Time Series Forecasting

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Structure of the Lecture

1. **Introduction to time series**
2. Modeling time series and evaluating forecasting performance
3. Exponential smoothing models
4. ARIMA models
5. Regression models and advanced forecasting models
6. Machine Learning and Deep Learning methods

Introduction to Time Series

What are time series?

- Collection of data points gathered over a period of time and ordered chronologically
- The data is typically measured at evenly spaced points in time
- Notation:

$$y = (y_t : t \in 1, \ldots T) = (y_1, y_2, \ldots, y_T),$$

  where t is the time index, y_t is the observation at time t and T is the length of the time series.
- Examples:
  - Monthly sales of a retail chain
  - Daily closing prices of a stock
  - Monthly rainfall in Vienna

Relevance

- Often, time series are the only type of data available to data analysts
- Time series allow to identify trends and systemic patterns over time as well as relationships between temporal developments of different variables
- Time series aid in forecasting future behavior and making informed decisions

Time series forecasting

- **Univariate time series forecasting methods** use only information on the variable to be forecast and extrapolate trend and seasonal patterns.

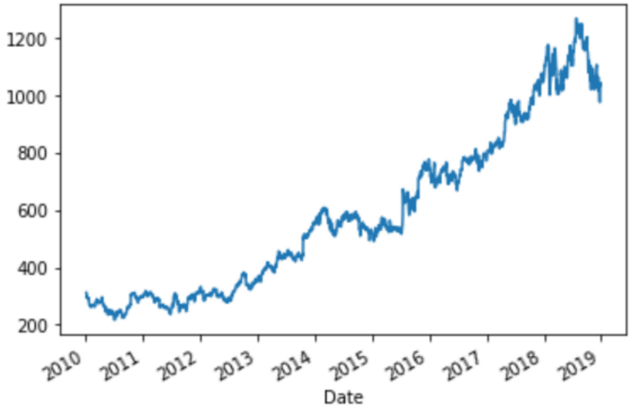
  Examples: decomposition, exponential smoothing and ARIMA models

- **Multivariate time series forecasting methods** use time series of different variables each measured at the same points in time to predict the development of the time series of interest. They take advantage of interrelations and temporal dependencies between variables.

  Examples: regression models and Prophet model

Time Series Visualization

Time plot: value of observations (y-axis) plotted against time of observation (x-axis), with consecutive observations joined by straight lines



Daily closing price of Alphabet Inc. stocks.

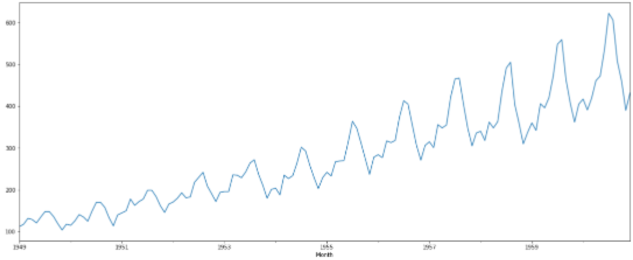
Time plot: value of observations (y-axis) plotted against time of observation (x-axis), with consecutive observations joined by straight lines

Economy class passengers Melbourne-Sydney.

Source: *FPP*

Time Series Visualization

Time plot: value of observations (y-axis) plotted against time of observation (x-axis), with consecutive observations joined by straight lines

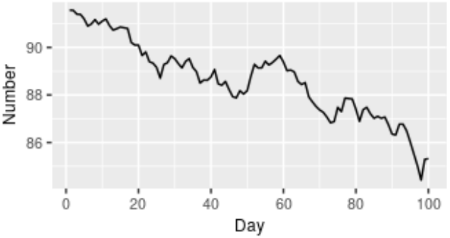


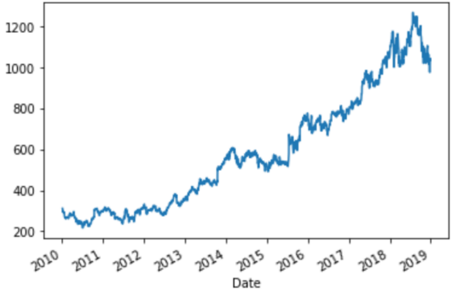
Monthly international airline passengers.

