Chapter 3
Rational Consumer Choice

Chapter Outline

- The budget constraint and the Feasible set
- What causes changes in the Budget constraint?
- Consumer Preferences
- The utility function
- Lagrange Multipliers
- Indifference Curves

GOAL

- We want to solve for the optimal bundle (a combination of goods) that a "rational consumer" will purchase.

METHODs

- Both Graphical and Mathematical

Retained Assumptions

1. Existence of a Feasible Set or Budget Constraint.
2. Existence of a Consumer's Preference described (mathematically) by his/her Utility Function.

Budget Constraint and Feasible Set

- Budget constraint:
  - It is the set of all bundles that can be purchased with a given level of income and prices, when all income is spent
- Feasible set:
  - The bundles on or within the budget triangle are referred to as the feasible set. In other words, the bundles for which the required expenditure at given prices is less or equal to the income available

Budget Constraint: Review

Consumers cannot **afford** all the goods and services they desire.
Consumers are limited by their income and the **prices** of goods.

Model Assumption: Consumers spend **all** their income (no savings).

Let:

- M=Income
- X and Y are two goods
- P_x=price of each unit of good X
- P_y = price of each unit of good Y

Budget constraint: numerical expression of which market baskets the consumer can afford.

<u>The Budget Constraint</u>:

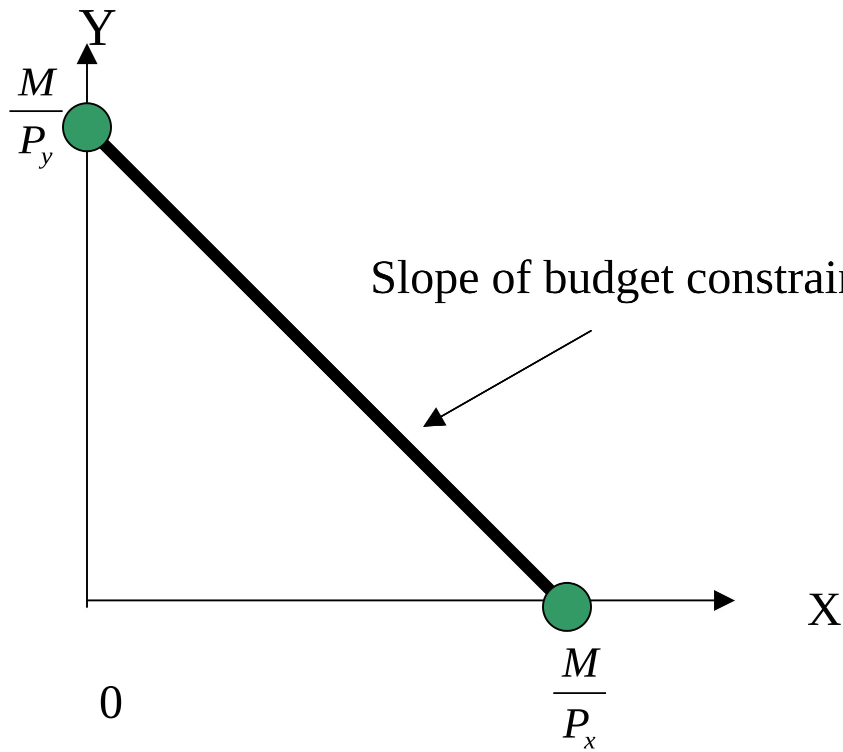
$$P_x X + P_y Y = M$$

Total expenditure = Income

Rearranging the above expression for the budget constraint, we are able to determine how many units of good Y we can consume for any given quantity of good X:

$$Y = \frac{M}{P_Y} - \frac{P_X}{P_Y} X$$

This equation of a straight line illustrates the trade-off between Y and X.



Slope of budget constraint is $-\frac{P_x}{P_y}$.

The slope of the equation $-\frac{P_X}{P_Y}$ is negative because in order to purchase more units of Y, the consumer must give up units of X.

Dividing income by the price of Y yields the maximum number of units of Y that can be purchased when no units of X are purchased.

Dividing income by the price of X yields the maximum number of units of X that can be purchased, when no units of Y are purchased.
⇨These are the intercepts of the budget line.

Shifts in the Budget Constraint

What happens to the budget constraint when: (1) income changes, (2) the price of X changes or (3) the price of Y changes?

