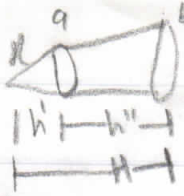
Perez de la Torre lan Axel



Ejercicio 27   1CV12

$$V=\int_a^b \pi\left(\frac{r}{H}x\right)^2 dx=\pi\int_a^b \frac{r^2}{H^2}x^2\,dx=\frac{\pi r^2}{H^2}\cdot\frac{x^3}{3}\Big]_a^b=\frac{\pi r^2}{H^2}\left(\frac{b^3}{3}-\frac{a^3}{3}\right)$$

$$\frac{\pi r^2}{3H^2}\left(b^3-a^3\right) \quad \ldots\ldots (1)$$

como $H=b$ y $h'=a$

$$\frac{\pi r^2}{3H^2}\left(H^3-(h')^3\right) \quad \ldots\ldots (2)$$

por semejanza

$$\frac{r}{H}=\frac{R}{h'} \Rightarrow h'=\frac{RH}{r} \quad \ldots\ldots (3)$$

(3) en (2)

$$\frac{\pi r^2}{3H^2}\left(H^3-\left(\frac{RH}{r}\right)^3\right)=\frac{\pi r^2}{3H^2}\cdot\left(H^3-\frac{R^3H^3}{r^3}\right)=$$

$$\frac{\pi r^2H^3}{3H^2}-\frac{R^3H^3\pi r^2}{3H^2r^3}=\frac{\pi r^2H^3}{3H^2}\left(1-\frac{R^3}{r^3}\right) \rightarrow (4)$$

$$\boxed{\frac{\pi r^2 H}{3}\left(1-\frac{R^3}{r^3}\right)}$$

Además, $H=h'+h''$, sustituyendo en (3)

$$H=h'+h''=h''+\left(\frac{RH}{r}\right)=\frac{h''r+RH}{r} \Rightarrow Hr=h''r\,RH$$

$$Hr-RH=h''r \quad (5)$$

$$\boxed{H=\frac{h''r}{r-R}}$$

ahora (5) en (4)

$$\frac{\pi r^2}{3}\left(\frac{h''r}{r-R}\right)\left(1-\frac{R^3}{r^3}\right)=\frac{\frac{\pi h''r^3}{r-R}}{3}\cdot\left(1-\frac{R^3}{r^3}\right)$$

$$=\frac{\pi h''r^3}{3(r-R)}\cdot\left(1-\frac{R^3}{r^3}\right)=\frac{\pi h''r^3}{3(r-R)}-\frac{\pi h''r^3R^3}{3r^3(r-R)}=$$

$$\boxed{=\frac{\pi h''}{3(r-R)}\left(r^3-R^3\right)} \quad \ldots\ldots (6)$$

fatorizando $(r^3 - R^3) = (r - R)(r^2 + R^2 + Rr)$

$$\frac{\pi h''}{3(r-R)}(r-R)(r^2+R^2+Rr)$$

$$V = \frac{\pi h''}{3}(r^2+R^2+Rr)$$