Math6450 Assignment2: Multiple Linear Regression

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1 Data Exploration

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(a) Descriptive Statistics for Continuous Variables
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Comprehensive Descriptive Statistics:
               Mean   Median  Std Dev  Minimum ␣
↪Maximum  Skewness  Kurtosis
claims       18.049   17.845    6.448     0.72    41.
↪39      0.254      0.095
deductible    2.490    1.905    1.942     0.51    10.
↪00      1.542      2.351
coverage    189.014  186.750   72.169    50.00   424.
↪50      0.145     -0.292
age          15.438   11.000   14.227     1.00    85.
↪00      1.869      4.496
premium       2.969    2.945    0.822     0.50     5.
↪78      0.245      0.030
```

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(b) Correlation Matrix for Continuous Variables
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Correlation Matrix:
             claims  deductible  coverage    age ␣
↪premium
claims        1.000      -0.265     0.761  0.199    0.
↪793
deductible   -0.265       1.000    -0.066  0.006   -0.
↪059
coverage      0.761      -0.066     1.000 -0.015    0.
↪723
age           0.199       0.006    -0.015  1.000    0.
↪314
premium       0.793      -0.059     0.723  0.314    1.
↪000

Variable with strongest linear relationship with␣
↪'claims':
Variable: premium
Correlation coefficient: 0.793
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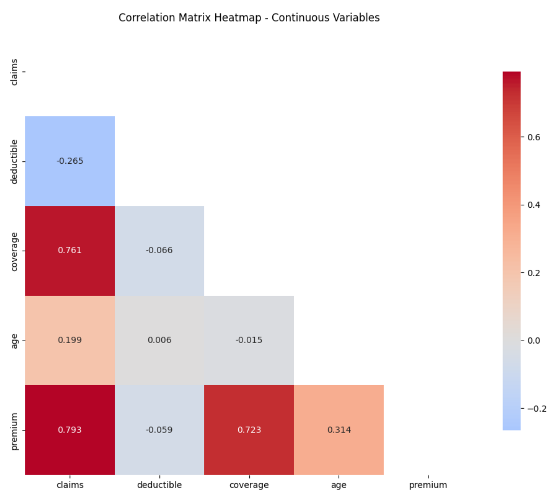


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(c) Skewness Analysis and Log Transformation␣
↪Assessment
--------------------------------------------------

Skewness Assessment:
Rule of thumb: |skewness| > 1 indicates highly skewed␣
↪distribution
Rule of thumb: 0.5 < |skewness| < 1 indicates␣
↪moderately skewed distribution

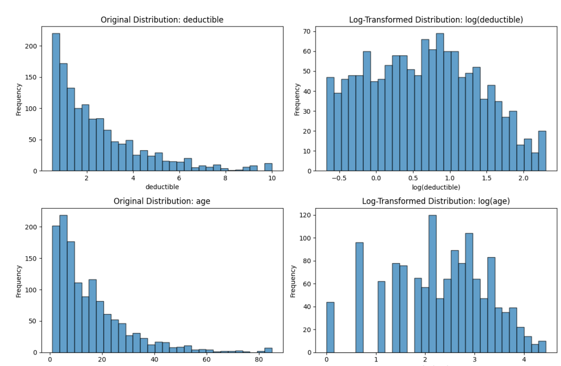
claims:
  Skewness: 0.254
  Assessment: Approximately symmetric

deductible:
  Skewness: 1.542
  Assessment: Highly skewed
  Log transformation skewness: 0.134
  Improvement from log transformation: 1.408
  Recommendation: Log transformation would improve␣
↪normality

coverage:
  Skewness: 0.145
  Assessment: Approximately symmetric

age:
  Skewness: 1.869
  Assessment: Highly skewed
  Log transformation skewness: -0.347
  Improvement from log transformation: 1.523
  Recommendation: Log transformation would improve␣
↪normality

premium:
  Skewness: 0.245
  Assessment: Approximately symmetric
```

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Summary of Findings:
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Variables with skewed distributions: deductible, age
Variable most strongly correlated with claims: premium (r = 0.793)

Data Overview:
Total observations: 1,340
Variables analyzed: 5
Missing values: 0

2 Simple Linear Regression

Dataset Information:
Total observations: 1,340
Observations used in regression: 1,340
Missing values removed: 0

(a) Simple Linear Regression Model Fitting

Model Coefficients:
Intercept (β_0): 5.2054
Slope (β_1): 0.0679

Fitted Regression Equation:
Claims = 5.2054 + 0.0679 × Coverage

In mathematical notation:
$\hat{y}$ = 5.2054 + 0.0679x
where $\hat{y}$ = predicted claims, x = coverage

(b) Interpretation of Slope Coefficient

Slope coefficient: 0.0679

Practical Interpretation:
- For every 1-unit increase in coverage, claims are expected to increase by
0.0679 units, on average.
- This indicates a positive relationship between coverage and claims.
- Properties with higher coverage amounts tend to have higher claims.

Alternative interpretation:
- For every 100-unit increase in coverage, claims change by 6.79 units, on
average.

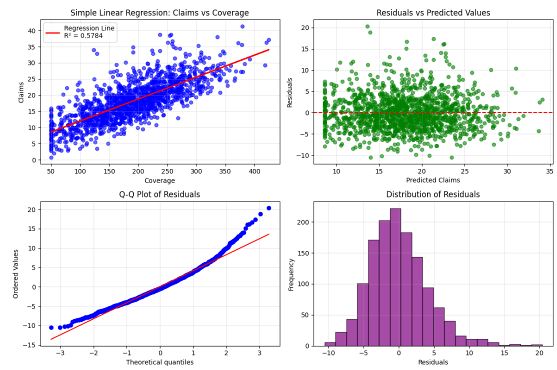
Example predictions:
- Coverage = 100: Predicted Claims = 12.00
- Coverage = 150: Predicted Claims = 15.40
- Coverage = 200: Predicted Claims = 18.80
- Coverage = 250: Predicted Claims = 22.19

(c) Coefficient of Determination (R^2) Analysis

Model Performance Metrics:
R^2 (Coefficient of Determination): 0.5784
R^2 as percentage: 57.84%
Correlation coefficient (r): 0.7605
Root Mean Square Error (RMSE): 4.1850

Interpretation of R^2:
- 57.84% of the variation in claims is explained by coverage.
- 42.16% of the variation in claims is due to other factors not included in the
model.
- The linear relationship between coverage and claims is moderate (R^2 = 0.5784).

Statistical Significance:
- t-statistic: 42.8442
- p-value: 0.0000
- Degrees of freedom: 1338
- The relationship is statistically significant at the 5% level.