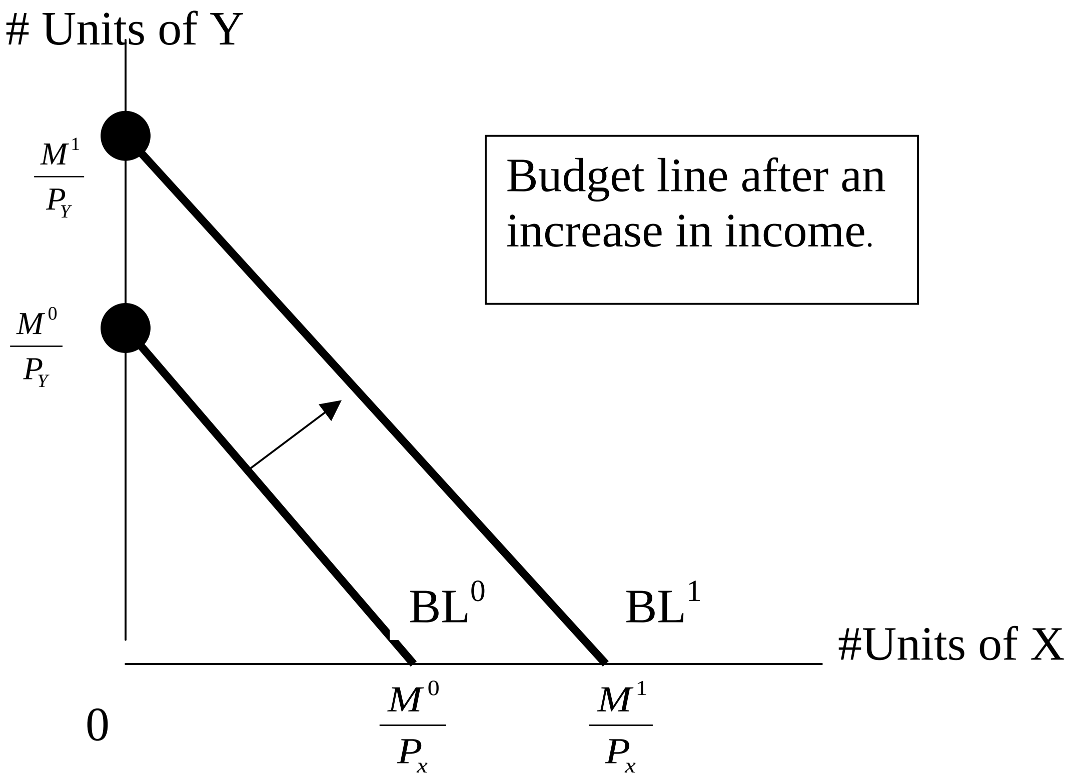
1) Income changes

Let M^0= initial income
M^1=new income

When income changes the Y intercept changes from $\frac{M^0}{P_Y}$ to $\frac{M^1}{P_Y}$ and the X intercept changes from $\frac{M^0}{P_X}$ to $\frac{M^1}{P_X}$.

The slope of the budget line remains unchanged at $-\frac{P_X}{P_Y}$.

Hence, a change in income is shown as a **parallel shift** inward or outward of the budget constraint.



(2) <u>A Change in the price of good X:</u>

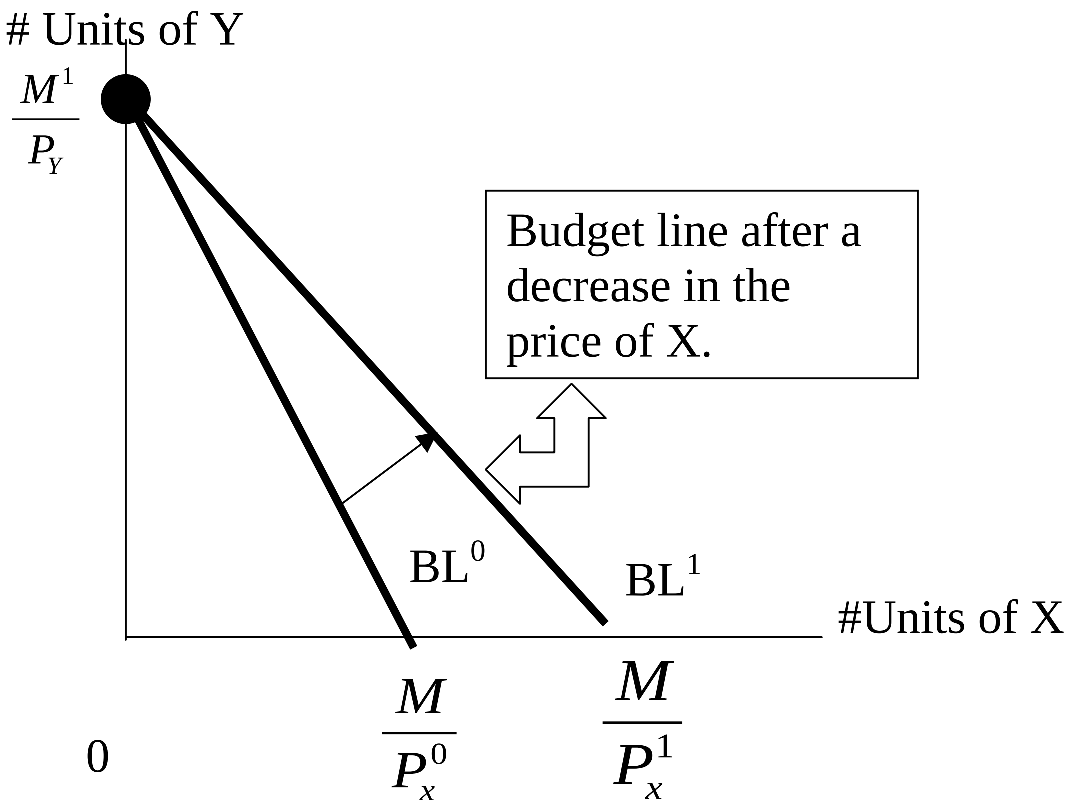
A change in the price of good X does not change the amount of good Y the consumer could buy if he or she spent all their income on Y. (i.e. the intercept remains the same.)

The budget line **<u>does</u>** change:

For price increases, the budget line becomes steeper

For price decreases, the budget line becomes flatter.

$$Y = \frac{M}{P_Y} - \frac{P_X}{P_Y} X$$



Let P_X^0 = the initial price of good X.

Let P_X^1 = the new (lower) price of X.

When the price of X decreases, the consumer can now purchase more of good X. The budget constraint swings outward and becomes flatter.

<u>(3) A Change in the Price of Good Y:</u>

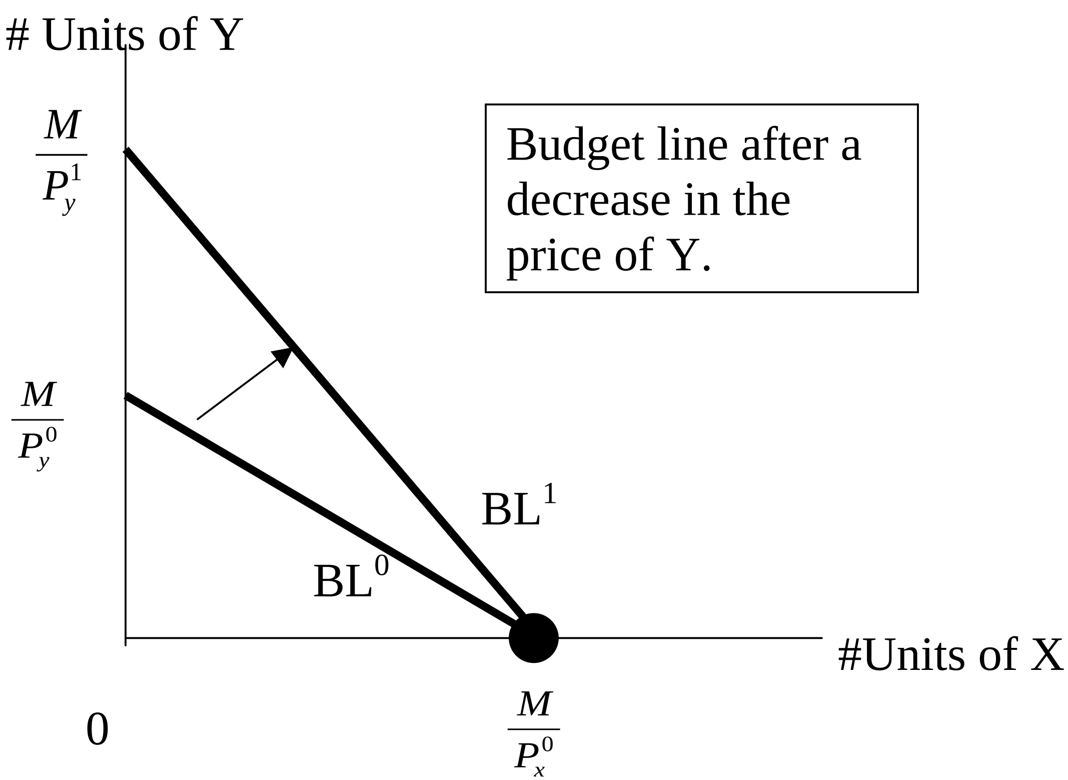
A change in the price of good Y does not change the amount of X the consumer could buy if he or she spent all their income on X. (I.e. the X intercept remains the same.)

