PEAK OF FLIGHT NEWSLETTER

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PEAK OF FLIGHT

Trapezoidal Fin Flutter Analysis Revisited

By John D Sahr

1 Introduction

Recently I have begun co-designing a two stage rocket that will exceed Mach 1 for the "high altitude" version and will approach Mach 2 for the "high speed" version. At such speeds it is important to have a good idea of fin strength requirements to defend against fin flutter. To this end I was grateful to find an article by Zachary Howard on the Apogee website:

https://www.apogeerockets.com/education/downloads/Newsletter291.pdf

Howard's article provides a very useful, semi-empirical model of fin flutter derived from more elaborate developments[1]. The key result is estimation of the "flutter velocity" V_f for a particular fin design. If a fin is flown faster than its flutter velocity, is is likely to fail, leading to subsequent unstable flight.

In working my way through Howard's article, I found a few things to be a little awkward. In particular the article employs imperial units rather than *SI* units. Also, the formula for the flutter velocity V_f can be rearranged in a way to simplify its application and illuminate important dependences that will aid in fin design.

The next section of the note explains my reasoning and method for manipulation of Howard's model. You may wish to skip immediately to section 3 to see the new, simpler (and hopefully more useful) formula that I present.

2 Recasting the flutter velocity equation

Howard's version of the Flutter Velocity equation can be rearranged to have the following completely equivalent form:

$$V_f = 1.223\, C_s \sqrt{\frac{G}{P}} \sqrt{\left(\frac{T}{B}\right)^3 \left(\frac{2+B}{1+\lambda}\right)} \qquad (1)$$

[1]c.f. http://www.ewp.rpi.edu/hartford/users/papers/engr/ernesto/kapust/EP/Other/ References%20-%20Rocket/Jon%20Champion%20docs%20from%20JMRC/Example of Fin Flutter.pdf

$$\begin{aligned}
& \underline{\text{fin parameters}} & \\
c_r &= \text{root chord}, L & (2)\\
c_t &= \text{tip chord}, L & (3)\\
b &= \text{fin height}, L & (4)\\
t &= \text{fin thickness}, L & (5)\\
G &= \text{fin material shear modulus}, R & (6)\\
& \underline{\text{atmospheric parameters}} & (7)\\
P(h) &= \text{pressure (function of altitude)}, R & (8)\\
C_s(h) &= \text{sound speed (function of altitude)}, V & (9)\\
& \underline{\text{derived parameters}} & \\
S &= \frac{h}{2}(c_r + c_t) \text{ fin area}, L^2 & (10)\\
\lambda &= \frac{c_t}{c_r} \quad \text{fin taper ratio} & (11)\\
B &= \frac{b^2}{S} \quad \text{aspect ratio} & (12)\\
T &= \frac{t}{c_r} \quad \text{normalized thickness} & (13)
\end{aligned}$$

In these expressions, L corresponds to a length unit, e.g. inches or mm; R corresponds to pressure units, psi or Pa, and V corresponds to velocity units, mph, fps, m/s. You can choose the units of the answer by selecting the sound speed in whatever units you prefer; you can use any length units for L as long as you always use the same length units; you can use any pressure units R as long as you use the same units for both P and G. The fin dimensions are illustrated in **Figure 1 on Page 3.**

I'll rewrite the formula once more and add some annotation:

$$V_f = 1.223 \underbrace{\left[C_s\sqrt{\frac{P_0}{P}}\right]}_{a} \underbrace{\left[\sqrt{\frac{G}{P_0}} \sqrt{\left(\frac{T}{B}\right)^3 \left(\frac{2+B}{1+\lambda}\right)}\right]}_{f} \qquad (14)$$

Here P_0 indicates the (constant) sea level pressure expressed in R pressure units. The underbrace a indicates that the term depends only upon properties of atmosphere; f indicates that the term depends only upon fin geometry and material. While it might be tempting to cancel the

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P_0 factor, the formula, as written, is almost completely free of dependence upon the system of units chosen by the analyst.

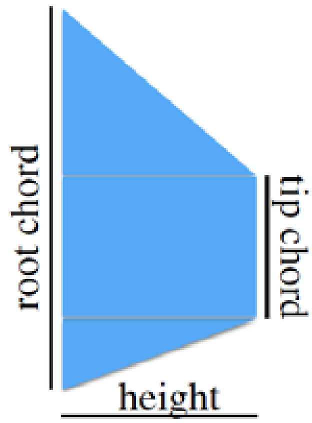

Figure 1: Definition of dimensions of a trapezoidal fin. The remaining dimension is the fin thickness.

2.1 *The atmospheric model for C_s and P*

Referring to Howard's article, and combining his expressions for temperature and pressure and sound speed vs height, we can re-arrange a bit to come up with two new formulas for pressure and sound speed versus altitude alone:

$$C_s(h) = C_{s0}\left(1 - 0.20\frac{h}{H}\right)^{1/2} \tag{15}$$

$$P(h) = P_0\left(1 - 0.190\frac{h}{H}\right)^{5.256} \tag{16}$$

Here H is the atmospheric scale height at sea level, which is about 8077 m or

26500 ft (in cold weather). Also, C_{S0} is the sound speed at sea level, and P_0 is the pressure at sea level. Notice that for rockets which don't exceed the altitude H, the two expressions are nearly constant.