Summary Table:

Metric	Value	Interpretation
Intercept (β_0)	5.2054	Expected claims when coverage = 0
Slope (β_1)	0.0679	Change in claims per unit increase in coverage
R^2	0.5784	57.8% of variance explained
Correlation (r)	0.7605	Linear association strength
RMSE	4.1850	Average prediction error
Observations	1340	Sample size

Key Findings Summary:
- Regression equation: Claims = 5.2054 + 0.0679 × Coverage
- Slope interpretation: Each additional unit of coverage is associated with a
0.0679 unit change in claims
- Model explains 57.8% of the variation in claims
- The relationship is statistically significant (p = 0.0000)

3 Multiple Regression Model

Dependent Variable: claims
Explanatory Variables: deductible, coverage, age, prior_claims, premium

Dataset Information:
Total observations: 1,340
Complete cases used: 1,340
Observations removed (missing data): 0
Number of explanatory variables: 5

(a) Fitted Regression Equation

Coefficient Estimates (rounded to 3 decimal places):
Intercept (β_0): 3.208
β_1 (deductible): -0.728

β_2 (coverage): 0.062
β_3 (age): 0.091
β_4 (prior_claims): 2.580
β_5 (premium): 0.495

Fitted Regression Equation:
Claims = 3.208 - 0.728 × deductible + 0.062 × coverage + 0.091 × age + 2.580 ×
prior_claims + 0.495 × premium

Compact Mathematical Form:
$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5$
$\hat{y} = 3.208 + -0.728 x_1 + 0.062 x_2 + 0.091 x_3 + 2.580 x_4 + 0.495 x_5$
where x_1=deductible, x_2=coverage, x_3=age, x_4=prior_claims, x_5=premium

(b) Standard Errors for Each Coefficient

Standard Errors:
Intercept (β_0): 0.3172
β_1 (deductible): 0.0394
β_2 (coverage): 0.0020
β_3 (age): 0.0068
β_4 (prior_claims): 0.1210
β_5 (premium): 0.2118

Additional Statistics (t-statistics and p-values):

Coefficient	Estimate	Std Error	t-stat	p-value	Significance
Intercept	3.208	0.3172	10.113	0.0000	***
deductible	-0.728	0.0394	-18.459	0.0000	***
coverage	0.062	0.0020	30.624	0.0000	***
age	0.091	0.0068	13.401	0.0000	***
prior_claims	2.580	0.1210	21.316	0.0000	***
premium	0.495	0.2118	2.338	0.0195	*

Significance codes: *** p<0.001, ** p<0.01, * p<0.05

(c) Model Performance Statistics

R^2 (Coefficient of Determination): 0.8130
Adjusted R^2: 0.8123
Residual Standard Deviation: 2.7938

Additional Model Statistics:
Multiple R (Correlation): 0.9016
Residual Sum of Squares (RSS): 10412.1409
Mean Squared Error (MSE): 7.8052
F-statistic: 1159.6202
F-statistic p-value: 0.000000
Overall model significance: Yes (α = 0.05)

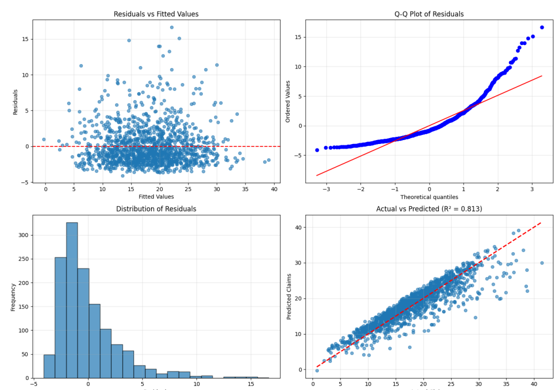
Degrees of Freedom:
Model: 5
Residual: 1334
Total: 1339

Summary Results Table:

	Variable	Coefficient	Std_Error	Coefficient_Rounded
0	Intercept	3.2078	0.3172	3.208
1	deductible	-0.7278	0.0394	-0.728
2	coverage	0.0621	0.0020	0.062
3	age	0.0906	0.0068	0.091
4	prior_claims	2.5797	0.1210	2.580
5	premium	0.4953	0.2118	0.495

Model Performance Table:

Statistic	Value
R^2	0.8130
Adjusted R^2	0.8123
Residual Std Deviation	2.7938
F-statistic	1159.6202
p-value (F-test)	0.000000
Observations	1340
Variables	5



Key Results Summary:
✓ Multiple regression equation fitted with 5 explanatory variables
✓ Model explains 81.3% of variance in claims (R^2 = 0.8130)
✓ Adjusted R^2 = 0.8123 (accounts for number of variables)
✓ Residual standard deviation = 2.7938
✓ Overall model is significant (F-test p-value = 0.000000)
✓ Standard errors calculated for all 6 coefficients

4 Statistical Inference

Multiple Linear Regression Model: Claims vs (Deductible, Coverage, Age,
Prior_Claims, Premium)
Model Summary:
Observations: 1340
Variables: 5
Degrees of freedom (residual): 1334
R^2: 0.8130
MSE: 7.8052

Coefficient Estimates:
Variable Coefficient Std Error t-statistic p-value

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deductible	-0.7278	0.0394	-18.4591	0.0000
coverage	0.0621	0.0020	30.6239	0.0000
age	0.0906	0.0068	13.4010	0.0000
prior_claims	2.5797	0.1210	21.3156	0.0000
premium	0.4953	0.2118	2.3382	0.0195

(a) Testing Significance of Age Coefficient
Hypothesis Test for Age Coefficient:

Null Hypothesis (H_0): β_age = 0
Alternative Hypothesis (H_1): β_age $\neq$ 0
Significance level (α): 0.05
Test type: Two-tailed t-test

Test Statistics:
Age coefficient (β_age): 0.0906
Standard error (SE): 0.0068
t-statistic: 13.4010
Degrees of freedom: 1334
p-value: 0.0000
Critical value (±): 1.9617

Decision Rule:
Reject H_0 if |t-statistic| > 1.9617 OR if p-value < 0.05

Conclusion:
✓ REJECT H_0: The coefficient for age IS statistically significant at the 5%
level.
  |t-statistic| = 13.4010 > 1.9617
  p-value = 0.0000 < 0.05
  Age has a statistically significant effect on claims.

(b) 95% Confidence Interval for Prior Claims Coefficient
Confidence Interval Calculation:
Coefficient (β_prior_claims): 2.5797
Standard error: 0.1210
Degrees of freedom: 1334
Confidence level: 95%

Confidence Interval Formula:
CI = β ± t_(α/2,df) × SE(β)
CI = 2.5797 ± 1.9617 × 0.1210
CI = 2.5797 ± 0.2374

95% Confidence Interval for Prior Claims Coefficient:
[2.3423, 2.8171]

Practical Interpretation:
- We are 95% confident that the true effect of having prior claims on current
claims
  is between 2.3423 and 2.8171 units.
- Since the entire interval is positive, prior claims consistently INCREASE
current claims.
- Properties with prior claims have significantly higher current claims than those without.
- The width of the interval (0.4748) indicates the precision of our estimate.

