Assignment 4 _Solutions_6637

11/19/2024

1) In the most recent (week10) class workspace, you will find a dataset called theories which represents data from a theories of intelligence scale. The items are at the end of this assignment.

Fit the generalized partial credit model to these data.

```
mirt(theories,1,"gpcm")->m1
```

a) Which item do you think is the most "difficult". Why? Check to see if it has the lowest raw rating of the 8 items.

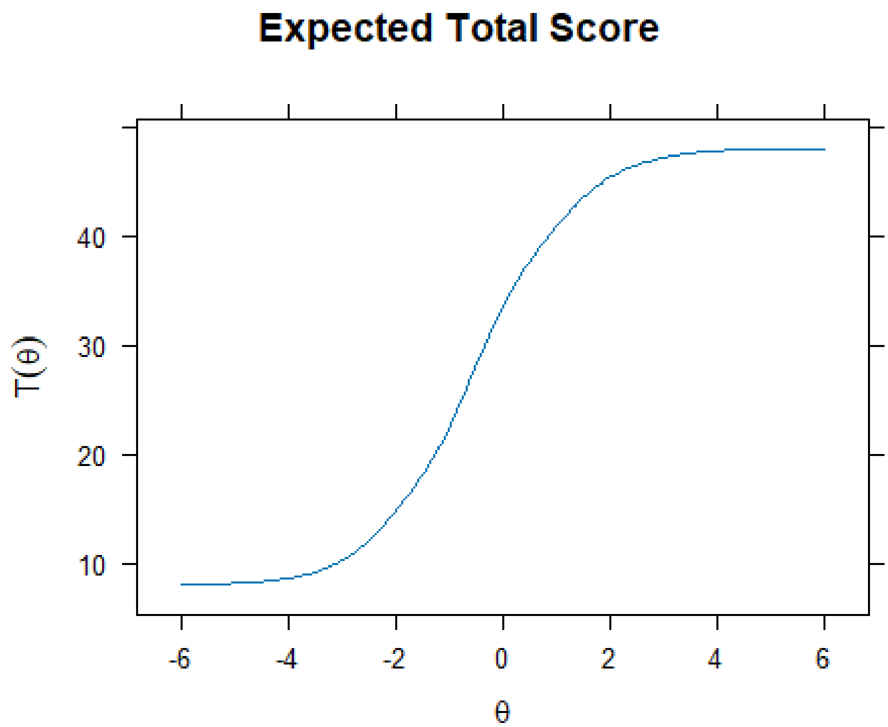
```
round(coef(m1, IRTpars=T, simplify = T)$items,2)

##             a    b1    b2    b3    b4   b5
## intell_1 2.00 -2.25 -1.04 -0.31 -0.18 1.29
## intell_2 2.49 -2.21 -1.11 -0.55 -0.11 1.16
## intell_3 1.85 -2.54 -1.12 -0.62  0.00 1.24
## intell_4 2.96 -2.20 -1.17 -0.67 -0.09 1.15
## intell_5 1.90 -2.38 -0.93 -0.51  0.32 1.52
## intell_6 1.42 -1.95 -0.69  0.08  0.47 2.32
## intell_7 1.17 -3.23 -1.59 -1.10 -0.04 1.75
## intell_8 1.48 -2.65 -1.04 -0.52  0.31 1.95
```

If we look at the thresholds in the IRT parameter format (b is a difficulty parameter), we see that thresholds for item 6 are farthest to the right. This means higher levels of θ are required to score in the higher categories as compared with the other items. Checking the item means (the column averages), we see that indeed item 6 has the lowest score.

```
round(colMeans(theories),2)

## intell_1 intell_2 intell_3 intell_4 intell_5 intell_6 intell_7 intell_8
##     4.07     4.18     4.15     4.24     3.91     3.49     4.27     3.90
```

b) Plot the overall score as a function of theta. (Hint –this the default for plot(model)) What is the approximate scale score for someone with a theta score of 0?

```
plot(m1)
```



The expected total test score at $\theta = 0$ seems to be roughly 30. (The exact answer is found by summing the 8 item response functions and their respective response option probabilities at $\theta = 0$, but that is a lengthy calculation.)

c) Here's some code to find the 20 worst fitting people to the model:

order(personfit(gpctheory)[,5])[1:20]

What is the likely issue with the data from the very worst fitting people?

```
theories[order(personfit(m1)[,5])[1:10],]

##      intell_1 intell_2 intell_3 intell_4 intell_5 intell_6 intell_7
intell_8
## 591         1        1        6        1        6        1        6
6
## 605         1        1        6        1        6        1        6
6
## 788         1        1        6        1        6        1        6
6
## 792         1        1        6        1        6        1        6
6
## 900         1        1        6        1        6        1        6
6
## 1437        1        1        6        1        6        1        6
6
## 582         3        1        6        1        6        1        6
6
```

```
## 951          6        1        1        5        6        6        4
5
## 852          6        6        1        2        2        1        6
2
## 1212         1        5        6        6        5        1        4
1
```

The most poorly fitting people seem to have some very contradictory responses. The worst six people disagree strongly with the idea that you can't change intelligence much (first two items) - but then they fully endorse that anyone can significantly change their intelligence (item 3). It is very likely that these people just answered down one side of the scale, indifferent to the fact that the items were worded in different directions, affirming and denying the growth mindset view. Remember that we see their responses shifting sides here, but that is *after* the reverse scoring.

d) Rerun the model, now eliminating these cases:

```
mirt(theories[-order(personfit(m1)[,5])[1:20],],1,'gpcm')->m1b
```

Give an overall comparison to the results from your previous model. Consider the following: model coefficients, information plot, item fit statistics, residual correlations.

There's a lot to go through here - but here's a quick rundown.

Coefficients: comparing the slopes and thresholds from the analysis with, and without, the 20 biggest outliers are similar in pattern but slightly higher signaling improved discrimination.

Information function: The plots of the information functions for the two models are similar, but slightly higher after removing outliers.

