# Time Series Forecasting

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# Organizational matters

# Structure of the lecture

1. Introduction to time series
2. Modeling time series and evaluating forecasting performance
3. Exponential smoothing models
4. ARIMA models
5. Regression models and advanced forecasting models
6. **Machine Learning and Deep Learning methods**

# Machine Learning and Deep Learning Methods

# Neural Network Models

- Neural networks are forecasting methods that allow for complex nonlinear relationships between outcome and predictors.
- To use neural networks for time series forecasting lagged values of the series can serve as inputs to the network (similar to ARIMA models). These models are referred to as neural network autoregression (NNAR) models.

# Neural Network Models

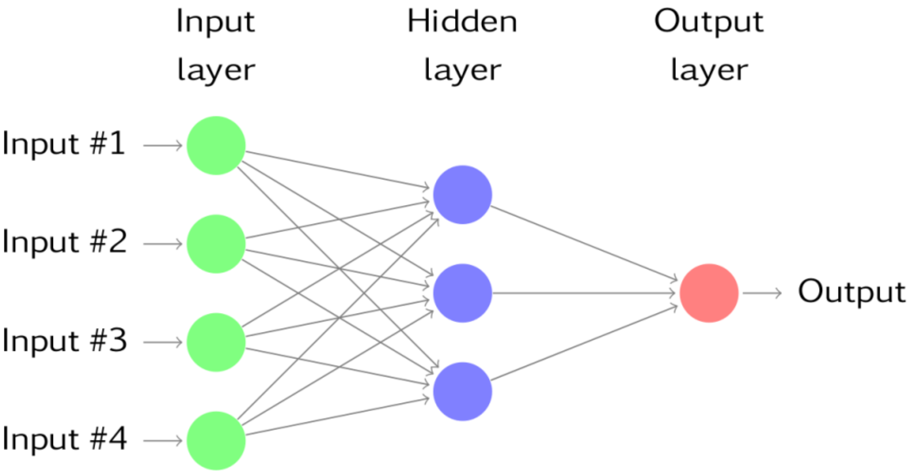


Figure: The inputs to the green nodes are combined using weighted linear combinations, then passed through a nonlinear function in the purple nodes, and finally, the output from the purple nodes is combined through another weighted linear combination to produce the outcome (orange).

# Neural Network Models

- The coefficients or weights are determined through a learning algorithm that minimizes a cost function such as Mean Squared Error (MSE). Initially, they are assigned random values, which are then adjusted based on the observed data during the training process.
- Here we focus on neural networks with one hidden layer in which the following transformation happens to the inputs $z$ that are fed into the hidden layer nodes:

$$s(z) = \frac{1}{1+e^{-z}},$$

  a function that transforms $z$ in a way that the effect of extreme values is reduced.

## Neural network autoregression models

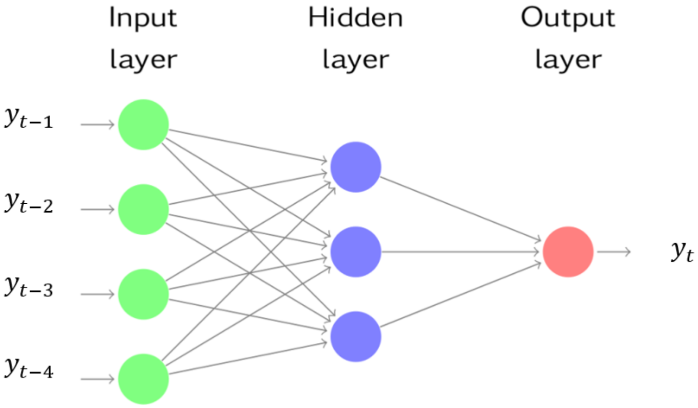


Figure: $y_{t-1}, \ldots, y_{t-4}$ are combined using weighted linear combinations, then passed through a nonlinear function in the purple nodes, and finally, the output from the purple nodes is combined through another weighted linear combination to produce the the forecast for $y_t$ (orange).

### Neural network autoregression models

- Neural network autoregression models are denoted as NNAR($p, k$), where $p$ is the number of lagged inputs ($y_{t-1}, ..., y_{t-p}$) and $k$ the number of nodes in the hidden layer.
- For seasonal data, one should also add the last observed values from the same season as inputs. Seasonal NNAR models are denoted as NNAR$(p, P, k)_m$, where $m$ is the frequency and $P$ the number of seasonal periods back that should be included in the model.
  - E.g. An NNAR$(2, 3, 7)_{12}$ model takes $y_{t-1}, y_{t-2}, y_{t-12}, y_{t-24}, y_{t-36}$ as inputs.

**Forecasting with neural network autoregression models**

- NNAR models are similar to ARIMA models (ARIMA$(p,0,0)$ and ARIMA$(p,0,0)(P,0,0)_m$ respectively) but ARIMA models assume linear relationship between lagged values and outcome and ARIMA models should only be used with stationary data, while NNAR models do not require time series to be stationary.
- For forecasting, the network is applied iteratively: The model is used to forecast $\hat{y}_{T+1|T}$ based on $y_T, \ldots, y_{T-p+1}, y_{T-m+1}, \ldots, y_{T-Pm+1}$. For forecasting $\hat{y}_{T+2|T}$, we use $\hat{y}_{T+1|T}, \ldots, y_{T-p+2}, y_{T-m+2}, \ldots, y_{T-Pm+2}$, etc.