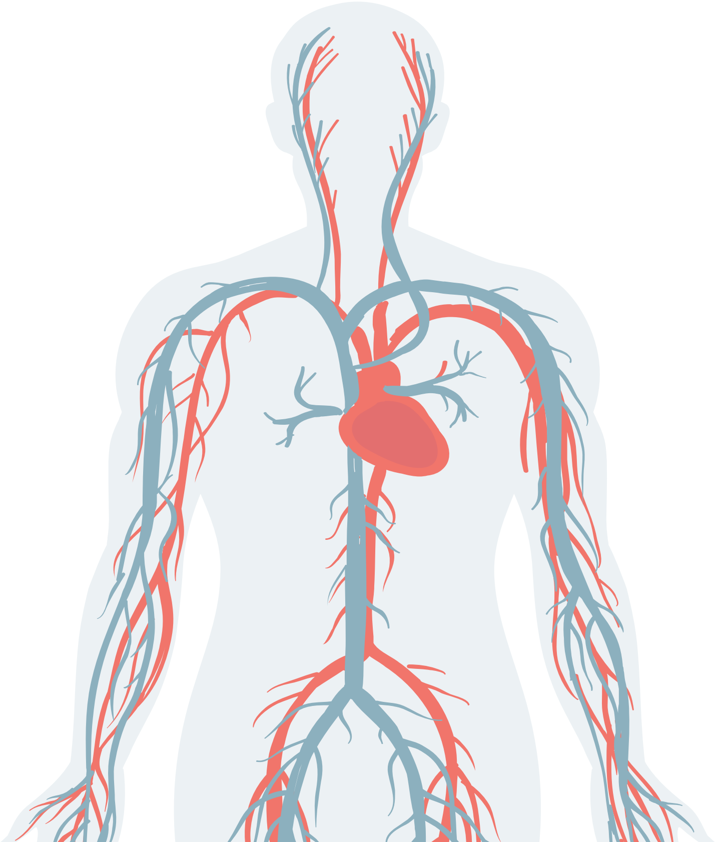
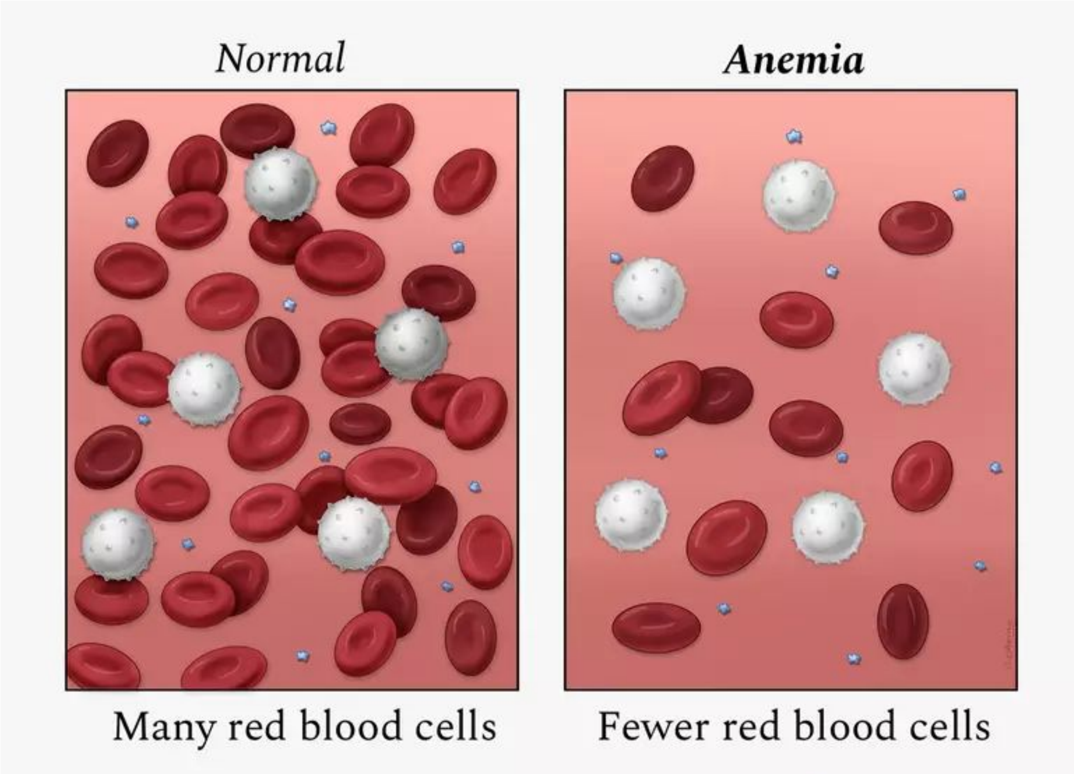
Hemorrhagic Shock Model

Ibrahim Al-Akash, Ruth Hong, Leanne Long, Riya Pagilla, Andrew Sun, Alice Tian, Sam Wu



Why Model Blood Flow?



(Britannica)

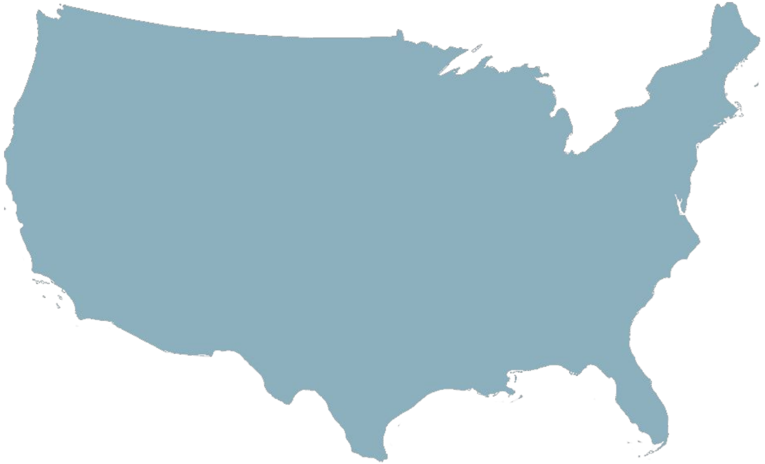
Hemorrhagic Shock

A decrease in intravascular volume from **blood loss** that compromises **cardiovascular function**.

(Hooper et al., 2022)

Hemorrhagic Shock: Motivation

1.5 Million Deaths Worldwide



3rd Leading Cause of Death in the US

(Hooper et al., 2022) (Longlo et al., 2018)

Hemorrhagic Shock: Class II

Hemorrhagic Shock in Blood Loss %

Class I: 0-15%

Class II: 15%-30%

Class III: 30%-40%

Class IV: over 40%

(Hooper et al., 2022)

