Problem 5 - Pringles stack ring

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Problem 5 - Pringles stack ring

Abstract

Structures by stacked Pringles on top of one another, in various configurations can be formed,the most distinct of them being the **Pringle stack ring**, which exhibits several interesting structural properties.In this paper,the physical parameters that allow the construction of different patterns are investigated, while also focusing on the features of the stack ring structure such as its size and structural integrity. Further more the ability of this structure to support external weight, is examined both theoretically and experimentally. In order to study the structures, an original,problem-specific notation has been developed based on "the stacking problem" for rhomboid stacks. Two different variable-weight experimental setups were constructed to perform press tests for a variety of load distributions. Experimental and theoretical results as well as future study processes and ideas are discussed.

1 Structural elements

1.1 Pringles

Pringles are saddle shaped chips, made to be stackable. For that purpose each one is almost identical to another.

The shape model of a Pringle is that of a hyperbolic paraboloid (1.1.1), where the constants have been evaluated based on a data set of precise measurements on a large amount of different pringles using both a caliper and pixel measurement analysis. Centimeters (S.I.) are used as units for every length that was calculated.

$$S = \begin{cases} 0.083x^2 - 0.257y^2 = z \\ 0.098x^2 + 0.204y^2 = 1 \end{cases} \qquad (1.1.1)$$

The wider arcs called z_1 and z_2 (Fig.1.) are given as

$$S_o = \begin{cases} \{z_1 = 0.083x^2 : -3.17 \le x \le 3.17\} \\ \{z_2 = -0.257y^2 : -2.22 \le y \le 2.22\} \end{cases} \qquad (1.1.2)$$

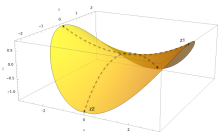

Fig.1: Pringle 3D model.

Contact between two Pringles. When two Pringles make contact with each other, forces occur. Using both carbon paper and paint tracing methods, it's been observed that two overlapping Pringles tend to touch in several discrete points that can also be viewed as a distribution. The equivalent replacement can be determined, using the symmetry of the shape, to be in the z_1 line. The symmetry reduces the problem to two dimensions and therefore Pringles can be seen as simple parabolic arcs.

1.1.1 Pringles deflection.

Several papers notice the strength of materials with the shape of a a hyperbolic paraboloid. For instance it's being observed from FE simulations[1] that there is no axis with a greater tendency to fail. Therefore Pringles tend to break into arbitrary shapes.

Adding load to the center of a Pringle will result z_1 to compression and tension on the z and on the x axis respectively. Experimentally it's been observed using tracing that the x-axis of a Pringle can withstand distortion of $\delta x \approx 0.01cm$. Using the formula, $\delta L \approx \frac{\partial L}{\partial a}\delta a + \frac{\partial L}{\partial b}\delta b$ for a parabolic arc with length $L = \frac{1}{2}\sqrt{b^2+16a^2} + \frac{b^2}{8a}ln\left(\frac{4a+\sqrt{b^2+16a^2}}{b}\right)$, height α, width b and for insignificant length changes, the variations are calculated as $\frac{\delta a}{\delta b} = -\frac{\partial_b L}{\partial_a L}$. That results to a new parabola $z = -4\frac{a-\delta a}{b+\delta b}x^2 + \mathcal{O}(z_0) \approx -(4\frac{a}{b} - \delta b\frac{a}{b^2} - \delta a\frac{1}{b})x^2 + \mathcal{O}(z_0)$ The variations for the two parabolas z_1, z_2 are $\delta a_1 = -0.044\delta b_1$, $\delta a_2 = -0.717\delta b_2$ respectively. Assuming $\delta a_1 \approx -\delta a_2$, the new critical shape before breaking can be calculated knowing $\delta b_2 \approx 0.01cm$, $z = f_1x^2 - f_2y^2$. The flexural rigidity EI can also be calculated from the moment-curvature equation for a parabolic arc as $k = \frac{M}{EI} \implies EI = \frac{M}{2 \cdot f_1}$, where M denotes the bending moment.