However, the <u>budget line</u> does change:

For price increases the budget constraint becomes flatter.

For price decreases the budget constraint becomes steeper. (I.e. the Y intercept increases.)

$$Y = \frac{M}{P_Y} - \frac{P_X}{P_Y} X$$



Let P_Y^0 = the initial price of good Y.

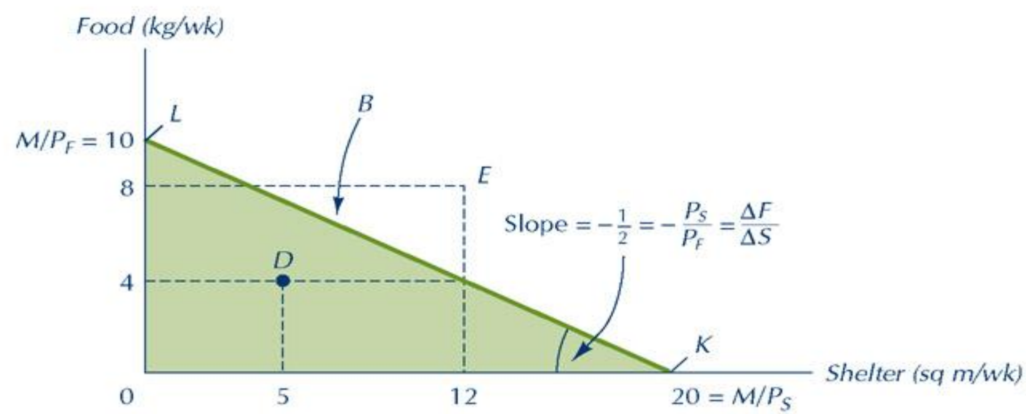
Let P_Y^1 =the new (lower) price of Y.

<u>**Note:**</u> If all prices and income change by the same proportion, the budget constraint is unaffected.

Graphically

FIGURE 3-2

The Budget Constraint and the Affordable Set With M = \$100/week, P_F = \$10/kg, and P_S = \$5/sq m, the shaded area (including the horizontal and vertical axes and the budget constraint, line B) defines the *affordable set* for the consumer. With this money income and these prices, she can afford any point (or *bundle*) in this triangular set. Line B describes the set of all bundles the consumer can purchase if all of income is spent at the given prices. Its slope is the negative of the price of shelter divided by the price of food. This slope is the opportunity cost of an additional unit of shelter along the budget constraint—the number of units of food that must be sacrificed in order to purchase one additional unit of shelter at market prices.



Mathematically

- The budget constraint must satisfy the following equation:

$$P_s S + P_F F = M$$

- Or equivalently:

$$F = \frac{M}{P_F} - \frac{P_S}{P_F} S$$

- Where $\frac{P_S}{P_F}$ is referred to as the marginal cost (in terms of good Y) of an additional unit of good X along the budget constraint and $\frac{M}{P_F}$ is the intercept.

What causes changes in the Budget constraint?

Two cases can happen:

1. Parallel shift inward or outward
2. Or, the slope can change

Case 1: Income Changes (Parallel Shift)

- The effect of a change in income is much like the effect of a proportional change in all prices. For example, cutting income by half has the same effect as doubling the prices of the goods.

Case 2: Relative price changes (Slope changes)

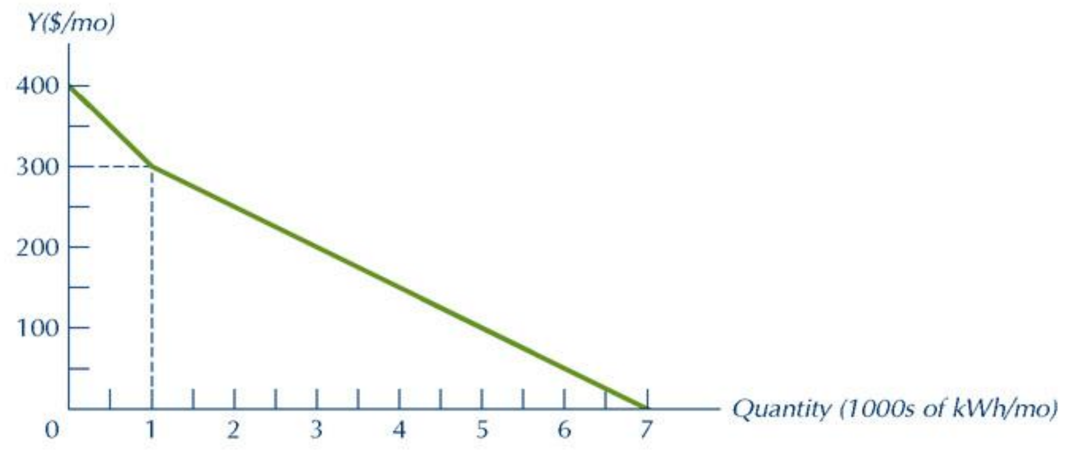
- A change in price of one good relative to other will rotate the budget constraint or in other words change the slope of the budget constraint.