Hemorrhagic Shock: Class II

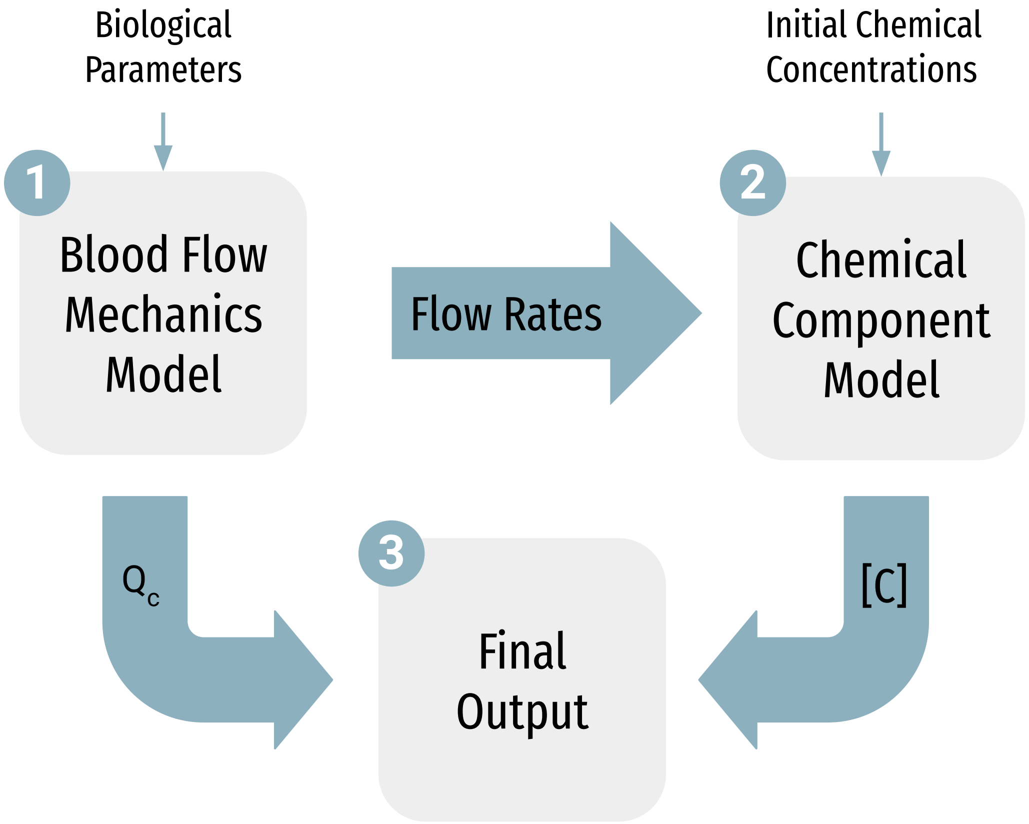
- Track **physiological changes** without overcomplications
- Guide treatment interventions **early on**

(Hooper et al., 2022)

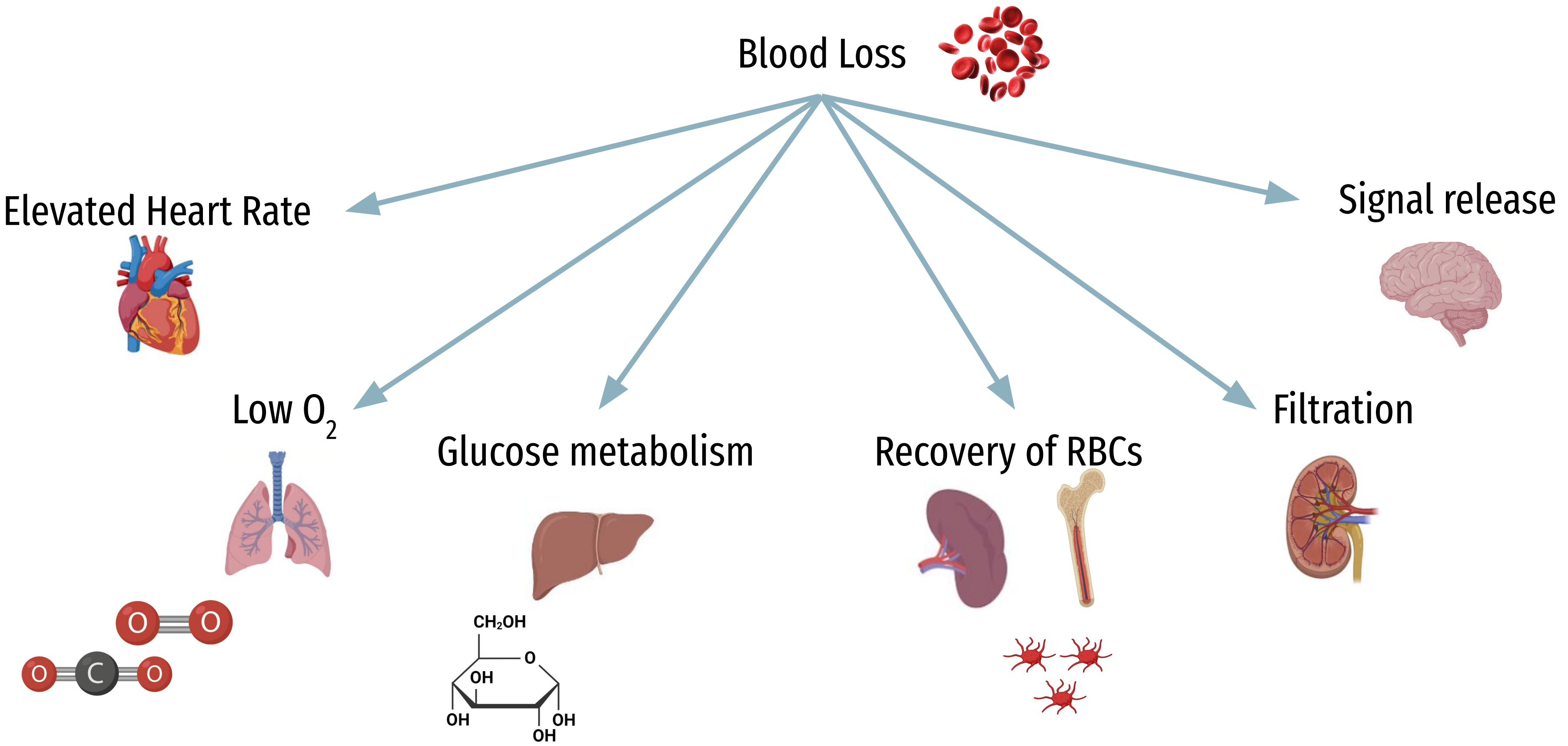
Model Objectives

- Determine blood flow characteristics
- Determine component concentrations
- Determine response to hemorrhagic shock

Model Development



Physiological Response



Organs and Components

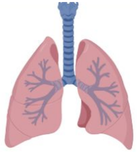
Organs

Heart

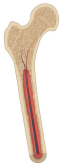
Liver

Brain

Kidneys



Lungs



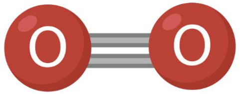
Other

Components

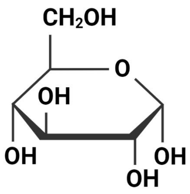
Red Blood Cells

Platelets

Carbon Dioxide



Oxygen



Glucose

The Model

Steady-state and Dynamic

Assumptions

- Model the heart as the left and right side
  - Left: splitter
  - Right: mixer
- No leaks in the organs
- Blood is well mixed
- The liver, kidneys, and brain receive fully oxygenated blood during steady state
- The only reactions in the body are aerobic cellular respiration and glycogenolysis

(Hooper et al., 2022)

Model Review

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Blood Flow Mechanics Model

Fundamental Equations

Navier-Stokes Equations

$$\frac{\partial \boldsymbol{u}}{\partial t} + \rho(\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u} + \boldsymbol{\nabla} P - \mathbf{div}(\mu \boldsymbol{D}(\boldsymbol{u})) = \boldsymbol{f}$$