Hence Pringles are observed to be stiff, the material parameters are assumed to have very low elastic modulus, fracture strain and plastic displacement. The x-axis variation δb is measured by applying loads on Pringles heavier than 70g on a carbon paper.

Friction. Assuming that near a Pringle's surface,the z_2 arc is an inclined plane, by placing a Pringle piece (with unknown mass), the dry static friction coefficient can be approximated for the Coulomb friction model,where $\mu = tan\phi = 2bx$. For several calculations μ is found to be $1.15 < \mu < 1.42$. A value above 1 merely implies that the force required for a Pringle to slide along the surface of another, is greater than the normal force of the surface. Comparing with known materials found in reference tables, Pr-Pr coefficient is similar to Al-Al but the material is less dense (average weight 2.0625 ± 0.1gr/piece). These factors make Pringle chips the ideal material for these structures.

1.2 Ring formation

Practically most rings structures, that are crafted, are formulaically indescribable. Consequently it is important for a formation to be able to be formally described. The structures are expected to have a vertical axis symmetry in order to balance with no external interferences. The following theoretical arrangement is based on the block-stacking problem, where blocks are flat and form a diamond stack. Pringles don't overlap so there are no imbalances, they are placed in their middles in order to weaken the torques and add extra small normal forces on their fine sides.

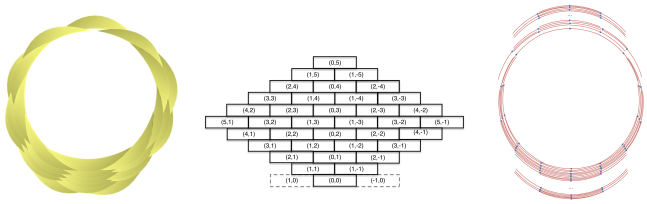

Fig.2: Ring with symmetric form derived from a rhomboid structure and the equivalent parabolic arcs ring.

2 Statics and Mechanics of Structures

2.1 Further notation and terminology.

The Pringles of the ring, denoted as "Pr(s)", can be arranged in i-columns and j-layers. Columns carry information about the tangent's angle at z_1 (at the saddle point) forms with the ground. Layers determine how many Prs are above (i,j) using the rhomboid equivalent. Signs are used to distinguish left from right Prs $\pm j$.

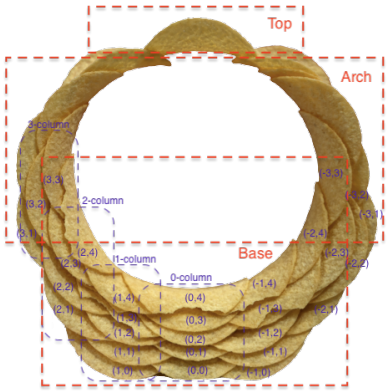

Fig. 3: Numbered ring.

The division of the greater construction into three main sub-parts makes it possible to distinguish interesting properties for its overall size and static properties.Each part will be examined individually.

Part 1-The Stacking Base Several bases can be crafted, with different widths and heights. All of them are stable stacks of Pringles, usually of odd columns so they don't have imbalances. When creating a base, a Pringle mold or two $(0, \pm 1)$ Prs are used so the ring won't swing. The forming of the ring starts from the (0,0)-Pr. Thus it can be separated from any base below it.

Part 2-The Arch The Arch is the set of Pringles whose tangent lines form an angle above $\frac{\pi}{2}$ with the ground. The Arch plays a great role when it comes to resistance against external loads.

The Top of the Ring

The top is the last Pringle of the structure to complete the circle and every Pringle above it. Forming a reverse stack. A correct placement of Pringles on the top can reinforce the construction. The Pringle in the maximum i-column, denoted as (m,1), is the most likely to turn when concentrated load is applied at the center-top of the ring. Equilibrium equations needs to be satisfied, $\sum F_{(m,1)\perp} = N_{(m-1,1)} + N_{(m-1,2)\perp} + m_{pr} g \cdot (N + cos\phi_m) \implies 0 = N_{(m-1,1)} + N_{(m-1,2)\perp} - \sum_{i=0}^{m-1} m_{pr} g \cdot cos\phi_i$, $\phi_i \approx i\frac{\pi}{m-1}$. In the case of an ordinary 6 column ring, this implies that it can maximum support 3-5 Pringles or $\approx 7gr$.

