MATH SL

TEST

DERIVATIVES

by Christos Nikolaidis

Name:_____

Date:_____

Marks:____/100

Grade: _____

Questions

1. Calculate the derivative $\frac{dy}{dx}$ for each of the following functions:

a) $y = 7x^3 - 2 - 5e^x - \frac{4\sin x}{5}$ ***b)*** $y = 1 + \frac{2}{x} + \frac{3}{x^2}$ ***c)*** $y = x^2(1 + \frac{2}{x} + \frac{3}{x^2})$

d) $y = 2\sqrt{x} + \ln x$ ***e)*** $y = 2\sqrt{x}\ln x$ ***f)*** $y = \frac{\ln x}{2\sqrt{x}}$

g) $y = \frac{2x+1}{3x-5}$ ***h)*** $y = \frac{1+x+x^2}{x^2}$ *[16 marks]*

2. Differentiate the following functions:

a) $f(x) = 2(x^2+5)^3 + x$ ***b)*** $f(x) = 2e^{x^2+1} + e$

c) $f(x) = 7e^{-x} + 8e^{\frac{x}{2}}$ ***d)*** $f(x) = 3\tan 3x + 3$

e) $f(x) = \ln(x^2+1) + 2\cos(\frac{\pi}{2}x)$ ***f)*** $f(x) = \sqrt{x^2+5} + \sqrt[3]{x}$

g) $f(x) = 2x + 5 + (2x+5)^2 + (2x+5)^3$ *[21 marks]*

3. Given the table

x	1	2	3	4
$f(x)$	5	4	–1	3
$g(x)$	1	–2	2	–5
$f'(x)$	5	6	0	7
$g'(x)$	–6	–4	–3	4

calculate

a) $\frac{d}{dx}[f(x)+g(x)]$ at x=1 ***b)*** $\frac{d}{dx}[3f(x)]$ at x=4

c) $\frac{d}{dx}[f(x)+x^2+2x]$ at x=2 *[7 marks]*

4. Let $f(x) = 2(3-x)^2 - 3x$.

a) Find the tangent and the normal line at $x = 3$ *[6 marks]*

b) Find the tangent line which is parallel to the line $y = 5x - 7$ *[4 marks]*

5. Let $f(x) = ax^3 + bx^2 + cx$

a) Find the first and second derivatives of $f(x)$ (in terms of a,b,c) *[2 marks]*

b) The graph has a local maximum at (1,4) (i.e. it passes through (1,4) and has a maximum at x=1), and a point of inflection at x=2. Write down three linear equations representing this information. *[3 marks]*

c) Hence find the values of a,b,c *[2 marks]*

d) The function has a local minimum at $x = d$. Find the value of d and justify that it is a minimum. *[4 marks]*

e) Sketch the graph of of $f(x)$ for $0 \le x \le 4$ and indicate the information given in questions ***b)*** an ***d)*** above. *[3 marks]*

6. Find $\frac{ds}{dt}$ for the following functions

a) $s = 2011\cos t$ ***b)*** $s = \cos(2011t)$ *[2+2 marks]*

c) $s = \cos^{2011} t$ ***d)*** $s = \cos(t^{2011})$ *[3+3 marks]*

7. The displacement s metres of a body, t minutes after leaving a fixed point A, is given by

$$s = 10t - 2.5t^2.$$

Calculate

a) the initial position and the initial velocity of the body. *[4 marks]*

b) the acceleration after 1 minute. *[3 marks]*

c) the displacement of the body from A when the velocity is zero. *[4 marks]*

d) the value of ***d*** if the body will be again at point A after ***d*** minutes. *[3 marks]*

8. Consider the derivative of a function $f'(x) = x^3 - 6x^2 + 9x - 2$,

a) Sketch the graph of $f'(x)$ for $0 \le x \le 4$ *[3 marks]*

b) Indicate on the graph (do not calculate the coordinates)

by A,B,C the three roots of $f'(x)$,

by M the local maximum of $f'(x)$ and

by N the local minimum of $f'(x)$.

For each of these points determine whether it corresponds to a local maximum, a local minimum or a point of inflection for $f(x)$. *[5 marks]*