A Special Case: Kinked Budget constraints

- This happens when the slope of the budget constraint is not constant across all bundles of goods.

FIGURE 3-7

A Quantity Discount Gives Rise to a Nonlinear Budget Constraint
Once electric power consumption reaches 1000 kWh/mo, the opportunity cost of additional power falls from $.10/kWh to $.05/kWh.



Mathematically

- We can view this situation as if there are two budget constraints.
- when $X < \overline{X}$, then $Y = \frac{M}{P_y} - \frac{P_x}{P_y} X$

otherwise,

$$Y = \frac{M}{P_y} - \frac{P_{x'}}{P_y} X$$

where, $P_x \neq P_{x'}$

Quantity Discount:

E.g. Fertilizer is $5 per pound.

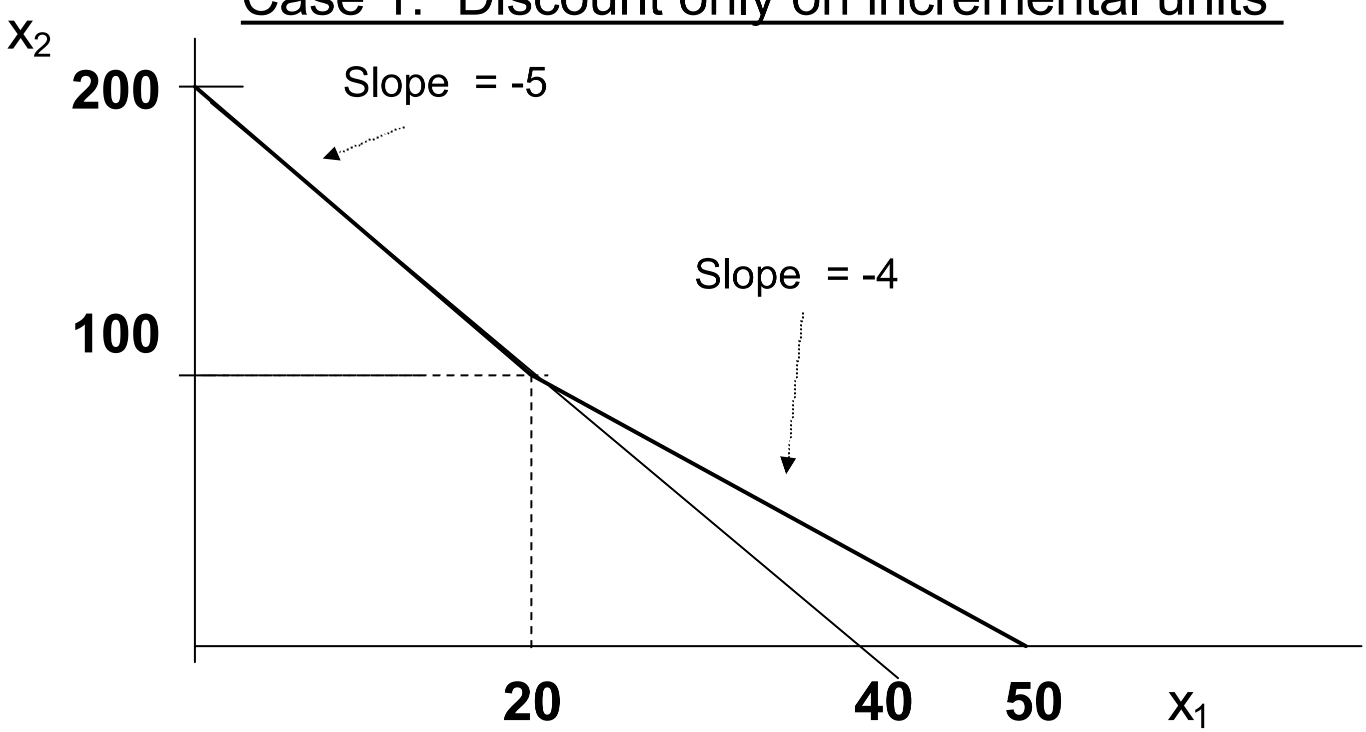
- If Buy more than 20 pounds, price is $4

Question: Does the discount apply:

(1) to just the fertilizer over the 20 pounds
(2) to all the fertilizer

Suppose you have $200 to spend on fertilizer, and all other goods. Let the price of all other goods (p_2) be $1.

Case 1: Discount only on incremental units



What is the budget line if no kink?

$$5x_1 + x_2 = 200$$

Where is the kink (i.e., at what point does the slope change?)

$$x_1 = 20 \Rightarrow x_2 = 100$$

What is the price beyond the kink?