2.2 Constructions under loads

The size of the construction can act as an external load as discussed before. Unlike the big base size, that can mostly increase the imbalances, an enhanced top can cause deflection of the ring similarly to external weight. From the other hand, a few extra Pringles can reinforce and protect it from external moments, as though they spread the normal compressive stresses along the arch. **Reinforced rings** with additional Pringles on top of the ring are used for all the experiments.

2.2.1 Deformation of the ring

The external load will cause the arch to shrink, that happens due to the fact that the compressed members of the rings arch instead of voussoirs are pringles and instead of normal forces on the thrust line the compression occurs due to friction forces. The ring will shrink until the opposite moments become proportional to the external weight. In order for this to happen the ring needs to be reinforced on top. The arch starts to separate from the base at the (i,j)-Prs of the arch, where the normal forces of the external weight act with the greatest relative angle. If the l.d. is concentrated enough, (i,j)-Prs might overturn. Otherwise if the distribution is a bit wider the only way for the structure to collapse is that a (i,j)-Pr (or it's supporting (i-1,j-1)-Pr) breaks. The video footage[additional experimental material] and fig.4-Right, shows the frames of a ring that collapses, under $623 \pm 2gr$, uniformly distributed load on a line of lenghth 5.93cm. The first frame shows how sound peaks when the Pringle that supports the arch breaks due to opposite moments. Based on the above, the (3,3) Pr is compressed on the edges of the z_1 line by a load $\approx 623/2 \cdot cos(24^o) = 284gr$. The break happens when the angle between the Pringles reaches a value big enough to cause a break, a longer load distribution wouldn't allow this to happen as soon, because it would require a greater weight for a Pringle to overturn, due to the opposite moments of the column.

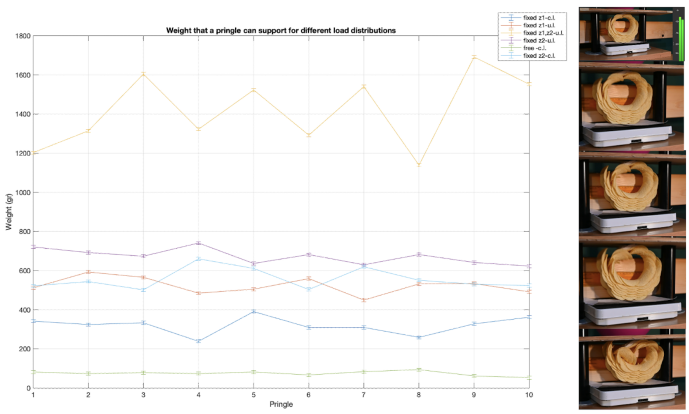

Fig. 4, Left: Load carrying capacity for different l.d. Right: Frames when ring collapses due to the breaking of a Pringle.

2.2.2 Load carrying capacity of a pringle

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Pringles are known to be brittle,so experiments took place to study the failure of Pringles under the action of external loads. The brittle failure depends on the load distribution (l.d.) and the deformation it creates.

By compressing Pringles with different ways,it was observed that the ratio of the different forces that act on the surface of the Pringle determines the failure. Three cases were tested.

[fig.4-Left] 1) The Pringle is free to bend on both z_1,z_2 thrust lines (as it was calculated above). 2) The Pringle is fixed in one line and free in the other, so the bending happens only in one line-family 3). The Pringle is fully fixed, so that we only have normal forces along parabolic arcs. The differences were enormous. A Pringle like the third case cannot break easily even if the load is above a kilogram. On the other hand the free Pringle under concentrated load breaks immediately. Finally the two last cases differ due to the fact that the greater arc (z_1) bends less for the same amount of weight, thus can support more weight. Uniformal distributions (u.l.) cause less damage compared to hats-triangular or concentrated (c.l.).

