confidence level requires a wider interval to capture the true parameter
- TRADE-OFF: More confidence (99% vs 95%) comes at the cost of precision (wider interval)

SIGNIFICANCE IMPLICATIONS:
95% CI contains zero: No
99% CI contains zero: No
- Since neither interval contains zero, the slope is significant at both levels
- This provides strong evidence for a real relationship between lstat and medv

3.4 TESTING SPECIFIC CLAIM
CLAIM TO TEST:
Someone claims that each 1% increase in lstat decreases median home value by exactly \$1000
In our units: β_1 = -1.0 (since medv is in thousands of dollars)

HYPOTHESES:
H_0: β_1 = -1.0 (the claim is correct)
H_1: $\beta_1 \neq$ -1.0 (the claim is incorrect)

TEST USING CONFIDENCE INTERVALS:
Observed slope coefficient: -0.950049
Claimed slope coefficient: -1.0

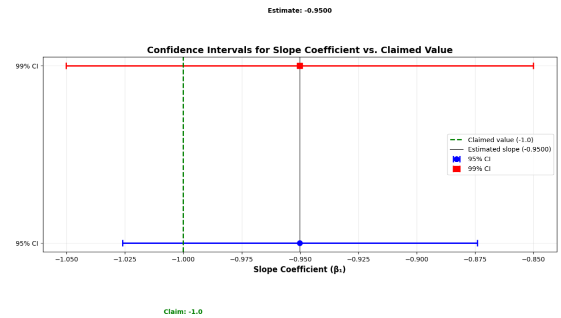
95% Confidence Interval Test:
- 95% CI: [-1.026148, -0.873951]
- Does the CI contain -1.0? Yes

99% Confidence Interval Test:
- 99% CI: [-1.050199, -0.849899]
- Does the CI contain -1.0? Yes

FORMAL T-TEST:
t-statistic = (observed - claimed) / SE = (-0.950049 - -1.0) / 0.038733
t-statistic = 1.2896
p-value (two-tailed): 0.1978

CONCLUSION:
FAIL TO REJECT the claim at 95% confidence level
- The claimed value (-1.0) IS within the 95% confidence interval
- Our regression results SUPPORT the claim
FAIL TO REJECT the claim at 99% confidence level
- The claimed value (-1.0) IS within the 99% confidence interval

STATISTICAL EVIDENCE:
- Our estimate: Each 1% increase in lstat decreases home value by \$950
- Claimed effect: Each 1% increase in lstat decreases home value by \$1000
- Difference: \$50
- The difference is not statistically significant (p = 0.1978 $\geq$ 0.05)
- Insufficient evidence to reject the claim



FINAL SUMMARY:
Hypotheses: H_0: β_1 = 0 vs H_1: $\beta_1 \neq$ 0
Test results: t = -24.5279, p = 5.08e-88
Conclusion: Reject H_0 - slope is significant
Confidence intervals:
95% CI: [-1.026148, -0.873951] (width: 0.152198)
99% CI: [-1.050199, -0.849899] (width: 0.200300)
99% CI is 31.6% wider than 95% CI
Claim test: The claim of exactly \$1000 decrease is SUPPORTED
Our estimate: \$950 decrease per 1% lstat increase
Statistical significance of difference: p = 0.1978

Part 4: Assumption Testing and Model Diagnostics

BOSTON HOUSING ASSUMPTION TESTING AND MODEL DIAGNOSTICS
MODEL SUMMARY:
Sample size: 506
Number of residuals: 506
Mean of residuals: 0.000000 (should be $\approx$ 0)
Standard deviation of residuals: 6.2096

4.1 SHAPIRO-WILK TEST FOR NORMALITY OF RESIDUALS
HYPOTHESIS TESTING:
H_0: Residuals follow a normal distribution
H_1: Residuals do not follow a normal distribution
Significance level: α = 0.05

TEST RESULTS:
Shapiro-Wilk test statistic (W): 0.878572
p-value: 0.000000

DECISION MAKING:
Decision rule: Reject H_0 if p-value < 0.05
Observed p-value: 0.000000

CONCLUSION AT 5% SIGNIFICANCE LEVEL:
REJECT H_0: Residuals do not follow a normal distribution
- Statistical evidence suggests departure from normality
- The normality assumption may be violated

