Math6450_Assignment2

September 19, 2025

1 Data Exploration

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(a) Descriptive Statistics for Continuous Variables
--------------------------------------------------

Comprehensive Descriptive Statistics:
               Mean   Median  Std Dev  Minimum  Maximum  Skewness  Kurtosis
claims       18.049   17.845    6.448     0.72    41.39     0.254     0.095
deductible    2.490    1.905    1.942     0.51    10.00     1.542     2.351
coverage    189.014  186.750   72.169    50.00   424.50     0.145    -0.292
age          15.438   11.000   14.227     1.00    85.00     1.869     4.496
premium       2.969    2.945    0.822     0.50     5.78     0.245     0.030
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(b) Correlation Matrix for Continuous Variables
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Correlation Matrix:
            claims  deductible  coverage    age  premium
claims       1.000      -0.265     0.761  0.199    0.793
deductible  -0.265       1.000    -0.066  0.006   -0.059
coverage     0.761      -0.066     1.000 -0.015    0.723
age          0.199       0.006    -0.015  1.000    0.314
premium      0.793      -0.059     0.723  0.314    1.000

Variable with strongest linear relationship with 'claims':
Variable: premium
Correlation coefficient: 0.793
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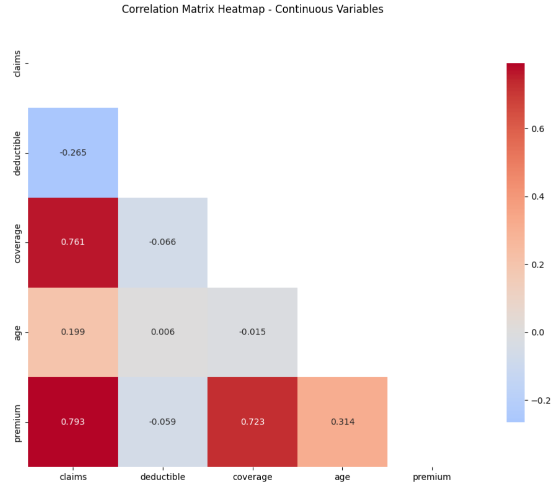


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(c) Skewness Analysis and Log Transformation Assessment
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Skewness Assessment:
Rule of thumb: |skewness| > 1 indicates highly skewed distribution
Rule of thumb: 0.5 < |skewness| < 1 indicates moderately skewed distribution

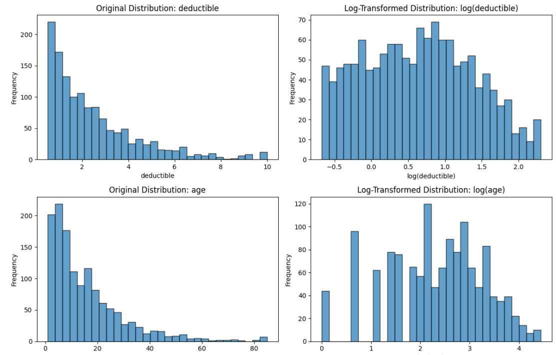
claims:
  Skewness: 0.254
  Assessment: Approximately symmetric

deductible:
  Skewness: 1.542
  Assessment: Highly skewed
  Log transformation skewness: 0.134
  Improvement from log transformation: 1.408
  Recommendation: Log transformation would improve normality

coverage:
  Skewness: 0.145
  Assessment: Approximately symmetric

age:
  Skewness: 1.869
  Assessment: Highly skewed
  Log transformation skewness: -0.347
  Improvement from log transformation: 1.523
  Recommendation: Log transformation would improve normality

