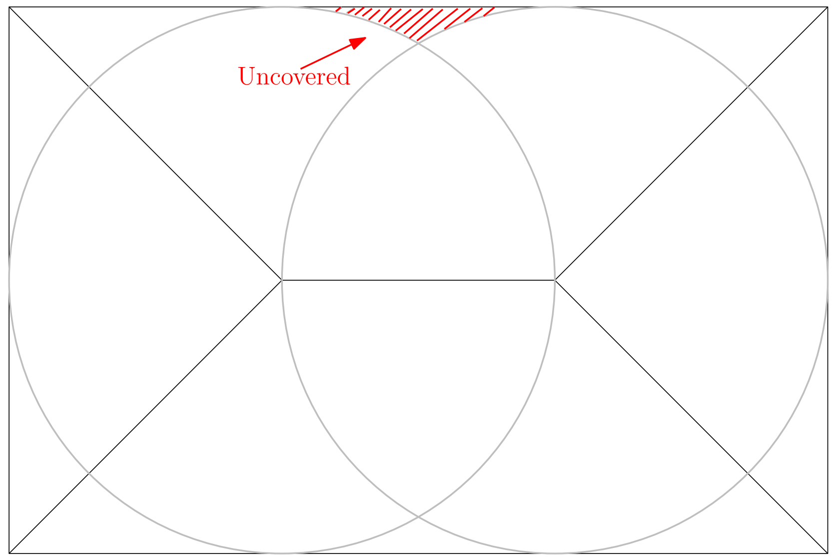
Sampling in 2D

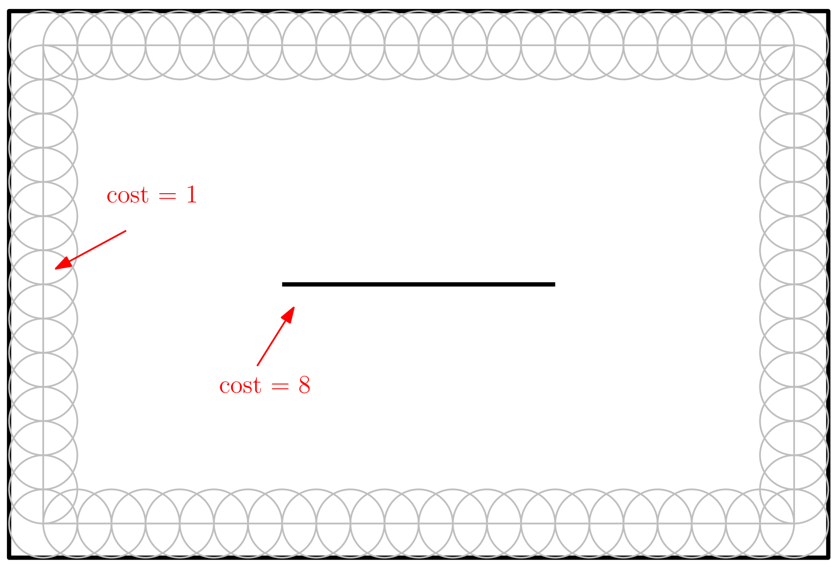
Specify the 2D space $D \in \mathbb{R}^2$ to be a square without obstacles. Let $P = \{p_1, p_2, ..., p_i, ...\}$ be a set of iso-cost contours where $\forall x \in p_i, clearance(x) = c_i$.

For $x \in D$, we define $B_D(x)$ to be the largest closed disc centered at x that is a subset of D, i.e., $B_D(x) = \overline{B}(x, \rho_D(x))$, where $\overline{B}(x, \rho_D(x))$ denotes the closed disc of radius $r \geq 0$, centered at x, and $\rho_D(x) = dist(x, \mathbb{R}^2 \setminus D)$ is the distance to the boundary for points inside S, and 0 for points outside D.

The media axis $MA(D)$ of S is defined to be the set of all points x of D whose $B_D(x)$ are maximal. [1]



