Small Time Optimality

1 Model

A Reed Shepp car model can be described by two variables $\boldsymbol{v}$ and $\boldsymbol{\omega}$ which are the speed and angular velocity. The turning radius is $r = \frac{\boldsymbol{v}}{\boldsymbol{\omega}}$.

We use (x, y, θ) to describe the state of a R.S car, (x, y) being the position of the car, while θ being the orientation.

2 Small Time Optimality

Theorem 2.1 *There exist a minimium lenth l for a R.S car to turn angle $\Delta\theta$.*

Proof: To turning a car with angle $\Delta\theta$ with the least amount of time, The control has to be $l^+l^-l^+l^-l^+$... or $r^+r^-r^+r^-r^+$... (keep turning counter-clockwisely or clockwisely)

Assume the $i-th$ operation turns $\Delta\theta_i$ angles, the number of operations is n ($\forall\Delta\theta_i \geq 0$ or $\forall\Delta\theta_i \leq 0$, and $\sum_{i=1}^{n}\Delta\theta_i = \Delta\theta$). The total path length of these operation is:

$\sum_{i=1}^{n} r\Delta\theta_i = r\sum_{i=1}^{n}\Delta\theta_i = r\Delta\theta.$
which has nothing to do with the number of operations, neither with the angle of each turn.

Theorem 2.2 *For a start configuration $s = (x_s, y_s, \theta_s)$ and a goal configuration $g = (x_g, y_g, \theta_g)$, and a path σ with length l. σ cannot be the optimal path*