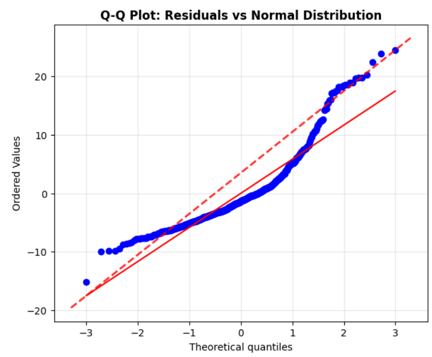
INTERPRETATION OF TEST STATISTIC:
- W = 0.878572
- W ranges from 0 to 1, with values closer to 1 indicating more normal-like data
- Our value suggests weak evidence of normality based on the test statistic alone

ADDITIONAL NORMALITY TESTS (for comparison):
D'Agostino's test: statistic = 137.0434, p-value = 0.000000
Jarque-Bera test: statistic = 291.3734, p-value = 0.000000

CONSENSUS: Tests show mixed results regarding normality

4.2 Q-Q PLOT ANALYSIS
Q-Q PLOT INTERPRETATION:
The Q-Q (Quantile-Quantile) plot compares residual quantiles to theoretical
normal quantiles
Q-Q plot correlation: 0.9373
(Values closer to 1 indicate better fit to normal distribution)

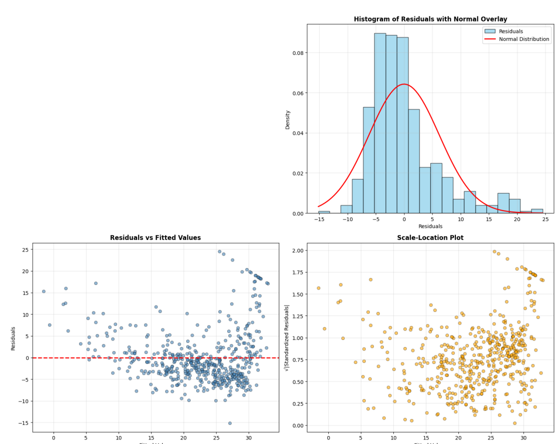
VISUAL ASSESSMENT:
- Good fit with minor deviations
- Look for points following the red diagonal line
- Systematic deviations suggest non-normality
- Graphed Q-Q plot backs up the previously observed weak evidence of normality based on the test statistic



4.3 HISTOGRAM WITH NORMAL DISTRIBUTION OVERLAY
SHAPE ANALYSIS:
Skewness: 1.4527
Kurtosis: 2.3191 (excess kurtosis)

SKEWNESS INTERPRETATION:
- Skewness = 1.4527 indicates highly skewed
- Distribution is skewed to the right

KURTOSIS INTERPRETATION:
- Excess kurtosis = 2.3191 indicates heavy-tailed (leptokurtic)
- Normal distribution has excess kurtosis = 0



DEPARTURES FROM NORMALITY:
Identified departures from normality:
1. Skewness (1.453)
2. Kurtosis (2.319)
3. Shapiro-Wilk test rejection
4. Q-Q plot deviations

4.2 VISUAL EVIDENCE VS STATISTICAL TEST COMPARISON:
Statistical test result (Shapiro-Wilk): Rejects normality
Visual evidence assessment: Shows deviations from normality
AGREEMENT: Visual evidence and statistical test both suggest departure from
normality

DETAILED VISUAL OBSERVATIONS:
Q-Q Plot:
- Systematic deviations from diagonal line (r = 0.9373)
- Visual evidence against perfect normality

Histogram:
- Notable departures from bell-shaped normal distribution
- Skewness and/or kurtosis concerns visible

PRACTICAL IMPLICATIONS FOR REGRESSION:
NORMALITY ASSUMPTION VIOLATED:
- Confidence intervals may be less reliable
- Consider robust standard errors
- Prediction intervals may be inaccurate
- Consider variable transformation

SAMPLE SIZE CONSIDERATIONS:
- Sample size: 506 observations
- Large sample: Central Limit Theorem helps with normality concerns
- Minor deviations from normality are less problematic

FINAL SUMMARY:
Shapiro-Wilk test: W = 0.878572, p = 0.000000
Conclusion: Residuals deviate from normality
Q-Q plot assessment: r = 0.9373
Visual evidence: Shows deviations from normality
Histogram analysis:
Skewness: 1.4527, Kurtosis: 2.3191
Shape: highly skewed, heavy-tailed (leptokurtic)
Overall normality assessment: VIOLATED

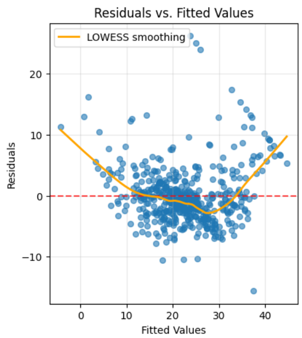
4.4: BREUSCH-PAGAN TEST RESULTS
Test Statistic: 4.1871
P-value: 0.0407
Degrees of Freedom: 1

Conclusion: Reject H0 at α = 0.05. Evidence of heteroscedasticity.

