Getting Eigenvalue and Eigenvectors of Symmetric Matrices by the Power Method and Hotelling Deflation:

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Hotelling's deflation:

Consider $\left(\mathbf{A}-\lambda_1\mathbf{u}_1\mathbf{u}_1^{\mathrm{T}}\right)\mathbf{u}_j = \mathbf{A}\mathbf{u}_j - \lambda_1\mathbf{u}_1\mathbf{u}_1^{\mathrm{T}}\mathbf{u}_j = \lambda_j\mathbf{u}_j - \lambda_1\mathbf{u}_1\left(\mathbf{u}_1^{\mathrm{T}}\mathbf{u}_j\right)$.

If $j=1$, then $\left(\mathbf{A}-\lambda_1\mathbf{u}_1\mathbf{u}_1^{\mathrm{T}}\right)\mathbf{u}_1 = \lambda_1\mathbf{u}_1 - \lambda_1\mathbf{u}_1\left(\mathbf{u}_1^{\mathrm{T}}\mathbf{u}_1\right) = 0\mathbf{u}_1$.
If $j\neq 1$, then $\left(\mathbf{A}-\lambda_1\mathbf{u}_1\mathbf{u}_1^{\mathrm{T}}\right)\mathbf{u}_j = \lambda_j\mathbf{u}_j - \lambda_1\mathbf{u}_1(0) = \lambda_j\mathbf{u}_j$.

thus, $\left(\mathbf{A}-\lambda_1\mathbf{u}_1\mathbf{u}_1^{\mathrm{T}}\right)$ has the same eigenvectors as **A**, and the same eigenvalues as **A** except that the largest one has been replaced by 0. thus we can use the power method to find the next biggest and so on.

```
function [eigvals, eigvecs, nit] = hoteldef(A, etol, maxit)
  % This function calculates the eigenvalues and the corresponding
  % eigenvectors for a symmetric matrix by the power iteration method. After
  % the getting the maximum eigenvalue and the corresponding eigenvector
  % the matrix is being deflated; deflation means modifying the matrix to
  % eliminate the influence of a given eigenvector, typically by setting
  % the associated eigenvalue to zero
  %
  % INPUTS:
  %     A = the matrix in question
  %     etol = the error tollerance
  %     maxit = the maximum number of iteration
  % OUTPUTS:
  %     eigvals = a digonal matrix where the diagonal is the eigenvalues
  %     eigvecs = the eigenvectors matrix; its columns is the eigenvectors

   n = size(A)(1);
   eigvals = zeros(n,n);
   eigvecs = zeros(n,n);
   nit = zeros(1,n);
   for i=n:-1:1
     [eigvals(i,i), eigvecs(:,i), nit(i)] = powerit(A, etol, maxit);
     A = A - eigvals(i,i)*(eigvecs(:,i)*eigvecs(:,i)');
   end
end
```

```
A =  6    4    3
     4   10    6
     3    6    8
```

```
>> [vals, vecs, nit] = hoteldef(A, 10^-6, 100)
vals =
    2.83921    0.00000    0.00000
    0.00000    3.87036    0.00000
    0.00000    0.00000   17.29044
vecs =
   0.25070   0.87913   0.40459
  -0.70022  -0.12358   0.70315
   0.66846  -0.46028   0.58471
nit =
    2   19    5
```