premium:
  Skewness: 0.245
  Assessment: Approximately symmetric
```

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Summary of Findings:
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Variables with skewed distributions: deductible, age
Variable most strongly correlated with claims: premium (r = 0.793)

Data Overview:
Total observations: 1,340
Variables analyzed: 5
Missing values: 0

2 Simple Linear Regression

Dataset Information:
Total observations: 1,340
Observations used in regression: 1,340
Missing values removed: 0

(a) Simple Linear Regression Model Fitting

Model Coefficients:
Intercept (β_0): 5.2054
Slope (β_1): 0.0679

Fitted Regression Equation:
Claims = 5.2054 + 0.0679 × Coverage

In mathematical notation:
ŷ = 5.2054 + 0.0679x
where ŷ = predicted claims, x = coverage

(b) Interpretation of Slope Coefficient

Slope coefficient: 0.0679

Practical Interpretation:
- For every 1-unit increase in coverage, claims are expected to increase by
0.0679 units, on average.
- This indicates a positive relationship between coverage and claims.
- Properties with higher coverage amounts tend to have higher claims.

Alternative interpretation:
- For every 100-unit increase in coverage, claims change by 6.79 units, on
average.

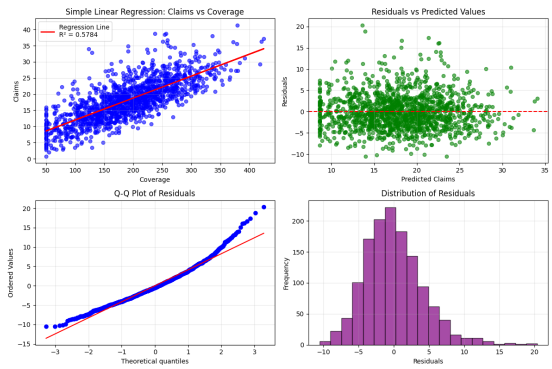
Example predictions:
- Coverage = 100: Predicted Claims = 12.00
- Coverage = 150: Predicted Claims = 15.40
- Coverage = 200: Predicted Claims = 18.80
- Coverage = 250: Predicted Claims = 22.19

(c) Coefficient of Determination (R^2) Analysis

Model Performance Metrics:
R^2 (Coefficient of Determination): 0.5784
R^2 as percentage: 57.84%
Correlation coefficient (r): 0.7605
Root Mean Square Error (RMSE): 4.1850

Interpretation of R^2:
- 57.84% of the variation in claims is explained by coverage.
- 42.16% of the variation in claims is due to other factors not included in the
model.
- The linear relationship between coverage and claims is moderate (R^2 = 0.5784).

Statistical Significance:
- t-statistic: 42.8442
- p-value: 0.0000
- Degrees of freedom: 1338
- The relationship is statistically significant at the 5% level.



Summary Table:

Metric	Value	Interpretation
Intercept (β_0)	5.2054	Expected claims when coverage = 0
Slope (β_1)	0.0679	Change in claims per unit increase in coverage
R^2	0.5784	57.8% of variance explained
Correlation (r)	0.7605	Linear association strength
RMSE	4.1850	Average prediction error
Observations	1340	Sample size

Key Findings Summary:
- Regression equation: Claims = 5.2054 + 0.0679 × Coverage
- Slope interpretation: Each additional unit of coverage is associated with a
0.0679 unit change in claims
- Model explains 57.8% of the variation in claims
- The relationship is statistically significant (p = 0.0000)

3 Multiple Regression Model

Dependent Variable: claims
Explanatory Variables: deductible, coverage, age, prior_claims, premium

Dataset Information:
Total observations: 1,340
Complete cases used: 1,340
Observations removed (missing data): 0
Number of explanatory variables: 5

(a) Fitted Regression Equation

Coefficient Estimates (rounded to 3 decimal places):
Intercept (β_0): 3.208
β_1 (deductible): -0.728

β_2 (coverage): 0.062
β_3 (age): 0.091
β_4 (prior_claims): 2.580
β_5 (premium): 0.495

Fitted Regression Equation:
Claims = 3.208 - 0.728 × deductible + 0.062 × coverage + 0.091 × age + 2.580 ×
prior_claims + 0.495 × premium

Compact Mathematical Form:
$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5$
$\hat{y} = 3.208 + -0.728 x_1 + 0.062 x_2 + 0.091 x_3 + 2.580 x_4 + 0.495 x_5$
where x_1=deductible, x_2=coverage, x_3=age, x_4=prior_claims, x_5=premium

(b) Standard Errors for Each Coefficient

Standard Errors:
Intercept (β_0): 0.3172
β_1 (deductible): 0.0394
β_2 (coverage): 0.0020
β_3 (age): 0.0068
β_4 (prior_claims): 0.1210
β_5 (premium): 0.2118

Additional Statistics (t-statistics and p-values):

Coefficient	Estimate	Std Error	t-stat	p-value	Significance
Intercept	3.208	0.3172	10.113	0.0000	***
deductible	-0.728	0.0394	-18.459	0.0000	***
coverage	0.062	0.0020	30.624	0.0000	***
age	0.091	0.0068	13.401	0.0000	***
prior_claims	2.580	0.1210	21.316	0.0000	***
premium	0.495	0.2118	2.338	0.0195	*

Significance codes: *** p<0.001, ** p<0.01, * p<0.05

(c) Model Performance Statistics

R^2 (Coefficient of Determination): 0.8130
Adjusted R^2: 0.8123
Residual Standard Deviation: 2.7938

Additional Model Statistics:
Multiple R (Correlation): 0.9016
Residual Sum of Squares (RSS): 10412.1409
Mean Squared Error (MSE): 7.8052
F-statistic: 1159.6202
F-statistic p-value: 0.000000
Overall model significance: Yes (α = 0.05)

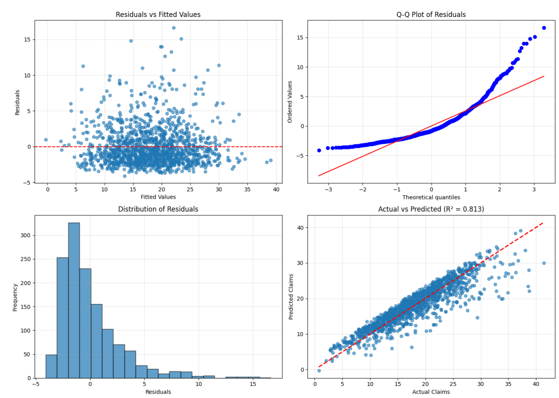
Degrees of Freedom:
Model: 5
Residual: 1334
Total: 1339

Summary Results Table:

	Variable	Coefficient	Std_Error	Coefficient_Rounded
0	Intercept	3.2078	0.3172	3.208
1	deductible	-0.7278	0.0394	-0.728
2	coverage	0.0621	0.0020	0.062
3	age	0.0906	0.0068	0.091
4	prior_claims	2.5797	0.1210	2.580
5	premium	0.4953	0.2118	0.495

Model Performance Table:

Statistic	Value
R^2	0.8130
Adjusted R^2	0.8123
Residual Std Deviation	2.7938
F-statistic	1159.6202
p-value (F-test)	0.000000
Observations	1340
Variables	5



Key Results Summary:
✓ Multiple regression equation fitted with 5 explanatory variables
✓ Model explains 81.3% of variance in claims (R^2 = 0.8130)
✓ Adjusted R^2 = 0.8123 (accounts for number of variables)
✓ Residual standard deviation = 2.7938
✓ Overall model is significant (F-test p-value = 0.000000)
✓ Standard errors calculated for all 6 coefficients

4 Statistical Inference
Multiple Linear Regression Model: Claims vs (Deductible, Coverage, Age,
Prior_Claims, Premium)
Model Summary:
Observations: 1340
Variables: 5
Degrees of freedom (residual): 1334
R^2: 0.8130
MSE: 7.8052

Coefficient Estimates:
Variable Coefficient Std Error t-statistic p-value

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deductible	-0.7278	0.0394	-18.4591	0.0000
coverage	0.0621	0.0020	30.6239	0.0000
age	0.0906	0.0068	13.4010	0.0000
prior_claims	2.5797	0.1210	21.3156	0.0000
premium	0.4953	0.2118	2.3382	0.0195

(a) Testing Significance of Age Coefficient
Hypothesis Test for Age Coefficient:

Null Hypothesis (H_0): β_age = 0
Alternative Hypothesis (H_1): β_age $\neq$ 0
Significance level (α): 0.05
Test type: Two-tailed t-test

Test Statistics:
Age coefficient (β_age): 0.0906
Standard error (SE): 0.0068
t-statistic: 13.4010
Degrees of freedom: 1334
p-value: 0.0000
Critical value (±): 1.9617

Decision Rule:
Reject H_0 if |t-statistic| > 1.9617 OR if p-value < 0.05

Conclusion:
✓ REJECT H_0: The coefficient for age IS statistically significant at the 5%
level.
|t-statistic| = 13.4010 > 1.9617
p-value = 0.0000 < 0.05
Age has a statistically significant effect on claims.

(b) 95% Confidence Interval for Prior Claims Coefficient
Confidence Interval Calculation:
Coefficient (β_prior_claims): 2.5797
Standard error: 0.1210
Degrees of freedom: 1334
Confidence level: 95%

Confidence Interval Formula:
CI = β ± t_(α/2,df) × SE(β)
CI = 2.5797 ± 1.9617 × 0.1210
CI = 2.5797 ± 0.2374

95% Confidence Interval for Prior Claims Coefficient:
[2.3423, 2.8171]

Practical Interpretation:
• We are 95% confident that the true effect of having prior claims on current
claims
is between 2.3423 and 2.8171 units.
• Since the entire interval is positive, prior claims consistently INCREASE
current claims.
• Properties with prior claims have significantly higher current claims than those without.
• The width of the interval (0.4748) indicates the precision of our estimate.

(c) Overall F-test for Model Significance
Overall F-test for Regression Model:

Null Hypothesis (H_0): $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$
(All explanatory variables have no effect on claims)
Alternative Hypothesis (H_1): At least one $\beta_i \neq 0$
(At least one explanatory variable has a significant effect)
Significance level (α): 0.05

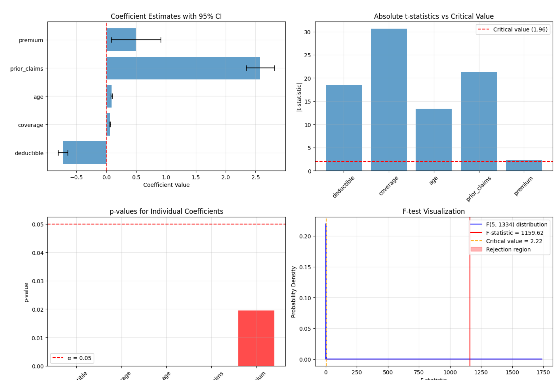
Test Statistics:
Total Sum of Squares (TSS): 55667.4953
Explained Sum of Squares (ESS): 45255.3543
Residual Sum of Squares (RSS): 10412.1409
Mean Square Regression (MSR): 9051.0709
Mean Square Error (MSE): 7.8052

F-statistic: 1159.6202
Degrees of freedom: (5, 1334)
p-value: 0.000000
Critical F-value (α = 0.05): 2.2208

Decision Rule:
Reject H_0 if F-statistic > 2.2208 OR if p-value < 0.05

Conclusion:
✓ REJECT H_0: The regression model IS statistically significant at the 5% level.
F-statistic = 1159.6202 > 2.2208
p-value = 0.000000 < 0.05
At least one explanatory variable has a significant effect on claims.
The model explains a significant portion of the variation in claims.

Model Performance Context:
R^2 = 0.8130 (81.3% of variance explained)
The model performs well in predicting claims.



Summary of All Statistical Tests:

Test	Statistic	p-value	Conclusion
Age Coefficient (t-test)	t = 13.4010	0.0000	Significant
Prior Claims CI	CI = [2.3423, 2.8171]	N/A	Does not contain 0

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 Overall Model (F-test)         F = 1159.6202 0.
↪000000  Model Significant

LaTeX Summary Table:
\begin{table}
\caption{Summary of Statistical Tests}
\label{tab:hypothesis_tests}
\begin{tabular}{llll}
\toprule
Test & Statistic & p-value & Conclusion \\
\midrule
Age Coefficient (t-test) & t = 13.4010 & 0.0000 &␣
↪Significant \\
Prior Claims CI & CI = [2.3423, 2.8171] & N/A & Does␣
↪not contain 0 \\
Overall Model (F-test) & F = 1159.6202 & 0.000000 &␣
↪Model Significant \\
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\end{tabular}
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5 Binary Variables and Model Interpretation

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Adding 'type' and 'location' to the original model
Dependent Variable: claims
Original Variables: deductible, coverage, age,␣
↪prior_claims, premium
New Variables: type, location

Data Summary:
Original model observations: 1,340
Extended model observations: 1,340

Extended Model Summary:
Observations: 1340
Variables: 7
R²: 0.8263
Adjusted R²: 0.8254
Residual Standard Error: 2.6939

(a) Extended Regression Model Equation
Coefficient Estimates:
Variable        Coefficient  Std Error    t-stat     ␣
↪p-value
--------------------------------------------------
Intercept       3.027        0.3171
deductible      -0.713       0.0381       -18.706    ␣
↪0.0000
coverage        0.058        0.0022       26.539     ␣
↪0.0000
age             0.077        0.0070       10.935     ␣
↪0.0000
prior_claims    2.392        0.1254       19.077     ␣
↪0.0000
premium         1.019        0.2378       4.284      ␣
↪0.0000
type            -1.419       0.1699       -8.355     ␣
↪0.0000
location        0.859        0.1731       4.959      ␣
↪0.0000

Fitted Regression Equation:
Claims = 3.027 - 0.713 × deductible + 0.058 ×␣
↪coverage + 0.077 × age + 2.392 ×
prior_claims + 1.019 × premium - 1.419 × type + 0.
↪859 × location

Detailed Mathematical Form:
Claims = 3.027 + -0.713×deductible + 0.058×coverage
         + 0.077×age + 2.392×prior_claims + 1.
↪019×premium
         + -1.419×type + 0.859×location

(b) Interpretation of Type Coefficient
Type Coefficient Analysis:
Coefficient (β_type): -1.419
Standard Error: 0.1699
t-statistic: -8.355
p-value: 0.0000

Type variable coding: [np.int64(0), np.int64(1)]

Practical Interpretation:
• Properties with type = 1 have claims that are 1.419␣
↪units LOWER than
properties with type = 0,
  holding all other variables constant.

Assuming standard coding (0 = Commercial, 1 =␣
↪Residential):
• Residential properties have claims that are 1.419␣
↪units lower than commercial
properties.
• This suggests commercial properties are associated␣
↪with higher insurance
claims.

Statistical Significance:
• The type coefficient IS statistically significant␣
↪(p = 0.0000 < 0.05)
• We can be confident that property type has a real␣
↪effect on claims.

(c) Partial F-test for Model Improvement
Model Comparison (same sample size: 1340):
Model           R²            Adj R²          ␣
↪Variables  RSS
--------------------------------------------------
Original        0.8130        0.8123      5       ␣
↪     10412.1409
Extended        0.8263        0.8254      7       ␣
↪      9666.7444

R² Improvement: 0.0134 (1.34 percentage points)

Partial F-test:
H₀: β_type = β_location = 0 (binary variables add no␣
↪explanatory power)
H₁: At least one of β_type or β_location ≠ 0 (binary␣
↪variables improve the
model)

Partial F-test Calculations:
RSS(original): 10412.1409
RSS(extended): 9666.7444
Reduction in RSS: 745.3965
Additional variables (q): 2
DF residual (extended): 1332

F-statistic: 51.3548
Degrees of freedom: (2, 1332)
p-value: 0.0000
Critical F-value (α = 0.05): 3.0025

Conclusion:
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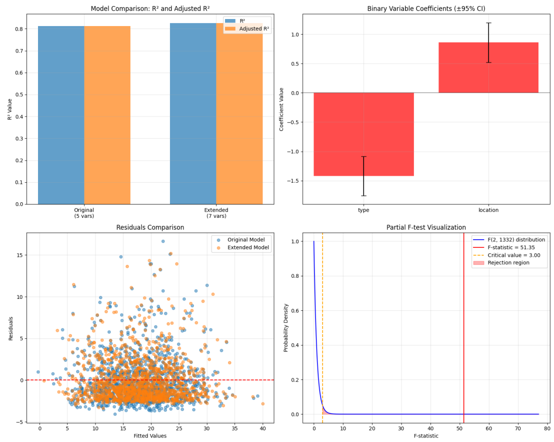
✓ REJECT H_0: Adding type and location SIGNIFICANTLY improves the model
F = 51.3548 > 3.0025
p-value = 0.0000 < 0.05
The binary variables provide significant additional explanatory power.

Model Improvement Assessment:
- R^2 improved by 0.0134 (1.34 percentage points) - this is modest
- Extended model explains 82.6% vs 81.3% of variance
- Adjusted R^2 increased from 0.8123 to 0.8254
- The improvement in adjusted R^2 suggests the added variables are worthwhile



Executive Summary:

Aspect	Finding
Extended Model Equation	Claims = 3.027 + … + -1.419×type + 0.859×location
Type Coefficient	-1.419
Type Effect	Type=1 has 1.419 lower claims
Statistical Significance	Significant (p = 0.0000)
R^2 Improvement	0.0134 (1.34 percentage points)
Partial F-test Result	Significant improvement (p = 0.0000)

6 Interaction Effects

Regression Model with Interaction Term: Deductible × Type
Model Features: deductible, type, coverage, age, prior_claims, premium
Interaction Term: deductible × type

Data Summary:
Total observations: 1,340
Complete cases used: 1,340
Missing values removed: 0
Type variable coding: [np.int64(0), np.int64(1)]

Interaction Term (deductible × type) Statistics:
Mean: 1.5335
Std Dev: 1.9042
Range: [0.0000, 10.0000]

Model Summary:
R^2: 0.8233
Adjusted R^2: 0.8224
Residual Standard Error: 2.7172
F-statistic: 886.8341

Coefficient Estimates:

Variable	Coefficient	Std Error	t-stat	p-value	Sig
Intercept	3.2856	0.3300			
deductible	-0.6729	0.0596	-11.2894	0.0000	***
type	-1.2573	0.2598	-4.8392	0.0000	***