Item fit statistics: In both models, the items have substantial amounts of misfit. When looking at the default S-X2 (see below), the degree of misfit is reduced after dropping the outliers on person fit.

```
itemfit(m1)

##       item     S_X2 df.S_X2 RMSEA.S_X2 p.S_X2
## 1 intell_1 117.736      69      0.020      0
## 2 intell_2 173.856      66      0.031      0
## 3 intell_3 203.631      72      0.032      0
## 4 intell_4 153.913      59      0.030      0
## 5 intell_5 151.717      71      0.026      0
## 6 intell_6 215.150      80      0.031      0
## 7 intell_7 281.867      82      0.037      0
## 8 intell_8 156.613      75      0.025      0

itemfit(m1b)

##       item     S_X2 df.S_X2 RMSEA.S_X2 p.S_X2
## 1 intell_1 105.043      66      0.019  0.002
```

```
## 2 intell_2 137.869      61      0.027  0.000
## 3 intell_3 166.609      66      0.030  0.000
## 4 intell_4 109.797      55      0.024  0.000
## 5 intell_5 109.600      65      0.020  0.000
## 6 intell_6 198.328      77      0.030  0.000
## 7 intell_7 202.197      77      0.031  0.000
## 8 intell_8 117.274      72      0.019  0.001
```

Residuals: Here's another place where there is substantial difference. Comparing the residual correlations ("Q3" statistic with residuals), they are slightly, but consistently, smaller in the model without the outliers. The LD (local dependence) X2 are huge for both models, but substantially smaller when the bad response patterns are dropped.

In summary - removing 1 percent of the poorest fitting data does seem to improve things a bit. The presence of outliers is impacting issues for item fit and residual analysis. It makes sense that these wildly out of character responses would contribute to local dependence. However, it is important also to note that the LD and item fit statistics are still signalling quite a few issues with fit even after the outliers are removed.

e) For kicks, what was the best fitting response pattern from the first model.

First, find out which pattern has the highest fit. We can reverse the ordering from the earlier command, and find the 10 best fits.

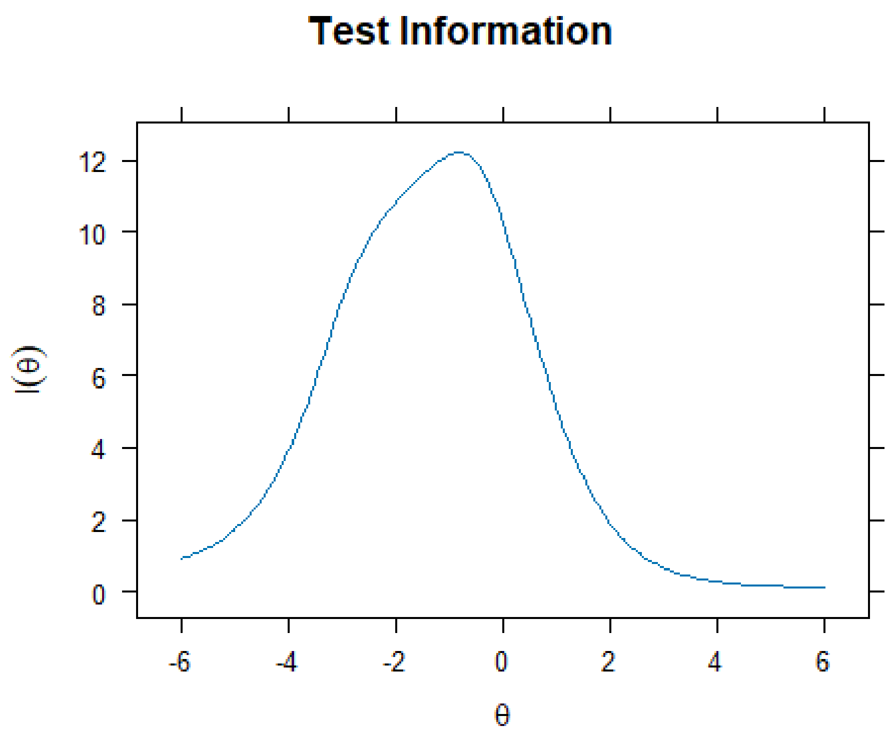
```
theories[order(personfit(m1)[,5], decreasing = T)[1:10],]

##      intell_1 intell_2 intell_3 intell_4 intell_5 intell_6 intell_7
intell_8
## 716         3        4        4        4        4        3        4
4
## 1122        3        4        4        4        4        3        4
4
## 123         2        2        2        2        2        2        2
2
## 206         2        2        2        2        2        2        2
2
## 299         2        2        2        2        2        2        2
2
## 349         2        2        2        2        2        2        2
2
## 628         2        2        2        2        2        2        2
2
## 634         2        2        2        2        2        2        2
2
## 748         2        2        2        2        2        2        2
2
## 815         2        2        2        2        2        2        2
2
```

The above list comes from the model with all students in it. It seems that the best fitting response patterns show a lot of continuity across the responses categories. No wild swings from side to side of the response scale.

2) Fit the nominal model to the fci data (Force Concept Inventory). To use the post-test items only, fit the nominal model this way: mirt(fci[,34:63],1,'nominal').

a) Plot the information function for your model. What is the approximate peak of the information curve.

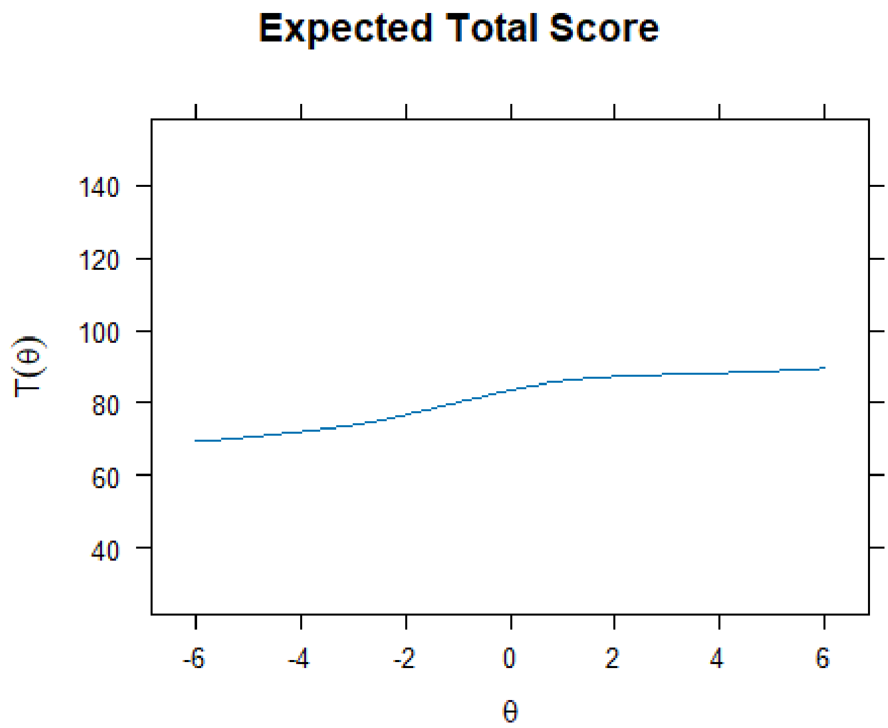
```
mirt(fci[,34:63],1,"nominal")->mQ2

plot(mQ2, type="info")
```



The information function peaks at around height of 12, somewhere in the $\theta = -1$ area.

b) Show the result of plot(model). What pattern do you see - does this make sense? Explain what you think is going on.

```
plot(mQ2)
```



This is a strange looking plot. In the previous question, we showed that the test total score as a function of θ had a strong monotonic relationship. Indeed, the test function is just the sum of all the item response functions. Here the plot seems flat, and stuck between a narrow range of points.

