Midterm Project Report

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Purpose:

This project aims to compare the results of time period length on the asset allocation determined the weight of a given asset in a portfolio by using different risk measure approaches, including Mean-Variance, Mean-Absolute Deviation (MAD), and Conditional Value at Risk (CVaR).

Method:

1. **Data Organization:** Begin by organizing the return data. Pull daily data for the Adjusted Closing Price from January 1, 2005, to April 1, 2024, for the following ETFs that represent different asset classes.

Market Segment	ETF Ticker
Large Cap U.S. Stocks	IWB
Small Cap U.S. Stocks	IWM
Dev. Mkts. nonUS Stocks	EFA
Emerging Mkt Stock	EEM
Global REIT	VNQ
Corp. Bonds	LQD
Short Treasury	SHY

2. **Data Segmentation:** Divide the return data into segments based on different time frames: daily, 5-day (weekly), 21-day (monthly), 62-day (quarterly), and 252-day (yearly).
3. **Risk Measure Weight Solver:** Create a solver using different risk measure approaches to find the weights needed to achieve the target annualized returns of 2%, 4%, and 6%.For each periodicity of the data (i.e., daily, weekly, monthly, quarterly, and yearly), determine portfolios with expected returns of 1/252 of the target, 5/252 of the target, 21/252 of the target, 62/252 of the target, and 1/252 of the target.
4. **Statistical Analysis:** Run the function to obtain statistics that compare the results from different periods.

Results:

1. **Daily:**
   The target annualized returns are 2%*(1/252) = 0.008%, 4%*(1/252) = 0.016%, and 6%*(1/252) = 0.024%.
   Statistics:

Model, Target Rate	Expected.Return	Volatility	MAD	VaR.90	CVaR.90
Mean Variance 0.008%	0.00008	0.00091	0.0006	0.00084	0.01632
MAD 0.008%	0.00008	0.00091	0.0006	0.00084	0.01632
CVaR 0.008%	0.00008	0.00091	0.0006	0.00084	0.01632
Mean Variance 0.016%	0.00016	0.00325	0.00203	0.00302	0.06062
MAD 0.016%	0.00016	0.00327	0.002	0.00297	0.06007
CVaR 0.016%	0.00016	0.00328	0.002	0.00298	0.06004
Mean Variance 0.024%	0.00024	0.00607	0.00363	0.00555	0.11308
MAD 0.024%	0.00024	0.00612	0.00359	0.00539	0.11154
CVaR 0.024%	0.00024	0.00617	0.00361	0.00538	0.11141

Weights:

Model, Target Rate	Weight.IWB	Weight.IWM	Weight.EFA	Weight.EEM	Weight.VNQ	Weight.LQD	Weight.SHY
Mean Variance 0.008%	0.0376	0	0	0	0	0	0.9624
MAD 0.008%	0.0376	0	0	0	0	0	0.9624
CVaR 0.008%	0.0376	0	0	0	0	0	0.9624
Mean Variance 0.016%	0.2389	0	0	0	0	0.1959	0.5652
MAD 0.016%	0.2215	0	0	0	0	0.2608	0.5177
CVaR 0.016%	0.2174	0	0	0	0	0.2759	0.5066
Mean Variance 0.024%	0.4321	0	0	0	0	0.4221	0.1458
MAD 0.024%	0.3943	0	0	0	0	0.5631	0.0426
CVaR 0.024%	0.3793	0	0	0	0	0.619	0.0017

2. **Weekly:**

   The target annualized returns are 2%*(5/252) = 0.04%, 4%*(5/252) = 0.079%, and 6%*(5/252)= 0.119%.

   Statistics:

Model, Target Rate	Expected.Return	Volatility	MAD	VaR.90	CVaR.90
Mean Variance 0.04%	0.0004	0.00189	0.00129	0.00151	0.031
MAD 0.04%	0.0004	0.00189	0.00129	0.00151	0.031
CVaR 0.04%	0.0004	0.00189	0.00129	0.00151	0.031
Mean Variance 0.079%	0.00079	0.00689	0.00456	0.00671	0.13083
MAD 0.079%	0.00079	0.00692	0.00457	0.00639	0.12989
CVaR 0.079%	0.00079	0.00694	0.00455	0.00633	0.12982
Mean Variance 0.119%	0.00119	0.01288	0.00847	0.01234	0.2488
MAD 0.119%	0.00119	0.013	0.00799	0.0118	0.24639
CVaR 0.119%	0.00119	0.01299	0.00801	0.01172	0.24626

   Weights:

Model, Target Rate	Weight.IWB	Weight.IWM	Weight.EFA	Weight.EEM	Weight.VNQ	Weight.LQD	Weight.SHY
Mean Variance 0.04%	0.0371	0	0	0	0	0	0.9629
MAD 0.04%	0.0371	0	0	0	0	0	0.9629
CVaR 0.04%	0.0371	0	0	0	0	0	0.9629
Mean Variance 0.079%	0.2451	0	0	0	0	0.1708	0.5841
MAD 0.079%	0.2274	0	0	0	0	0.237	0.5356
CVaR 0.079%	0.2233	0	0	0	0	0.2523	0.5245
Mean Variance 0.119%	0.4401	0	0	0	0	0.3899	0.1699
MAD 0.119%	0.3925	0	0	0	0	0.5676	0.0399
CVaR 0.119%	0.3957	0	0	0	0	0.5559	0.0485

3. **Monthly:**

   The target annualized returns are 2%*(21/252) = 0.167%, 4%*(21/252) = 0.333%, and 6%*(21/252) = 0.5%.

   Statistics:

Model, Target Rate	Expected.Return	Volatility	MAD	VaR.90	CVaR.90
Mean Variance 0.167%	0.00167	0.00382	0.00288	0.0023	0.05341
MAD 0.167%	0.00167	0.00382	0.00288	0.0023	0.05341
CVaR 0.167%	0.00167	0.00382	0.00288	0.0023	0.05341
Mean Variance 0.333%	0.00333	0.01338	0.00946	0.00969	0.25244
MAD 0.333%	0.00333	0.0134	0.00909	0.00991	0.25035
CVaR 0.333%	0.00333	0.01359	0.00849	0.01028	0.24885
Mean Variance 0.5%	0.005	0.02543	0.01788	0.02218	0.49126
MAD 0.5%	0.005	0.02548	0.01703	0.02161	0.48717
CVaR 0.5%	0.005	0.02599	0.0161	0.02091	0.47925

Weights:

Model, Target Rate	Weight.IWB	Weight.IWM	Weight.EFA	Weight.EEM	Weight.VNQ	Weight.LQD	Weight.SHY
Mean Variance 0.167%	0.0388	0	0	0	0	0	0.9612
MAD 0.167%	0.0388	0	0	0	0	0	0.9612
CVaR 0.167%	0.0388	0	0	0	0	0	0.9612
Mean Variance 0.333%	0.278	0	0	0	0	0.0612	0.6608
MAD 0.333%	0.2677	0	0	0	0	0.0994	0.6329
CVaR 0.333%	0.2457	0	0	0	0	0.181	0.5733
Mean Variance 0.5%	0.5018	0	0	0	0	0.1798	0.3184
MAD 0.5%	0.482	0	0	0	0	0.2534	0.2647
CVaR 0.5%	0.4297	0	0	0	0	0.4471	0.1232

4. **Quarterly:**

The target annualized returns are 2%*(62/252) = 492%, 4%*(62/252) = 0.984%, and 6%*(62/252) = 1.476%.

Statistics:

Model, Target Rate	Expected.Return	Volatility	MAD	VaR.90	CVaR.90
Mean Variance 0.492%	0.00492	0.00888	0.00514	0.00299	0.13021
MAD 0.492%	0.00492	0.00889	0.00523	0.00269	0.12809
CVaR 0.492%	0.00492	0.00889	0.00523	0.00269	0.12809
Mean Variance 0.984%	0.00984	0.02382	0.01904	0.025	0.46984
MAD 0.984%	0.00984	0.02383	0.01899	0.02416	0.47072
CVaR 0.984%	0.00984	0.02439	0.01807	0.02139	0.46623
Mean Variance 1.476%	0.01476	0.04503	0.0334	0.03948	0.89685
MAD 1.476%	0.01476	0.04508	0.03459	0.03634	0.90162
CVaR 1.476%	0.01476	0.04645	0.03532	0.04436	0.88373

Weights:

Model, Target Rate	Weight.IWB	Weight.IWM	Weight.EFA	Weight.EEM	Weight.VNQ	Weight.LQD	Weight.SHY
Mean Variance 0.492%	0.031	0.0095	0	0	0	0	0.9595
MAD 0.492%	0.038	0	0	0	0	0	0.962
CVaR 0.492%	0.038	0	0	0	0	0	0.962
Mean Variance 0.984%	0.2845	0	0	0	0	0.0272	0.6883
MAD 0.984%	0.2892	0	0	0	0	0.0096	0.7012
CVaR 0.984%	0.2327	0	0	0	0	0.2226	0.5446
Mean Variance 1.476%	0.4938	0	0	0	0	0.1951	0.3111
MAD 1.476%	0.5161	0	0	0	0	0.111	0.3729
CVaR 1.476%	0.3817	0	0	0	0	0.6183	0

5. **Yearly:**

The target annualized returns are 2%, 4%, and 6%.