coverage	0.0553	0.0021	25.9580	0.0000	***
age	0.0703	0.0070	10.1034	0.0000	***
prior_claims	2.2568	0.1234	18.2905	0.0000	***
premium	1.3647	0.2290	5.9595	0.0000	***
deductible_x_type	-0.0946	0.0779	-1.2151	0.2245	

Significance codes: *** p<0.001, ** p<0.01, * p<0.05

(a) Regression Function with Interaction Term
General Form:
Claims = β_0 + β_1×deductible + β_2×type + β_3×coverage + β_4×age + β_5×prior_claims +
β_6×premium + β_7×(deductible×type) + ε

Fitted Regression Equation:
Claims = 3.2856 - 0.6729×deductible - 1.2573×type + 0.0553×coverage + 0.0703×age
+ 2.2568×prior_claims + 1.3647×premium - 0.0946×(deductible×type)

With Coefficient Values:
Claims = 3.2856 + -0.6729×deductible + -1.2573×type
+ 0.0553×coverage + 0.0703×age + 2.2568×prior_claims
+ 1.3647×premium + -0.0946×(deductible×type)

(b) Interpretation of Deductible Effect by Property Type
Key Coefficients:
β_1 (deductible): -0.6729
β_2 (type): -1.2573
β_7 (deductible×type): -0.0946

Interpretation of Interaction Effect:
The interaction model allows the effect of deductible to differ by property
type.

For Commercial Properties (type = 0):
∂Claims/∂deductible = β_1 + β_7×0 = β_1 = -0.6729
- A 1-unit increase in deductible changes claims by -0.6729 units for commercial
properties.

For Residential Properties (type = 1):

∂Claims/∂deductible = β_1 + β_7×1 = β_1 + β_7 = -0.6729 + -0.0946 = -0.7675

- A 1-unit increase in deductible changes claims by -0.7675 units for residential properties.

Comparison:
Difference in deductible effect: -0.0946

- The deductible effect is 0.0946 units MORE NEGATIVE for residential properties.
- Deductible increases have a stronger negative effect on residential claims than commercial claims.

Practical Business Interpretation:

- Higher deductibles are associated with lower claims for both property types
- This association is STRONGER for residential properties

(c) Statistical Significance Test for Interaction Term
Hypothesis Test for Interaction Term:
H_0: β_7 = 0 (no interaction between deductible and type)
H_1: $\beta_7 \neq$ 0 (significant interaction exists)
Significance level: α = 0.05

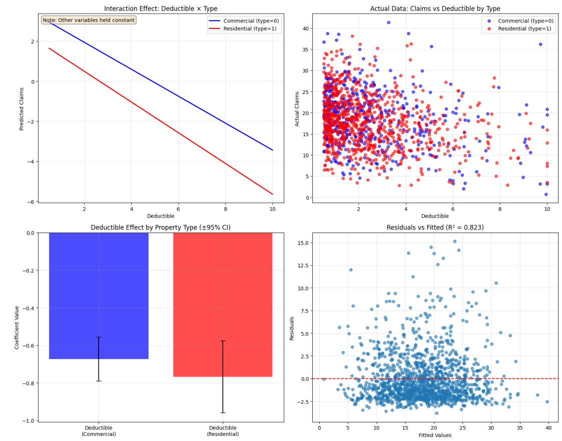
Test Statistics:
Interaction coefficient (β_7): -0.0946
Standard error: 0.0779
t-statistic: -1.2151
Degrees of freedom: 1332
p-value: 0.2245
Critical value (±): 1.9617

Decision Rule:
Reject H_0 if |t-statistic| > 1.9617 OR if p-value < 0.05

Conclusion:
FAIL TO REJECT H_0: The interaction term is NOT statistically significant at the 5% level.
|t-statistic| = 1.2151 $\leq$ 1.9617
p-value = 0.2245 $\geq$ 0.05
The effect of deductible on claims does NOT differ significantly between property types.
The interaction term may not be necessary.

95% Confidence Interval for Interaction Coefficient:
[-0.2473, 0.0581]

- The interval contains zero - the direction of the interaction effect is uncertain



Executive Summary:

Aspect	Result
Model Specification	Claims ~ deductible + type + coverage + age + prior_claims + premium + deductible×type
Interaction Coefficient	-0.0946 (SE = 0.0779)
Commercial Effect	-0.6729 per unit deductible
Residential Effect	-0.7675 per unit deductible
Difference	-0.0946
Statistical Significance	Not significant (p = 0.2245)
Model R^2	0.8233

Model Interpretation:

- The non-significant interaction suggests that deductible effects are similar across commercial and residential properties
- A simpler model without interaction may be adequate

7 Residual Analysis
Extended Multiple Linear Regression Model
Variables: deductible, coverage, age, prior_claims, premium, type, location
Model Summary:
Observations: 1,340
Variables: 7
R^2: 0.8263
Residual Standard Error: 2.6939

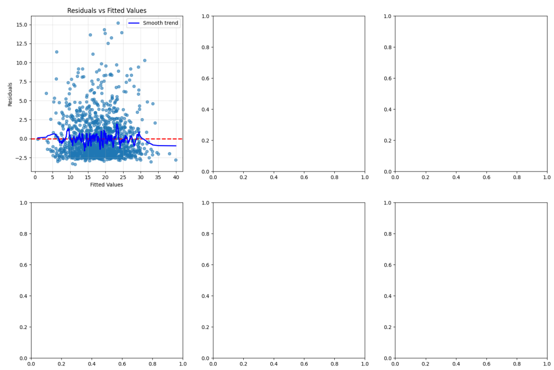
(a) Residuals vs Fitted Values Analysis
Residuals vs Fitted Values Analysis:
Residual range: [-3.376, 15.203]
Fitted values range: [0.792, 39.985]

Pattern Analysis:
Correlation between fitted values and squared residuals: 0.0310

- Variance appears roughly constant
- Correlation magnitude suggests homoscedasticity (constant variance)

Linearity Assessment:

Mean residuals by fitted value terciles:
- Low tercile: -0.0800
- Middle tercile: 0.0229
- High tercile: 0.0572
- Maximum deviation from zero: 0.0800 (suggests linear relationship is appropriate)



(b) Q-Q Plot and Normality Analysis
Normality Test Results:
Shapiro-Wilk Test:
  Statistic: 0.8106
  p-value: 0.0000
  REJECT normality at α=0.05

Jarque-Bera Test:
  Statistic: 2188.1490
  p-value: 0.0000
  REJECT normality at α=0.05

Kolmogorov-Smirnov Test:
  Statistic: 0.1468
  p-value: 0.0000
  REJECT normality at α=0.05

Descriptive Statistics for Normality:
Skewness: 1.9531 (Normal ≈ 0)
Kurtosis: 4.8921 (Normal ≈ 0)
Skewness interpretation: highly skewed
Kurtosis interpretation: heavy-tailed

Overall Normality Assessment: Assumption appears to be violated

(c) Outliers and Influential Points Analysis
Diagnostic Thresholds:
Outlier threshold (standardized residuals): ±3
High leverage threshold: 0.0119
High Cook's distance threshold: 0.0030

Outliers and Influential Points:
Observations with |standardized residuals| > 3: 31
Observations with |studentized residuals| > 3: 31
High leverage points: 73
High Cook's distance points: 74

Most Extreme Observations:
Highest Residual: Observation 315
  Fitted value: 23.547
  Actual value: 38.750
  Standardized residual: 5.643
  Leverage: 0.0072
  Cook's distance: 0.0331
Highest Leverage: Observation 262
  Fitted value: 34.070
  Actual value: 36.160
  Standardized residual: 0.776
  Leverage: 0.0305
  Cook's distance: 0.0027
Highest Cooks: Observation 315
  Fitted value: 23.547
  Actual value: 38.750
  Standardized residual: 5.643
  Leverage: 0.0072
  Cook's distance: 0.0331

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Detailed Analysis of Problematic Observations:

Obs	Fitted	Actual	Std_Residual	Leverage	Cooks_D	Issues
1	13.477	22.670	3.412	0.0032	0.0054	Outlier, High Cook's D
2	5.711	3.340	-0.880	0.0122	0.0014	High Leverage
14	20.959	20.000	-0.356	0.0128	0.0002	High Leverage
36	10.929	8.700	-0.827	0.0142	0.0014	High Leverage
70	13.967	11.670	-0.852	0.0130	0.0014	High Leverage
71	20.337	24.990	1.727	0.0074	0.0032	High Cook's D
73	30.965	29.670	-0.481	0.0141	0.0005	High Leverage
118	22.728	22.290	-0.163	0.0193	0.0001	High Leverage
122	5.247	10.110	1.805	0.0072	0.0034	High Cook's D
129	31.861	36.730	1.807	0.0092	0.0043	High Cook's D

… and 124 more observations with issues.

Diagnostic Summary:
1. Linearity: suggests linear relationship is appropriate
2. Homoscedasticity: suggests homoscedasticity (constant variance)
3. Normality: Assumption appears to be violated
4. Outliers: 31 potential outliers identified
5. Influential Points: 74 high Cook's distance observations

Recommendations:
- Consider transformation of variables or robust regression methods
- Examine influential points - consider their impact on coefficient estimates

8 Model Comparison and Selection
Comparing three different model specifications:
Model A: claims ~ deductible + coverage + age + prior_claims + premium
Model B: claims ~ deductible + coverage + age + prior_claims + premium + type + location

