Finite Fourier series approximation of low-thrust lunar ballistic trajectories

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1 Introduction

Shape-Based Approximations to defining an initial guess for trajectory optimization have been extensively studied for the two-body problem. Along these lines, analogous studies have not been performed on the circular restricted three body problem (CR3BP). This is partly because of the problem's higher level of complexity but also because of the more limited applications of this case. Finite Fourier series (FFS) do not necessarily have any constraint that does not allow their use on such applications, yet the experimentation with this option has had limited attention.

2 GSOC Proposal Prospects

The present proposal tries to give attention to the CR3BP in its potential applications for Earth-Moon transfers, as well as propose an alternative method to start any general optimization with a tool that has been proven very powerful for the two body problem. The mathematics involved cannot be fully presented in this proposal since although Fourier analysis and the CR3BP both have known properties and may be found easily on the literature, the conjunction has not been proposed or at least the literature does not appear to have it reflected as such.

In particular, the project will consist on developing briefly the mathematics, adapting the existing structure in MOLTO-3BP suit for the introduction of a FFS-based initial guess production code segment. It would also provide quantitative tools to assess the quality of the guesses and finally an easy visualization with the help of Matlab built-in functions.

3 Discussion on possible problems

The trajectories in the CR3BP expressed in the sidereal reference frame (inertial reference system) are expected to result in shapes too complex to be approximated with Fourier series. Furthermore, it is reported that these shapes have singularities when the trajectory tries to represent near misses or collision courses. The way that this can be worked around is by making use of the synodic reference frame (non-inertial). The mathematics involved in this conversion are already well posed and allow the dynamical system to be presented by means of a potential. This representation of the problem increases the ability to visualize possible solutions and aggregate them into trajectory families, as they are known in the literature.

There already exist several solvers that provide the constants of the Fourier series by means of the boundary conditions and the thrust constraints. This process is expected to be moderately simple to be adapted into the synodic presentation of the CR3BP. An additional advantage of this particular shape is that it can be configured in such a way that some unknowns are left to particularize the shape depending on an extra constraint, see the maximum thrust force.

This study's particular application in lunar transfers should be straightforward from this point. A preemptive study of the literature reveals that there are plenty of examples to feed the solver with. These examples could help validate the code or at least provide a good idea on the limitations of this solving method, either by computing time or precision. On these lines, the CR3BP is known to present a relatively low but still chaotic behavior for values of m_1 and m_2, considering these as the masses of the primaries. This could mean that precision will need to be much finer than that of the two body method. Although this may discourage the use of the presented approximation as Fourier series, this might be accounted for by the posterior optimizer since this is only providing the initial guess.

4 Timeline

The time frame contemplated in GSOC stipulates that there should be about 2.75 months. The intention is to develop the mathematics necessary for the further evolution prior to the start of the code segment in June. By then, and along the time disposed by GSOC, everything will be prepared to start working on the code itself. The first steps involve creating a generic finite Fourier series for the 2 body problem. Then this will improve into a 3 body problem method. The work continues on by proving the stability of this method on either the synodic or sidereal reference frames. Finally the code is validated against existing examples. In order to finish up the work there will be a presentation of the results provided by the visualization tools developed through this process.