Statistics:

Model, Target Rate	Expected.Return	Volatility	MAD	VaR.90	CVaR.90
Mean Variance 2%	0.02	0.02206	0.017	-0.00091	0.4243
MAD 2%	0.02149	0.02263	0.0157	-0.00128	0.44748
CVaR 2%	0.02	0.02228	0.01748	-0.00228	0.39777
Mean Variance 4%	0.04	0.0508	0.02953	0.07868	0.87595
MAD 4%	0.04	0.05099	0.03222	0.07123	0.93631
CVaR 4%	0.04	0.05079	0.02936	0.07921	0.87132
Mean Variance 6%	0.06	0.093	0.06298	0.13372	1.9635
MAD 6%	0.06	0.0941	0.04983	0.15542	1.74704

Weights:

Model, Target Rate	Weight.IWB	Weight.IWM	Weight.EFA	Weight.EEM	Weight.VNQ	Weight.LQD	Weight.SHY
Mean Variance 2%	0	0.0154	0	0	0.0406	0	0.944
MAD 2%	0.0307	0	0	0	0.0414	0	0.9279
CVaR 2%	0.0231	0	0	0	0.0226	0	0.9543
Mean Variance 4%	0.2852	0	0	0	0	0.071	0.6439
MAD 4%	0.2585	0	0	0	0	0.1631	0.5784
CVaR 4%	0.2871	0	0	0	0	0.0642	0.6486
Mean Variance 6%	0.4862	0	0	0	0	0.3014	0.2124
MAD 6%	0.4009	0	0	0	0	0.5959	0.0032
CVaR 6%	0.3996	0	0	0	0	0.6004	0

Analysis:

From the results, we can observe that most of the models do not assign any weight to IWM, EFA, and EEM. Conversely, most of the models include IWB, LQD, and SHY. The primary reason the models predominantly incorporate IWB, excluding IWM, EFA, and EEM, is that IWB offers the highest expected return while maintaining the lowest volatility compared to these three ETFs (IWM, EFA, and EEM).

Most of the models targeting a 2% or 4% annualized return allocate a significant portion of weight to SHY, as it has the lowest volatility among the ETFs. This characteristic enables SHY to efficiently assist the models in achieving their goal by reducing volatility, Mean Absolute Deviation (MAD), or Conditional Value at Risk (CVaR).

When the target annualized return is 6%, some LQD is present in the models because SHY, having the lowest return, contributes less toward reaching the target return. Consequently, LQD acts as a substitute, offering the second-lowest volatility with a higher return than SHY.

From a weight allocation perspective, the weights are predominantly balanced between IWB, LQD, and SHY based on the annualized return target. A lower annualized return target corresponds to a higher proportion of SHY, while a higher target leads to a greater portion of IWB. LQD is allocated proportionally between these two to modulate volatility and achieve the desired annualized return target.

Additionally, the CVaR models include the highest portion of LQD compared to the other two types of models when targeting a 6% annualized return. This preference indicates that CVaR models are inclined to hold LQD in the portfolio. I believe this is due to LQD having the second-highest 10th quantile, which implies that its Value at Risk

(VaR) at the 90th percentile is comparatively low. However, LQD also offers a much higher return than SHY, making it more suitable for reaching a 6% target rate.

Data periodicities appear to have a significant impact on the models' weight allocations. Apart from the 6% target rate and the CVaR model, most models generally have a smaller proportion of LQD and SHY as the data periodicity shifts from daily to yearly. This trend might be attributable to the fact that variance and MAD are more affected by changes in data periodicity.

Moreover, as the periodicity changes from daily to yearly, the weight allocations differ more markedly. I theorize that this is because smaller data sizes, like yearly, introduce greater variability in risk measurement estimation, which in turn leads to a broader divergence in weight allocations.