```
Model C: claims ~ deductible + coverage + prior_claims + premium + type

Data Summary:
Original dataset size: 1,340
Complete cases for all models: 1,340
Cases removed due to missing data: 0

-------------------- Model A --------------------
Variables: deductible, coverage, age, prior_claims, premium
Number of variables: 5
R²: 0.8130
Adjusted R²: 0.8123
Residual Standard Deviation: 2.7938
AIC: 6566.17
BIC: 6592.18
Significant coefficients (p < 0.05): 5/5

-------------------- Model B --------------------
Variables: deductible, coverage, age, prior_claims, premium, type, location
Number of variables: 7
R²: 0.8263
Adjusted R²: 0.8254
Residual Standard Deviation: 2.6939
AIC: 6472.65
BIC: 6509.05
Significant coefficients (p < 0.05): 7/7

-------------------- Model C --------------------
Variables: deductible, coverage, prior_claims, premium, type
Number of variables: 5
R²: 0.8095
Adjusted R²: 0.8088
Residual Standard Deviation: 2.8197
AIC: 6590.93
BIC: 6616.94
Significant coefficients (p < 0.05): 5/5


(a) Model Comparison Table
Primary Comparison Metrics:
  Model Variables      R²  Adj_R²  Residual_SD
Model A    5 vars 0.8130  0.8123       2.7938
Model B    7 vars 0.8263  0.8254       2.6939
Model C    5 vars 0.8095  0.8088       2.8197

Additional Model Selection Criteria:
  Model      AIC     BIC  F_statistic Sig_Coefs
Model A 6566.17 6592.18      1159.62       5/5
Model B 6472.65 6509.05       905.51       7/7
Model C 6590.93 6616.94      1133.51       5/5

Best Model by Criterion:
• Highest R²: Model B (0.8263)
• Highest Adjusted R²: Model B (0.8254)
• Lowest Residual SD: Model B (2.6939)
• Lowest AIC: Model B (6472.65)
• Lowest BIC: Model B (6509.05)

