$$u_i^{n-1} - c\left(u_i^{n-1} + \Psi_{i+1/2}\frac{1-c}{2}\left(u_{i+1}^{n-1} - u_i^{n-1}\right) - u_{i-1}^{n-1} - \Psi_{i-1/2}\frac{1-c}{2}\left(u_i^{n-1} - u_{i-1}^{n-1}\right)\right) \tag{1}$$

$$\begin{aligned} u_i^{n-1} - c\left(u_i^{n-1}\left(1 - \Psi_{i+1/2}\frac{1-c}{2}\right) + \Psi_{i+1/2}\frac{1-c}{2}u_{i+1}^{n-1}\right) \\ -c\left(u_{i-1}^{n-1}\left(1 - \Psi_{i-1/2}\frac{1-c}{2}\right) + \Psi_{i-1/2}\frac{1-c}{2}u_i^{n-1}\right) \end{aligned} \tag{2}$$

$$u_i^{n-1} - c\left(u_i^{n-1} - u_{i-1}^{n-1}\right) + c\left(\Psi_{i+1/2}\frac{1-c}{2}\left(u_{i+1}^{n-1} - u_i^{n-1}\right) - \Psi_{i-1/2}\frac{1-c}{2}\left(u_i^{n-1} - u_{i-1}^{n-1}\right)\right) \tag{3}$$

$$u_i^{n-1} - c\left(u_i^{n-1} - u_{i-1}^{n-1}\right) + c\Psi_i\left(\frac{1-c}{2}\left(u_{i+1}^{n-1} - u_i^{n-1}\right) - \frac{1-c}{2}\left(u_i^{n-1} - u_{i-1}^{n-1}\right)\right) \tag{4}$$

$$u_i^n = u_i^{n-1} - c\left(u_i^{n-1} - u_{i-1}^{n-1}\right) + c\Psi_i\left(\frac{1-c}{2}\left(u_{i+1}^{n-1} - u_i^{n-1}\right) - \frac{1-c}{2}\left(u_i^{n-1} - u_i^{n-1}\right)\right)$$

Let $u_{i-1}^{n-1} < u_i^{n-1} < u_{i+1}^{n-1}$.

$$\begin{aligned} u_{i-1}^{n-1} \leq & u_i^{n-1} - c\left(u_i^{n-1} - u_{i-1}^{n-1}\right) \\ & + c\Psi_i\left(\frac{1-c}{2}\left(u_{i+1}^{n-1} - u_i^{n-1}\right) - \frac{1-c}{2}\left(u_i^{n-1} - u_i^{n-1}\right)\right) \leq u_i^{n-1} \end{aligned} \tag{5}$$

$$\begin{aligned} 0 \leq & (1-c)\left(u_i^{n-1} - u_{i-1}^{n-1}\right) \\ & + c\Psi_i\left(\frac{1-c}{2}\left(u_{i+1}^{n-1} - u_i^{n-1}\right) - \frac{1-c}{2}\left(u_i^{n-1} - u_i^{n-1}\right)\right) \leq u_i^{n-1} - u_{i-1}^{n-1} \end{aligned} \tag{6}$$

$$0 \leq 1 + \frac{c\Psi_i}{2}\left(\frac{1}{r_i} - 1\right) \leq \frac{1}{1-c} \tag{7}$$

$$-1 \leq \frac{c\Psi_i}{2}\left(\frac{1}{r_i} - 1\right) \leq \frac{1}{1-c} - 1 \tag{8}$$

$$-1 \leq \frac{c\Psi_i}{2}\left(\frac{1}{r_i} - 1\right) \leq \frac{c}{1-c} \tag{9}$$

$$-\frac{2}{c} \leq \Psi_i\left(\frac{1}{r_i} - 1\right) \leq \frac{2}{1-c}. \tag{10}$$

$$-\frac{2}{c} \leq \Psi_i\left(\frac{1-r_i}{r_i}\right) \leq \frac{2}{1-c}.$$

If $r_i < 1$

$$-\frac{2r_i}{c\left(1-r_i\right)} \leq \Psi_i \leq \frac{2r_i}{(1-c)\left(1-r_i\right)}.$$

If $r_i > 1$

$$\frac{2r_i}{c\left(r_i-1\right)} \geq \Psi_i \geq -\frac{2r_i}{(1-c)\left(r_i-1\right)}.$$