Conservation of Linear Momentum

$$\mathbf{div}\boldsymbol{u} = 0$$

Conservation of Mass

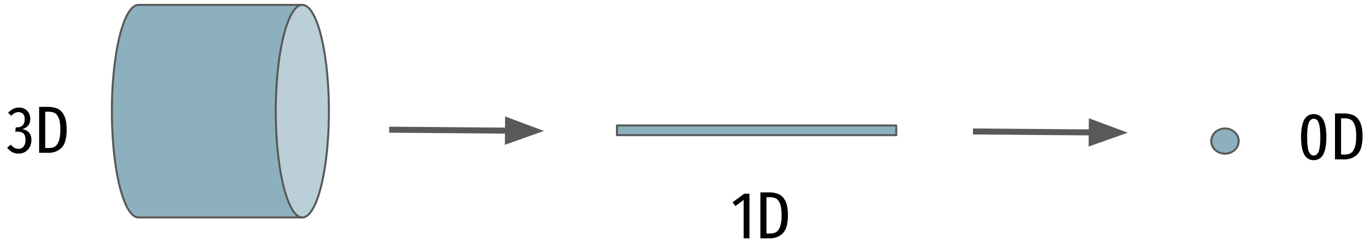
0D Lumped Parameter Simplification

$$L\frac{d\hat{Q}}{dt} + R\hat{Q} + P_2 - P_1 = 0$$

Conservation of Linear Momentum

$$C\frac{d\hat{p}}{dt} + Q_2 - Q_1 = 0$$

Conservation of Mass



(Formaggia et al., 2009)

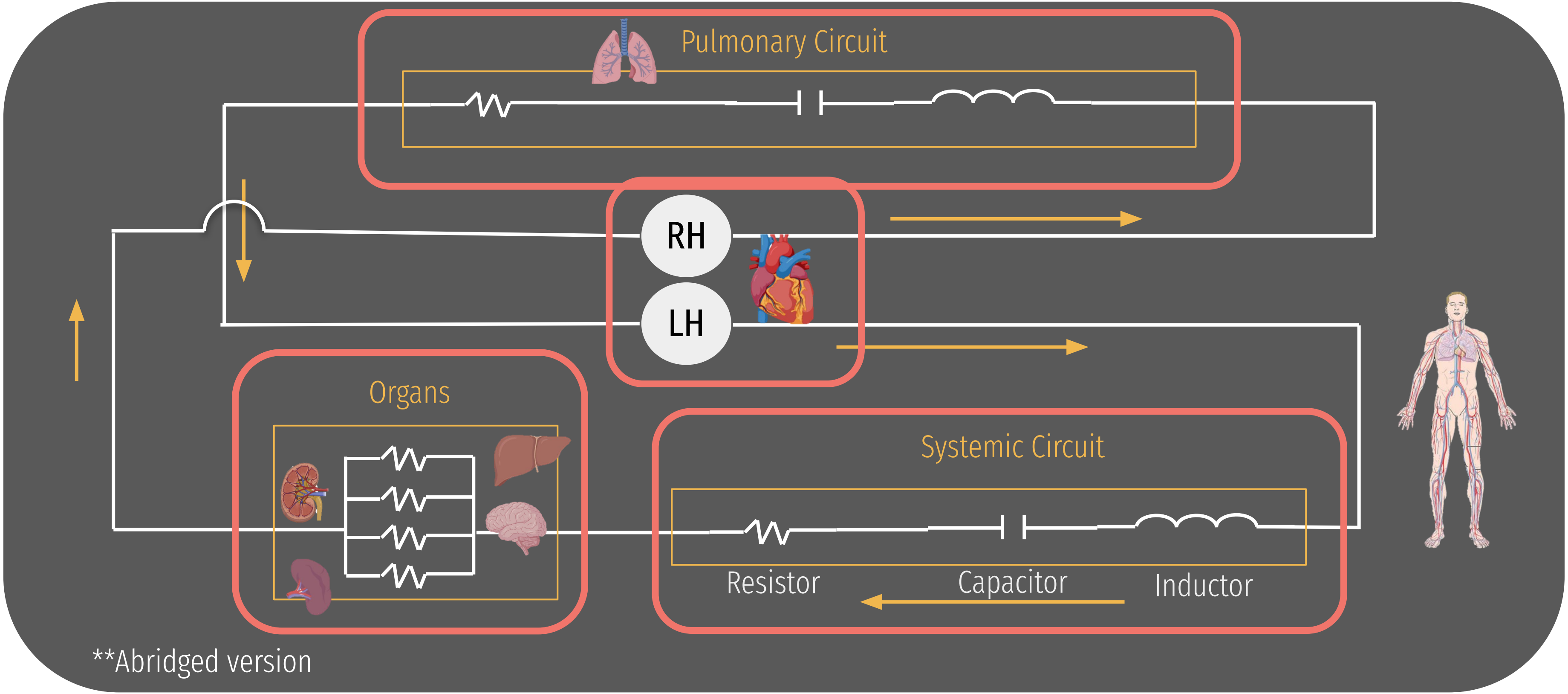
The Circuit Model

Hydraulic	Electric Analog
Pressure (P)	Voltage (V)
Flow Rate (Q)	Current (I)
Blood Volume (V)	Charge (Q)
Blood Viscosity	Resistance *R*
Blood Inertia	Inductance *L*
Mass Storage	Capacitance *C*

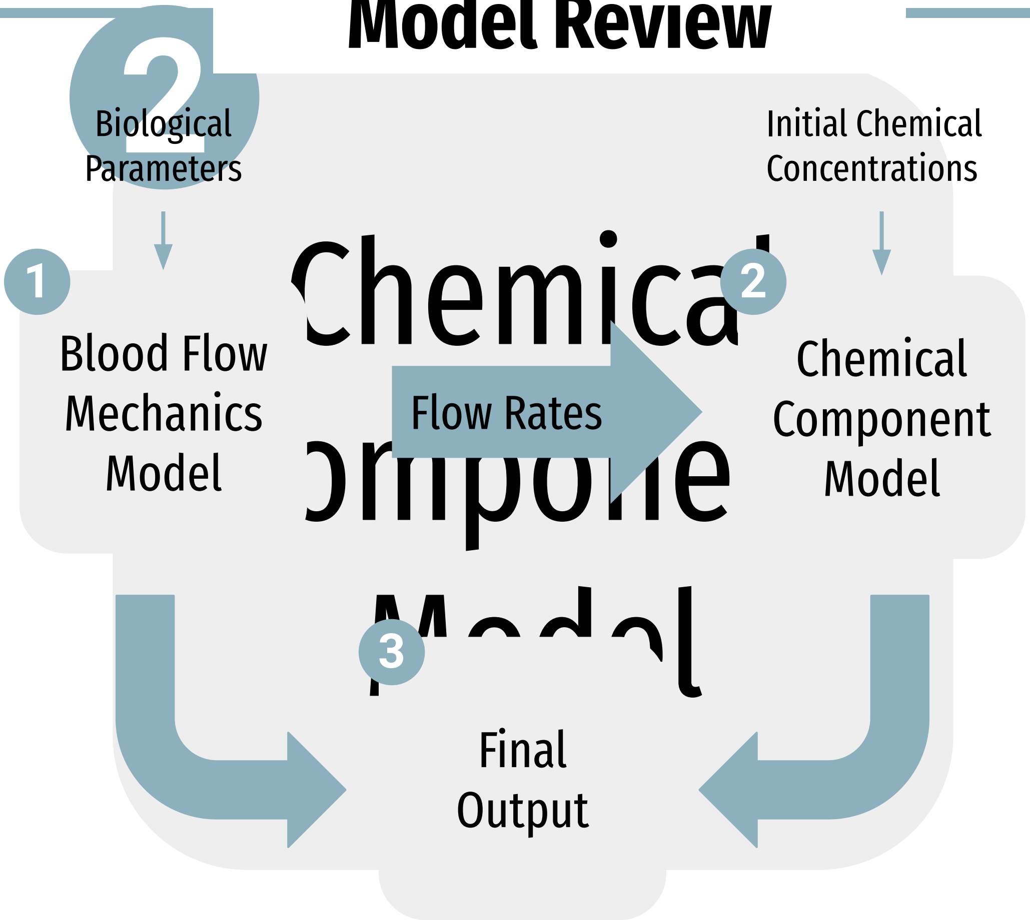
Recall: $$\begin{cases} L\dfrac{d\hat{Q}}{dt} + R\hat{Q} + P_2 - P_1 = 0 \\ C\dfrac{d\hat{p}}{dt} + Q_2 - Q_1 = 0 \end{cases}$$