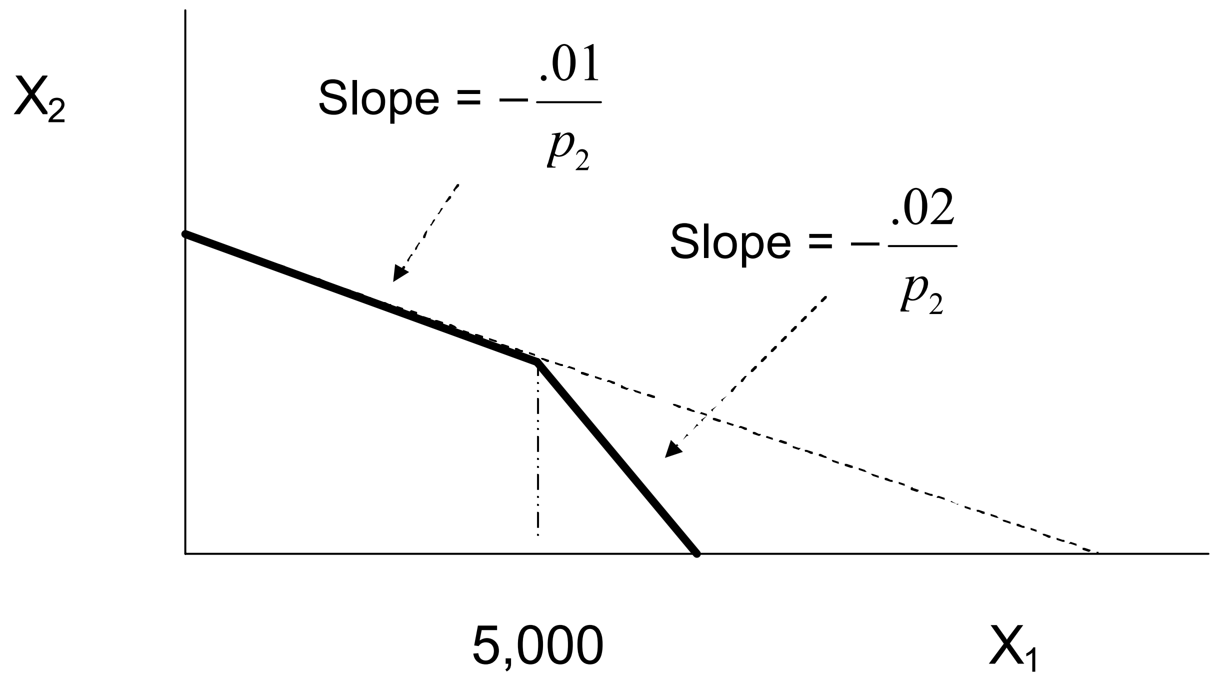
$$p'_1 = 4$$

What is the maximum number of pounds?

$$x_1(\text{max}) = 50$$

Quantity Surcharge:

E.g. Suppose that electricity costs $.01 per kwh if use up to 5,000, but $.02 per kwh for all kwh over 5,000.



Consumer's Preference.

- Consumer's preference is used to rank different bundles.
  - Example: (based on a preference) a person can say:
    - he/she prefers bundle A over bundle B or
    - he/she prefers bundle B more than bundle A or
    - he/she is indifferent between A and B.

Consumer's Preference.

- Assumptions of the Consumer's Preference:
  - ***Completeness*** – Consumer is able to rank all possible bundles.
  - ***Non-satiation*** – Consumer always want more of a product ceteris paribus.
  - ***Transitivity*** – If bundle A is better than bundle B and bundle B is better than bundle C then bundle A must be better than bundle C.
  - ***Convexity*** – Consumer prefers a little of anything.

Consumer's Preference

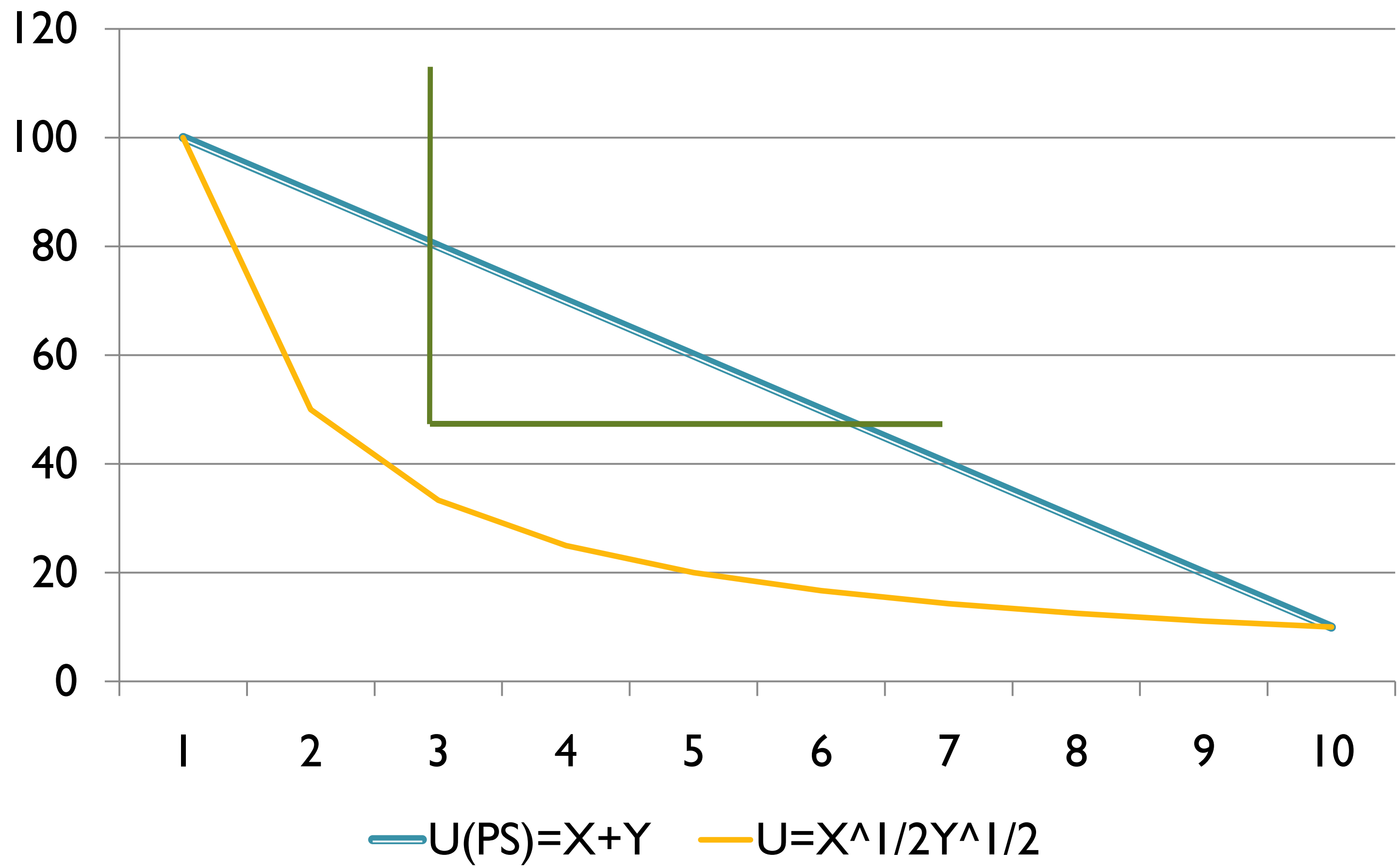
- Consumer's Preference is graphically expressed by the Indifference Curve.
- Indifference curve is a set of bundles which are equally preferred.
  - Most of the time, we deal with the case where the indifference curve is convex toward the origin.
  - Slope of the Indifference Curve at a point is called the Marginal Rate of Substitution (MRS) – how many (vertical) good a person is willing to give up to get an additional (horizontal) good.
  - The MRS gives the marginal benefit of good X in terms of good Y.