The issue is that the individual item responses, of 1 through 5, are *nominal* and do not have a low to high ordinal, let alone interval, interpretation. For some questions 1 is the right answer, and for some it is 5. Adding together the item responses just gets you a meaningless mess - and that's reflected in the plot. The plot "makes sense" in that it is understandable why it looks flat - but it has no actual interpretation because summing nominal responses for a total score doesn't make sense.

c) On this version of the FCI – items 5 and 18 are actually almost identical in the specific concept they assess. They also have the same correct response (choice 2 for both) and same main distractor (choice 4 for both). Examine the data and your model diagnostics to find evidence consistent with the claim that they are highly similar items.

If items 5 and 18 are almost identical, both in content, and in the correct and most popular incorrect nominal answers, then they should have some shared features. They will have an excess similarity, more than can be explained simply due to their common dependence on θ.

As a starting point, we can make a table of the raw response options.

```r
table(fci[,c("Post.Q5","Post.Q18")])

##        Post.Q18
## Post.Q5    1    2    3    4    5
##       1   27  103    6   17    7
##       2   31 1252   30  108   45
##       3   10  118   38  105   42
##       4   14  242   24  426   88
##       5    5   72   21  125   92
```

Here we see two large cells - the people who got both items correct (1252 people in the 2-2 cell), and people who got the answer wrong in the same way (426 people in the 4-4 cell). It's possibly interesting that more than half of the 3048 students fall into two of the possible 25 cells in the table. However, this pattern could also happen for very easy items, or very difficult items where there are a lot of similar responses due to floor and ceiling effects.

A second place we could look could be the correlation between items 5 and 18 in terms of correct/incorrect. We can't do this in the nominal data set for the reasons discussed above. But we can look to the 0 = wrong, 1 = correct data set. Interestingly, if we look at all the correlations among the 30 items, the correlation between 5 and 18 is highest.

```r
cor(fcirw[,c(34:63)])->temp
diag(temp)<-0
max(temp)

## [1] 0.5233228

temp[5,18]

## [1] 0.5233228
```

That's pretty telling. Usually the highest correlating items on a scale are somehow excessively similar. They might have similar wording, or be asking about the same aspect of a broad construct.

An important way to evaluate this "extra similarity" is to see if the model residuals for items 5 and 18 have a high correlation. That is, after fitting the model and fully conditioning on θ - do the responses for items 5 and 18 still correlate highly? Under the assumption that the items are conditionally *independent* given θ, they are not expected to correlate at all, let alone highly.