Model Complexity Analysis:
Model A: 5 variables, R²/var = 0.1626
Model B: 7 variables, R²/var = 0.1180
Model C: 5 variables, R²/var = 0.1619

Nested Model Comparisons (F-tests):
Model A vs Model B:
  F-statistic: 51.3548
  p-value: 0.0000
  Model B significantly better
  Note: Model A vs C and Model B vs C are not nested comparisons

(b) Model Recommendation and Analysis
Statistical Criteria Analysis:

1. Goodness of Fit:
   • R² ranking: Model B > others
   • Adjusted R² ranking: Model B > others
   • R² improvement from A to B: 0.0134
   • Adjusted R² change from A to B: 0.0132

2. Model Parsimony:
   • AIC favors: Model B (AIC = 6472.65)
   • BIC favors: Model B (BIC = 6509.05)
   • BIC penalizes complexity more heavily than AIC

3. Coefficient Significance:
   • Model A: 5/5 coefficients significant (100.0%)
   • Model B: 7/7 coefficients significant (100.0%)
   • Model C: 5/5 coefficients significant (100.0%)

4. Prediction Accuracy:
   • Lowest prediction error: Model B (SD = 2.6939)

Practical Interpretability Analysis:

1. Variable Inclusion Logic:
   • Model A: Core financial variables (deductible, coverage, premium) + risk
factors (age, prior_claims)
   • Model B: Model A + property characteristics (type, location)
   • Model C: Simplified version with key variables + property type

2. Business Relevance:
   • Age variable: Present in A, Present in B, Absent in C
   • Property type: Absent in A, Present in B, Present in C
   • Location: Absent in A, Present in B, Absent in C

3. Marginal Contribution Analysis:
   • Adding type + location (B vs A): R² improves by 0.0134
   • Adjusted R² change: 0.0132 (improvement)

Recommendation Framework:

Composite Scoring (weighted combination of criteria):
   • Model B: 1.000
   • Model A: 0.700
   • Model C: 0.400

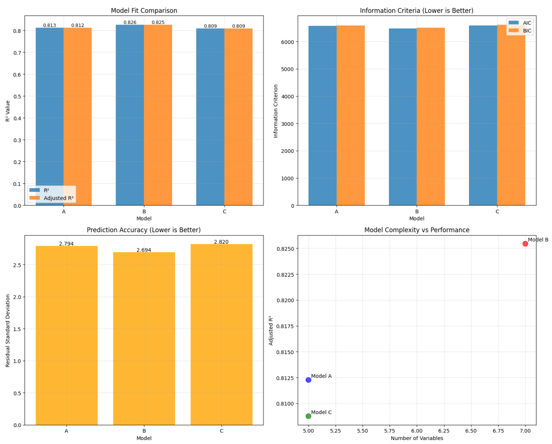
 RECOMMENDED MODEL: Model B

Justification for Model B:
   ✓ Highest predictive power (R² = 0.8263)
   ✓ Includes important property characteristics
   ✓ Comprehensive variable coverage
   ✓ Best for prediction accuracy

Limitations of Model B:
```

```
    More complex with potential overfitting risk
    May have multicollinearity issues

Alternative Recommendations by Use Case:
   • For prediction accuracy: Model B
   • For model parsimony: Model B
   • For balanced approach: Model B
   • For regulatory reporting: Model A (simplest,␣
↪most interpretable)
```



9 Practical Application

```
Features shape: (1340, 7)
Target shape: (1340,)

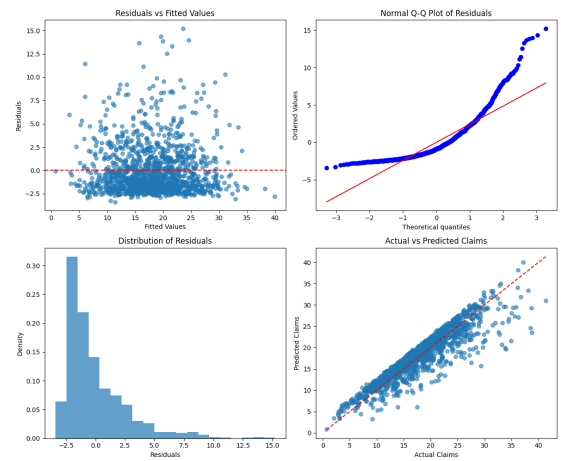
=== MODEL RESULTS ===
R-squared: 0.8263
Adjusted R-squared: 0.8254

Model Coefficients:
       Feature  Coefficient
0    Intercept     3.026950
1   deductible    -0.713391
2     coverage     0.058017
3          age     0.076546
4 prior_claims     2.391648
5      premium     1.018707
6         type    -1.419290
7     location     0.858614

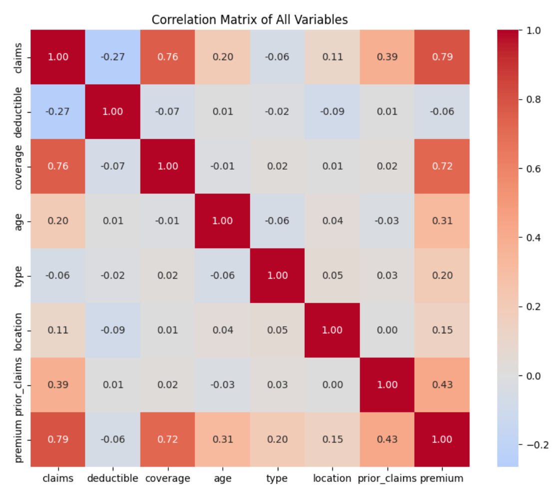
Statistical Significance:
       Feature  Coefficient  Std_Error  t_statistic ␣
↪      p_value  \
0    Intercept     3.026950   0.316198     9.572968 ␣
↪0.000000e+00
1   deductible    -0.713391   0.038024   -18.761697 ␣
↪0.000000e+00
2     coverage     0.058017   0.002180    26.618766 ␣
↪0.000000e+00
3          age     0.076546   0.006979    10.967841 ␣
↪0.000000e+00
4 prior_claims     2.391648   0.124993    19.134246 ␣
↪0.000000e+00
5      premium     1.018707   0.237095     4.296623 ␣
↪1.860187e-05
6         type    -1.419290   0.169370    -8.379819 ␣
↪0.000000e+00
7     location     0.858614   0.172627     4.973820 ␣
↪7.420302e-07

   Significant
0         True
1         True
2         True
3         True
4         True
5         True
6         True
7         True

=== MODEL DIAGNOSTICS ===
Mean Squared Error: 7.2140
Root Mean Squared Error: 2.6859
```