(Formaggia et al., 2009)

Circulatory Circuit

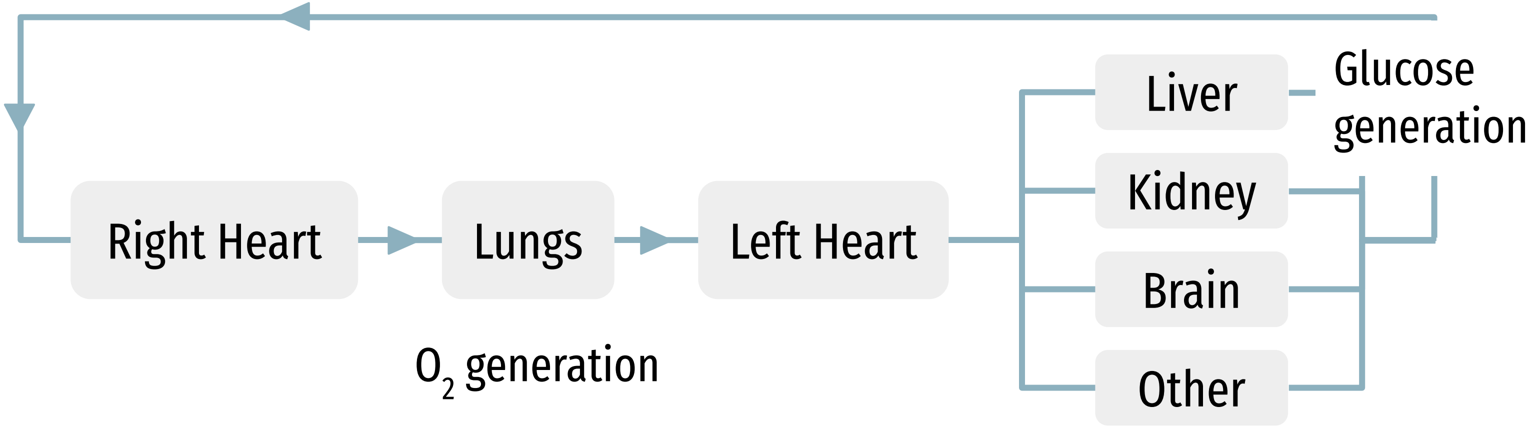


Model Review



Biochemical Model

Venous component concentrations are mixed before recycling into the circuit



$$[C]_{vein} = [C]_{art} \pm \frac{\dot{m}_C}{Q}\Delta t$$

OUT = IN ± CONS/GEN

CO_2 generation
O_2 consumption
Glucose consumption

Modeling Perturbation

Same basic equation for O_2, CO_2, and glucose generation/consumption

Used to inform
- Changes in cardiac output
- Blood flow distribution

Change in Consumption Rate

$$\dot{C}_{cons} = k * \dot{C}_{cons,0}$$

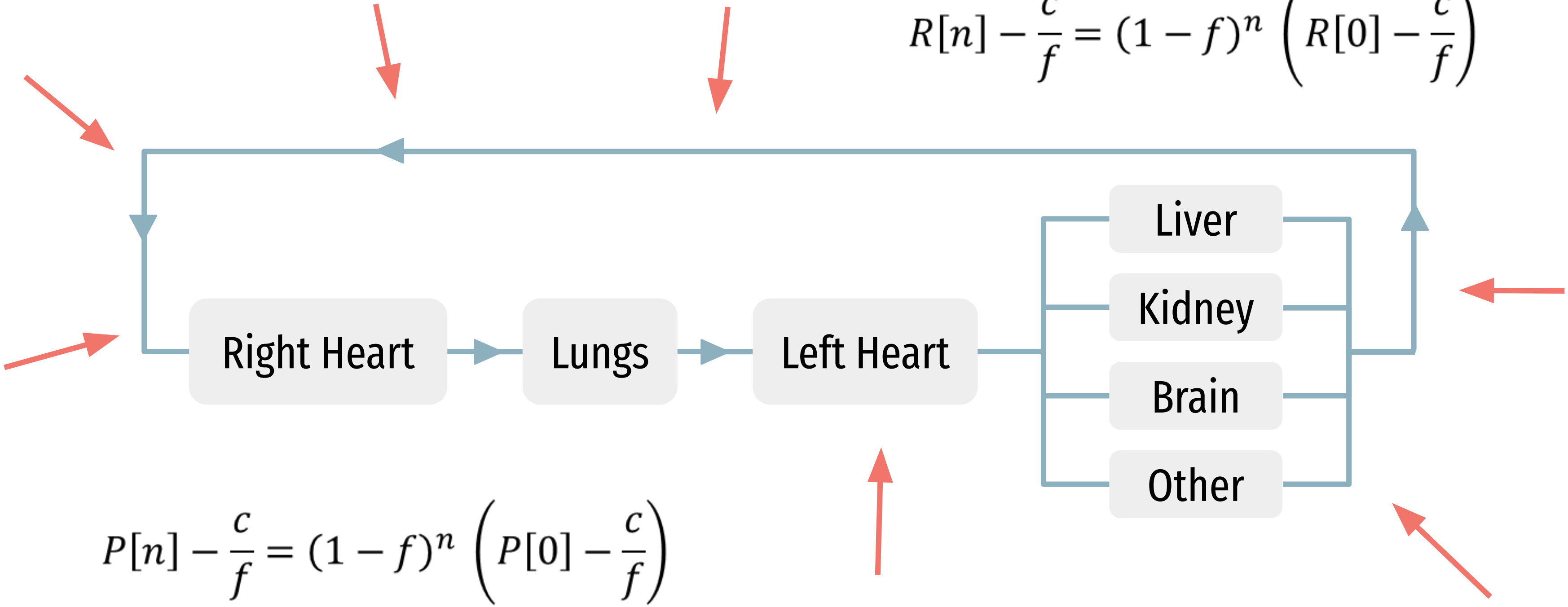
Scaling Factor Calculation

$$k = m \frac{V_{blood}}{V_{blood,0}} + b$$

Different for each organ & component modeled

Biological Model

$$R[n] - \frac{c}{f} = (1-f)^n \left(R[0] - \frac{c}{f}\right)$$



$$P[n] - \frac{c}{f} = (1-f)^n \left(P[0] - \frac{c}{f}\right)$$

(Otto and Day, 2007)