Furthermore, using the following theorem from calculus,

$$\lim_{\alpha \to 0}(1-\alpha)^{\beta} \to \exp(-\alpha\beta) \tag{17}$$

we can rewrite the sounds speed and pressure formulas,

$$C_s(h) \approx C_{s0}\exp\left(-0.1\frac{h}{H}\right) \tag{18}$$

$$P(h) \approx P_0\exp\left(-\frac{h}{H}\right) \tag{19}$$

The simplicity of the pressure function is no accident, as this represents the exact solution for pressure versus altitude of an isothermal atmosphere[2].

[2]https://en.wikipedia.org/wiki/Scale height

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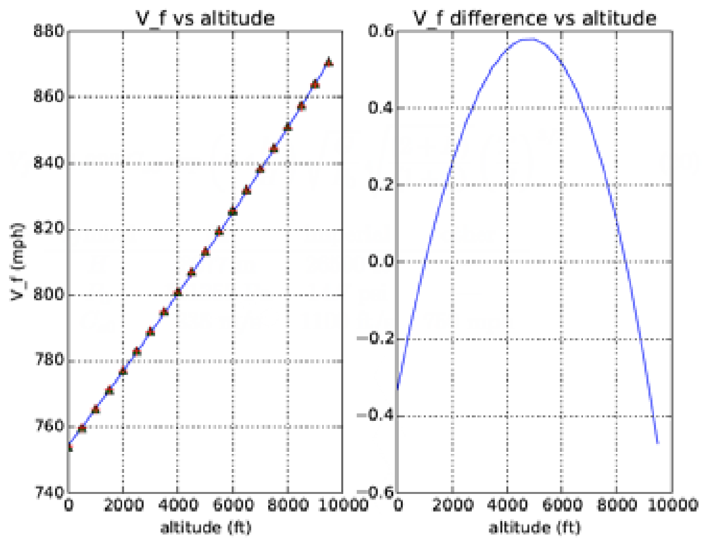

Figure 2: (Left) a plot of Zach Howard's fin flutter velocity (solid), with an overlay of his formula refactored eq. 1, and the new expression eq. 20. (Right) a plot of the difference between the new formula (eq. 20) and Zach Howard's fin flutter velocity formula. The difference is less than 1 mph below 10000 ft.

3 A simpler, more general formula

Returning to the fin flutter formula we can put it all together:

$$V_f = 1.223\, C_{s0} \exp\left(0.4\frac{h}{H}\right)\sqrt{\frac{G}{P_0}}\sqrt{\frac{(2+B)}{(1+\lambda)}}\left(\frac{T}{B}\right)^{3/2} \qquad (20)$$

symbol	SI	imperial	other
H	8077 m	26500 ft	—
P_0	101352 Pa	14.7 psi	—
C_{s0}	335 m/s	1100 ft/s	750 mph

Howard tests his model with a particular fin with root chord 9.75", tip chord 3.75", height 4.75", and thickness 0.125", and with a shear modulus of 380000 psi. I was able to duplicate his results quite closely especially when using a scale height H = 26500 ft. A comparison is shown in **Figure 2.**

This formula, which is nearly identical to Howard's, has the following nice properties:

1. It handles all systems of units. As long as all lengths and pressures use the same units, the formula will yield the correct answer. You can choose the units of the answer by putting in the appropriate value for the sea level sound speed C_{0S}.

2. It shows a gradual increase in the fin flutter velocity with altitude, because the pressure drops more quickly than the sound speed.

3. It shows fairly weak dependence on fin aspect ratio B and taper ratio λ — that term will always have a value near √2 for reasonably shaped rocket fins.

4. It shows very strong dependence on the fin thickness T.

5. It should be straightforward to incorporate this formula into rocket modeling software such as RockSim[3] and OpenRocket[4], to cause warnings when fin flutter velocity is exceeded; it is straightforward to add such a check in spreadsheet analysis of RockSim and OpenRocket numerical output.

The limitations of this formula are the same as of Howard's presentation:

1. The formula is only valid for trapezoidal fins, although one would expect the formula to work for "nearly trapezoidal" fins.

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2. The formula is only valid in the troposphere, for $h < H$.

3. Even for trapezoidal fins the formula is *not* exact, but rather an attempt to capture the main physics without doing a full blown numerical model of the fin. One should not be surprised if a rocket travelling at $0.9V_f$ loses its fins due to flutter — and of course fins can fail other ways.

John D. Sahr is a professor of Electrical Engineering at the University of Washington in Seattle. He dedicates this note to the memory of Professor R. D. Middlebrook of CalTech.

[3]https://www.apogeerockets.com/Rocket Software/RockSim

[4]http://openrocket.sourceforge.net