Distributed loading

The above makes apparent that the load distributions is in general a crucial parameter. Four main different cases are examined.

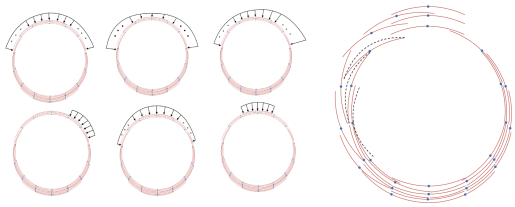

Fig. 5, Right: Ring's parabolic arcs equivalent, showing how opposite moments occur due to reinforced top and how the deformation happens. Left: Different load distributions.

The different distributed loads throughout the arch that can be supported by the ring are mostly uniformly distributed loads(UDL) and concentrated distributions. Also Triangular-hat type distributions are similar to concentrated loads. Asymmetrical loads of any kind will destroy or overturn the structure and so they are avoided. All l.d. can be simulated by scalar fields $f(l)$ along arch's thrust line.

3 Size of the Ring

For the purpose of studying the size, it is necessary to clarify what defines it. **The amount** of pringles, **the total shape** and the **radius** of the ring are considered as size-parameters. **By tacking and adding Pringles** it is possible to increase the size of the structure by starting to form the ring from a high stacking base. Also it is possible to increase the top [Fig.8]. This way the amount of Pringles can also be increased. Experimentally it's observed that a ring can support more than 200gr (80-100 additional Pringles) of external weight, while the stacking base can support more than 1.1kg even for stacks formed as $\{(0,0),(\pm1,0),(0,1),(\pm1,1)\}$. In this manner the structure will collapse most likely, due to imbalances in the third dimension.

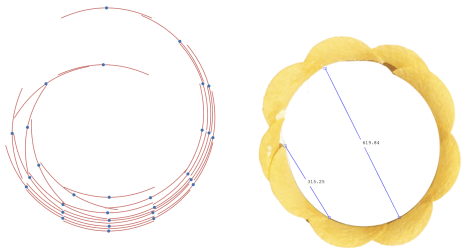

Fig.6, Right: Matlab distance measure, using pixels for the maximum ring. Left: Rings Parabolic arc equivalent. Shaping half a ring with fully adjoining Prs and half a ring with slightly shifted Prs, taking an elliptical-oval shape, smaller than the ring.

3.1 Ring Radius

In order to maximize the radius, Pringles need to touch smoothly along their surfaces.

The radius of the circle that the Pringles form it's not equal to the radius of curvature, instead assuming that the Pringles at the final layer of the ring are placed smooth, the radius reaches the maximum value. Using silicone and stick, several crafts are made [fig.6]. The maximum radius is evaluated as $R = 6.79 \pm 0.02cm$.

Two successive equal arcs of the same parabola are connected, so that the connections are perfectly smooth (the tangent lines of the parabolas are perfectly matching at the connecting points). Their width is $2x_0$, the one is rotated by $\phi = 2arctan(2ax_0)$. Assuming the edge points of each parabolic segment correspond to the successive corners of a canonical polygon, the corresponding polygon will have N edges with $N = \dfrac{2\pi}{\phi}$ This number will not be an integer, in general. Assuming this is an integer, though, the radius of the circumscribing circle will be

$$R = \frac{x_0}{sin(\phi/2)} = \frac{\sqrt{1+(2ax_0)^2}}{2a} \qquad (2.1)$$

From this formula, radius takes the value $R = 6.81cm$, which is in very good agreement with the experimental value of the ideal ring perimeter, as described before.

3.1.1 Experimental confirmation

Radius sizes have been measured for a big amount of different structures, using **Pixel analysis** methods and traditional measuring devices.