Model Review

3

Final
Output

1

Blc
Me

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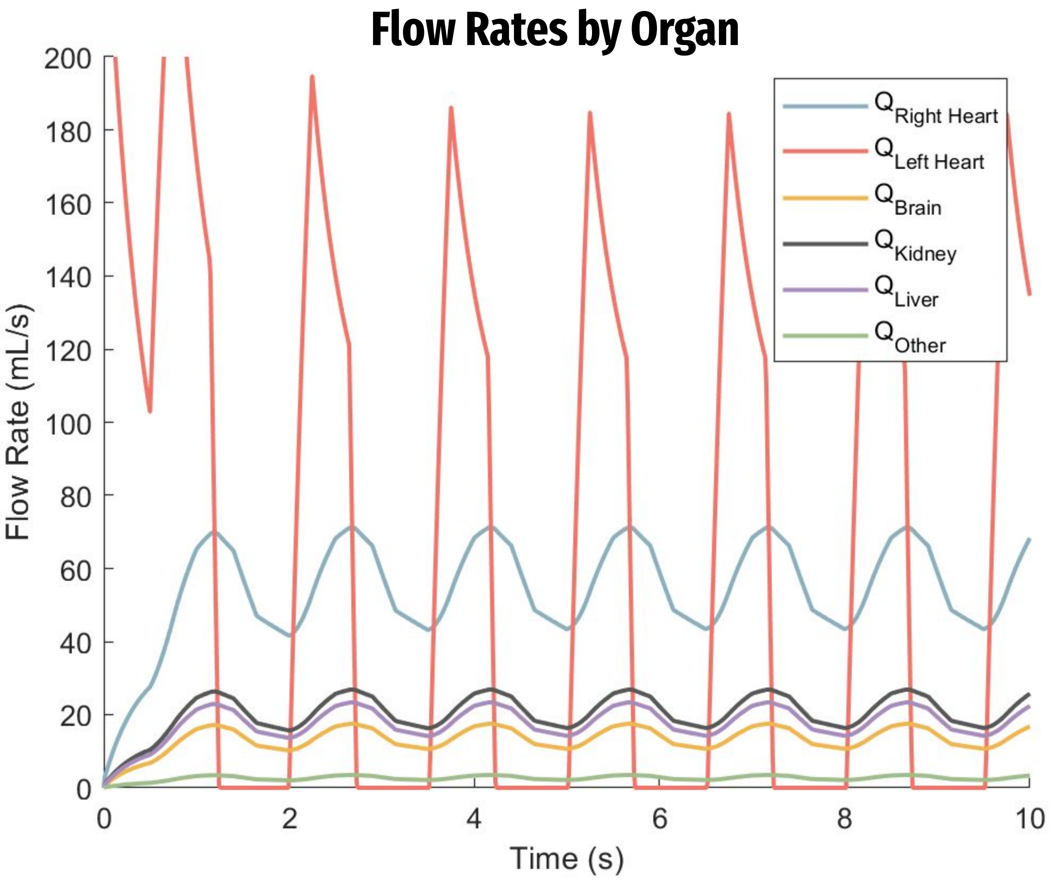
nt

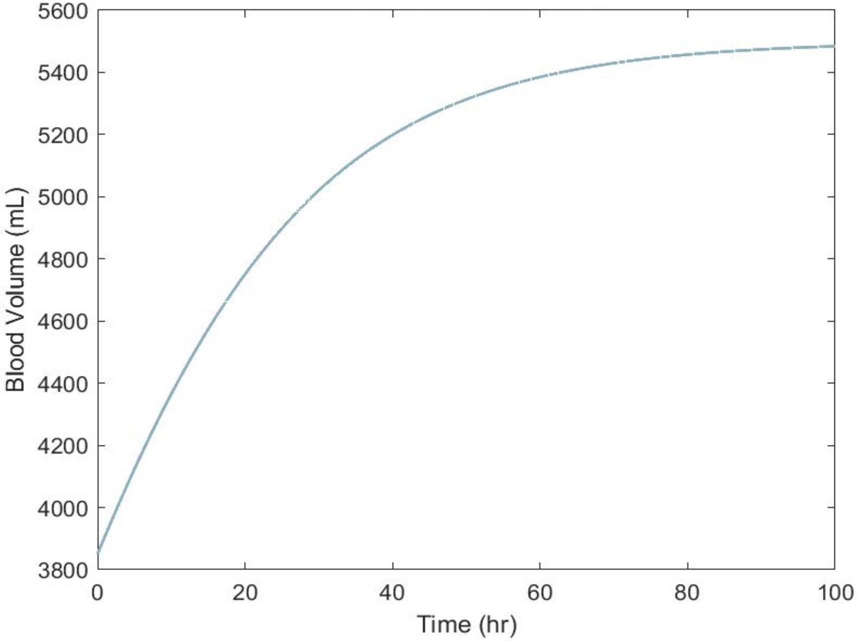
Healthy Output



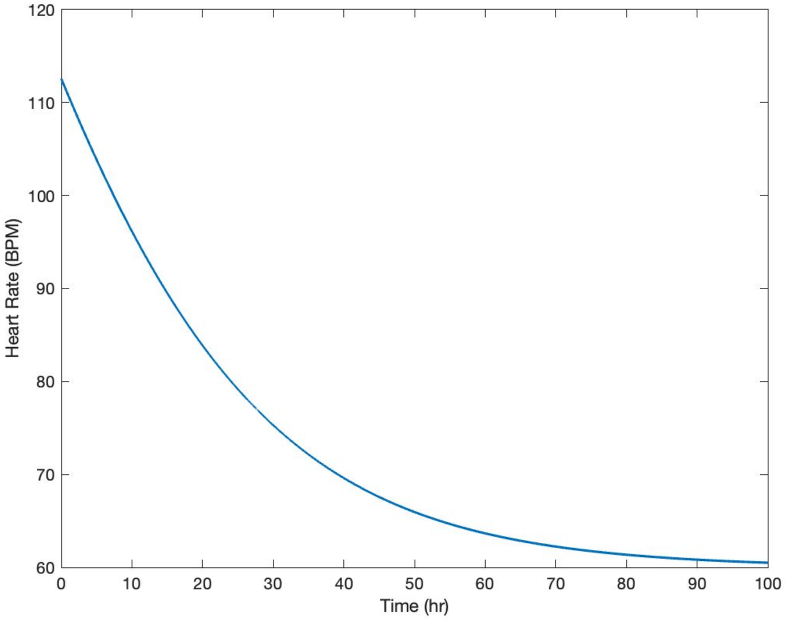
Flow rate is <u>pulsatile</u>, meaning it changes with each beat of the heart.