Overall, the investment models typically exclude IWM, EFA, and EEM, preferring IWB, LQD, and SHY instead, due to IWB's superior expected return and SHY's lower volatility. For modest annualized return targets of 2% or 4%, SHY is heavily weighted for its low volatility, aiding in achieving lower risk metrics. As the target return increases to 6%, LQD becomes more prominent, offering a balance of higher returns and controlled volatility. Allocation between IWB, LQD, and SHY varies according to the desired return, with SHY preferred for lower targets and IWB for higher. CVaR models distinctly prefer LQD at the 6% target due to its better return than SHY but slightly worse 10th quantile performance than SHY. Additionally, the model's weightings are considerably influenced by data periodicity. The variance and MAD are more sensitive to changes in data periodicity.

Review of the Predictions:

My prediction was partially correct regarding the results, as they mostly assigned a high weight to SHY. Initially, I thought that SHY, with its very low volatility, would significantly reduce the portfolio's volatility and other risk measurements. However, I forgot to consider the target annualized return rate, and I overlooked SHY's low expected return. With a higher target rate, SHY holds a relatively lower portion than it does with a low target rate. Consequently, in the results, IWB and LQD also hold a significant portion in the portfolio. Therefore, when considering the weighting of a portfolio to reduce risk measurements, the trade-off between volatility and expected return should be considered.

Additional Insight:

After calculating and observing the Sharpe ratio for each model, it becomes apparent that models with a low return target, such as a 2% annualized return, exhibit a better Sharpe ratio than models targeting a higher return, like a 6% annualized return. This could be attributed to the models' higher allocation to SHY, which possesses the highest Sharpe ratio among the ETFs. Therefore, we may conclude that the individual Sharpe ratios significantly influence the overall Sharpe ratio of the portfolio.

Code:

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#Package

```
library(quantmod)
```

```
## Loading required package: xts
```

```
## Loading required package: zoo
```

```
##
## Attaching package: 'zoo'
```

```
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

```
## Loading required package: TTR
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```
library(PerformanceAnalytics)
```

```
##
## Attaching package: 'PerformanceAnalytics'
```

```
## The following object is masked from 'package:graphics':
##
##     legend
```

```
library(CVXR)
```

```
##
## Attaching package: 'CVXR'
```

```
## The following object is masked from 'package:stats':
##
##     power
```

```
library(moments)
```

```
##
## Attaching package: 'moments'
```

```
## The following objects are masked from 'package:PerformanceAnalytics':
##
##     kurtosis, skewness
```

```
library(readxl)
```

```
## Warning: package 'readxl' was built under R version 4.0.5
```

```
library(dplyr)
```

```
##
## Attaching package: 'dplyr'
```

```
## The following object is masked from 'package:CVXR':
##
##     id
```

```
## The following objects are masked from 'package:xts':
##
##     first, last
```

```
## The following objects are masked from 'package:stats':
##
##     filter, lag
```

```
## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

#Solver

```
#Mean_Variance
slvr_mv<-function(return_matrix,target_rate){
  cov_mat<- cov(return_matrix)
  X <- Variable(7)
  constraint_full_investment <- sum(X) == 1.0
  constraint_non_negativity <- X >= 0
  constraint_return_limit <- mean(return_matrix%*%X) >= target_rate
  prob_return_constraint <- Problem(Minimize(quad_form(X, cov_mat)),
                                    list(constraint_full_investment,
                                         constraint_non_negativity ,
                                         constraint_return_limit))
  result_return_constraint <- solve(prob_return_constraint)
  weight<-result_return_constraint$getValue(X)
  return(round(weight,digits = 4))
}

#MAD
slvr_mad<-function(return_matrix,target_rate){
  mu<- matrix(colMeans(return_matrix),nrow =7)
  X <- Variable(7)
  constraint_full_investment <- sum(X) == 1.0
  constraint_non_negativity <- X >= 0
  constraint_return_limit <- mean(return_matrix%*%X) >= target_rate
  prob_return_constraint <- Problem(Minimize(mean(abs(return_matrix%*%X-t(mu)
%*%X))),
                                    list(constraint_full_investment,
```

```
                                    constraint_non_negativity ,
                                    constraint_return_limit))
  result_return_constraint <- solve(prob_return_constraint,solver="GLPK")
  weight<-result_return_constraint$getValue(X)
  return(round(weight,digits = 4))
}

#CVaR
slvr_cvar<-function(return_matrix,target_rate){
  X <- Variable(7)
  gamma <- Variable(1) #gamma is equivalent to VaR
  L <- Variable(nrow(return_matrix)) #losses
  constraint_full_investment <- sum(X) == 1.0
  constraint_non_negativity <- X >= 0
  constraint_return_limit <- mean(return_matrix%*%X) >= target_rate
  L_value<- L >= -return_matrix %*% X - gamma
  L_greater <- L >= 0
  prob_return_constraint <- Problem(Minimize(gamma + mean(L) / 0.1),
                                    list(constraint_full_investment,
                                         constraint_non_negativity ,
                                         constraint_return_limit,
                                         L_value,
                                         L_greater))
  result_return_constraint <- solve(prob_return_constraint,solver="GLPK")
  weight<-result_return_constraint$getValue(X)
  return(round(weight,digits = 4))
}
```