Consumer's Preference.

- Mathematical expression of a Consumer's Preference – the Utility Function.
  - Additive utility functions (if goods are **perfect substitutes**) – Linear utility function.
    - Example 1: U(x , y)=x + y
    - Example 2: U(x , y)=ax+by
  - Multiplicative utility functions – Cobb Douglas. Convex to the origin utility function.
    - Example 1: U(x , y)=xy
    - Example 2: U(x , y)= $x^a y^b$
  - Of the form min *of* (ax , by) (if goods are **perfect complements** – Leontief. L-shape utility function.
    - Example 1: U(x , y)=min[x , y]
    - Example 2: U(x , y)=min[2x , 3y]

Consumer's Preference

- Graphical Presentation of the Consumer's Preference.



Solve the Rational Consumer's Problem.

- Method 1 (for Cobb Douglas Utility).
  - From the budget constraint, isolate one of the good as the function of income, prices and the other good.

$$Y = \frac{M}{P_Y} - \frac{P_X}{P_Y} X$$

  - Substitute the expression of Y into the utility function U(X,Y).
  - Maximize the utility function which is now a function of only one of the good (good X in this case).
    - The F.O.C yields the solution for good X.
  - Substitute the value of good X onto the budget constraint to get the solution for good Y.

Solve the Rational Consumer's Problem.

- Advanced Mathematical Method.
  - Lagrangean Multipliers (for Cobb Douglas Utility)..

$$\mathcal{L}(X,Y) = U(X,Y) + \lambda(M - P_x X - P_y Y)$$

- Set $\frac{\partial \mathcal{L}}{\partial X} = 0 \; (1)$ and $\frac{\partial \mathcal{L}}{\partial Y} = 0 \; (2)$

And let (1)/(2) → $MRS = \frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$

- Similar equilibrium condition as the graphical method.
- Solve for the quantity of two goods given prices and income level.

Utility Maximization Under Constraint
2 Methods

Question:

Tom spends all his $100 weekly income on two goods, X and Y.

His utility function is given by U(X, Y) = XY.

If P_x= $4/ unit and P_y=$10/ unit, how much of each good should he buy to maximize utility?

The budget constraint equals:

$$\boxed{\begin{array}{c} P_x X + P_y Y = M \\ 4X + 10Y = 100 \end{array}}$$

Method 1: Lagrangean Multipliers:

First, transform the constrained maximization problem into the following unconstrained maximization problem:

$$\underset{X,Y,\lambda}{Max}\ \mathcal{L} = U(X,Y) - \lambda(P_x X + P_y Y - M)$$

The lagrangean multiplier's (λ) role is to assure that the budget constraint is satisfied.

The first order conditions for a maximum of $\mathcal{L}$ are obtained by taking the first partial derivatives of $\mathcal{L}$ with respect to X, Y and λ, and equating them to zero:

$$\underset{X,Y,\lambda}{Max}\ \mathcal{L} = XY - \lambda(4X + 10Y - 100)$$

$$\frac{\partial\mathcal{L}}{\partial X} = Y - 4\lambda = 0$$

$$\frac{\partial\mathcal{L}}{\partial Y} = X - 10\lambda = 0$$

$$\frac{\partial\mathcal{L}}{\partial\lambda} = -1(4X + 10Y - 100 = 0$$

Now: Using the ratio of the first two FOCs:

$$\frac{Y - 4\lambda}{X - 10\lambda} = 0 \Rightarrow \frac{Y}{X} = \frac{4\lambda}{10\lambda} = 0.4$$

$$Y = 0.4X$$

Expression for Y

Inserting the expression for Y into the budget constraint:

$$4X + 10Y = 100$$

$$4X + 10(0.4X) = 100$$

$$8X = 100$$

$$X = 12.5$$

Substituting X=12.5 into the expression for Y:

$$Y = 0.4X$$

$$Y = 0.4(12.5) = 5$$

The bundle X=12.5 and Y=5 will maximize utility while constrained at M=100.

Method 2: Solve the budget constraint for Y in terms of X and substitute the result wherever Y appears in the utility function. Utility then becomes a function X alone and we can maximize it by taking its first derivative with respect to X and equating that to zero. The value of X that solves that equation is the optimal value of X, which can them be substituted back into the budget constraint to find the optimal value of Y.

$$P_x X + P_y Y = M \Leftarrow \text{budget constraint}$$

$$4X + 10Y = 100$$

rearrange:

$$Y = 10 - 0.4X$$

The utility function:

$$U(X,Y) = XY$$

$$U(X,Y) = X(10 - 0.4X) = 10X - 0.4X^2$$

Max Utility wrt X:

$$\frac{\partial U}{\partial X} = 10 - 0.8X = 0$$

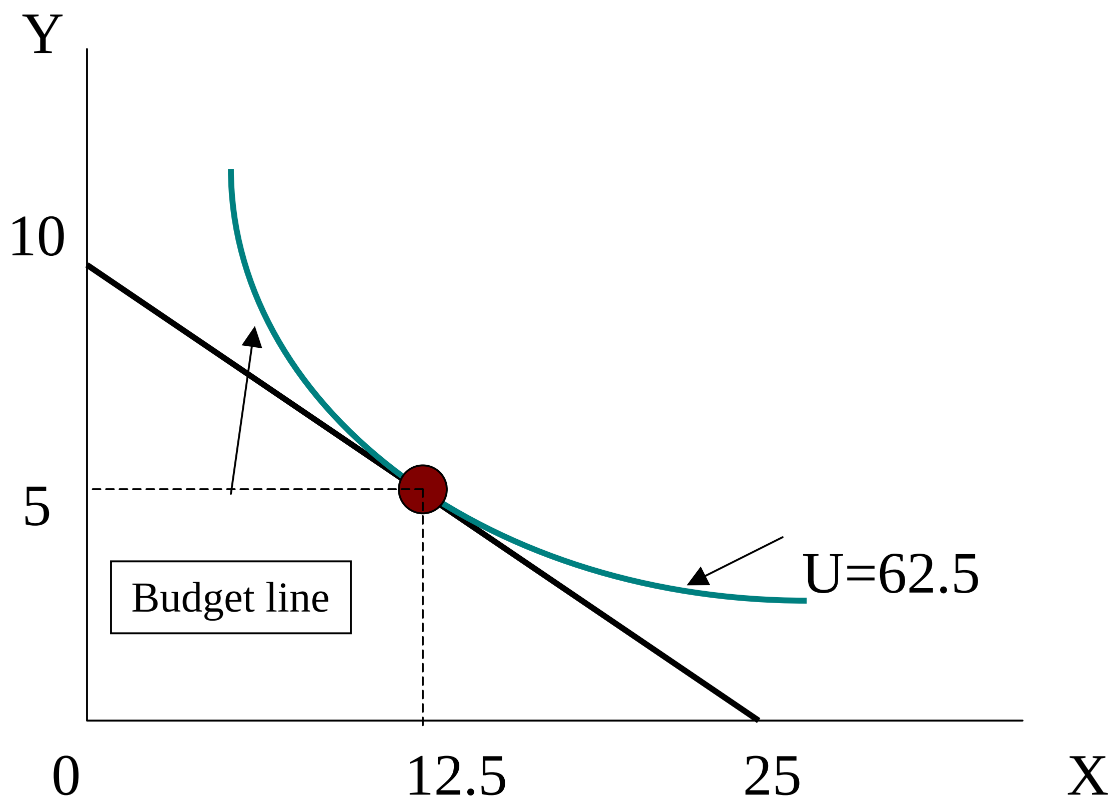
$$10 = 0.8X$$

$$X = 12.5$$

$$Y = 10 - 0.4X = 10 - 0.4(12.5) = 5$$

Same answer: (X=12.5, Y=5)

Graphically:



Which leads us to indifference curves:

Solve the Rational Consumer's Problem.

- For other types of utility functions:
  - Perfect Substitute Linear Utility Function
    - The slope of the Utility Function, MRS, is a constant.
    - It is either equal, greater, or less than the slope of the budget line $\frac{P_X}{P_Y}$ (good Y is on the vertical axis).
      - If equal: Multiple optimal bundles along the indifference curve/budget line.
      - If greater: Corner solution where the optimal bundle is at the intersection between the budget line and the horizontal axis (consuming only good X).
      - If less: Corner solution where the optimal bundle is at the intersection between the budget line and the vertical axis (consuming only good Y).

Solve the Rational Consumer's Problem.

- Perfect Substitute Linear Utility Function
  - Example: Problem 10 p.82
    - For Dan, coffee and tea are perfect-substitutes: one cup of coffee is equivalent to one cup of tea. Suppose Dan has $90 per month to spend on these beverages, and coffee costs $0.9/cup while tea costs $1.2/cup. Find Dan's best affordable/optimal bundle of tea and coffee. How much could the price of a cup of coffee rise without harming her standard of living? Similarly, how much could the price of a cup of tea rise?

Solve the Rational Consumer's Problem.

- For Perfect Complement L-shape Utility Function.
  - Notice U=min(aX,bY)
    - This implies the optimal ratio is always aX=bY.
  - Use the optimal ratio and the budget constraint to solve for the optimal solution for X and Y.

Solve the Rational Consumer's Problem.

- For Perfect Complement L-shape Utility Function.
  - Example: Problem 16 p.83
    - Carlo budgets \$9/week for his coffee and milk. He likes it only if it is prepared with 4 parts coffee, 1 part milk or U=min(C,4M). Coffee costs \$1/unit, milk \$0.5/unit. How much coffee and how much milk will Carlo buy per week? How will your answers change if the price of coffee rises to \$3.25/unit?

Properties of indifference curves

1. Non-satiation: IC's slope downward
2. Non-satiation + Transitivity: IC's cannot cross each other
3. Non-satiation: IC's that are father from the origin represent higher levels of utility
4. Convexity: IC's are bowed inward (they are convex)

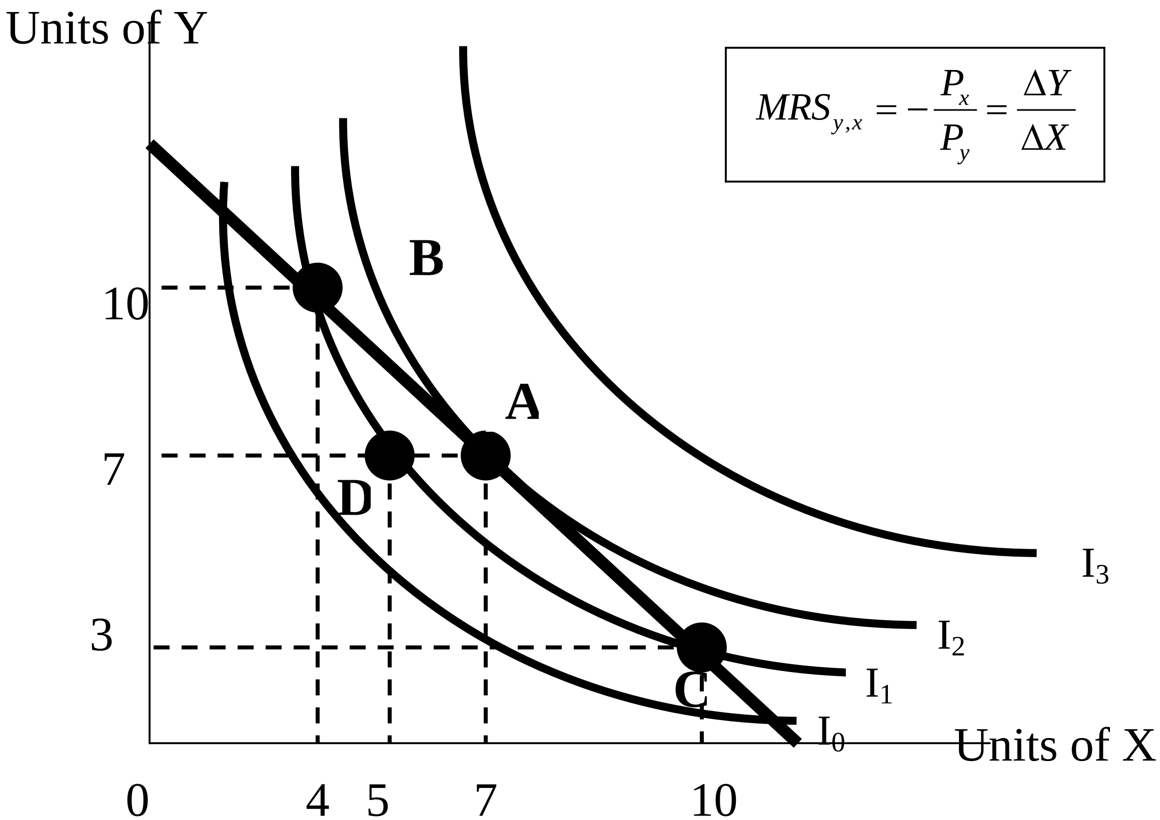
The Consumption Decision:

The objective of the consumer is to maximize satisfaction subject to his or her budget (income) constraint. We must bring together the concepts of budget constraint and preference to determine the consumer's consumption decision.