Blood Loss and Recovery

Blood Volume Recovery

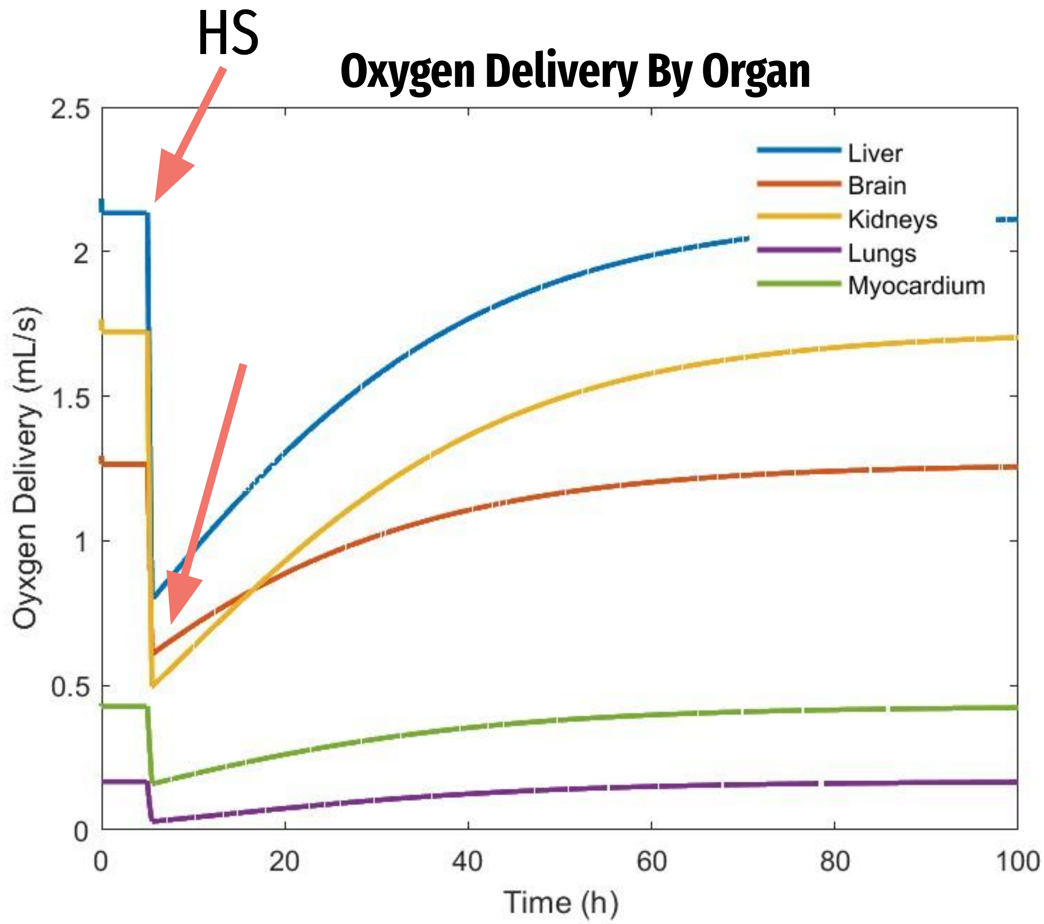


Heart Rate Recovery



Blood volume and heart rate recover after <u>72-100 hours</u>.

O2 Perfusion to Organs

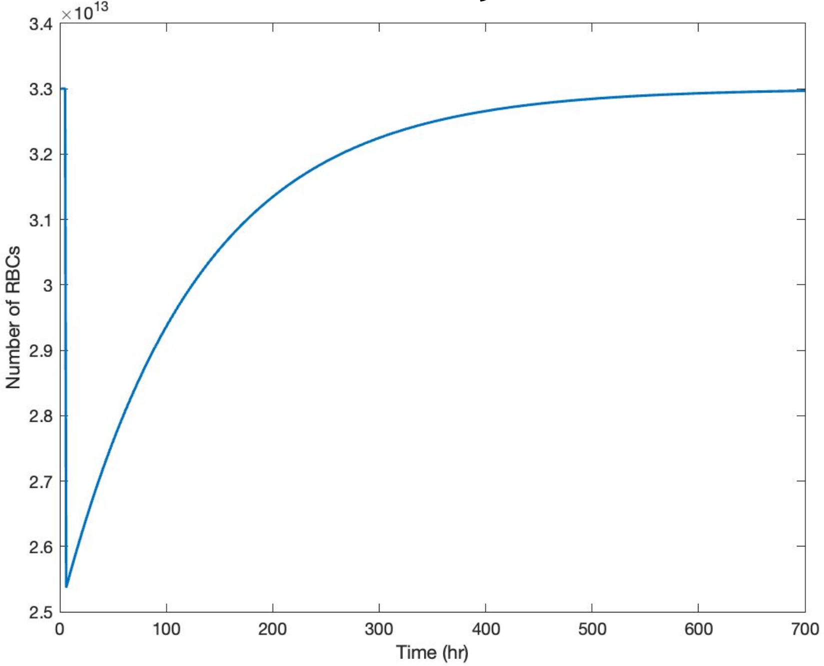


At <u>5 hours</u>, hemorrhagic shock occurs.

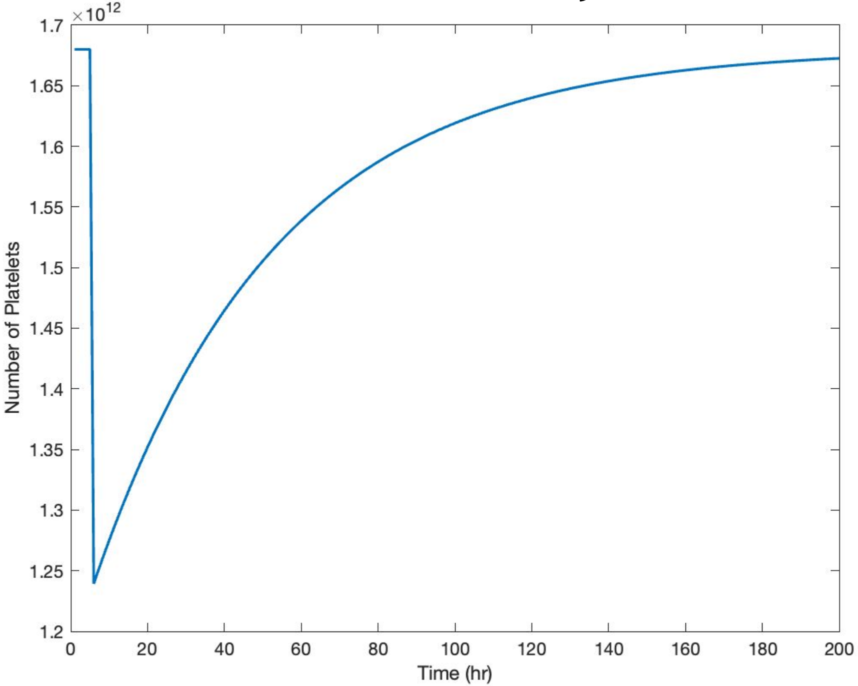
The brain <u>loses a smaller percentage</u> of O_2 as compared to the liver and kidneys.