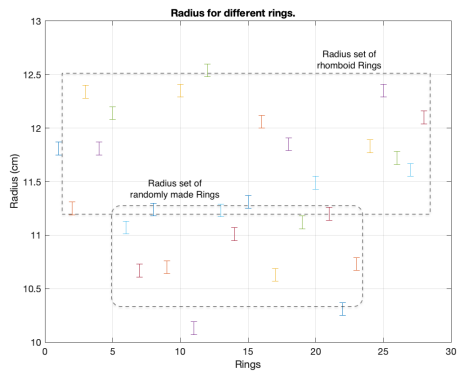

Fig.7: Experimental values of the ring radius for different constructions. Rings based on the rhomboid formula have on average greater radius.

3.1.2 Correction of the model

As it's observed experimentally, rings diverge more than 5% from the theoretical value. The divergence happens because the Pringles cannot be placed so their surfaces are exactly smooth. That is similar for every column having a slightly different z_1 parabolic line. That difference δa_i, for column-i, is significant,since (2.1)it is highly sensitive to any variation of the parabola's opening. The total correction of the radius is given by the following formula.

$$\delta R = -\sum_i \frac{1}{2a_i^2\sqrt{4x_i^2a_i^2+1}}\delta a_i \qquad (3.1)$$

A specific correction is calculated for a 5-layers ring and for δa_i estimated by a big amount of rings, $\delta R \approx 2,33cm$. Similar corrections for a given ring can be calculated to study the

divergence of the radius and compare it with other values like the amount of Pringles that was used or the weight support of the larger structures.

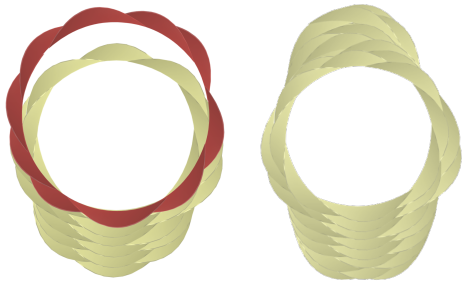

Fig.8: Different rings, compared to the theoretical structure of Prs touching smooth (red). The right structure also shows how a ring can be increased in size by adding Pringles on the base or top.

Equilibrium equations

An (i,j)-Pr of the ring, is in contact with a set of Prs, like $\{(i-1,j+1),(i+1,j+1),(i+1,j-1),(i-1,j-1)\}$, using the equivalent point for the parabolic arcs implies that the forces are in the z_1 arc and concentrated on one point.

Changing the frame of reference and the coordinates, so that the (i,j)-Pr is now rotated by the Pr-ground angle $\phi_{(i,j)}$ and the new coordinates are $(\hat{f},\hat{l})$ such that the forces can be separated into those that act parallel to the surface-line and those that act perpendicular to the surface-line, like friction and normal forces respectively. For instance, parabolic coordinates can be used or with a good approximation for the arcs, polar-coordinates as well. Then the equilibrium equations are,

$$\begin{cases} \sum_{(i',j')}\{F_{(i',j')\perp}\cdot\hat{f}+F_{(i',j')\parallel}\cdot\hat{l}\}+\vec{W}=0 \\ \sum_{(i',j')}\{F_{(i',j')\perp}\hat{f}+F_{(i',j')\parallel}\cdot\hat{l}\}\cdot\vec{r}_{(i,j)}=0 \end{cases} \tag{3.2}$$

Due to formation of the ring weight-moments can be ignored since the most likely point for a Pringle to rotate is it's middle. $\vec{W}=m_{pr}g\cdot(cos(\phi_{(i,j)}\cdot\hat{l}+sin(\phi_{(i,j)})\cdot\hat{f}$ The two equations,

$\sum_{(i',j')}\{N_{(i',j')}(\hat{l}+\mu\cdot\hat{f})\}+\vec{W}=0$, $\sum_{(i',j')}N_{(i',j')}\hat{f}\cdot\vec{r}_{(i,j)}=0$

need to be satisfied. In order to calculate the maximum m_{ext} for a given l.d. $f(l)$ the most weighted Pr (i,j) will receive opposite moments $M_1=-M_2$ such

$\sum_{i'_-,j'_-}\{m_{(i'_-,j'_-)}g\cdot cos(\phi(i'_-,j'_-))\cdot x_0\}+\sum_{i'_-,j'_-}\{m_{ext}g\cdot\int_{c_1}f(l)dl\cdot cos(\phi(i'_-,j'_-))\cdot x_0\}=-m_{ext}g\cdot\int_{c_2}f(l)dl\cdot cos(\phi(i'_+,j'_+))\cdot x_0$

for c_1,c_2 left and right i-column's lines where l.d. is applied. Using this formula and estimates for the flexural rigidity, comparisons can be performed with the experimental data to predict the structures endurance.