We can look at the residuals for the nominal model. The residuals indicate an extra correlation among the responses to items 5 and 18, even after controlling for ability.

```r
residuals(mQ2) ->modresiduals

## LD matrix (lower triangle) and standardized values.
##
## Upper triangle summary:
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
##  -0.171  -0.059   0.050   0.015   0.067   0.224 
## 
##          Post.Q1 Post.Q2 Post.Q3 Post.Q4 Post.Q5 Post.Q6 Post.Q7 Post.Q8
## Post.Q1       NA   0.116  -0.048  -0.049   0.044  -0.044   0.047   0.039
## Post.Q2  163.944      NA  -0.071  -0.084   0.074   0.085  -0.043   0.096
## Post.Q3   27.569  61.947      NA  -0.059   0.054   0.060   0.051   0.047
## Post.Q4   29.544  85.514  42.293      NA  -0.072  -0.056   0.059  -0.068
## Post.Q5   23.806  67.585  34.929  63.666      NA   0.080   0.057   0.064
## Post.Q6   24.077  87.980  44.217  38.366  78.515      NA   0.203   0.082
## Post.Q7   27.127  22.393  31.726  42.918  39.275 501.927      NA  -0.070
## Post.Q8   18.608 112.501  27.041  56.776  50.171  81.842  59.687      NA
## Post.Q9   23.958  37.517  26.300  43.211  67.340  27.440  28.342 375.896
## Post.Q10  37.761  64.395  33.083  54.752  46.859  64.819  24.891 110.457
## Post.Q11  70.757  62.483  26.156  41.437 219.847  52.902  32.928  45.311
## Post.Q12  38.351  73.529  41.096  32.530  53.630  32.030  33.282  40.212
## Post.Q13  36.816  72.408  66.433  24.529 190.007  83.648  41.373  52.981
## Post.Q14  35.453  28.633  20.596  82.021  25.190  95.059  47.041 134.250
## Post.Q15  67.650  51.124  22.466 275.144  64.981  38.523  49.644  70.747
## Post.Q16  20.486  27.216  43.170  51.238  45.384  53.923  25.587  42.152
## Post.Q17  41.537  31.436  59.794  81.953  55.512  99.014  24.076  61.304
## Post.Q18  44.315  75.070  29.652  46.846 612.591  66.709  38.102  59.626
## Post.Q19  15.146  22.156  26.296  25.990  44.984  63.153  33.992  24.728
## Post.Q20  17.003  27.468  19.456  14.369  60.145  42.036  16.475  24.543
## Post.Q21  14.046  37.070  16.048  55.263  61.788  39.568  41.203 278.674
## Post.Q22  48.103  45.888  64.332  98.131  57.504  44.529  27.912  33.263
## Post.Q23  15.451  46.288  11.479  43.393  31.463  52.860  28.941 163.655
## Post.Q24  41.229  29.999  63.312  72.501  50.985  63.231  66.300  63.617
## Post.Q25  21.474  47.981  41.820  59.847  88.735  40.785  47.612  55.415
## Post.Q26  30.270  70.556  18.120  65.539  45.244  45.612  17.773  52.246
## Post.Q27  22.515  31.994  28.730  68.018  38.823  32.703  28.653  46.969
## Post.Q28  23.821  51.180  25.095 266.815  96.768  42.363  36.128  38.865
## Post.Q29  13.566  48.879  32.631  33.209  78.529  28.612  13.863  14.666
## Post.Q30  37.346  41.672  21.000 105.275 178.357  80.525  36.228  88.569
##          Post.Q9 Post.Q10 Post.Q11 Post.Q12 Post.Q13 Post.Q14 Post.Q15
Post.Q16
## Post.Q1    0.044    0.056    0.076    0.056   -0.055   -0.054    0.074
0.041
## Post.Q2    0.055    0.073    0.072    0.078   -0.077   -0.048    0.065   -
0.047
## Post.Q3   -0.046    0.052   -0.046    0.058   -0.074   -0.041    0.043
0.060
## Post.Q4   -0.060    0.067   -0.058    0.052   -0.045   -0.082   -0.150   -
0.065
## Post.Q5    0.074    0.062   -0.134    0.066   -0.125    0.045    0.073
0.061
## Post.Q6   -0.047    0.073   -0.066    0.051   -0.083    0.088    0.056
0.067
## Post.Q7   -0.048    0.045    0.052    0.052   -0.058   -0.062    0.064
0.046
```

```
## Post.Q8    0.176    0.095   -0.061    0.057   -0.066    0.105    0.076    
0.059
## Post.Q9       NA    0.052   -0.075    0.069   -0.075   -0.056    0.113    
0.049
## Post.Q10  32.462       NA   -0.046    0.059    0.052    0.075   -0.078   -
0.048
## Post.Q11  67.821   25.807       NA    0.124    0.130   -0.035    0.090    
0.056
## Post.Q12  57.659   42.902  186.020       NA    0.076   -0.077    0.104    
0.059
## Post.Q13  68.571   33.168  205.535   70.084       NA   -0.066   -0.123    
0.069
## Post.Q14  37.802   68.556   14.554   72.099   53.183       NA    0.065    
0.056
## Post.Q15 156.844   74.376   98.895  132.240  185.231   52.070       NA   -
0.112
## Post.Q16  29.831   28.231   38.841   42.518   58.397   38.015  153.271    
NA
## Post.Q17  30.328   62.754   38.718   30.172   66.268   77.360  159.397  
131.625
## Post.Q18  60.095   39.638  226.892   77.695  169.792   76.689   66.466   
47.410
## Post.Q19  33.864   37.778   61.007   35.937   50.939   47.504   78.602   
38.482
## Post.Q20  18.346   21.450   22.340   32.224   24.978   49.826   63.046   
40.665
## Post.Q21  84.133   39.083   44.772   28.581   57.422  141.772   42.646   
46.062
## Post.Q22  35.135  127.016   57.249   39.048   50.007   52.985   85.599   
81.384
## Post.Q23  46.735   24.431   33.717   43.353   78.319   50.396   40.360   
25.213
## Post.Q24  40.419  277.774   98.802   87.911  114.040  168.865  238.105   
97.574
## Post.Q25  30.997   44.128  142.093   68.114   70.838   38.653   40.021   
49.371
## Post.Q26  21.633   44.083   48.365   42.090   18.418   57.719   59.225   
48.478