#Statistical tools

```
#Expected return
expected_return <-function(weight,return_matrix){
  mu<- matrix(colMeans(return_matrix),ncol  = 7)
  return(mu%*%weight)
}
#Standard deviation
standard_deviation <-function(weight,return_matrix){
  cov_mat<- cov(return_matrix)
  return(sqrt(t(weight)%*%cov_mat%*%weight))
}
#MAD
mad_function <-function(weight,return_matrix){
  return(mad(return_matrix%*%weight))
}
#VaR
var_function <-function(weight,return_matrix){
  VaR90 <- quantile(-return_matrix %*% weight, probs = 0.9, type = 1)
  return(VaR90)
}
```

```r
#CVaR
cvar_function <-function(weight,return_matrix){
  VaR90 <- quantile(-return_matrix %*% weight, probs = 0.9, type = 1)
  val <- -return_matrix %*% weight
  CVaR90 <- mean(val[val>VaR90])/(0.1) + VaR90
  return(CVaR90 )
}
#Statistical summary
show_stat <- function(weight,return_matrix){
    stats <- c(
    expected_return  = expected_return(weight,return_matrix),
    standard_deviation = standard_deviation(weight,return_matrix),
    mad=mad_function(weight,return_matrix),
    var=var_function(weight,return_matrix),
    cvar=cvar_function(weight,return_matrix),
    weight_IWB=weight[1],
    weight_IWM=weight[2],
    weight_EFA=weight[3],
    weight_EEM=weight[4],
    weight_VNQ=weight[5],
    weight_LQD=weight[6],
    weight_SHY=weight[7]
    )
  return(round(stats,digits=5))
}
```

#Organizer

```r
#Data frame organizer
slvrs_stats<- function(return_matrix,target_rate){
  w1<- slvr_mv(return_matrix,target_rate)
  w2<- slvr_mad(return_matrix,target_rate)
  w3<- slvr_cvar(return_matrix,target_rate)
  result_mv<-show_stat(w1,return_matrix)
  result_mad<-show_stat(w2,return_matrix)
  result_cvar<-show_stat(w3,return_matrix)
  df<- data_frame(result_mv,result_mad,result_cvar)
  colnames(df)<-c(paste0("Mean Variance ", round(target_rate,digits = 5)*100,
 "%"),
                  paste0("MAD ", round(target_rate,digits = 5)*100, "%"),
                  paste0("CVaR ", round(target_rate,digits = 5)*100, "%")
                  )

  return(df)
}
```

#Data

```r
#Get data
daily_return <- read_excel("Project1Data.xlsx", sheet ="days_1" )
weekly_return <- read_excel("Project1Data.xlsx", sheet ="days_5" )
```

```r
monthly_return <- read_excel("Project1Data.xlsx", sheet ="days_21" )
quarterly_return <- read_excel("Project1Data.xlsx", sheet ="days_62" )
yearly_return <- read_excel("Project1Data.xlsx", sheet ="days_252" )

#Transfer into matrix
return_matrix_daily<-data.matrix(daily_return)[,2:8]
return_matrix_weekly<-data.matrix(weekly_return)[,2:8]
return_matrix_monthly<-data.matrix(monthly_return)[,2:8]
return_matrix_quarterly<-data.matrix(quarterly_return)[,2:8]
return_matrix_yearly<-data.matrix(yearly_return)[,2:8]
```