4 Experimental investigation

4.1 Experimental Setup

A simple yet effective setup was devised in order to measure the weight support a ring can have before collapse. The press consisted of cardboard or thin wood sheet which could be lowered or raised and support weight without bending significantly. A kitchen scale with one gram accuracy, 2kg of water or rice for precise weight measurements were utilised. The top plank could move steadily using either two columns or drawer slides, sprayed with lubricant to minimize frictions. Contributing to the setup, two cameras, recording at 240 fps and taking photos with telephoto lens, so that the footage could be later analyzed. Different surfaces where used, made by plasticine, white clay or sponge to distribute loads as desired.

Fig. 9: The experimental setup. Top left: Press that uses columns. Bottom Left: Press that uses drawer slides and sponge. Top and bottom right: Three frames of the rings under load, for the two different setups.

4.2 Experimental Investigation

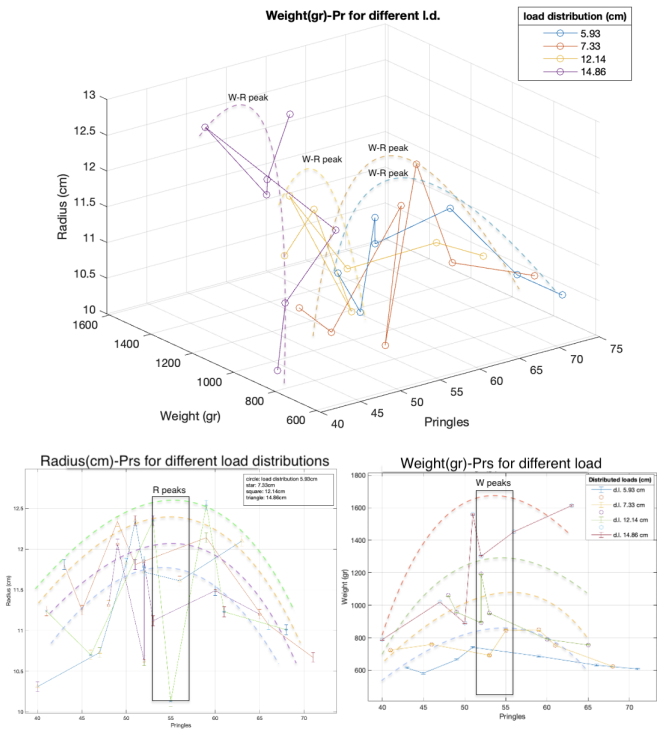

Fig. 10 , Top: 3D graph for all the parameters. Bottom right: Graph for Pringles amount-Weight(gr) that it can support for different l.d. Bottom left: Graph for Pringles amount-Radius(cm) for different l.d.

For several different widths, uniformly distributed loads and concentrated loads (UDL or simple u.l.) the ring's weight support was tested. The following experiments measured the number of Pringles, the radius of the ring (using matlabs pixel measure) and the maximum weight under limited time that the structure can support before it collapses. Besides the ring, Pringles, stacks and archs were measured separately.

Peaks in data. It is noticed that graphs present peaks for a certain amount of Pringles, that's due to the fact that the forming of the ring happens with a certain formula, thus the extra Pringles are placed either at the base or the top. Most of the structures had extra Pringles on top, which reinforces the arch up to the point the extra layers can be supported by Pringle layers of the base. That is approximately three layers (10-15 Prs). Any extra layer above those, when it's compressed doesn't have a springer-Pr for counterbalance, thus extra Pringles behave as external weight rather than reinforce layers, or even if they are placed slightly asymmetrically, they cause moment forces making the thrust line to move outside the arch geometry.

