Linear Algebra

The Nullspace of a Matrix

$$A=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$$

By definition, the nullspace of A consists of all vectors $\mathbf{x}$ such that $A\mathbf{x} = \mathbf{0}$. Perform the following elementary row operations on A,

$$\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix} \xrightarrow{r_1 \leftrightarrow r_2} \begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix} \xrightarrow{-2r_1 \text{ added to } r_2} \begin{bmatrix}1 & 2\\ 0 & -3\end{bmatrix}$$

to conclude that $A\mathbf{x} = \mathbf{0}$ is equivalent to the simpler system

$$\begin{bmatrix}1 & 2\\ 0 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

The second row implies that $x_2 = 0$, and back-substituting this into the first row implies that $x_1 = 0$ also. Since the only solution of $A\mathbf{x} = \mathbf{0}$ is $\mathbf{x} = \mathbf{0}$, the nullspace of A consists of the zero vector alone. This subspace, $\{\mathbf{0}\}$, is called the **trivial subspace** (of $\mathbf{R}^2$).

Example 4: Find the nullspace of the matrix

$$B=\begin{bmatrix}2 & 1\\ -4 & -2\end{bmatrix}$$

Subspaces of R^n

NEXT
The Rank Plus Nullity Theorem >