(c) Overall F-test for Model Significance
Overall F-test for Regression Model:

Null Hypothesis (H_0): β_1 = β_2 = β_3 = β_4 = β_5 = 0
  (All explanatory variables have no effect on claims)
Alternative Hypothesis (H_1): At least one $\beta_i \neq 0$
  (At least one explanatory variable has a significant effect)
Significance level (α): 0.05

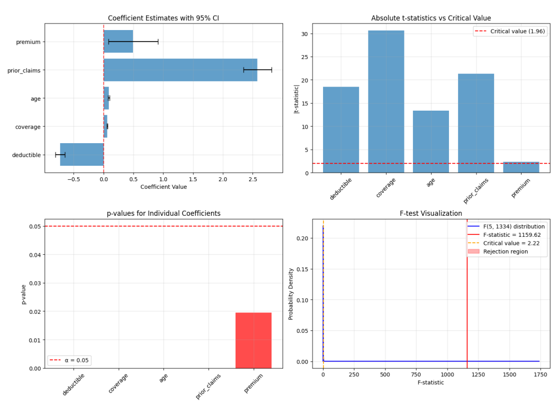
Test Statistics:
Total Sum of Squares (TSS): 55667.4953
Explained Sum of Squares (ESS): 45255.3543
Residual Sum of Squares (RSS): 10412.1409
Mean Square Regression (MSR): 9051.0709
Mean Square Error (MSE): 7.8052

F-statistic: 1159.6202
Degrees of freedom: (5, 1334)
p-value: 0.000000
Critical F-value (α = 0.05): 2.2208

Decision Rule:
Reject H_0 if F-statistic > 2.2208 OR if p-value < 0.05

Conclusion:
✓ REJECT H_0: The regression model IS statistically significant at the 5% level.
  F-statistic = 1159.6202 > 2.2208
  p-value = 0.000000 < 0.05
  At least one explanatory variable has a significant effect on claims.
  The model explains a significant portion of the variation in claims.

Model Performance Context:
R^2 = 0.8130 (81.3% of variance explained)
The model performs well in predicting claims.



Summary of All Statistical Tests:

Test	Statistic	p-value	Conclusion
Age Coefficient (t-test)	t = 13.4010	0.0000	Significant
Prior Claims CI	CI = [2.3423, 2.8171]	N/A	Does not contain 0

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 Overall Model (F-test)         F = 1159.6202 0.000000  Model Significant

LaTeX Summary Table:
\begin{table}
\caption{Summary of Statistical Tests}
\label{tab:hypothesis_tests}
\begin{tabular}{llll}
\toprule
Test & Statistic & p-value & Conclusion \\
\midrule
Age Coefficient (t-test) & t = 13.4010 & 0.0000 & Significant \\
Prior Claims CI & CI = [2.3423, 2.8171] & N/A & Does not contain 0 \\
Overall Model (F-test) & F = 1159.6202 & 0.000000 & Model Significant \\
\bottomrule
\end{tabular}
\end{table}
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5 Binary Variables and Model Interpretation

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Adding 'type' and 'location' to the original model
Dependent Variable: claims
Original Variables: deductible, coverage, age, prior_claims, premium
New Variables: type, location

Data Summary:
Original model observations: 1,340
Extended model observations: 1,340

Extended Model Summary:
Observations: 1340
Variables: 7
R²: 0.8263
Adjusted R²: 0.8254
Residual Standard Error: 2.6939

(a) Extended Regression Model Equation
Coefficient Estimates:
Variable        Coefficient  Std Error    t-stat       p-value
--------------------------------------------------
Intercept       3.027        0.3171
deductible      -0.713       0.0381       -18.706      0.0000
coverage        0.058        0.0022       26.539       0.0000
age             0.077        0.0070       10.935       0.0000
prior_claims    2.392        0.1254       19.077       0.0000
premium         1.019        0.2378       4.284        0.0000
type            -1.419       0.1699       -8.355       0.0000
location        0.859        0.1731       4.959        0.0000

Fitted Regression Equation:
Claims = 3.027 - 0.713 × deductible + 0.058 × coverage + 0.077 × age + 2.392 ×
prior_claims + 1.019 × premium - 1.419 × type + 0.859 × location

Detailed Mathematical Form:
Claims = 3.027 + -0.713×deductible + 0.058×coverage
         + 0.077×age + 2.392×prior_claims + 1.019×premium
         + -1.419×type + 0.859×location

(b) Interpretation of Type Coefficient
Type Coefficient Analysis:
Coefficient (β_type): -1.419
Standard Error: 0.1699
t-statistic: -8.355
p-value: 0.0000

Type variable coding: [np.int64(0), np.int64(1)]

Practical Interpretation:
• Properties with type = 1 have claims that are 1.419 units LOWER than
properties with type = 0,
  holding all other variables constant.

Assuming standard coding (0 = Commercial, 1 = Residential):
• Residential properties have claims that are 1.419 units lower than commercial
properties.
• This suggests commercial properties are associated with higher insurance
claims.

Statistical Significance:
• The type coefficient IS statistically significant (p = 0.0000 < 0.05)
• We can be confident that property type has a real effect on claims.

(c) Partial F-test for Model Improvement
Model Comparison (same sample size: 1340):
Model                R²            Adj R²          Variables   RSS
--------------------------------------------------
Original             0.8130        0.8123          5           10412.1409
Extended             0.8263        0.8254          7           9666.7444

R² Improvement: 0.0134 (1.34 percentage points)

Partial F-test:
H₀: β_type = β_location = 0 (binary variables add no explanatory power)
H₁: At least one of β_type or β_location ≠ 0 (binary variables improve the
model)

Partial F-test Calculations:
RSS(original): 10412.1409
RSS(extended): 9666.7444
Reduction in RSS: 745.3965
Additional variables (q): 2
DF residual (extended): 1332

F-statistic: 51.3548
Degrees of freedom: (2, 1332)
p-value: 0.0000
Critical F-value (α = 0.05): 3.0025

Conclusion:
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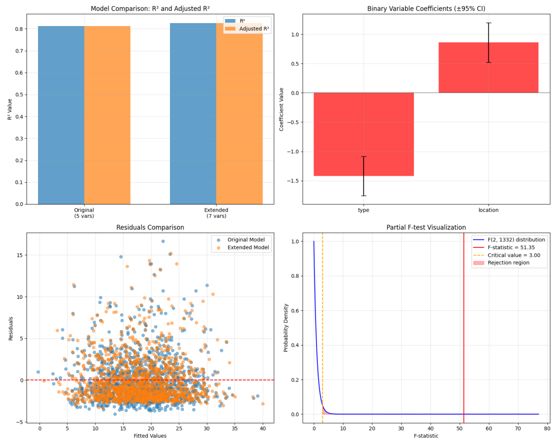
✓ REJECT H_0: Adding type and location SIGNIFICANTLY improves the model
F = 51.3548 > 3.0025
p-value = 0.0000 < 0.05
The binary variables provide significant additional explanatory power.