#Computation

```r
#Yearly
return_matrix<-return_matrix_yearly
day<-252
total_df <- data.frame(matrix(ncol = 0, nrow = 12))
for (i in c(0.02,0.04,0.06)){
  df_target<-slvrs_stats(return_matrix,i*(day/252))
  total_df <- cbind(total_df, df_target)
}

## Warning: `data_frame()` was deprecated in tibble 1.1.0.
## ℹ Please use `tibble()` instead.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_lifecycle_warnings()` to see where this warning was
## generated.

stat_names <- c("Expected Return","Volatility", "MAD", "VaR 90", "CVaR 90",
                "Weight IWB", "Weight IWM", "Weight EFA", "Weight EEM",
                "Weight VNQ", "Weight LQD", "Weight SHY")
rownames(total_df) <- stat_names
total_df_yearly<-data.frame(t(total_df))

#Quarterly
return_matrix<-return_matrix_quarterly
day<-62
total_df <- data.frame(matrix(ncol = 0, nrow = 12))
for (i in c(0.02,0.04,0.06)){
  df_target<-slvrs_stats(return_matrix,i*(day/252))
  total_df <- cbind(total_df, df_target)
}
stat_names <- c("Expected Return","Volatility", "MAD", "VaR 90", "CVaR 90",
                "Weight IWB", "Weight IWM", "Weight EFA", "Weight EEM",
                "Weight VNQ", "Weight LQD", "Weight SHY")
rownames(total_df) <- stat_names
total_df_quarterly<-data.frame(t(total_df))

#Monthly
return_matrix<-return_matrix_monthly
day<-21
```

```r
total_df <- data.frame(matrix(ncol = 0, nrow = 12))
for (i in c(0.02,0.04,0.06)){
  df_target<-slvrs_stats(return_matrix,i*(day/252))
  total_df <- cbind(total_df, df_target)
}
stat_names <- c("Expected Return","Volatility", "MAD", "VaR 90", "CVaR 90",
                "Weight IWB", "Weight IWM", "Weight EFA", "Weight EEM",
                "Weight VNQ", "Weight LQD", "Weight SHY")
rownames(total_df) <- stat_names
total_df_monthly<-data.frame(t(total_df))

#Weekly
return_matrix<-return_matrix_weekly
day<-5
total_df <- data.frame(matrix(ncol = 0, nrow = 12))
for (i in c(0.02,0.04,0.06)){
  df_target<-slvrs_stats(return_matrix,i*(day/252))
  total_df <- cbind(total_df, df_target)
}
stat_names <- c("Expected Return","Volatility", "MAD", "VaR 90", "CVaR 90",
                "Weight IWB", "Weight IWM", "Weight EFA", "Weight EEM",
                "Weight VNQ", "Weight LQD", "Weight SHY")
rownames(total_df) <- stat_names
total_df_weekly<-data.frame(t(total_df))

#Daily
return_matrix<-return_matrix_daily
day<-1
total_df <- data.frame(matrix(ncol = 0, nrow = 12))
for (i in c(0.02,0.04,0.06)){
  df_target<-slvrs_stats(return_matrix,i*(day/252))
  total_df <- cbind(total_df, df_target)
}
stat_names <- c("Expected Return","Volatility", "MAD", "VaR 90", "CVaR 90",
                "Weight IWB", "Weight IWM", "Weight EFA", "Weight EEM",
                "Weight VNQ", "Weight LQD", "Weight SHY")
rownames(total_df) <- stat_names
total_df_daily<-data.frame(t(total_df))
```