RBC and Platelet Recovery

RBC Recovery

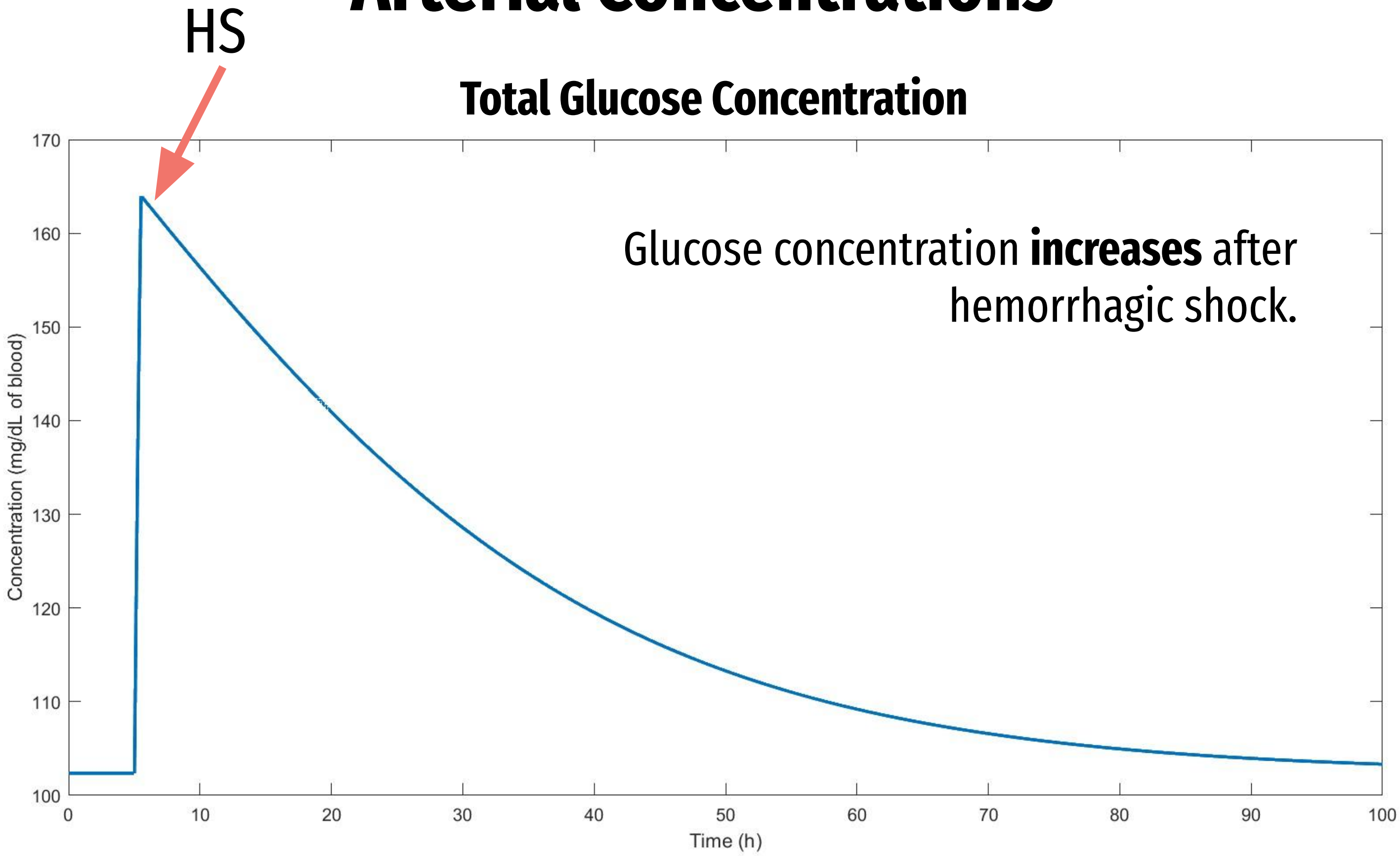


Platelet Recovery

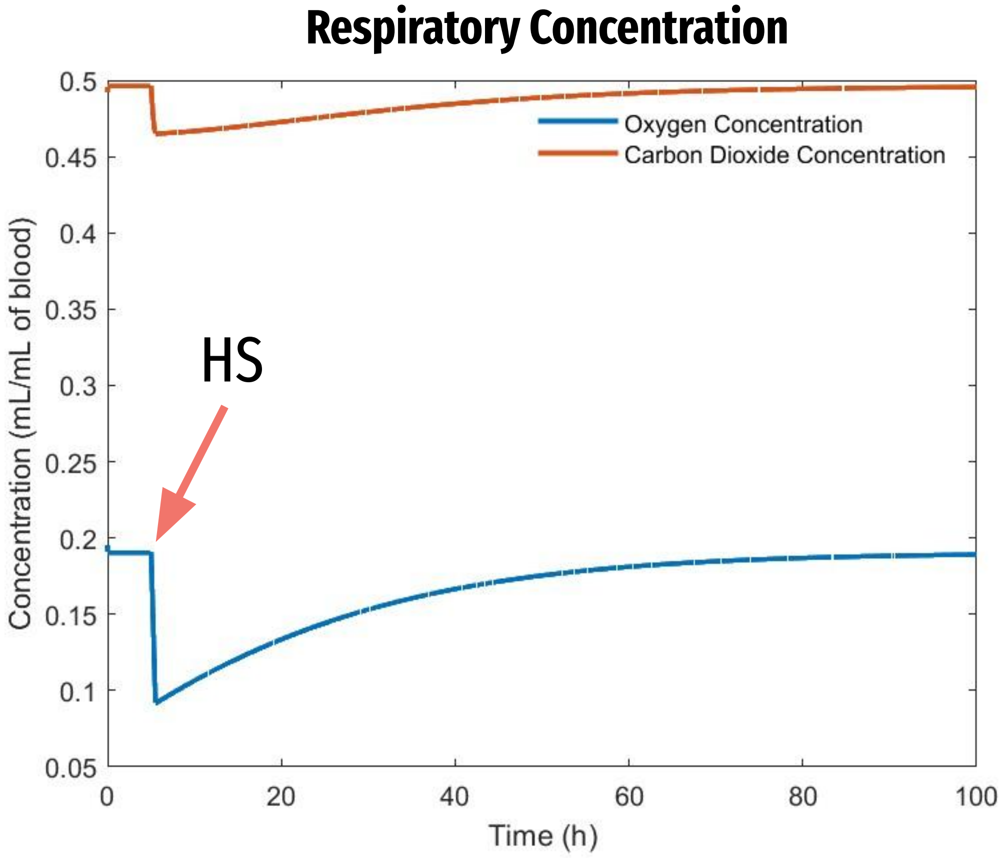


RBC and platelet recovery take longer than other components

Arterial Concentrations



Arterial Concentrations



Respiratory Quotient <u>increases</u> after hemorrhagic shock.

This is reflected in the <u>steeper drop</u> in O_2 concentration.

Results

Interpretation and Importance

Limitations and Future Improvements

- Improve accuracy of flow model
- Human models and further research
- Further work to include all classes of hemorrhagic shock

Conclusions

- Recovery time for almost all components within 100 hrs
- 98% accuracy in blood flow rates compared to literature values
- Blood flow characteristics
    - Body recovers by diverting blood to critical organs
- Component characteristics
    - Rise in glucose levels
    - Platelets and RBC recovery takes longer
- Guide treatment with this information

Acknowledgements

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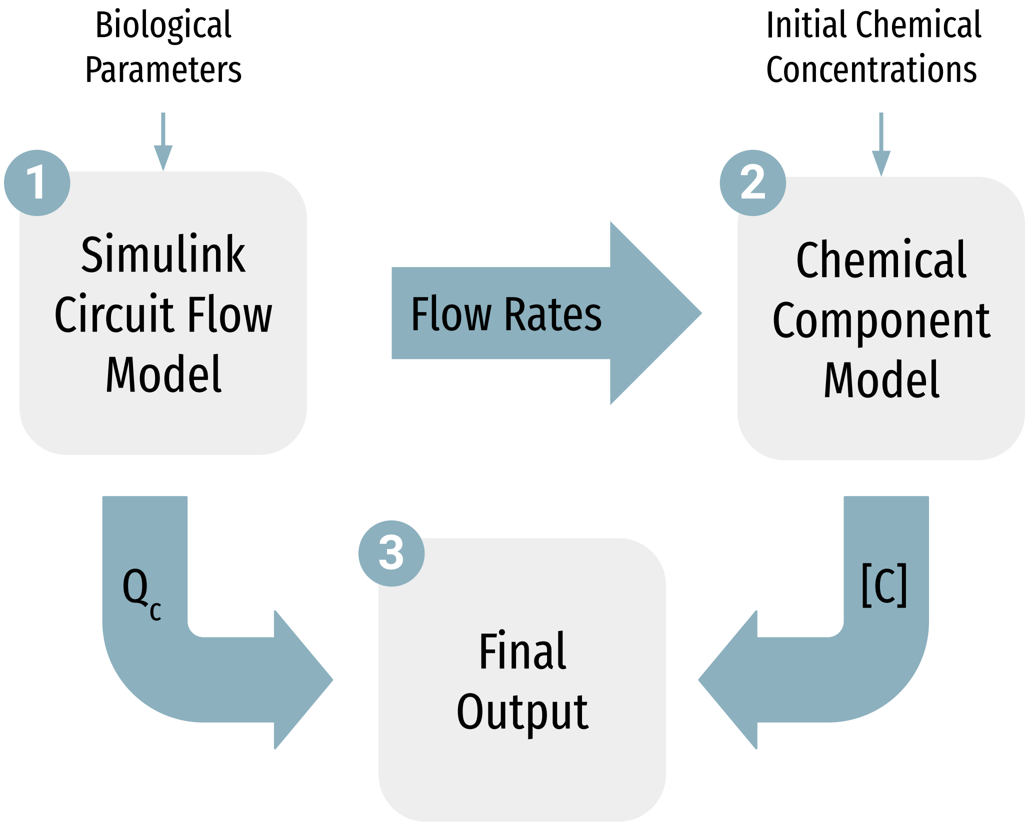
Model in Summary