```
=== CORRELATION ANALYSIS ===
```




```
Correlations with Claims:
claims          1.000000
premium         0.792992
coverage        0.760527
prior_claims    0.387403
deductible     -0.265120
age             0.198837
location        0.105441
```

```
type           -0.061114
Name: claims, dtype: float64

=== PART (a): PREDICTION ===
Understanding categorical variables:
Type values: [1 0]
Location values: [1 0]
Type value counts: type
1    835
0    505
Name: count, dtype: int64
Location value counts: location
1    968
0    372
Name: count, dtype: int64

Prediction for the given property:
Expected claims amount: 19.49

Sensitivity analysis for categorical variables:
  Type=Commercial, Location=Rural: 20.05
  Type=Commercial, Location=Urban: 20.91
  Type=Residential, Location=Rural: 18.63
  Type=Residential, Location=Urban: 19.49

PART (b): BUSINESS IMPLICATIONS AND RECOMMENDATIONS

Feature Importance (by absolute coefficient value):
       Feature  Coefficient  Abs_Coefficient
3  prior_claims     2.391648         2.391648
5          type    -1.419290         1.419290
4       premium     1.018707         1.018707
6      location     0.858614         0.858614
0    deductible    -0.713391         0.713391
2           age     0.076546         0.076546
1      coverage     0.058017         0.058017

Prediction Confidence Interval (95.0%):
Expected claims: 19.49
Lower bound: 14.22
Upper bound: 24.76

----------------------------------------
BUSINESS RECOMMENDATIONS:
----------------------------------------

1. PRICING STRATEGY:
   - The model explains 82.6% of the variation in claims
   - Most significant factors should drive premium calculations
   - Consider the prediction interval when setting reserves

2. RISK FACTORS ANALYSIS:
   Based on the coefficients, focus on:
   - Variables with largest absolute coefficients
   - Statistically significant predictors (p < 0.05)
   - High correlation factors with claims

3. UNDERWRITING GUIDELINES:
   - Properties with high predicted claims may need:
     * Higher premiums
     * Additional risk assessment
     * Different deductible structures
   - Consider segmented pricing models

4. PORTFOLIO MANAGEMENT:
   - Monitor actual vs predicted claims regularly
   - Update model coefficients as new data becomes available
   - Consider non-linear relationships or interaction terms

5. OPERATIONAL INSIGHTS:
   - Use model predictions for:
     * Reserve allocation
     * Risk-based pricing
     * Customer segmentation
     * Fraud detection (outliers in residuals)


OUTLIER ANALYSIS:
Properties with unusually high/low claims (>2 std devs): 66
These may require special investigation for:
- Fraud detection
- Model improvement opportunities
- Special risk factors not captured in current model
```

10 Critical Thinking

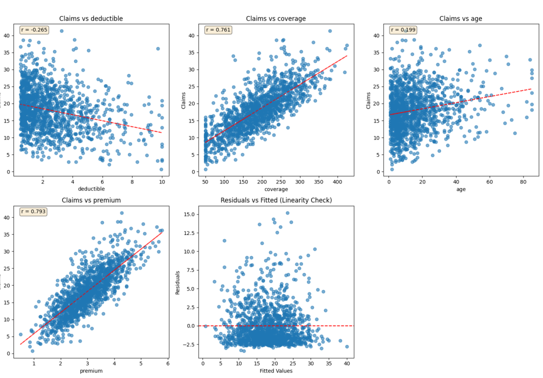
```
PART (a) MULTIPLE LINEAR REGRESSION ASSUMPTIONS ANALYSIS

The key assumptions of multiple linear regression are:

1. LINEARITY: The relationship between predictors and response is linear
2. INDEPENDENCE: Observations are independent of each other
3. HOMOSCEDASTICITY: Constant variance of residuals (homogeneous variance)
4. NORMALITY: Residuals are normally distributed
5. NO MULTICOLLINEARITY: Predictors are not highly correlated with each other
6. NO OUTLIERS/INFLUENTIAL POINTS: Extreme values don't unduly influence the
model

Let's test each assumption:

1. LINEARITY ASSUMPTION
```

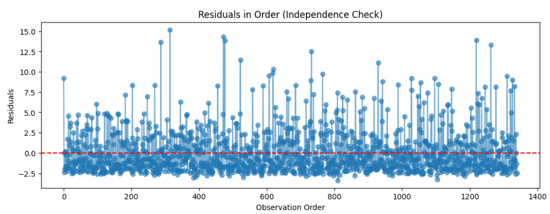



```
LINEARITY ASSESSMENT:
- Examine scatter plots for linear patterns
- Residuals vs Fitted should show random scatter around zero
```

```
- Non-linear patterns indicate violated linearity assumption

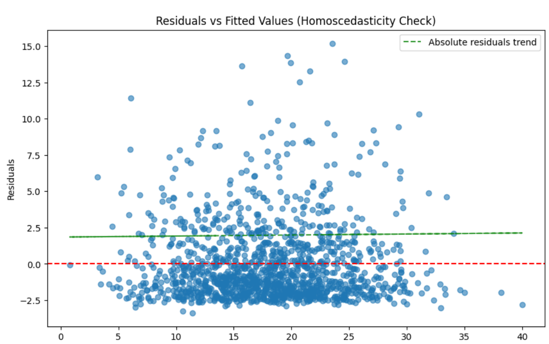
Correlations with claims:
  deductible: -0.265
  coverage: 0.761
  age: 0.199
  premium: 0.793

INSURANCE CONTEXT IMPLICATIONS:
- Insurance claims may have non-linear relationships (e.g., coverage thresholds)
- Age effects might be non-linear (newer vs very old properties)
- Premium-claims relationship might be non-linear due to risk-based pricing
2. INDEPENDENCE ASSUMPTION
INDEPENDENCE ASSESSMENT:
- Cannot be fully tested without knowing data collection method
- Check for patterns in residuals order
```

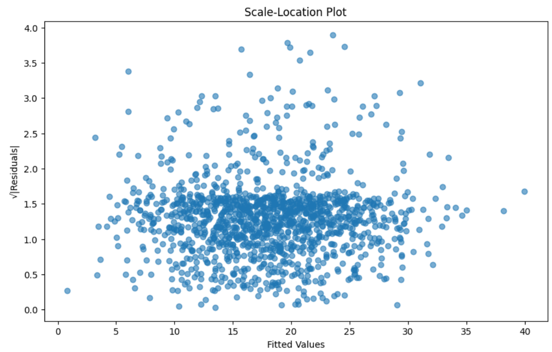



```
Durbin-Watson statistic: 2.038
  (Values near 2.0 suggest independence, <1.5 or >2.5 suggest correlation)