## Post.Q27  31.384   41.381   43.985   90.801   45.923  111.762   69.836   
65.561
## Post.Q28  67.236   48.338  160.665   44.002   91.519   51.271  351.089   
53.412
## Post.Q29  39.558   42.775  110.009   46.065   71.753   72.270  138.456   
69.636
## Post.Q30  30.334   48.799  257.573  109.969  266.360   45.897   47.223   
47.025
##          Post.Q17 Post.Q18 Post.Q19 Post.Q20 Post.Q21 Post.Q22 Post.Q23
## Post.Q1    -0.058    0.060   -0.035    0.037    0.034    0.063    0.036
## Post.Q2    -0.051    0.078    0.043    0.047    0.055    0.061    0.062
## Post.Q3    -0.070    0.049   -0.046   -0.040   -0.036   -0.073    0.031
```

```
## Post.Q4    -0.082   -0.062   -0.046   -0.034   -0.067   -0.090    0.060
## Post.Q5    -0.067    0.224    0.061   -0.070    0.071    0.069   -0.051
## Post.Q6     0.090    0.074   -0.072   -0.059   -0.057    0.060   -0.066
## Post.Q7     0.044    0.056   -0.053    0.037   -0.058    0.048   -0.049
## Post.Q8     0.071    0.070   -0.045    0.045    0.151    0.052    0.116
## Post.Q9    -0.050    0.070    0.053    0.039    0.083    0.054   -0.062
## Post.Q10    0.072    0.057    0.056    0.042    0.057    0.102    0.045
## Post.Q11    0.056   -0.136   -0.071   -0.043   -0.061   -0.069    0.053
## Post.Q12    0.050    0.080    0.054    0.051    0.048    0.057    0.060
## Post.Q13    0.074   -0.118   -0.065   -0.045   -0.069   -0.064    0.080
## Post.Q14   -0.080    0.079   -0.062    0.064    0.108   -0.066   -0.064
## Post.Q15    0.114    0.074    0.080    0.072    0.059    0.084    0.058
## Post.Q16    0.104    0.062    0.056    0.058    0.061    0.082    0.045
## Post.Q17       NA   -0.087   -0.076   -0.048   -0.054    0.069    0.064
## Post.Q18   91.244       NA    0.067    0.058    0.070    0.065    0.057
## Post.Q19   70.833   54.089       NA    0.073    0.050    0.050    0.038
## Post.Q20   28.440   41.578   65.600       NA   -0.044   -0.065   -0.038
## Post.Q21   35.752   59.891   30.101   24.075       NA    0.134    0.215
## Post.Q22   57.805   52.082   31.003   51.094  219.196       NA    0.076
## Post.Q23   50.127   40.201   17.804   17.698  563.648   70.683       NA
## Post.Q24  113.795   58.662   72.553   72.300   76.165  140.697  361.952
## Post.Q25  355.480  109.393   30.200   56.312   38.509   57.699   26.886
## Post.Q26  111.948   54.963   19.641   14.150   28.125  103.507   25.511
## Post.Q27   72.164   41.201   37.204   55.946   36.018   40.969   35.207
## Post.Q28   53.919   65.994   31.726   42.641   44.475   65.307   43.193
## Post.Q29   88.266   99.954   42.044   27.548   38.315   56.692   54.054
## Post.Q30  112.997  254.884   33.984   20.654   52.582   29.688   29.766
##          Post.Q24 Post.Q25 Post.Q26 Post.Q27 Post.Q28 Post.Q29 Post.Q30
## Post.Q1     0.058    0.042    0.050    0.043   -0.044   -0.033    0.055
## Post.Q2     0.050    0.063   -0.076    0.051   -0.065    0.063    0.058
## Post.Q3     0.072   -0.059   -0.039    0.049   -0.045   -0.052    0.042
## Post.Q4    -0.077   -0.070   -0.073   -0.075    0.148   -0.052   -0.093
## Post.Q5     0.065    0.085   -0.061   -0.056   -0.089    0.080    0.121
## Post.Q6     0.072   -0.058    0.061    0.052   -0.059    0.048    0.081
## Post.Q7    -0.074    0.062    0.038   -0.048    0.054    0.034    0.055
## Post.Q8     0.072    0.067    0.065    0.062   -0.056    0.035    0.085
## Post.Q9    -0.058    0.050    0.042    0.051   -0.074    0.057    0.050
## Post.Q10    0.151   -0.060    0.060    0.058    0.063    0.059    0.063
## Post.Q11   -0.090   -0.108   -0.063   -0.060   -0.115   -0.095   -0.145
## Post.Q12    0.085    0.075    0.059    0.086    0.060    0.061    0.095
## Post.Q13   -0.097   -0.076    0.039   -0.061   -0.087    0.077   -0.148
## Post.Q14   -0.118   -0.056    0.069    0.096   -0.065    0.077   -0.061
## Post.Q15   -0.140    0.057    0.070    0.076   -0.170    0.107    0.062
## Post.Q16    0.089    0.064    0.063    0.073   -0.066    0.076    0.062
## Post.Q17    0.097   -0.171    0.096    0.077   -0.067    0.085   -0.096
## Post.Q18   -0.069    0.095   -0.067    0.058    0.074    0.091    0.145
## Post.Q19   -0.077    0.050   -0.040   -0.055   -0.051   -0.059    0.053
## Post.Q20   -0.077    0.068    0.034   -0.068    0.059   -0.048   -0.041
## Post.Q21   -0.079    0.056    0.048    0.054   -0.060    0.056   -0.066
## Post.Q22    0.107   -0.069    0.092    0.058   -0.073    0.068    0.049
```

```
## Post.Q23    0.172   -0.047    0.046    0.054    0.060    0.067    0.049
## Post.Q24       NA   -0.106   -0.082    0.125   -0.103    0.086   -0.082
## Post.Q25  136.407       NA   -0.142   -0.069    0.085    0.075    0.099
## Post.Q26   82.880  245.167       NA    0.076   -0.048    0.059   -0.075
## Post.Q27  189.312   57.556   69.867       NA   -0.066    0.059   -0.064
## Post.Q28  130.240   87.272   28.546   53.904       NA   -0.087   -0.095
## Post.Q29   90.237   69.376   42.181   42.803   93.274       NA    0.093
## Post.Q30   81.010  119.804   68.003   50.183  110.269  106.414       NA

modresiduals[5,18]

## [1] 0.2241546

modresiduals[18,5]

## [1] 612.5905
```