Verification (statsmodels function): Stat = 65.1218, P-value = 0.0000

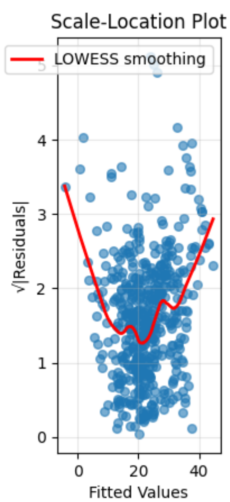
4.5: RESIDUALS VS. FITTED VALUES ANALYSIS
Pattern interpretation:
- HOMOSCEDASTICITY: Points should be randomly scattered around the horizontal line at y=0
- HETEROSCEDASTICITY indicators:
  * Funnel shape (variance increases or decreases with fitted values)
  * Curved patterns in the smoothing line
  * Clear clustering or systematic patterns

Variance in lowest third of fitted values: 17.2703
Variance in highest third of fitted values: 31.7984
Variance ratio (high/low): 1.8412
Interpretation: Ratio > 2 or < 0.5 suggests heteroscedasticity



4.6: SCALE-LOCATION PLOT ANALYSIS
Evidence of changing variance:
- CONSTANT VARIANCE: Smoothing line should be roughly horizontal
- CHANGING VARIANCE indicators:
  * Upward or downward trend in smoothing line
  * Clear patterns or curves in the line

Correlation between fitted values and |residuals|: 0.1507
Interpretation:
  * Moderate correlation suggests possible heteroscedasticity



COMPREHENSIVE HOMOSCEDASTICITY ASSESSMENT

TEST RESULTS SUMMARY:
1. Breusch-Pagan Test: Statistic = 4.1871, P-value = 0.0407
   → Reject H0 at α = 0.05. Evidence of heteroscedasticity.

2. Variance Ratio Analysis: 1.8412
   → Suggests homoscedasticity

3. Scale-Location Correlation: 0.1507
   → Moderate evidence of heteroscedasticity

FINAL SUMMARY
Evidence suggests heteroscedasticity
Explore different model specifications

4.7: DURBIN-WATSON TEST RESULTS
Durbin-Watson Statistic: 1.0784
First-order autocorrelation (ρ): 0.4608

INTERPRETATION:
→ Evidence of positive autocorrelation. Independence assumption may be violated.

Durbin-Watson Guidelines:
- DW $\approx$ 2.0: No autocorrelation (ideal)
- DW < 1.5: Strong positive autocorrelation
- DW > 2.5: Strong negative autocorrelation
- 1.5 $\leq$ DW $\leq$ 2.5: Acceptable range

4.8: COOK'S DISTANCE ANALYSIS
Maximum Cook's Distance: 0.1657
Mean Cook's Distance: 0.0030
Standard Deviation: 0.0112

INFLUENTIAL OBSERVATIONS CRITERIA:
- Threshold 4/n = 4/506 = 0.0079
- Conservative threshold = 1.0

RESULTS:
- Observations with Cook's D > 4/n: 30 (5.9%)
- Observations with Cook's D > 1.0: 0 (0.0%)

CONCLUSION: Moderate Cook's distance values. Some observations may be
influential but not necessarily problematic.

TOP 5 MOST INFLUENTIAL OBSERVATIONS:
1. Observation 368: Cook's D = 0.1657
2. Observation 372: Cook's D = 0.0941
3. Observation 364: Cook's D = 0.0694
4. Observation 365: Cook's D = 0.0672
5. Observation 369: Cook's D = 0.0553

4.9: HIGH LEVERAGE ANALYSIS
Number of parameters (p): 14
Sample size (n): 506
High leverage threshold (2p/n)
: 2 $\times$ 14 / 506 = 0.0553

HIGH LEVERAGE RESULTS:
- Observations with high leverage: 36
- Percentage of total sample: 7.1%
- Maximum leverage value: 0.3060
- Mean leverage value: 0.0277

TOP 5 HIGHEST LEVERAGE OBSERVATIONS:
1. Observation 380: Leverage = 0.3060
2. Observation 418: Leverage = 0.1901
3. Observation 405: Leverage = 0.1564
4. Observation 410: Leverage = 0.1247
5. Observation 365: Leverage = 0.0985

FINAL SUMMARY
36 observations exceed the leverage threshold, meaning they have unusually strong influence on the fitted values.
The 7.1% figure is a bit higher than you'd expect under ideal conditions since leverage values tend to be low and centered around the mean of 0.0277
The maximum leverage of 0.3060 is quite high-this observation (380) likely has a strong impact on the regression line and should be examined for potential outlier behavior or undue influence.