That is, the basket of goods the consumer eventually purchases is determined by individual taste and affordability.

We know that all market baskets on the budget constraint are attainable because they are affordable. The consumer will choose the basket of goods that is on his or her highest attainable indifference curve.

Graphically we have the following:



The consumer maximizes utility at market basket A, where the MRS equals the slope of the budget constraint.

A necessary condition for the consumer to maximize utility subject to the budget constraint is:

$$\boxed{MRS_{YX} = -\frac{P_X}{P_Y}}$$

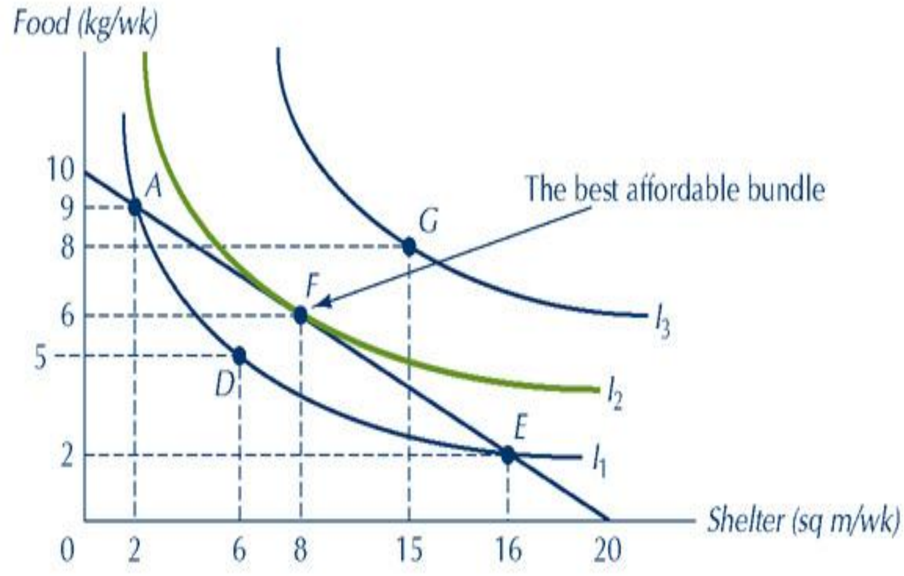
The consumer divides total income between the two goods so that the MRS between the two goods equals the negative of the price ratio which equals the slope of the budget constraint.

"When selecting a market basket containing both goods, a consumer maximizes utility by equating his or her marginal rate of substitution with the market's marginal rate of substitution."

FIGURE 3-16

The Best Affordable Bundle

The best the consumer can do is to choose the bundle on the budget constraint that lies on the highest attainable indifference curve. Here, that is bundle *F*, which lies at a tangency between the indifference curve and the budget constraint.



Derivation of the slope of ICs

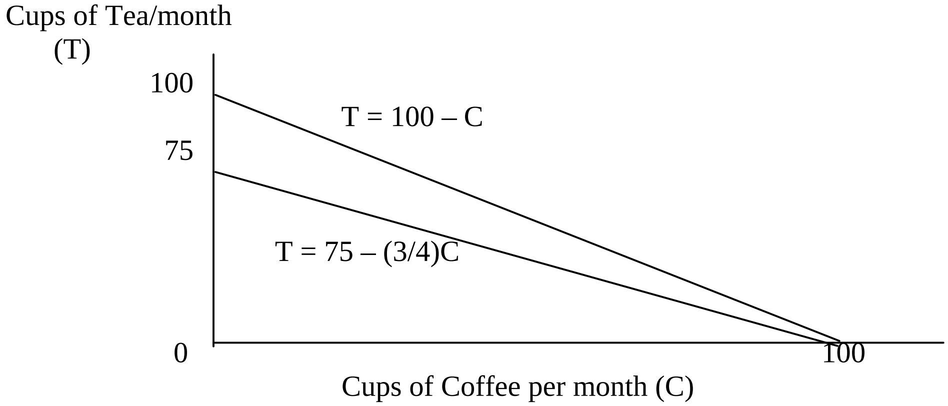
- Recall: slope IC = –MRS

U(x,y)=constant $\Rightarrow \frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy = 0$

$$SlopeIC = \frac{dy}{dx} = -\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} = -\frac{MU_x}{MU_y}$$

In other words, MRS of y for x= $\frac{MU_x}{MU_y}$

10. Alexi's budget constraint is T = 75 – (3/4)C. Her perfect substitute preferences yield linear indifference curves with slope equal to negative one, such as T = 75 – C and T = 100 – C. By consuming 90/0.90 = 100 cups of coffee each month, she reaches a higher indifference curve than consuming 90/1.20 = 75 cups of tea (or any affordable mixture of coffee and tea). Thus Alexi buys 100 cups of coffee and no tea. Any increase in the price of coffee would force Alexi to a lower indifference curve, and thus lower her standard of living. An increase in the price of tea, in contrast, would have no effect on her standard of living.



16. Let C = coffee (units/week) and M = milk (units/week). Because of Carlo's preferences, C = 4M. At the original prices we have the budget line as follows:
C + (0.5)M = 4M +(0.5)M = 4.5M = 9, and so M = 2 units/week and C = 8 units/week.
Let M' and C' be the new values of milk and coffee. Again, we know that C' = 4M'. With the new prices we have the budget line as follows:
(3.25) C' + (0.5)M' = (3.25)(4 M') + (0.5)M' = (13.5)M' = 9, and so M' = 2/3 units/week and C' = 8/3 units/week.