Relative to all the item pairs (except 21 and 23 which also have high similarity on LD), items 5 and 18 are showing large evidence for local dependence.

If we look at the residual correlations, we see that items 5 and 18 have a 0.4 positive correlation.

```
residuals(mQ2, "Q3") ->modresiduals

## Q3 summary statistics:
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  -0.235  -0.006   0.025   0.025   0.052   0.404
##
##           Post.Q1 Post.Q2 Post.Q3 Post.Q4 Post.Q5 Post.Q6 Post.Q7 Post.Q8
## Post.Q1     1.000   0.124   0.029   0.003   0.042   0.017   0.053   0.014
## Post.Q2     0.124   1.000  -0.040  -0.019   0.050  -0.011  -0.010   0.053
## Post.Q3     0.029  -0.040   1.000  -0.045   0.022   0.032  -0.011   0.025
## Post.Q4     0.003  -0.019  -0.045   1.000  -0.051   0.014   0.058  -0.012
## Post.Q5     0.042   0.050   0.022  -0.051   1.000   0.062   0.042   0.041
## Post.Q6     0.017  -0.011   0.032   0.014   0.062   1.000   0.286   0.070
## Post.Q7     0.053  -0.010  -0.011   0.058   0.042   0.286   1.000  -0.032
## Post.Q8     0.014   0.053   0.025  -0.012   0.041   0.070  -0.032   1.000
## Post.Q9     0.036   0.043   0.004  -0.065   0.095   0.021  -0.014   0.132
## Post.Q10    0.034   0.024   0.039   0.052   0.015   0.066   0.002   0.133
## Post.Q11    0.039   0.057   0.000  -0.049   0.018   0.007   0.021  -0.017
## Post.Q12    0.100   0.048   0.058   0.023   0.005   0.061   0.008   0.082
## Post.Q13    0.010   0.029   0.000  -0.040  -0.019   0.025   0.028   0.020
## Post.Q14   -0.022  -0.012  -0.019  -0.036   0.072   0.076  -0.002   0.060
## Post.Q15    0.072   0.079  -0.009  -0.235   0.095   0.027  -0.005   0.008
## Post.Q16    0.004  -0.009   0.038  -0.051   0.040   0.059  -0.018   0.030
## Post.Q17   -0.014  -0.006   0.037  -0.004   0.008   0.073   0.017   0.030
## Post.Q18    0.056   0.056   0.035  -0.017   0.404  -0.002   0.007   0.008
## Post.Q19   -0.008   0.046   0.003  -0.061   0.086  -0.010  -0.047   0.026
## Post.Q20    0.051   0.029  -0.005  -0.038   0.047  -0.007   0.027   0.027
## Post.Q21    0.027   0.065  -0.010  -0.103   0.114  -0.019  -0.016   0.128
```

```
## Post.Q22   0.072   0.063   0.012  -0.029   0.047   0.008   0.020   0.067
## Post.Q23   0.037   0.021   0.003   0.030  -0.042  -0.016  -0.015   0.108
## Post.Q24   0.044   0.002   0.033   0.045  -0.061   0.036  -0.028   0.032
## Post.Q25   0.014   0.043  -0.013  -0.038   0.120  -0.002   0.024  -0.002
## Post.Q26   0.044   0.022  -0.004  -0.041   0.003   0.061   0.032   0.054
## Post.Q27   0.020   0.020   0.026  -0.055   0.019   0.058  -0.033   0.078
## Post.Q28   0.017   0.038  -0.034   0.210   0.061  -0.018   0.046   0.005
## Post.Q29  -0.002   0.031   0.007  -0.006   0.072   0.050   0.001   0.022
## Post.Q30   0.058   0.018   0.054  -0.021   0.179   0.024   0.008   0.013
##           Post.Q9 Post.Q10 Post.Q11 Post.Q12 Post.Q13 Post.Q14 Post.Q15
Post.Q16
## Post.Q1    0.036    0.034    0.039    0.100    0.010   -0.022    0.072
0.004
## Post.Q2    0.043    0.024    0.057    0.048    0.029   -0.012    0.079   -
0.009
## Post.Q3    0.004    0.039    0.000    0.058    0.000   -0.019   -0.009
0.038
## Post.Q4   -0.065    0.052   -0.049    0.023   -0.040   -0.036   -0.235   -
0.051
## Post.Q5    0.095    0.015    0.018    0.005   -0.019    0.072    0.095
0.040
## Post.Q6    0.021    0.066    0.007    0.061    0.025    0.076    0.027
0.059
## Post.Q7   -0.014    0.002    0.021    0.008    0.028   -0.002   -0.005   -
0.018
## Post.Q8    0.132    0.133   -0.017    0.082    0.020    0.060    0.008
0.030
## Post.Q9    1.000    0.068   -0.051    0.065   -0.065   -0.010    0.084
0.067
## Post.Q10   0.068    1.000   -0.008    0.068    0.085    0.019   -0.028   -
0.016
## Post.Q11  -0.051   -0.008    1.000    0.103    0.014   -0.029    0.061
0.007
## Post.Q12   0.065    0.068    0.103    1.000    0.069   -0.002    0.029
0.016
## Post.Q13  -0.065    0.085    0.014    0.069    1.000   -0.050    0.005
0.036
## Post.Q14  -0.010    0.019   -0.029   -0.002   -0.050    1.000    0.013
0.033
## Post.Q15   0.084   -0.028    0.061    0.029    0.005    0.013    1.000   -
0.038
## Post.Q16   0.067   -0.016    0.007    0.016    0.036    0.033   -0.038
1.000
## Post.Q17  -0.011    0.033    0.005    0.030    0.015    0.014    0.032
0.077
## Post.Q18   0.097   -0.010    0.004    0.021   -0.047    0.053    0.041   -
0.006
## Post.Q19   0.026    0.071    0.013    0.059   -0.030   -0.017    0.052
0.017
## Post.Q20   0.015    0.071    0.005    0.048   -0.010    0.001    0.023
```

```
0.045
## Post.Q21   0.028    0.085   -0.030    0.049   -0.088    0.074    0.091
0.020
## Post.Q22   0.073    0.148    0.028    0.044    0.001   -0.008    0.037
0.056
## Post.Q23   0.007    0.068    0.040    0.042    0.031   -0.013    0.002
0.035
## Post.Q24   0.049    0.176    0.036    0.044    0.057   -0.001   -0.039
0.023
## Post.Q25   0.067    0.017    0.051    0.028    0.021   -0.008    0.061
0.035
## Post.Q26  -0.006    0.071   -0.096    0.052   -0.025   -0.017    0.062
0.031
## Post.Q27   0.058    0.061   -0.042    0.046   -0.071    0.051    0.049
0.062
## Post.Q28  -0.041    0.084   -0.011    0.099   -0.084   -0.068   -0.112   -
0.023
## Post.Q29   0.028    0.030    0.000    0.029    0.049    0.036    0.027
0.012
## Post.Q30   0.074    0.015    0.035    0.027   -0.130    0.029    0.028
0.027
##            Post.Q17 Post.Q18 Post.Q19 Post.Q20 Post.Q21 Post.Q22 Post.Q23
## Post.Q1    -0.014    0.056   -0.008    0.051    0.027    0.072    0.037
## Post.Q2    -0.006    0.056    0.046    0.029    0.065    0.063    0.021
## Post.Q3     0.037    0.035    0.003   -0.005   -0.010    0.012    0.003
## Post.Q4    -0.004   -0.017   -0.061   -0.038   -0.103   -0.029    0.030
## Post.Q5     0.008    0.404    0.086    0.047    0.114    0.047   -0.042
## Post.Q6     0.073   -0.002   -0.010   -0.007   -0.019    0.008   -0.016
## Post.Q7     0.017    0.007   -0.047    0.027   -0.016    0.020   -0.015
## Post.Q8     0.030    0.008    0.026    0.027    0.128    0.067    0.108
## Post.Q9    -0.011    0.097    0.026    0.015    0.028    0.073    0.007
## Post.Q10    0.033   -0.010    0.071    0.071    0.085    0.148    0.068
## Post.Q11    0.005    0.004    0.013    0.005   -0.030    0.028    0.040
## Post.Q12    0.030    0.021    0.059    0.048    0.049    0.044    0.042
## Post.Q13    0.015   -0.047   -0.030   -0.010   -0.088    0.001    0.031
## Post.Q14    0.014    0.053   -0.017    0.001    0.074   -0.008   -0.013
## Post.Q15    0.032    0.041    0.052    0.023    0.091    0.037    0.002
## Post.Q16    0.077   -0.006    0.017    0.045    0.020    0.056    0.035
## Post.Q17    1.000   -0.053   -0.041    0.002    0.002    0.047    0.042
## Post.Q18   -0.053    1.000    0.113    0.066    0.130    0.033   -0.027
## Post.Q19   -0.041    0.113    1.000    0.038    0.050    0.041    0.015
## Post.Q20    0.002    0.066    0.038    1.000   -0.011    0.021    0.014
## Post.Q21    0.002    0.130    0.050   -0.011    1.000    0.122    0.081
## Post.Q22    0.047    0.033    0.041    0.021    0.122    1.000    0.066
## Post.Q23    0.042   -0.027    0.015    0.014    0.081    0.066    1.000
## Post.Q24    0.074   -0.080    0.009    0.025    0.042    0.062    0.266
## Post.Q25   -0.054    0.129    0.085    0.074    0.057   -0.009   -0.022
## Post.Q26    0.100   -0.006   -0.032   -0.016   -0.029    0.044    0.062
## Post.Q27    0.030    0.020   -0.001   -0.027    0.058    0.044    0.052
## Post.Q28   -0.039    0.072    0.007    0.054   -0.067   -0.001    0.046
```