#Results

```r
print(total_df_yearly)
```

```
##                  Expected.Return Volatility     MAD   VaR.90 CVaR.90 Weigh
t.IWB
## Mean Variance 2%         0.02000    0.02206 0.01700 -0.00091 0.42430     0
.0000
## MAD 2%                   0.02149    0.02263 0.01570 -0.00128 0.44748     0
.0307
## CVaR 2%                  0.02000    0.02228 0.01748 -0.00228 0.39777     0
.0231
```

```
## Mean Variance 4%         0.04000    0.05080 0.02953  0.07868 0.87595     0
.2852
## MAD 4%                   0.04000    0.05099 0.03222  0.07123 0.93631     0
.2585
## CVaR 4%                  0.04000    0.05079 0.02936  0.07921 0.87132     0
.2871
## Mean Variance 6%         0.06000    0.09300 0.06298  0.13372 1.96350     0
.4862
## MAD 6%                   0.06000    0.09410 0.04983  0.15542 1.74704     0
.4009
## CVaR 6%                  0.06000    0.09413 0.04876  0.15575 1.74375     0
.3996
##                   Weight.IWM Weight.EFA Weight.EEM Weight.VNQ Weight.LQD
## Mean Variance 2%     0.0154          0          0     0.0406     0.0000
## MAD 2%               0.0000          0          0     0.0414     0.0000
## CVaR 2%              0.0000          0          0     0.0226     0.0000
## Mean Variance 4%     0.0000          0          0     0.0000     0.0710
## MAD 4%               0.0000          0          0     0.0000     0.1631
## CVaR 4%              0.0000          0          0     0.0000     0.0642
## Mean Variance 6%     0.0000          0          0     0.0000     0.3014
## MAD 6%               0.0000          0          0     0.0000     0.5959
## CVaR 6%              0.0000          0          0     0.0000     0.6004
##                   Weight.SHY
## Mean Variance 2%     0.9440
## MAD 2%               0.9279
## CVaR 2%              0.9543
## Mean Variance 4%     0.6439
## MAD 4%               0.5784
## CVaR 4%              0.6486
## Mean Variance 6%     0.2124
## MAD 6%               0.0032
## CVaR 6%              0.0000

print(total_df_quarterly)

##                       Expected.Return Volatility     MAD  VaR.90 CVaR.90
## Mean Variance 0.492%          0.00492    0.00888 0.00514 0.00299 0.13021
## MAD 0.492%                    0.00492    0.00889 0.00523 0.00269 0.12809
## CVaR 0.492%                   0.00492    0.00889 0.00523 0.00269 0.12809
## Mean Variance 0.984%          0.00984    0.02382 0.01904 0.02500 0.46984
## MAD 0.984%                    0.00984    0.02383 0.01899 0.02416 0.47072
## CVaR 0.984%                   0.00984    0.02439 0.01807 0.02139 0.46623
## Mean Variance 1.476%          0.01476    0.04503 0.03340 0.03948 0.89685
## MAD 1.476%                    0.01476    0.04508 0.03459 0.03634 0.90162
## CVaR 1.476%                   0.01476    0.04645 0.03532 0.04436 0.88373
##                       Weight.IWB Weight.IWM Weight.EFA Weight.EEM Weight.VN
Q
## Mean Variance 0.492%     0.0310     0.0095          0          0
0
## MAD 0.492%               0.0380     0.0000          0          0
```

```
0
## CVaR 0.492%              0.0380     0.0000          0          0
0
## Mean Variance 0.984%     0.2845     0.0000          0          0
0
## MAD 0.984%               0.2892     0.0000          0          0
0
## CVaR 0.984%              0.2327     0.0000          0          0
0
## Mean Variance 1.476%     0.4938     0.0000          0          0
0
## MAD 1.476%               0.5161     0.0000          0          0
0
## CVaR 1.476%              0.3817     0.0000          0          0
0
##                       Weight.LQD Weight.SHY
## Mean Variance 0.492%     0.0000     0.9595
## MAD 0.492%               0.0000     0.9620
## CVaR 0.492%              0.0000     0.9620
## Mean Variance 0.984%     0.0272     0.6883
## MAD 0.984%               0.0096     0.7012
## CVaR 0.984%              0.2226     0.5446
## Mean Variance 1.476%     0.1951     0.3111
## MAD 1.476%               0.1110     0.3729
## CVaR 1.476%              0.6183     0.0000
```

```
print(total_df_monthly)
```

```
##                       Expected.Return Volatility     MAD  VaR.90 CVaR.90
## Mean Variance 0.167%          0.00167    0.00382 0.00288 0.00230 0.05341
## MAD 0.167%                    0.00167    0.00382 0.00288 0.00230 0.05341
## CVaR 0.167%                   0.00167    0.00382 0.00288 0.00230 0.05341
## Mean Variance 0.333%          0.00333    0.01338 0.00946 0.00969 0.25244
## MAD 0.333%                    0.00333    0.01340 0.00909 0.00991 0.25035
## CVaR 0.333%                   0.00333    0.01359 0.00849 0.01028 0.24885
## Mean Variance 0.5%            0.00500    0.02543 0.01788 0.02218 0.49126
## MAD 0.5%                      0.00500    0.02548 0.01703 0.02161 0.48717
## CVaR 0.5%                     0.00500    0.02599 0.01610 0.02091 0.47925
##                       Weight.IWB Weight.IWM Weight.EFA Weight.EEM Weight.VN
Q
## Mean Variance 0.167%     0.0388          0          0          0
0
## MAD 0.167%               0.0388          0          0          0
0