Media Axis



Iso-cost contour

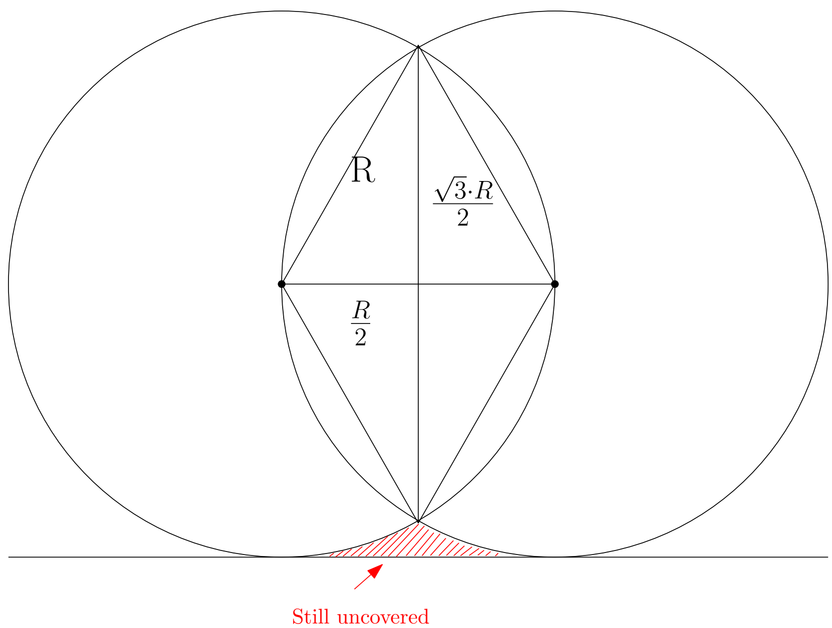
1 Some Observations

1. Continuously sample all points $x \in MA(D)$ totally covers D. $S_{c_{ma}} = \{B_D(x) | \forall x \in MA(D)\}$, $C(S_{c_{ma}}) = D$, where $C(S) = \cup_{\forall b \in S}$.

2. Continuously sample all points $x \in P$ totally covers D, too. $S_{c_{iso}} = \{B_D(x) | \forall x \in P\}$, $C(S_{c_{iso}}) = D$.

3. Even if we can continuously sample in $MA(D)$, but restrict new balls to center outside existing ones, some areas near obstacles will be uncovered.

4. Use iso-cost contours to sample small balls can cover these left areas. Questions: How many contours do we need? What are the costs for these contours?



Assume a series of discs with the same radius R and center on each other's boundary. Connect the centers, points within $\frac{\sqrt{3}R}{2}$ from the connecting line are guaranteed to be covered in the discs chain.

If we want to cover all points in the left areas, assume the point farthest from obstacles has clearance C_{max}, we requires $(\frac{\sqrt{3}}{2}+1) \cdot R = C_{max}$. $R = \frac{2 \cdot C_{max}}{\sqrt{3}+2}$

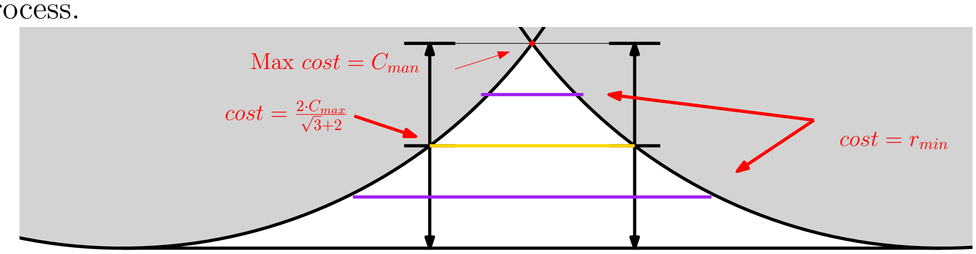
Now there are three possible situations:

1. $R = r_{min}$.
2. $R < r_{min}$.
3. $R > r_{min}$.

If $R = r_{min}$, fine, just sample the contour with cost $c_i = r_{min}$.

If $R < r_{min}$, still sample the contour with cost $c_i = r_{min}$.

If $R > r_{min}$, sample the contour with cost $c_i = R$. Then repeat the process.



2 Sampling on Media Axis

In [1], the author proposed an algorithm to sample on the media axis:

Algorithm 3.1 MAPRM in 2D

Preprocessing:

Input. N, the number of nodes to generate.
Output. N nodes in F connected into a roadmap graph.

1: **repeat**
2:     Generate a uniformly random point p in C.
3:     Find the nearest point q on ∂F to p.
4:     **if** p is free **then**
5:         Take the retraction direction $\vec{v}$ to be $\overrightarrow{qp}$, and let the start point s be p.
6:     **else**
7:         Take the retraction direction $\vec{v}$ to be $\overrightarrow{pq}$, and let the start point s be q.
8:     **end if**
9:     Using bisection, move s in the direction $\vec{v}$ until q is not the unique nearest point of ∂F to s. This moves s onto the medial axis of F
10: **until** N nodes have been generated
11: For each pair of nodes: if the pair can be connected with a straight line, insert an edge into the graph connecting them.

By changing the terminal condition in line 9, we can sample discs with any radius.

References

[1] Steven A. Wilmarth, Nancy M. Amato, Peter F. Stiller. "MAPRM: A Probabilistic Roadmap Planner with Sampling on the Medial Axis of the Free Space", In Proc. IEEE Int. Conf. Robot. Autom. (ICRA), pp. 1024-1031, Detroit, MI, May 1999. Also, Technical Report, TR98-0022, Department of Computer Science and Engineering, Texas A & M University, Nov 1998.