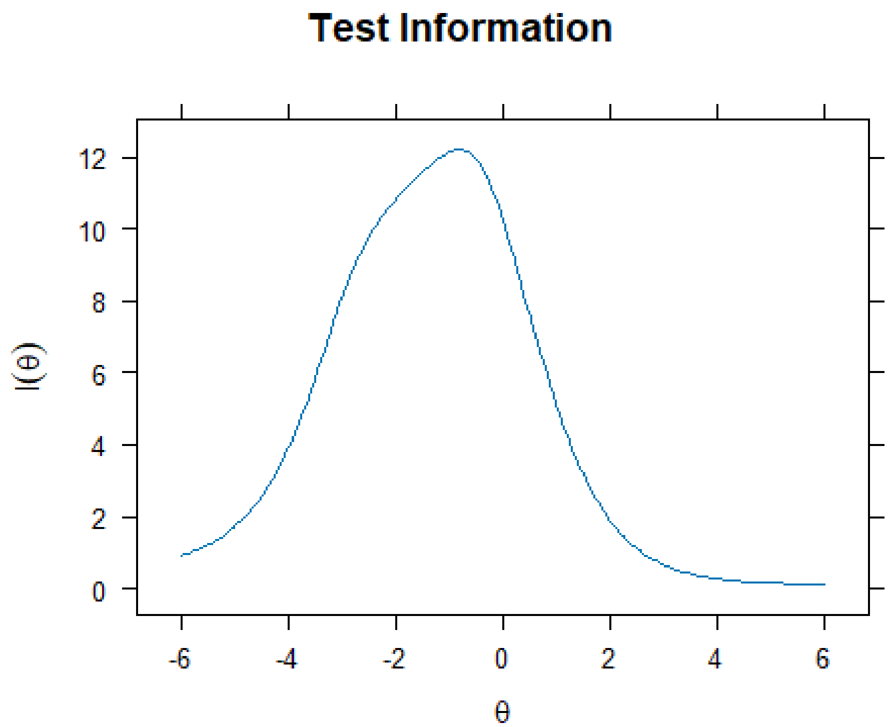
```
## Post.Q29    0.098    0.077    0.024    0.010    0.033    0.012    0.024
## Post.Q30   -0.013    0.230    0.083    0.036    0.056    0.010    0.024
##           Post.Q24 Post.Q25 Post.Q26 Post.Q27 Post.Q28 Post.Q29 Post.Q30
## Post.Q1     0.044    0.014    0.044    0.020    0.017   -0.002    0.058
## Post.Q2     0.002    0.043    0.022    0.020    0.038    0.031    0.018
## Post.Q3     0.033   -0.013   -0.004    0.026   -0.034    0.007    0.054
## Post.Q4     0.045   -0.038   -0.041   -0.055    0.210   -0.006   -0.021
## Post.Q5    -0.061    0.120    0.003    0.019    0.061    0.072    0.179
## Post.Q6     0.036   -0.002    0.061    0.058   -0.018    0.050    0.024
## Post.Q7    -0.028    0.024    0.032   -0.033    0.046    0.001    0.008
## Post.Q8     0.032   -0.002    0.054    0.078    0.005    0.022    0.013
## Post.Q9     0.049    0.067   -0.006    0.058   -0.041    0.028    0.074
## Post.Q10    0.176    0.017    0.071    0.061    0.084    0.030    0.015
## Post.Q11    0.036    0.051   -0.096   -0.042   -0.011    0.000    0.035
## Post.Q12    0.044    0.028    0.052    0.046    0.099    0.029    0.027
## Post.Q13    0.057    0.021   -0.025   -0.071   -0.084    0.049   -0.130
## Post.Q14   -0.001   -0.008   -0.017    0.051   -0.068    0.036    0.029
## Post.Q15   -0.039    0.061    0.062    0.049   -0.112    0.027    0.028
## Post.Q16    0.023    0.035    0.031    0.062   -0.023    0.012    0.027
## Post.Q17    0.074   -0.054    0.100    0.030   -0.039    0.098   -0.013
## Post.Q18   -0.080    0.129   -0.006    0.020    0.072    0.077    0.230
## Post.Q19    0.009    0.085   -0.032   -0.001    0.007    0.024    0.083
## Post.Q20    0.025    0.074   -0.016   -0.027    0.054    0.010    0.036
## Post.Q21    0.042    0.057   -0.029    0.058   -0.067    0.033    0.056
## Post.Q22    0.062   -0.009    0.044    0.044   -0.001    0.012    0.010
## Post.Q23    0.266   -0.022    0.062    0.052    0.046    0.024    0.024
## Post.Q24    1.000   -0.037    0.055    0.040    0.018    0.028   -0.044
## Post.Q25   -0.037    1.000   -0.128   -0.008    0.068    0.020    0.118
## Post.Q26    0.055   -0.128    1.000    0.083   -0.065    0.035   -0.012
## Post.Q27    0.040   -0.008    0.083    1.000   -0.061    0.057    0.014
## Post.Q28    0.018    0.068   -0.065   -0.061    1.000   -0.033    0.105
## Post.Q29    0.028    0.020    0.035    0.057   -0.033    1.000    0.031
## Post.Q30   -0.044    0.118   -0.012    0.014    0.105    0.031    1.000
```

```r
modresiduals[5,18]
```

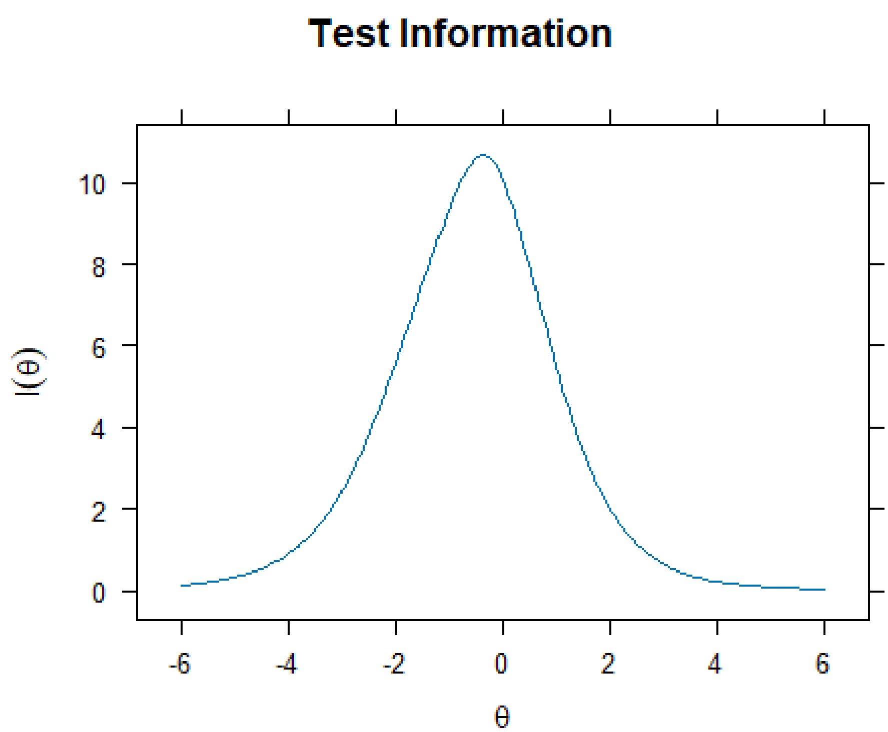
```
## [1] 0.4043104
```

d) Compare the information function from the standard 2PLfit to the fcirw (correct/incorrect) dataset to the information function from the nominal model above.

```r
mirt(fcirw[,34:63],1,"2PL")->m2q2PL

plot(mQ2, type="info")
```




```
plot(m2q2PL, type = "info")
```