Model Improvement Assessment:
- R^2 improved by 0.0134 (1.34 percentage points) - this is modest
- Extended model explains 82.6% vs 81.3% of variance
- Adjusted R^2 increased from 0.8123 to 0.8254
- The improvement in adjusted R^2 suggests the added variables are worthwhile



Executive Summary:

Aspect	Finding
Extended Model Equation	Claims = 3.027 + … + -1.419×type + 0.859×location
Type Coefficient	-1.419
Type Effect	Type=1 has 1.419 lower claims
Statistical Significance	Significant (p = 0.0000)
R^2 Improvement	0.0134 (1.34 percentage points)
Partial F-test Result	Significant improvement (p = 0.0000)

6 Interaction Effects

Regression Model with Interaction Term: Deductible × Type
Model Features: deductible, type, coverage, age, prior_claims, premium
Interaction Term: deductible × type

Data Summary:
Total observations: 1,340
Complete cases used: 1,340
Missing values removed: 0
Type variable coding: [np.int64(0), np.int64(1)]

Interaction Term (deductible × type) Statistics:
Mean: 1.5335
Std Dev: 1.9042
Range: [0.0000, 10.0000]

Model Summary:
R^2: 0.8233
Adjusted R^2: 0.8224
Residual Standard Error: 2.7172
F-statistic: 886.8341

Coefficient Estimates:

Variable	Coefficient	Std Error	t-stat	p-value	Sig
Intercept	3.2856	0.3300			
deductible	-0.6729	0.0596	-11.2894	0.0000	***
type	-1.2573	0.2598	-4.8392	0.0000	***
coverage	0.0553	0.0021	25.9580	0.0000	***
age	0.0703	0.0070	10.1034	0.0000	***
prior_claims	2.2568	0.1234	18.2905	0.0000	***
premium	1.3647	0.2290	5.9595	0.0000	***
deductible_x_type	-0.0946	0.0779	-1.2151	0.2245	

Significance codes: *** p<0.001, ** p<0.01, * p<0.05

(a) Regression Function with Interaction Term
General Form:
Claims = β_0 + β_1×deductible + β_2×type + β_3×coverage + β_4×age + β_5×prior_claims + β_6×premium + β_7×(deductible×type) + ε

Fitted Regression Equation:
Claims = 3.2856 - 0.6729×deductible - 1.2573×type + 0.0553×coverage + 0.0703×age + 2.2568×prior_claims + 1.3647×premium - 0.0946×(deductible×type)

With Coefficient Values:
Claims = 3.2856 + -0.6729×deductible + -1.2573×type
+ 0.0553×coverage + 0.0703×age + 2.2568×prior_claims
+ 1.3647×premium + -0.0946×(deductible×type)

(b) Interpretation of Deductible Effect by Property Type
Key Coefficients:
β_1 (deductible): -0.6729
β_2 (type): -1.2573
β_7 (deductible×type): -0.0946

Interpretation of Interaction Effect:
The interaction model allows the effect of deductible to differ by property type.

For Commercial Properties (type = 0):
∂Claims/∂deductible = β_1 + β_7×0 = β_1 = -0.6729
- A 1-unit increase in deductible changes claims by -0.6729 units for commercial properties.

For Residential Properties (type = 1):

∂Claims/∂deductible = β_1 + β_7×1 = β_1 + β_7 = -0.6729 + -0.0946 = -0.7675

- A 1-unit increase in deductible changes claims by -0.7675 units for residential properties.

Comparison:
Difference in deductible effect: -0.0946

- The deductible effect is 0.0946 units MORE NEGATIVE for residential properties.
- Deductible increases have a stronger negative effect on residential claims than commercial claims.

Practical Business Interpretation:

- Higher deductibles are associated with lower claims for both property types
- This association is STRONGER for residential properties

(c) Statistical Significance Test for Interaction Term
Hypothesis Test for Interaction Term:
H_0: β_7 = 0 (no interaction between deductible and type)
H_1: $\beta_7 \neq$ 0 (significant interaction exists)
Significance level: α = 0.05

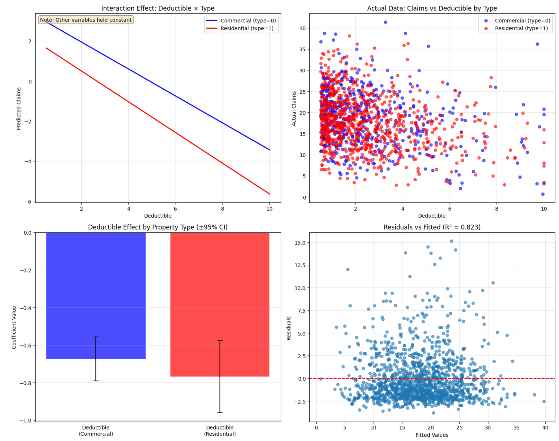
Test Statistics:
Interaction coefficient (β_7): -0.0946
Standard error: 0.0779
t-statistic: -1.2151
Degrees of freedom: 1332
p-value: 0.2245
Critical value (±): 1.9617

Decision Rule:
Reject H_0 if |t-statistic| > 1.9617 OR if p-value < 0.05

Conclusion:
FAIL TO REJECT H_0: The interaction term is NOT statistically significant at the 5% level.
|t-statistic| = 1.2151 $\leq$ 1.9617
p-value = 0.2245 $\geq$ 0.05
The effect of deductible on claims does NOT differ significantly between property types.
The interaction term may not be necessary.