For the purposes of those experiments, the material fails because of delays are neglected. The experiments were done with minimum delays in the loading.

5 Conclusions and Future work

5.1 Conclusion

The number of Pringle that are used in the construction increase the radius and the endurance up to a peak-point. The maximum support and size is achieved at this point. The ability of the structure to support weight has been tested for several different scenarios of loading and described theoretically.The radius of the structure has also been calculated precisely theoretically and experimentally. A specific formalism has been developed to study the equilibrium equations and can describe the deformation of the ring in a good agreement between theory and experiment.

The following table summarizes the experimental data.

Average peak values for load distributions.			
LD(cm)	Prs	Weight(gr)	Radius(cm)
5.93	55	647gr±2	11,58±0.06
7.33	55	749gr±2	11,31±0.06
12.14	54	943gr±2	11,41±0.06
14.86	51	1232gr±2	11,52±0.06

Table 1. Average experimental values for different width uniformly distributed loads (u.l.).

5.2 Further work

5.2.1 Structural elements

Different Shape

In the classic stacking problem, with flat blocks, it is known that a 5-block tall diamond structure is unstable[2].

Fig.11, Right: Different stack-types, where D denotes double Prs. Left: Flat and curved pringles stacked in their middles.

It's interesting that changing the curvature of a block, this rhomboid structures become more stable. The radius of the ring as shown is affected by the curvature of the arc. For instance, using the z_2 arcs to construct rings makes them much smaller. Future work needs to be done to determine the minimum curvature Pringles could have so the ring has an enclosed shape. This curvature would also correspond to the maximum radius. Similarly, different arcs could be studied, like **circular arcs** or **longer Pringles** on the z_1 line. For that purpose 3D pringles can be used, as well some graphical representations.

Different materials. Changing the surface of a Pringle will effect the structures shape and strength. The endurance and the friction of the components can change. For instance other flavors of Pringles where tested and it was observed that the weight resistance of the structure wouldn't exceed 200gr, with Pringles having smaller friction coefficient and less endurance overall. Some additional experiments using Pringles painted with nail polish also took place but further work needs to be done to test how much the friction can effect the structure.

Side surface. Another way to increase the stability of the structures would be flattening the z_2 side, since is quite likely a Pringle in the structure to rotate and cause imbalances.

5.2.2 Ring formation

A different approach to study the endurance of the structure is being discussed, separating the problem into substructures and testing the endurance of them. As fig.5 shows, stack-types are arranged as T.0: (0,0)-Pr, T.1: $\{(0,0)$ to $(0,1)\}$.

Additional Experiments Additional experiments need to completed for separate parts of the ring. The arch specifically was measured by fixing it to the ground with clay, but it was noticed that is more likely the springer-Pr to break under loads below 200gr, confirming that the structure has a unique ability to support weight due to its ability to shrink along the thrust line (friction becomes kinetic and Pringles slide).

Different structures The curvature and the material of the Prs makes it possible to look for other structures besides the ring. Though none of the other structures that were tested, both theoretically and in practices, have presented an interesting differentiation. The reason is that all of them tend to stack until unsteadiness takes over. Otherwise they reach a point where they need to curve in order to differ from a stack, forming eventually a ring. However, it is interesting to look for other structures that at this point haven't been considered.

Finite element simulation Using FEM can be helpful for capturing local effects and having a simulation to compare the experimental results and confirm the theoretical solutions. For this purpose a FEM model is being developed.

6 Bibliography

[1]: I.Karnovsky, O.Lebed, 2010: Advanced Methods of Structural Analysis, Springer.

[2]: R. Richards Jr, 2000: Principles of Solid Mechanics, CRC.

Refrences

[1]: Youngbin LIM, 2022: FE simulation for Pringles.

[2]: Paterson and others, 2009: Maximum overhang, MAA.