The nominal model has used all the available data by considering all response options. It makes sense that it has slightly higher maximum information, and that it can give more information at the lower end of the distribution by considering the specific patterns of errors.

3) The textbook describes the condom dataset, a 6 item questionnaire on condom attitudes. Fit a generalized partial credit model to the condom data. We can also fit a standard one dimension factor analysis model.

a) Conduct a factor analysis of the data. Using the psych package, you can do

fa(condom,1)->famodel

The factor model loadings are found by: famodel$loadings

Compare to the slopes from the gpcm.

```
library(psych)

fa(condom,1)->famodel
mirt(condom,1,"gpcm") ->q3gpcm

famodel$loadings

##
## Loadings:
##    MR1
## I1 0.462
## I2 0.626
## I3 0.621
## I4 0.342
## I5 0.411
## I6 0.376
##
##                  MR1
## SS loadings    1.417
## Proportion Var 0.236

coef(q3gpcm, simplify = T)$items[,1]

##        I1        I2        I3        I4        I5        I6
## 0.4546532 0.9208777 0.9676019 0.3108090 0.3753964 0.3696279
```

We can also correlate the loadings from the FA with the slopes from the gpcm.

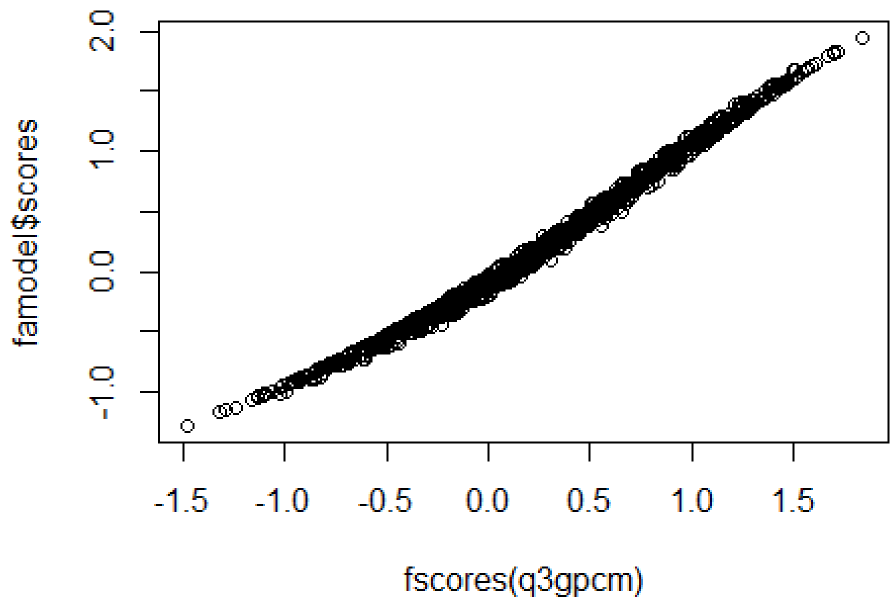
```
cor(famodel$loadings,coef(q3gpcm,simplify=T)$items[,1])

##         [,1]
## MR1 0.981589
```

Clearly there is a high correspondence here between the two approaches.

b) Compare the factor scores from the two models. (For fa take famodel$scores) What pattern do you see?

We can plot the fscores of each model. Note the high level of correspondence, and the slight non-linearity due to model assumptions.

```
plot(famodel$scores ~fscores(q3gpcm))
```



And here's the correlation:

```
cor(famodel$scores,fscores(q3gpcm))

##            F1
## MR1 0.9948293
```

Again, a super high similarity in the ability estimates.

c) Discuss the information available in var(condom) and unique(condom).

The factor model and gpcm each use one of what you just calculated. Explain.

```
round(var(condom),2)

##       I1   I2   I3   I4   I5   I6
## I1 1.61 0.41 0.37 0.25 0.29 0.43
## I2 0.41 1.44 0.68 0.23 0.37 0.25
## I3 0.37 0.68 1.34 0.25 0.39 0.20
## I4 0.25 0.23 0.25 1.47 0.32 0.31
```

```
## I5 0.29 0.37 0.39 0.32 1.77 0.27
## I6 0.43 0.25 0.20 0.31 0.27 1.40
```

```r
dim(unique(condom))
```

```
## [1] 1209    6
```

The traditional factor analysis model treats the item responses as continuous. There are 3473 cases but the FA only works with the 6 by 6 covariance matrix of the items.

The factor analysis is derived entirely from the covariances of the items. Once the covariance matrix is calculated, the rest of the data can be discarded. The algorithm to estimate factor loadings and fit the model uses only the covariance matrix.

With regard to IRT - there are 1209 unique data patterns in the data, unique combinations of scores for six items and four response options on each item. The maximum number of unique combinations, incidentally, is $4^6 = 4096$. With a sample size of 3473 it isn't even possible to represent them all, and many are extremely unlikely given the underlying model. The IRT analysis proceeds using only the observed frequencies of the unique data patterns.

The factor model and the IRT model are working with different data, but of course they are aligned. The FA treats the variables as continuous, and fits a linear regression on a latent factor to fit the covariance matrix. The IRT treats the variables as categorical and ordinal, and fits a non-linear response probability model based on a latent factor.

Finally, the 1209 unique response patterns represent a limit. If two people have exactly the same response vector, there is nothing else to differentiate them. Therefore, 1209 must be the maximum resolution that either model can give in terms of scores. So let's just see how many unique factor scores there are for each model.

```r
length(unique(fscores(q3gpcm)))
```

```
## [1] 1209
```

```r
length(unique(famodel$scores))
```

```
## [1] 1209
```