95% Confidence Interval for Interaction Coefficient:
[-0.2473, 0.0581]

- The interval contains zero - the direction of the interaction effect is uncertain



Executive Summary:

Aspect	Result
Model Specification	Claims ~ deductible + type + coverage + age + prior_claims + premium + deductible×type
Interaction Coefficient	-0.0946 (SE = 0.0779)
Commercial Effect	-0.6729 per unit deductible
Residential Effect	-0.7675 per unit deductible
Difference	-0.0946
Statistical Significance	Not significant (p = 0.2245)
Model R^2	0.8233

Model Interpretation:

- The non-significant interaction suggests that deductible effects are similar across commercial and residential properties
- A simpler model without interaction may be adequate

7 Residual Analysis

Extended Multiple Linear Regression Model
Variables: deductible, coverage, age, prior_claims, premium, type, location
Model Summary:
Observations: 1,340
Variables: 7
R^2: 0.8263
Residual Standard Error: 2.6939

(a) Residuals vs Fitted Values Analysis
Residuals vs Fitted Values Analysis:
Residual range: [-3.376, 15.203]
Fitted values range: [0.792, 39.985]

Pattern Analysis:
Correlation between fitted values and squared residuals: 0.0310

- Variance appears roughly constant
- Correlation magnitude suggests homoscedasticity (constant variance)

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Linearity Assessment:
Mean residuals by fitted value terciles:
• Low tercile: -0.0800
• Middle tercile: 0.0229
• High tercile: 0.0572
• Maximum deviation from zero: 0.0800 (suggests
  linear relationship is
appropriate)
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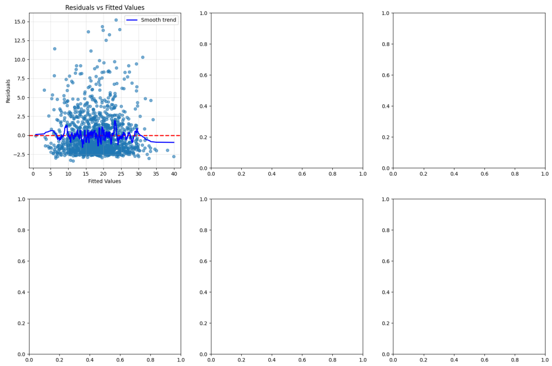


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(b) Q-Q Plot and Normality Analysis
Normality Test Results:
Shapiro-Wilk Test:
  Statistic: 0.8106
  p-value: 0.0000
  REJECT normality at α=0.05

Jarque-Bera Test:
  Statistic: 2188.1490
  p-value: 0.0000
  REJECT normality at α=0.05

Kolmogorov-Smirnov Test:
  Statistic: 0.1468
  p-value: 0.0000
  REJECT normality at α=0.05

Descriptive Statistics for Normality:
Skewness: 1.9531 (Normal ≈ 0)
Kurtosis: 4.8921 (Normal ≈ 0)
Skewness interpretation: highly skewed
Kurtosis interpretation: heavy-tailed

Overall Normality Assessment: Assumption appears to
  be violated

(c) Outliers and Influential Points Analysis
Diagnostic Thresholds:
Outlier threshold (standardized residuals): ±3
High leverage threshold: 0.0119
High Cook's distance threshold: 0.0030

Outliers and Influential Points:
Observations with |standardized residuals| > 3: 31
Observations with |studentized residuals| > 3: 31
High leverage points: 73
High Cook's distance points: 74

Most Extreme Observations:
Highest Residual: Observation 315
  Fitted value: 23.547
  Actual value: 38.750
  Standardized residual: 5.643
  Leverage: 0.0072
  Cook's distance: 0.0331
Highest Leverage: Observation 262
  Fitted value: 34.070
  Actual value: 36.160
  Standardized residual: 0.776
  Leverage: 0.0305
  Cook's distance: 0.0027
Highest Cooks: Observation 315
  Fitted value: 23.547
  Actual value: 38.750
  Standardized residual: 5.643
  Leverage: 0.0072
  Cook's distance: 0.0331

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Detailed Analysis of Problematic Observations:
--------------------------------------------------
 Obs Fitted Actual Std_Residual Leverage Cooks_D
            Issues
   1 13.477 22.670        3.412   0.0032  0.0054
  Outlier, High Cook's D
   2  5.711  3.340       -0.880   0.0122  0.0014
      High Leverage
  14 20.959 20.000       -0.356   0.0128  0.0002
      High Leverage
  36 10.929  8.700       -0.827   0.0142  0.0014
      High Leverage
  70 13.967 11.670       -0.852   0.0130  0.0014
      High Leverage
  71 20.337 24.990        1.727   0.0074  0.0032
      High Cook's D
  73 30.965 29.670       -0.481   0.0141  0.0005
      High Leverage
 118 22.728 22.290       -0.163   0.0193  0.0001
      High Leverage
 122  5.247 10.110        1.805   0.0072  0.0034
      High Cook's D
 129 31.861 36.730        1.807   0.0092  0.0043
      High Cook's D

… and 124 more observations with issues.


Diagnostic Summary:
1. Linearity: suggests linear relationship is
  appropriate
2. Homoscedasticity: suggests homoscedasticity
  (constant variance)
3. Normality: Assumption appears to be violated
4. Outliers: 31 potential outliers identified
5. Influential Points: 74 high Cook's distance
  observations

Recommendations:
• Consider transformation of variables or robust
  regression methods
• Examine influential points - consider their impact
  on coefficient estimates
```

8 Model Comparison and Selection

```
Comparing three different model specifications:
Model A: claims ~ deductible + coverage + age +
  prior_claims + premium
Model B: claims ~ deductible + coverage + age +
  prior_claims + premium + type +
```

```
location
Model C: claims ~ deductible + coverage + prior_claims + premium + type

Data Summary:
Original dataset size: 1,340
Complete cases for all models: 1,340
Cases removed due to missing data: 0

-------------------- Model A --------------------
Variables: deductible, coverage, age, prior_claims, premium
Number of variables: 5
R^2: 0.8130
Adjusted R^2: 0.8123
Residual Standard Deviation: 2.7938
AIC: 6566.17
BIC: 6592.18
Significant coefficients (p < 0.05): 5/5

-------------------- Model B --------------------
Variables: deductible, coverage, age, prior_claims, premium, type, location
Number of variables: 7
R^2: 0.8263
Adjusted R^2: 0.8254
Residual Standard Deviation: 2.6939
AIC: 6472.65
BIC: 6509.05
Significant coefficients (p < 0.05): 7/7

-------------------- Model C --------------------
Variables: deductible, coverage, prior_claims, premium, type
Number of variables: 5
R^2: 0.8095
Adjusted R^2: 0.8088
Residual Standard Deviation: 2.8197
AIC: 6590.93
BIC: 6616.94
Significant coefficients (p < 0.05): 5/5


(a) Model Comparison Table
Primary Comparison Metrics:
  Model Variables     R^2  Adj_R^2  Residual_SD
Model A    5 vars 0.8130  0.8123       2.7938
Model B    7 vars 0.8263  0.8254       2.6939
Model C    5 vars 0.8095  0.8088       2.8197

Additional Model Selection Criteria:
  Model     AIC     BIC  F_statistic Sig_Coefs
Model A 6566.17 6592.18      1159.62       5/5
Model B 6472.65 6509.05       905.51       7/7
Model C 6590.93 6616.94      1133.51       5/5