Hemodynamic Model

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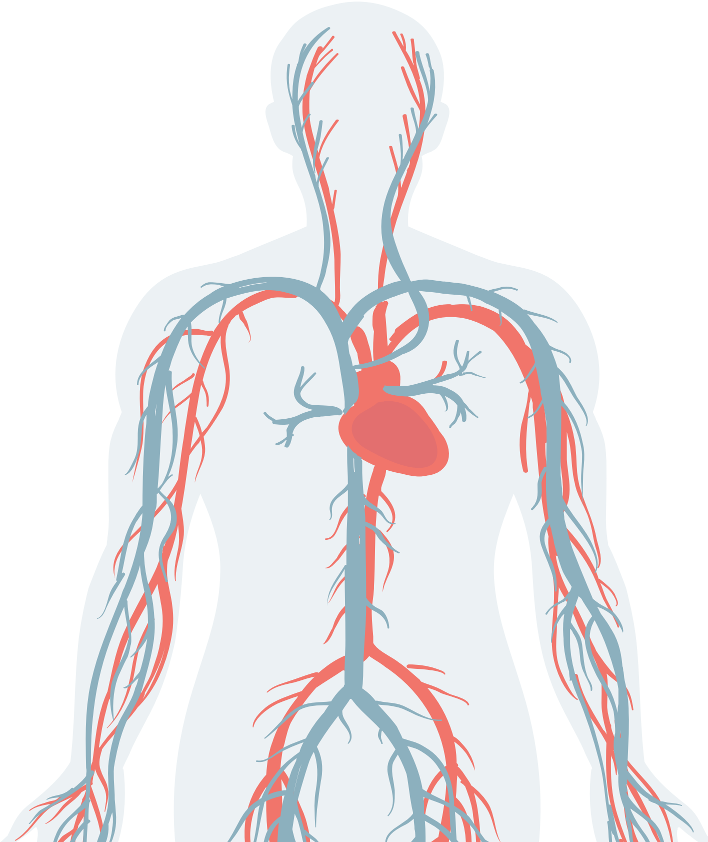
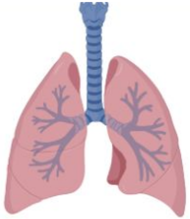


Appendix

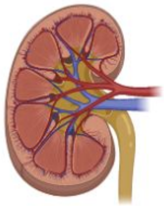
Organs

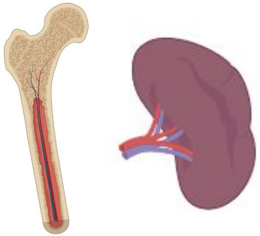


	Heart	Brain	Lungs
Function	Blood flow rate	Nervous control	Exchange of O_2 and CO_2
Relevance	Central pump	Major consumer of components	Supplies O_2
Response	100 BPM → 120 BPM	Directs response	20 → 24 breaths per minute

(Hooper et. al, 2022) (Silverthorn et. al, 2019) (Armentano et. al, 2019)

Organs

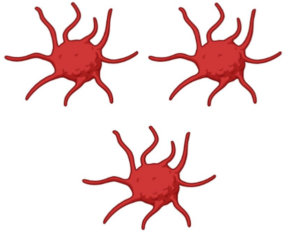




	Liver	Kidneys	Other
Function	Extracts nutrients	Filters waste products	Generates/consumes components
Relevance	Glucose metabolism	Blood filtration	Generates/consumes RBCs & platelets
Response	Increased glucose production	Increased HR → Decreased glomerular filtration rate (GFR)	Increased production and release of RBCs & platelets

(Holzrichter et. al, 1987) (Lautt, 2009) (National Kidney Foundation) (Tantisattamo et. al, 2021) (Arias 2017) (Marieb & Hoehn, 2013)

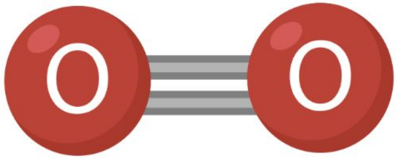
Components

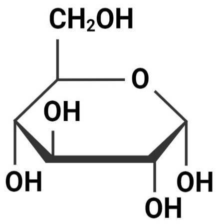


	Red Blood Cells	Platelets
Function	45% of blood by volume	<1% of blood by volume
Relevance	Defines severity of shock	Crucial due to blood clotting abilities
Response	**30% Blood volume loss**	Initial loss then increase in platelets

(Marieb et. al, 2013) (Wannberg et. al 2020) (Pretini et. al 2019)

Components





	O2	CO2	Glucose
Function	Inhaled from surroundings	Exhaled as waste product	Main source of energy
Relevance	Aerobic cellular respiration	Aerobic cellular respiration	Used in metabolic processes
Response	Respiratory rate increases	Respiratory quotient increases	Hypermetabolic response → rise in blood glucose levels

(Wright et. al) (Winkelmann et. al) (Hooper et. al)

Simplifying Assumptions

Assuming Poiseuille Flow and Perfectly Cylindrical Vessel

$$L\frac{d\hat{Q}}{dt} + R\hat{Q} + P_2 - P_1 = 0$$

$$C\frac{d\hat{p}}{dt} + Q_2 - Q_1 = 0$$

4 Unknowns: $\hat{Q}, \hat{p}, P_1$ and Q_2
2 Equations: **Underspecified**

$$R = \frac{8\mu l}{\pi r_0^4}$$

Resistance induced to the flow by the blood viscosity

$$L = \frac{\rho l}{\pi r_0^2}$$

Inertial term in the momentum equation

$$C = \frac{3\pi r_0^3 l}{2Eh_0}$$

Mass storage term in the mass conservation law due to compliance of vessel

$$\hat{p} \approx P_1, \quad \hat{Q} \approx Q_2$$

Eliminates two unknowns to become a correctly specified system of equations

$$C\frac{dP_1}{dt} + Q_2 = Q_1,$$
$$L\frac{dQ_2}{dt} + RQ_2 - P_1 = P_2$$

Reduced equations
2 Unknowns: P_1 and Q_2
2 Equations: **Correctly Specified**

41
41

Biochemical Dynamics

Venous Concentration of Chemical Species

$$[C]_v = \sum_{i \in \{sm,sp,o\}} \frac{\dot{Q}_i}{\dot{Q}_a} [C]_{v,i}$$

V- tissue volume,
A- stoichiometric coefficient matrix
ψ- reaction rate vector
c- concentration of the species

Arterial Concentration of Chemical Species

$$V_i \frac{\mathrm{d}\mathbf{c}_i}{\mathrm{d}t} = \mathbf{A}\boldsymbol{\psi}_i(\mathbf{c}_i, t) + \mathbf{b}_i(\mathbf{c}_i, \mathbf{c}_{a,i}, \dot{Q}_i, t)$$

$$\mathbf{b}_i(\mathbf{c}_i, \mathbf{c}_{a,i}, \dot{Q}_i, t) = \dot{Q}_i(\mathbf{c}_{a,i} - \boldsymbol{\sigma}_i.\mathbf{c}_i)$$

b- mass transfer term
$\dot{Q}$- blood flow rate
σ- partition coefficient
i- the compartment of interest, such as kidneys, liver, brain, spleen, etc.

(D’Angelo et al., 2005)