4.10: COMPREHENSIVE MODEL VALIDATION SUMMARY

LINEAR REGRESSION ASSUMPTIONS ASSESSMENT:
1. LINEARITY:
    Test method: Shapiro-Wilk test
    Result: W = 0.878572
    Status: VIOLATED

2. INDEPENDENCE OF RESIDUALS:
    Test method: Durbin-Watson test
    Result: DW = 1.0784
    Status: VIOLATED

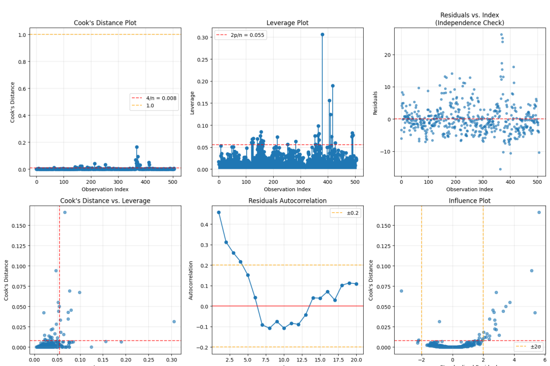
3. HOMOSCEDASTICITY (Constant Variance):

Test method: Breusch-Pagan test, residuals plots
Result: Test Statistic: 4.1871
P-value: 0.0407
Degrees of Freedom: 1
Status: VIOLATED

4. NORMALITY OF RESIDUALS:
Test method: Shapiro-Wilk, Q-Q plots, histograms
Result: Residuals deviate from normality
Q-Q plot assessment: r = 0.9373
Visual evidence: Shows deviations from normality
Histogram analysis:
Skewness: 1.4527, Kurtosis: 2.3191
Shape: highly skewed, heavy-tailed (leptokurtic)
Status: VIOLATED

5. NO EXCESSIVE INFLUENTIAL OBSERVATIONS:
Test method: Cook's distance, leverage analysis
Cook's D max: 0.1657
High leverage obs: 36 (7.1%)
Status: MARGINAL - Some influential observations present

OVERALL MODEL VALIDITY FOR STATISTICAL INFERENCE:
CURRENT ASSESSMENT (based on available tests):
- Assumptions checked: 5
- Assumptions satisfied: 0



Part 5: Predictions and Intervals

PREDICTIONS AND INTERVALS ANALYSIS

DATASET OVERVIEW
Dataset shape: (506, 14)
Column names: ['crim', 'zn', 'indus', 'chas', 'nox', 'rm', 'age', 'dis', 'rad', 'tax', 'ptratio', 'b', 'lstat', 'medv']

Using 'medv' as target variable
Using 'lstat' as predictor variable (lstat)

SIMPLE LINEAR REGRESSION MODEL
Model: medv ~ lstat
R-squared: 0.5441
Regression equation: medv = 34.5538 + -0.9500 × lstat

5.1: PREDICTION FOR LSTAT = 10%
CALCULATION:
$\hat{y} = \beta_0 + \beta_1 \times X$
$\hat{y} = 34.5538 + -0.9500 \times 10.0$
$\hat{y} = 25.0533$

Predicted median home value for lstat = 10%: $25.05k

5.2: 95% CONFIDENCE INTERVAL FOR MEAN RESPONSE
CALCULATION DETAILS:
- Predicted value: 25.0533
- Standard error of mean: 0.2948
- t-critical (α=0.05, df=504.0): 1.9647
- Margin of error: 0.5792

95% CONFIDENCE INTERVAL: [24.4741, 25.6326]
In dollars: [$24.47k, $25.63k]

INTERPRETATION:
We are 95% confident that the mean median home value for all neighborhoods
with lstat = 10% is between $24.47k and $25.63k.