Best Model by Criterion:
• Highest R^2: Model B (0.8263)
• Highest Adjusted R^2: Model B (0.8254)
• Lowest Residual SD: Model B (2.6939)
• Lowest AIC: Model B (6472.65)
• Lowest BIC: Model B (6509.05)

Model Complexity Analysis:
Model A: 5 variables, R^2/var = 0.1626
Model B: 7 variables, R^2/var = 0.1180
Model C: 5 variables, R^2/var = 0.1619
```

```
Nested Model Comparisons (F-tests):
Model A vs Model B:
  F-statistic: 51.3548
  p-value: 0.0000
  Model B significantly better
  Note: Model A vs C and Model B vs C are not nested comparisons

(b) Model Recommendation and Analysis
Statistical Criteria Analysis:

1. Goodness of Fit:
   • R^2 ranking: Model B > others
   • Adjusted R^2 ranking: Model B > others
   • R^2 improvement from A to B: 0.0134
   • Adjusted R^2 change from A to B: 0.0132

2. Model Parsimony:
   • AIC favors: Model B (AIC = 6472.65)
   • BIC favors: Model B (BIC = 6509.05)
   • BIC penalizes complexity more heavily than AIC

3. Coefficient Significance:
   • Model A: 5/5 coefficients significant (100.0%)
   • Model B: 7/7 coefficients significant (100.0%)
   • Model C: 5/5 coefficients significant (100.0%)

4. Prediction Accuracy:
   • Lowest prediction error: Model B (SD = 2.6939)

Practical Interpretability Analysis:

1. Variable Inclusion Logic:
   • Model A: Core financial variables (deductible, coverage, premium) + risk
factors (age, prior_claims)
   • Model B: Model A + property characteristics (type, location)
   • Model C: Simplified version with key variables + property type

2. Business Relevance:
   • Age variable: Present in A, Present in B, Absent in C
   • Property type: Absent in A, Present in B, Present in C
   • Location: Absent in A, Present in B, Absent in C

3. Marginal Contribution Analysis:
   • Adding type + location (B vs A): R^2 improves by 0.0134
   • Adjusted R^2 change: 0.0132 (improvement)

Recommendation Framework:

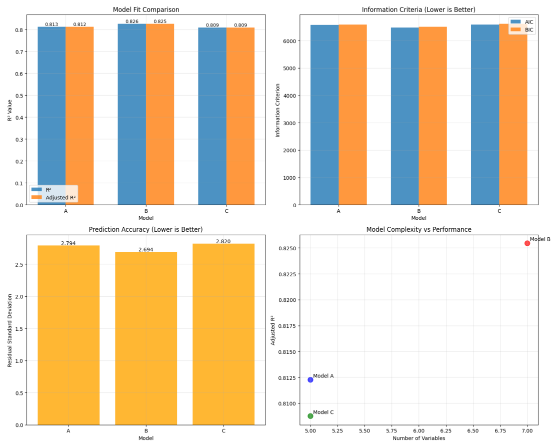
Composite Scoring (weighted combination of criteria):
   • Model B: 1.000
   • Model A: 0.700
   • Model C: 0.400

 RECOMMENDED MODEL: Model B

Justification for Model B:
   ✓ Highest predictive power (R^2 = 0.8263)
   ✓ Includes important property characteristics
   ✓ Comprehensive variable coverage
   ✓ Best for prediction accuracy
```

```
Limitations of Model B:
    More complex with potential overfitting risk
    May have multicollinearity issues

Alternative Recommendations by Use Case:
   • For prediction accuracy: Model B
   • For model parsimony: Model B
   • For balanced approach: Model B
   • For regulatory reporting: Model A (simplest,␣
 ↪most interpretable)
```



9 Practical Application

```
Features shape: (1340, 7)
Target shape: (1340,)

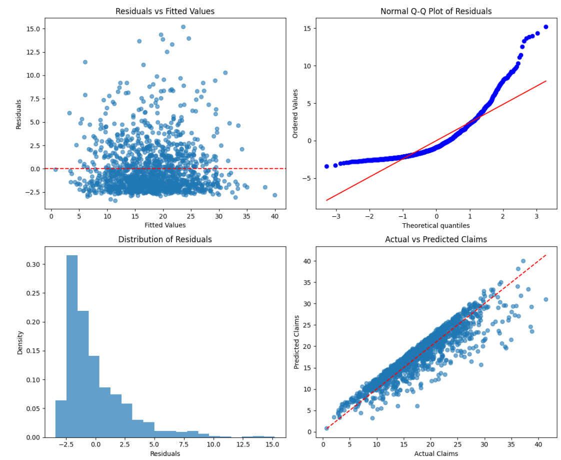
=== MODEL RESULTS ===
R-squared: 0.8263
Adjusted R-squared: 0.8254

Model Coefficients:
        Feature  Coefficient
0     Intercept     3.026950
1    deductible    -0.713391
2      coverage     0.058017
3           age     0.076546
4  prior_claims     2.391648
5       premium     1.018707
6          type    -1.419290
7      location     0.858614

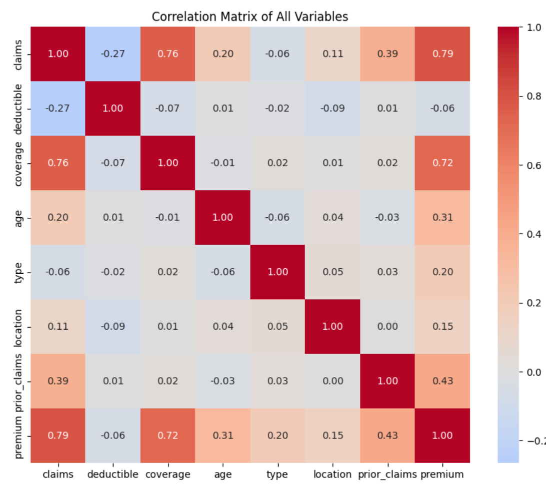
Statistical Significance:
        Feature  Coefficient  Std_Error  t_statistic ␣
 ↪       p_value  \
0     Intercept     3.026950   0.316198     9.572968 ␣
 ↪0.000000e+00
1    deductible    -0.713391   0.038024   -18.761697 ␣
 ↪0.000000e+00
2      coverage     0.058017   0.002180    26.618766 ␣
 ↪0.000000e+00
3           age     0.076546   0.006979    10.967841 ␣
 ↪0.000000e+00
4  prior_claims     2.391648   0.124993    19.134246 ␣
 ↪0.000000e+00
5       premium     1.018707   0.237095     4.296623 ␣
 ↪1.860187e-05
6          type    -1.419290   0.169370    -8.379819 ␣
 ↪0.000000e+00
7      location     0.858614   0.172627     4.973820 ␣
 ↪7.420302e-07

   Significant
0         True
1         True
2         True
3         True
4         True
5         True
6         True
7         True