Source: FPP

Time Series Patterns

Trend: long-term increase or decrease in the data



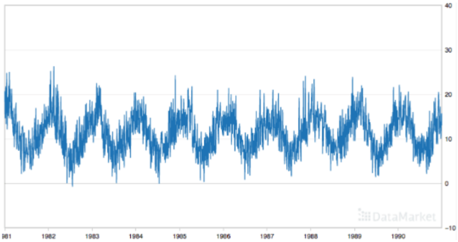


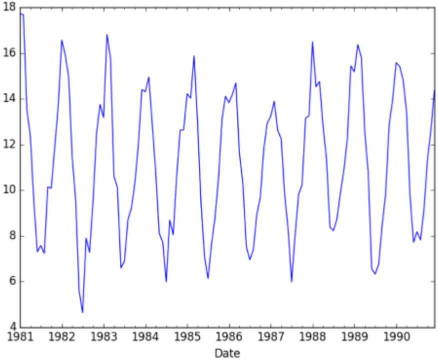
Daily count of US treasury bill contracts (left) and Alphabet Inc. stock closing price (right).

A trend is said to be "**changing direction**" when it transitions from an increasing trend to a decreasing trend, or vice versa.

Source (left figure): FPP

Seasonality





Weekly minimum temperature (left) and minimum monthly temperature (right).

Source: `https://machinelearningmastery.com/`.

Time Series Patterns

Seasonality

- Any type of recurring behavior of the time series where the **frequency of the behavior is known and constant** over time.
- Time series can exhibit seasonality at different frequencies at the same time, e.g. human behavior tends to have:
  - Daily seasonality (e.g., regular sleep-wake rhythm)
  - Weekly seasonality (e.g., increased social activities on weekends)
  - Yearly seasonality (e.g., higher travel rates during summer vacation months)

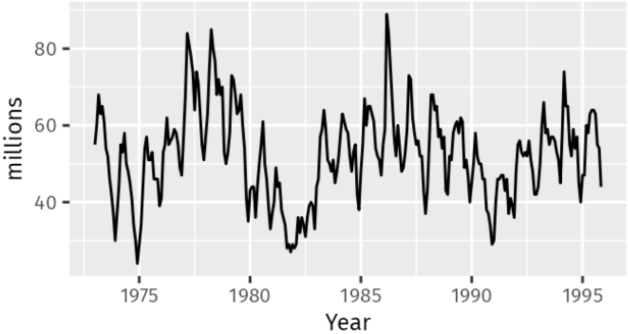
Seasonality

- The **frequency** of a seasonal pattern is the number of observations before the seasonal pattern repeats.
- **Examples:**
  - For daily data, weekly seasonality has frequency 7 and annual seasonality frequency 365.25.
  - For weekly data, annual seasonality has frequency $365.25/7 \approx 52.18$.
- Notation: The frequency of a seasonal pattern is denoted as m. E.g., daily seasonality in hourly data has the frequency $m = 24$. Put differently, there are m seasons in each day.

Cycles

- A cyclical pattern exists when the data exhibits **fluctuations, i.e. rises and falls, that are not of a fixed frequency**.
- In business data, cyclical patterns often stem from economic conditions, they are tied to the "business cycle" and usually last at least two years.

Cycles



Sales of new 1-family-homes in the U.S.

Source: FPP.