5.3: 95% PREDICTION INTERVAL FOR INDIVIDUAL RESPONSE
CALCULATION DETAILS:
- Predicted value: 25.0533
- Standard error of prediction: 6.4803
- t-critical (α=0.05, df=504.0): 1.9647
- Margin of error: 12.7316

95% PREDICTION INTERVAL: [12.3217, 37.7850]
In dollars: [$12.32k, $37.78k]

INTERVAL COMPARISON:
- Confidence interval width: 1.1584
- Prediction interval width: 25.4633
- Prediction interval is 21.98x wider than confidence interval

5.4: CONFIDENCE VS PREDICTION INTERVALS
CONCEPTUAL DIFFERENCES:

CONFIDENCE INTERVAL:
- Estimates uncertainty about the MEAN response for a given X value
- Answers: 'What is the average Y for all observations with this X?'
- Accounts for uncertainty in estimating the population mean
- Gets narrower as sample size increases
- Narrower interval (less uncertainty)

PREDICTION INTERVAL:
- Estimates uncertainty about an INDIVIDUAL response for a given X value
- Answers: 'What might Y be for a single new observation with this X?'
- Accounts for both estimation uncertainty AND individual variation
- Includes natural scatter around the regression line
- Wider interval (more uncertainty)

WHEN TO USE EACH:

USE CONFIDENCE INTERVAL when:
- Estimating average outcomes for policy/planning
- Comparing mean responses between groups
- Making statements about population parameters
- Example: 'What's the average home value in 10% lstat neighborhoods?'

USE PREDICTION INTERVAL when:
- Predicting outcomes for specific individuals/cases
- Setting bounds for individual forecasts
- Risk assessment for single observations
- Example: 'What might this specific house be worth?'

5.5: PREDICTIONS AT MULTIPLE LSTAT VALUES
POINT PREDICTIONS:

lstat = 5%:
- → Predicted value: $29.80k
- → 95% CI: [$29.01k, $30.60k]
- → 95% PI: [$16.63k, $42.98k]

lstat = 10%:
- → Predicted value: $25.05k
- → 95% CI: [$24.47k, $25.63k]
- → 95% PI: [$12.32k, $37.78k]

lstat = 15%:
- → Predicted value: $20.30k
- → 95% CI: [$19.73k, $20.87k]
- → 95% PI: [$7.58k, $33.02k]

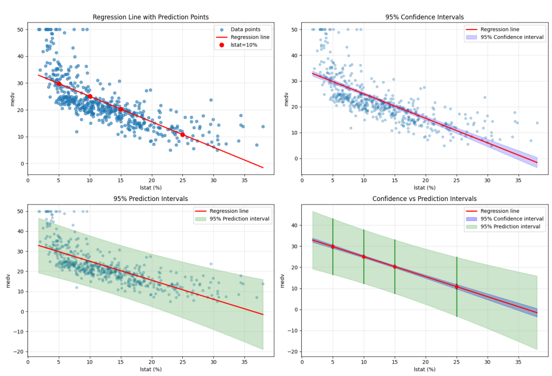
lstat = 25%:
- → Predicted value: $10.80k
- → 95% CI: [$9.72k, $11.89k]
- → 95% PI: [$-3.15k, $24.75k]

RELATIONSHIP ANALYSIS:

Model slope (β_1): -0.9500
Interpretation: For each 1% increase in lstat, median home value
decreases by $0.95k on average

CHANGES BETWEEN LSTAT LEVELS:
- 5.0% → 10.0%: Change = $-4.75k
  Rate: $-0.95k per 1% lstat increase
- 10.0% → 15.0%: Change = $-4.75k
  Rate: $-0.95k per 1% lstat increase
- 15.0% → 25.0%: Change = $-9.50k
  Rate: $-0.95k per 1% lstat increase

COMMENTS ON RELATIONSHIP:
- The relationship shows moderate negative association
- Linear relationship assumed constant across all lstat levels
- Higher lstat (more lower status population) associated with lower home values



PREDICTIONS SUMMARY TABLE

DETAILED PREDICTIONS TABLE:

lstat	prediction	ci_lower	ci_upper	pi_lower	pi_upper	ci_width	pi_width	width_ratio
5	29.804	29.007	30.600	16.627	42.980	1.592	26.353	16.550
10	25.053	24.474	25.633	12.322	37.785	1.158	25.463	21.981
15	20.303	19.732	20.875	7.585	33.021	1.143	25.436	22.254
25	10.803	9.717	11.888	-3.148	24.754	2.170	27.902	12.856

KEY INSIGHTS:
- As lstat increases, predicted home values decrease
- Prediction intervals are consistently 18.4x wider than confidence intervals
- The linear relationship appears moderate (R^2 = 0.544)