MATH SL

TEST

INTEGRALS

(WITH GDC)

by Christos Nikolaidis

Name:_____

Date:_____

Marks:____/100

Grade: _____

Questions

1. Calculate the following integrals

a) $\int (7x^3 - 2 - 5e^x - \frac{4\sin x}{5})dx$ **b)** $\int (1 + \frac{2}{x} + \frac{3}{x^2})dx$ **c)** $\int x^2 (1 + \frac{2}{x} + \frac{3}{x^2})dx$

[4+3+3 marks]

2. Let $f'(x) = 5(3 - x^2)$. Find $f(x)$

a) given that $f(0) = 2$ *[5 marks]*

b) if the graph of the function passes through the point (3,-2) *[3 marks]*

3. Calculate the following integrals

a) $\int \frac{10}{2x+5}dx$ **b)** $\int \frac{2x+5}{10}dx$ **c)** $\int \frac{2x+5}{x}dx$ *[4+4+4 marks]*

4. Evaluate the following integrals *[you may use a GDG]*

a) $\int_0^2 \frac{2x+5}{2x-5}dx$ **b)** $\int_0^{\pi} 2x\sin^2 x dx$ *[3+3 marks]*

5. Calculate the following integrals in terms of *e* and *π [do not use GDG]*

a) $\int_0^{\pi} (e^{-5x} + 5)dx$ **b)** $\int_2^e 9\sin(6 - 3x)dx$ **c)** $\int_0^e \frac{2}{e + 2x}dx$ *[4+4+4 marks]*

6. Let $\int_0^2 f(x)dx = 7$, $\int_2^5 f(x)dx = -1$, $\int_0^2 g(x)dx = 2$, $g(0)=5$, $g(2)=10$. Find

a) $\int_0^5 f(x)dx$ **b)** $\int_5^2 2f(x)dx$ **c)** $\int_0^2 g'(x)dx$

d) $\int_0^2 [2f(x) + 3g(x)]\ dx$ **e)** $\int_0^2 (g(x) + 2x + 1)dx$ *[2+2+2+2+4 marks]*

7. A body is moving on a straight line and its velocity at time t seconds is given by the formula

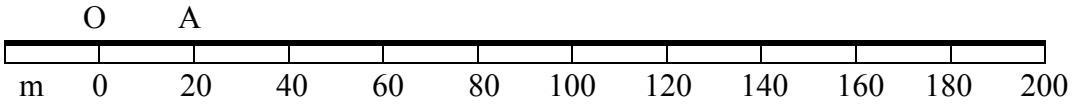
$$v = 120 - 30t^2 \text{ ms}^{-1}$$

a) Find its acceleration when $t = 2$. *[3 marks]*

b) Let s be the displacement of the body from a fixed point O. Find an expression for s in terms of t, given that the initial displacement is 20 meters. *[4 marks]*

c) The body is stationary only once during its movement. Find the displacement of the body at that time! *[4 marks]*

d) The body is initially at point A, stationary at point B, and 3 seconds after the beginning at point C. Indicate points B and C on the line below:

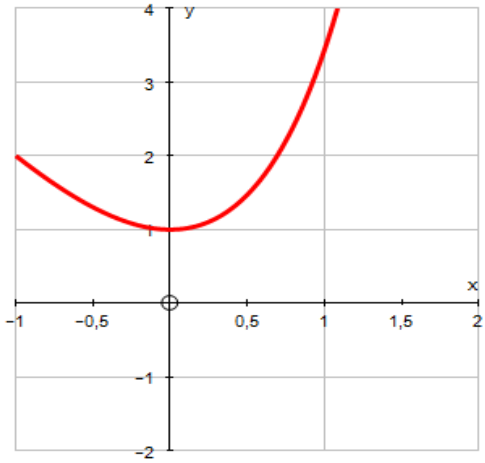


[3 marks]

e) Find the distance travelled by the body in the first 2 seconds. Write down an integral that expresses this distance. *[4 marks]*

f) Find the distance travelled by the body in the first 3 seconds. *[2 marks]*

8. The diagram shows the graph of the curve $f(x) = (x+1)e^x - 2x$



a) Shade the region $\boldsymbol{A}$ enclosed by the curve $y = f(x)$, the two axes and the vertical line x=1. Express the area $\boldsymbol{A}$ by a definite integral. *[3 marks]*

b) The area $\boldsymbol{A}$ is rotated through 360° about the x-axis. Write down an expression for the volume $\boldsymbol{V}$ obtained. *[2 marks]*

c) Show that $\frac{d}{dx}(xe^x - x^2) = f(x)$ *[3 marks]*

d) Find the area $\boldsymbol{A}$ in terms of e. *[Do not use a GDC]* *[4 marks]*

e) Find the volume $\boldsymbol{V}$. *[Use a GDG]* *[4 marks]*

f) Find the area enclosed by the graph $y = f(x)$ above and the line $y = 2x + 1$ *[Use a GDG]* *[4 marks]*