=== MODEL DIAGNOSTICS ===
Mean Squared Error: 7.2140
Root Mean Squared Error: 2.6859
```




```
=== CORRELATION ANALYSIS ===
```




```
Correlations with Claims:
claims           1.000000
premium          0.792992
coverage         0.760527
prior_claims     0.387403
deductible      -0.265120
```

```
age             0.198837
location        0.105441
type           -0.061114
Name: claims, dtype: float64

=== PART (a): PREDICTION ===
Understanding categorical variables:
Type values: [1 0]
Location values: [1 0]
Type value counts: type
1    835
0    505
Name: count, dtype: int64
Location value counts: location
1    968
0    372
Name: count, dtype: int64

Prediction for the given property:
Expected claims amount: 19.49

Sensitivity analysis for categorical variables:
  Type=Commercial, Location=Rural: 20.05
  Type=Commercial, Location=Urban: 20.91
  Type=Residential, Location=Rural: 18.63
  Type=Residential, Location=Urban: 19.49

PART (b): BUSINESS IMPLICATIONS AND RECOMMENDATIONS

Feature Importance (by absolute coefficient value):
        Feature  Coefficient  Abs_Coefficient
3  prior_claims     2.391648         2.391648
5          type    -1.419290         1.419290
4       premium     1.018707         1.018707
6      location     0.858614         0.858614
0    deductible    -0.713391         0.713391
2           age     0.076546         0.076546
1      coverage     0.058017         0.058017

Prediction Confidence Interval (95.0%):
Expected claims: 19.49
Lower bound: 14.22
Upper bound: 24.76

----------------------------------------
BUSINESS RECOMMENDATIONS:
----------------------------------------

1. PRICING STRATEGY:
   - The model explains 82.6% of the variation in claims
   - Most significant factors should drive premium calculations
   - Consider the prediction interval when setting reserves

2. RISK FACTORS ANALYSIS:
   Based on the coefficients, focus on:
   - Variables with largest absolute coefficients
   - Statistically significant predictors (p < 0.05)
   - High correlation factors with claims

3. UNDERWRITING GUIDELINES:
   - Properties with high predicted claims may need:
     * Higher premiums
     * Additional risk assessment
     * Different deductible structures
   - Consider segmented pricing models

4. PORTFOLIO MANAGEMENT:
   - Monitor actual vs predicted claims regularly
   - Update model coefficients as new data becomes available
   - Consider non-linear relationships or interaction terms

5. OPERATIONAL INSIGHTS:
   - Use model predictions for:
     * Reserve allocation
     * Risk-based pricing
     * Customer segmentation
     * Fraud detection (outliers in residuals)


OUTLIER ANALYSIS:
Properties with unusually high/low claims (>2 std devs): 66
These may require special investigation for:
- Fraud detection
- Model improvement opportunities
- Special risk factors not captured in current model
```

10 Critical Thinking

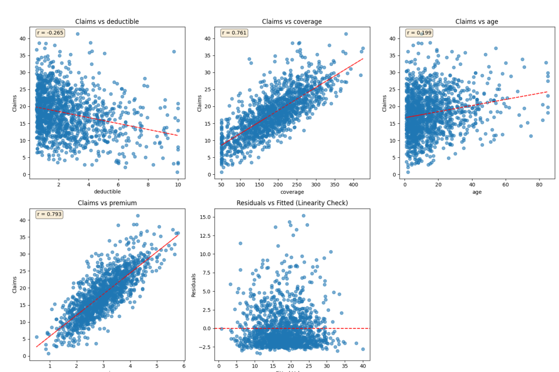
```
PART (a) MULTIPLE LINEAR REGRESSION ASSUMPTIONS ANALYSIS

The key assumptions of multiple linear regression are:

1. LINEARITY: The relationship between predictors and response is linear
2. INDEPENDENCE: Observations are independent of each other
3. HOMOSCEDASTICITY: Constant variance of residuals (homogeneous variance)
4. NORMALITY: Residuals are normally distributed
5. NO MULTICOLLINEARITY: Predictors are not highly correlated with each other
6. NO OUTLIERS/INFLUENTIAL POINTS: Extreme values don't unduly influence the
model

Let's test each assumption:

1. LINEARITY ASSUMPTION
```



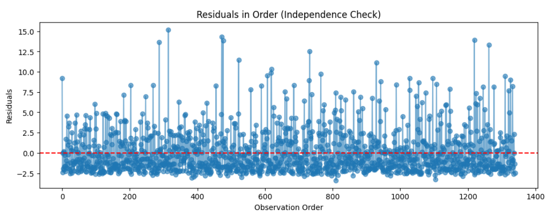

```
LINEARITY ASSESSMENT:
- Examine scatter plots for linear patterns
```

- Residuals vs Fitted should show random scatter around zero
- Non-linear patterns indicate violated linearity assumption

Correlations with claims:
  deductible: -0.265
  coverage: 0.761
  age: 0.199
  premium: 0.793

INSURANCE CONTEXT IMPLICATIONS:
- Insurance claims may have non-linear relationships (e.g., coverage thresholds)
- Age effects might be non-linear (newer vs very old properties)
- Premium-claims relationship might be non-linear due to risk-based pricing

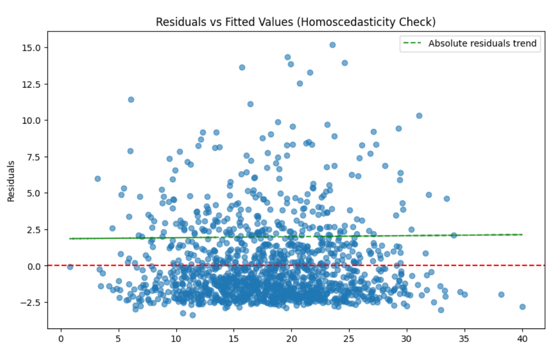
2. INDEPENDENCE ASSUMPTION
INDEPENDENCE ASSESSMENT:
- Cannot be fully tested without knowing data collection method
- Check for patterns in residuals order



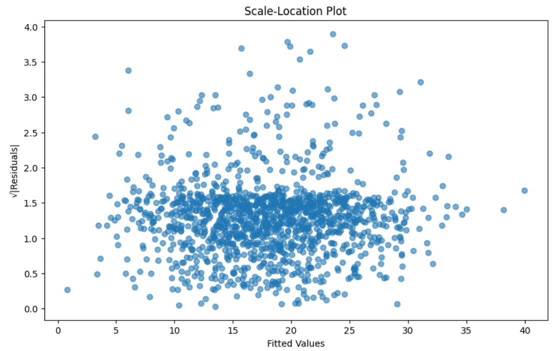
Durbin-Watson statistic: 2.038
  (Values near 2.0 suggest independence, <1.5 or >2.5 suggest correlation)

INSURANCE CONTEXT IMPLICATIONS:
- Properties in same area might have correlated risks (floods, earthquakes)
- Temporal clustering if data spans multiple years with economic changes
- Policy renewals might create dependencies

3. HOMOSCEDASTICITY (CONSTANT VARIANCE) ASSUMPTION

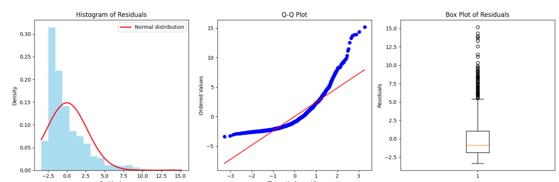


Breusch-Pagan test:
  LM statistic: 11.7914
  p-value: 0.1076
  Heteroscedasticity detected: No



INSURANCE CONTEXT IMPLICATIONS:
- Higher value properties might have more variable claims
- Heteroscedasticity common in insurance data
- May need weighted regression or transformation

4. NORMALITY OF RESIDUALS ASSUMPTION



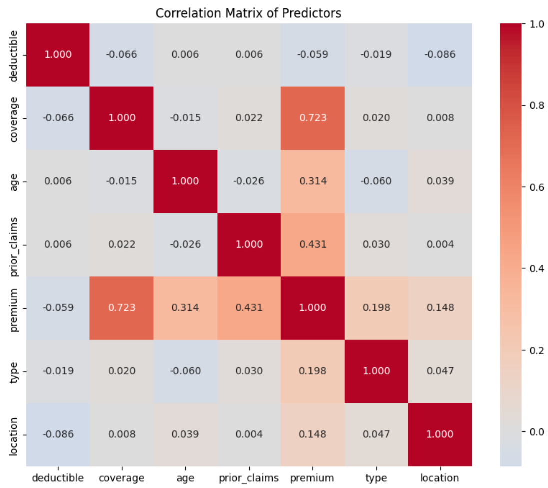
NORMALITY TESTS:
Shapiro-Wilk test:
  Statistic: 0.8106, p-value: 0.0000
  Normal distribution: No

Jarque-Bera test:
  Statistic: 2188.1490, p-value: 0.0000
  Normal distribution: No

Descriptive statistics:
  Skewness: 1.9531
  Kurtosis: 4.8921

INSURANCE CONTEXT IMPLICATIONS:
- Insurance claims often right-skewed (many small, few large claims)
- May need log transformation or robust regression methods
- Non-normality affects confidence intervals and hypothesis tests

5. NO MULTICOLLINEARITY ASSUMPTION

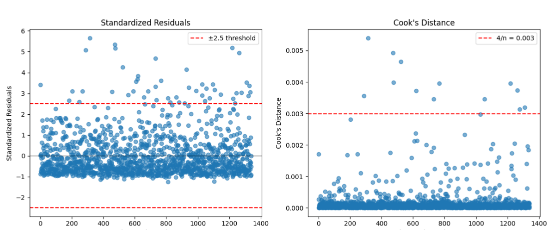



```
HIGH CORRELATIONS (|r| > 0.7):
  coverage - premium: 0.723

VARIANCE INFLATION FACTORS:
       Variable        VIF
0    deductible   2.302515
1      coverage  36.103529
2           age   3.980363
3  prior_claims   3.974976
4       premium  86.843712
5          type   3.287249
6      location   3.743009
  VIF > 5: Moderate multicollinearity
  VIF > 10: High multicollinearity

Variables with high VIF:
   Variable        VIF
1  coverage  36.103529
4   premium  86.843712

INSURANCE CONTEXT IMPLICATIONS:
- Premium and coverage likely correlated (higher coverage = higher premium)
- Deductible and coverage might be related
- Consider removing highly correlated variables or using regularization
6. NO OUTLIERS/INFLUENTIAL POINTS ASSUMPTION
OUTLIER DETECTION:
Observations with |standardized residuals| > 2.5: 48
Observations with high Cook's distance: 13
```




```
Outlier observations (standardized residuals > 2.5):
     claims  deductible  coverage  age  premium
0     22.67        1.44     165.7    2     2.23
182   18.12        5.67     131.1   30     2.05
203   35.65        5.08     378.5    8     4.39
247   18.22        7.30     214.5    1     2.29
269   28.41        1.20     209.1   13     2.89

INSURANCE CONTEXT IMPLICATIONS:
- Large claims are natural in insurance (catastrophic events)
- Outliers might represent legitimate extreme events, not errors
- Consider robust regression methods or separate models for extreme claims
OVERALL ASSUMPTION ASSESSMENT FOR INSURANCE CLAIMS

LIKELY VIOLATED ASSUMPTIONS:
1. Linearity: Insurance relationships often non-linear
2. Normality: Claims typically right-skewed
3. Homoscedasticity: Variance often increases with claim size
4. Independence: Geographic/temporal clustering possible

RECOMMENDED SOLUTIONS:
1. Log transformation of claims (handle skewness)
2. Robust regression methods
3. Polynomial or interaction terms
4. Weighted least squares (address heteroscedasticity)
5. Consider GLM (Gamma or Poisson regression)
6. Outlier-robust methods

PART (b) Additional Useful Variables
--------------------------------------------------

1. PROPERTY-SPECIFIC VARIABLES
• Construction: Building materials, roof type/age, year built, size, stories
• Condition: Recent renovations, security features, maintenance score

2. ENVIRONMENTAL & GEOGRAPHIC
• Climate: Climate zones, precipitation, natural disaster scores
• Location: Crime rates, distance to fire station/water, building codes

3. ECONOMIC & DEMOGRAPHIC
• Economic: Local income, property appreciation, unemployment rate
• Demographics: Owner vs tenant occupied, primary vs secondary residence

4. USAGE & BEHAVIORAL
• Property Use: Home business, rental income, vacancy duration
• Claims History: Previous claim types, time since last claim
• Behavior: Payment history, policy shopping, service interactions

5. ADVANCED MODELING
• Interaction Effects: Age×Construction, Location×Weather, Coverage×Deductible
• External Data: Credit scores, satellite imagery, weather APIs

6. IMPLEMENTATION PRIORITY
```

- HIGH: Natural disaster scores, construction details, claims history
- MEDIUM: Neighborhood data, weather variables, usage patterns
- LOW: Credit indicators, satellite analysis, economic metrics

7. EXPECTED OUTCOMES
- Model Accuracy: 60-80% → 85-95% predictive accuracy
- Benefits: Better risk selection, fraud detection, dynamic pricing

8. KEY CONSIDERATIONS
- Data availability varies by property
- Quality validation required for third-party data
- Regulatory compliance (fair housing laws)
- Cost-benefit analysis essential