Time Series Decomposition

- Time series decomposition methods aim at breaking down a time series into its components:
  - Trend-cycle component: T_t
  - Seasonal component: S_t
  - Remainder component: R_t
- The components can either be assumed to enter the time series y_t additively:

$$y_t = T_t + S_t + R_t$$

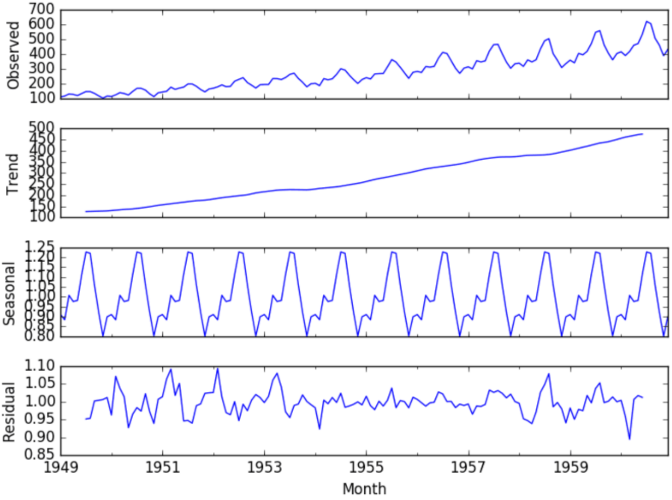
or multiplicatively:

$$y_t = T_t \times S_t \times R_t$$

Time Series Decomposition

Classical decomposition approach

1. Smooth the time series to obtain $\hat{T}_t$
2. Calculate the de-trended series as $y_t - \hat{T}_t$ in case of additive decomposition or $y_t / \hat{T}_t$ in case of multiplicative decomposition
3. Average the observations of the de-trended series for each season $1, \ldots, m$ separately to obtain $\hat{S}_t$
4. Calculate the remainder component as $\hat{R}_t = y_t - \hat{T}_t - \hat{S}_t$ in case of additive decomposition (or $\hat{R}_t = y_t / (\hat{T}_t \hat{S}_t)$ in case of multiplicative decomposition)



Multiplicative decomposition of monthly international air passenger load.

Source: *https://machinelearningmastery.com/decompose-time-series-data-trend-seasonality/*

Time Series Decomposition

Getting a deeper understanding of a time series through decomposition

- The **trend-cycle component** T_t helps identify long-term patterns and turning points in the series.
- **Seasonally adjusted data** ($y_t - \hat{S}_t$ or $y_t/\hat{S}_t$, respectively): Removing the seasonal component from a series provides insights into the underlying behavior beyond regular fluctuations.

  Example: economic indicators like the unemployment rate are usually seasonally adjusted

Data Transformations and Adjustments

Changing the resolution/granularity of time series

The resolution of a time series can only be sensibly decreased, not increased. Through:

- **Aggregation:** Aggregate observations falling into evenly spaced time intervals.

  E.g.: Aggregating hourly data into daily, weekly or monthly data by taking the average, median, maximum, minimum, etc. within these intervals.

- **Downsampling:** Select a subset of observations at regular intervals.

  E.g.: When temperature is measured every minute, it may be sensible to include only every 60th observation in the sample.

Data Transformations and Adjustments

When to change the resolution of time series

- Decreasing the resolution may be beneficial when the high-resolution series contains too much detail or noise
  $\Rightarrow$ helps reduce computational burden and focus on broader trends.

  E.g.: To predict the sales of an online retailer for the next 3 days, it may be sensible to aggregate hourly sales data into daily totals $\Rightarrow$ removes burden of seasonality

- **Multivariate time series forecasting:** if different variables are measured with different frequency, bring all series to the same (the lowest) resolution.

Data Transformations and Adjustments

Calendar adjustments:

- Potential calendar-related issues:
  - The number of (working) days per month can differ across months and years (different number of days per month, calendrical week-day shifts, Easter, leap years)
  - The number of (working) days per week may differ due to holidays

  $\Rightarrow$ Assess average outcome per (working) day instead of weekly/monthly/quarterly outcome
- Other potentially necessary adjustments:
  - Population adjustments
  - Inflation adjustments