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Tools for Stat Theory HW2

Lemma 4.1.2

Let B represent an m $\times$ n matrix, and let V represent a linear space of m$\times$n matrices. Then, for any matrix A in V, $A+B \in V$ if and only if $B \in V$.

Part 1: Suppose A in V and $B \in V$ $\rightarrow$ Show $A+B \in V$

Since A and B both are in V, by definition its sum will also be in V. B can be represented as $B = A+B-B$

Part 2: Suppose A in V and $A+B \in V$ $\rightarrow$ Show $B \in V$

Linear combinations of matrices in V are still in V by definition. Suppose $A+B = C$ The linear combination $C - A \in V$ so $(B+A) - A \in V$ and therefore $B \in V$.

Lemma 4.2.2 (for column space)

For any m $\times$ n matrix A and m $\times$ p matrix B, $C(B) \subset C(A)$ if and only if there exists an n $\times$ p matrix F such that $B = AF$.

Part 1: If F exists such that $B = AF$, show $C(B) \subset C(A)$.

$B = [a_1, ..., a_k][f_1, ..., f_k] = Af_i$ Since B can be represented as a linear combination of A, it must be a subspace of A.

This notation is weird.

Part 2: If $C(B) \subset C(A)$, show F exists such that $B = AF$. By definition, if $C(B) \subset C(A)$, B can be represented by A in a linear combination, so F must exist.

It's correct, but you should write you detailed proof.

Lemma 4.2.2 (for row space)

For any m $\times$ n matrix A and q $\times$ n matrix C, $R(C) \subset R(A)$ if and only if there exists a q $\times$ m matrix L such that $C = LA$.

Part 1: If L exists such that $C = LA$, show $R(C) \subset R(A)$.

$C = [l_1^T, ..., l_k^T][a_1^T, ..., a_k^T] = Al_i^T$ Since C can be represented as a linear combination of A, it must be a subspace of A.

This notation is weird.

Part 2: If $R(C) \subset R(A)$, show L exists such that $C = LA$. By definition, if $R(C) \subset R(A)$, C can be represented by A in a linear combination, so L must exist.