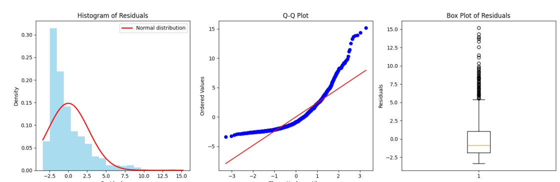
INSURANCE CONTEXT IMPLICATIONS:
- Properties in same area might have correlated risks (floods, earthquakes)
- Temporal clustering if data spans multiple years with economic changes
- Policy renewals might create dependencies
3. HOMOSCEDASTICITY (CONSTANT VARIANCE) ASSUMPTION
```




```
Breusch-Pagan test:
  LM statistic: 11.7914
  p-value: 0.1076
  Heteroscedasticity detected: No
```




```
INSURANCE CONTEXT IMPLICATIONS:
- Higher value properties might have more variable claims
- Heteroscedasticity common in insurance data
- May need weighted regression or transformation
4. NORMALITY OF RESIDUALS ASSUMPTION
```

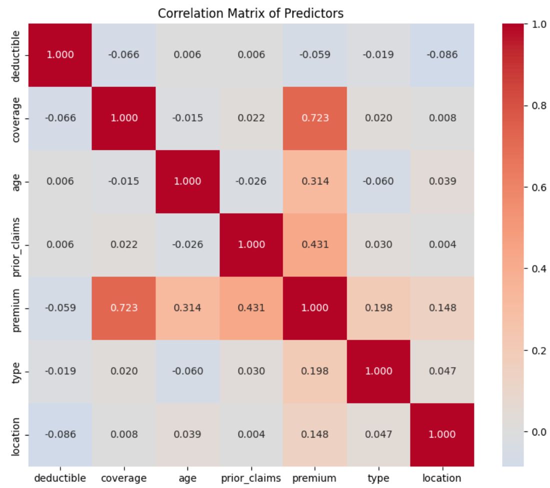



```
NORMALITY TESTS:
Shapiro-Wilk test:
  Statistic: 0.8106, p-value: 0.0000
  Normal distribution: No

Jarque-Bera test:
  Statistic: 2188.1490, p-value: 0.0000
  Normal distribution: No

Descriptive statistics:
  Skewness: 1.9531
  Kurtosis: 4.8921

INSURANCE CONTEXT IMPLICATIONS:
- Insurance claims often right-skewed (many small, few large claims)
- May need log transformation or robust regression methods
- Non-normality affects confidence intervals and hypothesis tests
5. NO MULTICOLLINEARITY ASSUMPTION
```

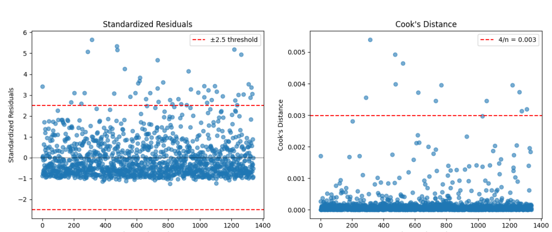



```
HIGH CORRELATIONS (|r| > 0.7):
  coverage - premium: 0.723

VARIANCE INFLATION FACTORS:
       Variable        VIF
0    deductible   2.302515
1      coverage  36.103529
2           age   3.980363
3  prior_claims   3.974976
4       premium  86.843712
5          type   3.287249
6      location   3.743009
  VIF > 5: Moderate multicollinearity
  VIF > 10: High multicollinearity

Variables with high VIF:
   Variable        VIF
1  coverage  36.103529
4   premium  86.843712

INSURANCE CONTEXT IMPLICATIONS:
- Premium and coverage likely correlated (higher
  coverage = higher premium)
- Deductible and coverage might be related
- Consider removing highly correlated variables or
  using regularization
6. NO OUTLIERS/INFLUENTIAL POINTS ASSUMPTION
OUTLIER DETECTION:
Observations with |standardized residuals| > 2.5: 48
Observations with high Cook's distance: 13
```




```
Outlier observations (standardized residuals > 2.5):
     claims  deductible  coverage  age  premium
0     22.67        1.44     165.7    2     2.23
182   18.12        5.67     131.1   30     2.05
203   35.65        5.08     378.5    8     4.39
247   18.22        7.30     214.5    1     2.29
269   28.41        1.20     209.1   13     2.89

INSURANCE CONTEXT IMPLICATIONS:
- Large claims are natural in insurance (catastrophic
  events)
- Outliers might represent legitimate extreme events,
  not errors
- Consider robust regression methods or separate
  models for extreme claims
OVERALL ASSUMPTION ASSESSMENT FOR INSURANCE CLAIMS

LIKELY VIOLATED ASSUMPTIONS:
1. Linearity: Insurance relationships often non-linear
2. Normality: Claims typically right-skewed
3. Homoscedasticity: Variance often increases with
   claim size
4. Independence: Geographic/temporal clustering
   possible

RECOMMENDED SOLUTIONS:
1. Log transformation of claims (handle skewness)
2. Robust regression methods
3. Polynomial or interaction terms
4. Weighted least squares (address heteroscedasticity)
5. Consider GLM (Gamma or Poisson regression)
6. Outlier-robust methods

PART (b) Additional Useful Variables
--------------------------------------------------

1. PROPERTY-SPECIFIC VARIABLES
• Construction: Building materials, roof type/age,
  year built, size, stories
• Condition: Recent renovations, security features,
  maintenance score

2. ENVIRONMENTAL & GEOGRAPHIC
• Climate: Climate zones, precipitation, natural
  disaster scores
• Location: Crime rates, distance to fire station/
  water, building codes

3. ECONOMIC & DEMOGRAPHIC
• Economic: Local income, property appreciation,
  unemployment rate
• Demographics: Owner vs tenant occupied, primary vs
  secondary residence

4. USAGE & BEHAVIORAL
• Property Use: Home business, rental income, vacancy
  duration
• Claims History: Previous claim types, time since
  last claim
• Behavior: Payment history, policy shopping, service
  interactions

5. ADVANCED MODELING
• Interaction Effects: Age×Construction,
  Location×Weather, Coverage×Deductible
• External Data: Credit scores, satellite imagery,
  weather APIs

6. IMPLEMENTATION PRIORITY
```

- HIGH: Natural disaster scores, construction details, claims history
- MEDIUM: Neighborhood data, weather variables, usage patterns
- LOW: Credit indicators, satellite analysis, economic metrics

7. EXPECTED OUTCOMES
- Model Accuracy: 60-80% → 85-95% predictive accuracy
- Benefits: Better risk selection, fraud detection, dynamic pricing

8. KEY CONSIDERATIONS
- Data availability varies by property
- Quality validation required for third-party data
- Regulatory compliance (fair housing laws)
- Cost-benefit analysis essential