## CVaR 0.167%              0.0388          0          0          0
0
## Mean Variance 0.333%     0.2780          0          0          0
0
## MAD 0.333%               0.2677          0          0          0
0
```

```
## CVaR 0.333%              0.2457          0          0          0
0
## Mean Variance 0.5%       0.5018          0          0          0
0
## MAD 0.5%                 0.4820          0          0          0
0
## CVaR 0.5%                0.4297          0          0          0
0
##                       Weight.LQD Weight.SHY
## Mean Variance 0.167%     0.0000     0.9612
## MAD 0.167%               0.0000     0.9612
## CVaR 0.167%              0.0000     0.9612
## Mean Variance 0.333%     0.0612     0.6608
## MAD 0.333%               0.0994     0.6329
## CVaR 0.333%              0.1810     0.5733
## Mean Variance 0.5%       0.1798     0.3184
## MAD 0.5%                 0.2534     0.2647
## CVaR 0.5%                0.4471     0.1232
```

```
print(total_df_weekly)
```

```
##                       Expected.Return Volatility     MAD  VaR.90 CVaR.90
## Mean Variance 0.04%           0.00040    0.00189 0.00129 0.00151 0.03100
## MAD 0.04%                     0.00040    0.00189 0.00129 0.00151 0.03100
## CVaR 0.04%                    0.00040    0.00189 0.00129 0.00151 0.03100
## Mean Variance 0.079%          0.00079    0.00689 0.00456 0.00671 0.13083
## MAD 0.079%                    0.00079    0.00692 0.00457 0.00639 0.12989
## CVaR 0.079%                   0.00079    0.00694 0.00455 0.00633 0.12982
## Mean Variance 0.119%          0.00119    0.01288 0.00847 0.01234 0.24880
## MAD 0.119%                    0.00119    0.01300 0.00799 0.01180 0.24639
## CVaR 0.119%                   0.00119    0.01299 0.00801 0.01172 0.24626
##                       Weight.IWB Weight.IWM Weight.EFA Weight.EEM Weight.VN
Q
## Mean Variance 0.04%      0.0371          0          0          0
0
## MAD 0.04%                0.0371          0          0          0
0
## CVaR 0.04%               0.0371          0          0          0
0
## Mean Variance 0.079%     0.2451          0          0          0
0
## MAD 0.079%               0.2274          0          0          0
0
## CVaR 0.079%              0.2233          0          0          0
0
## Mean Variance 0.119%     0.4401          0          0          0
0
## MAD 0.119%               0.3925          0          0          0
0
## CVaR 0.119%              0.3957          0          0          0
```

```
##                       Weight.LQD Weight.SHY
## Mean Variance 0.04%       0.0000     0.9629
## MAD 0.04%                 0.0000     0.9629
## CVaR 0.04%                0.0000     0.9629
## Mean Variance 0.079%      0.1708     0.5841
## MAD 0.079%                0.2370     0.5356
## CVaR 0.079%               0.2523     0.5245
## Mean Variance 0.119%      0.3899     0.1699
## MAD 0.119%                0.5676     0.0399
## CVaR 0.119%               0.5559     0.0485
```

```
print(total_df_daily)
```

```
##                       Expected.Return Volatility     MAD  VaR.90 CVaR.90
## Mean Variance 0.008%          0.00008    0.00091 0.00060 0.00084 0.01632
## MAD 0.008%                    0.00008    0.00091 0.00060 0.00084 0.01632
## CVaR 0.008%                   0.00008    0.00091 0.00060 0.00084 0.01632
## Mean Variance 0.016%          0.00016    0.00325 0.00203 0.00302 0.06062
## MAD 0.016%                    0.00016    0.00327 0.00200 0.00297 0.06007
## CVaR 0.016%                   0.00016    0.00328 0.00200 0.00298 0.06004
## Mean Variance 0.024%          0.00024    0.00607 0.00363 0.00555 0.11308
## MAD 0.024%                    0.00024    0.00612 0.00359 0.00539 0.11154
## CVaR 0.024%                   0.00024    0.00617 0.00361 0.00538 0.11141
##                       Weight.IWB Weight.IWM Weight.EFA Weight.EEM Weight.VN
Q
## Mean Variance 0.008%      0.0376          0          0          0
0
## MAD 0.008%                0.0376          0          0          0
0
## CVaR 0.008%               0.0376          0          0          0
0
## Mean Variance 0.016%      0.2389          0          0          0
0
## MAD 0.016%                0.2215          0          0          0
0
## CVaR 0.016%               0.2174          0          0          0
0
## Mean Variance 0.024%      0.4321          0          0          0
0
## MAD 0.024%                0.3943          0          0          0
0
## CVaR 0.024%               0.3793          0          0          0
0
##                       Weight.LQD Weight.SHY
## Mean Variance 0.008%      0.0000     0.9624
## MAD 0.008%                0.0000     0.9624
## CVaR 0.008%               0.0000     0.9624
## Mean Variance 0.016%      0.1959     0.5652
## MAD 0.016%                0.2608     0.5177
```

```
## CVaR 0.016%              0.2759     0.5066
## Mean Variance 0.024%     0.4221     0.1458
## MAD 0.024%               0.5631     0.0426
## CVaR 0.024%              0.6190     0.0017
```