CENG4513 MODELLING & SILMULATION

LECTURE #2

4. Systems, Models & Simulation

Discrete-event Simulation

Arrival time vs interarrival time

Part Number	Arrival Time	Interarrival Time
1	0.00	1.73
2	1.73	1.35
3	3.08	0.71
4	3.79	0.62
5	4.41	14.28
6	18.69	0.70
7	19.39	15.52
8	34.91	3.15
9	38.06	1.76
10	39.82	1.00
11	40.82	.
.	.	.
.	.	.

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Discrete-event Simulation

Manual Discrete Simulation of Cashier Service

Cus No.	Random IAT Since Last Arrival (min)	Random Service Time (min)	Customer Arrival Time	Service Starts	Service Ends	Customer Wait Time (min)	Cashier Idle Time (min)
1	-	1	0:00	0:00	0:01	1	0
2	3	4	0:03	0:03	0:07	4	2
3	7	4	0:10	0:10	0:14	4	3
4	3	2	0:13	0:14	0:16	3	0
5	9	1	0:22	0:22	0:23	1	6
6	10	5	0:32	0:32	0:37	5	9
7	6	4	0:38	0:38	0:42	4	1
8	8	6	0:46	0:46	0:52	6	4
9	8	1	0:54	0:54	0:55	1	2
10	8	3	1:02	1:02	1:05	3	7
11	7	5	1:09	1:09	1:14	5	4
12	3	5	1:12	1:14	1:19	7	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
						TotalWT	TotalIT

IAT – Inter Arrival Time

Average Waiting Time = TotalWT / N
% Cashier Idle = (100 x TotalIT / T) %

N = Number of customers simulated
T = Simulation duration time

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Discrete-event Simulation Example

- In this example the cars form in queues to be served by the charging bay resource.
- Modeling the scenario entails quantifying the car arrival times and their respective charging (service) timesav

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Discrete-event Simulation Example

- Interarrival times 2, 8, 7, 2, 11, 3, 15, 9
- Charging duration 11, 8, 5, 8, 8, 5, 10, 12

Assumptions:

- Cars are positioned to the station as soon as previous one's charging is completed.

- No technical problem preventing charging occurs.

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Discrete-event Simulation Example

Analysis parameters

- Waiting time
- Queue length
- Resource utilization

- How many customers are served in one hour?
- Total idle-time for the charging bay?
- Minimum and maximum queue length?
- Total queue time and average waiting time?

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Discrete-event Simulation Example

Car #	Inter-arrival Time	Charging Time	Clock Time	Charging Start	Charging End	Wait Time	Charger Idle Time	Queue Length Start	Queue Length End
1	2	11	2	2	13	0	2	0	1
2	8	8	10	13	21	3	0	1	2
3	7	5	17	21	26	4	0	2	1
4	2	8	19	26	34	7	0	1	2
5	11	8	30	34	42	4	0	2	2
6	3	5	33	42	47	9	0	2	0
7	15	10	48	48	58	0	1	0	1
8	9	12	57	58	70	1	0	1	0

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Discrete-event Simulation Example

0-2

Clock	Server	Queue	Statistics
00:00			Cars Entered = 0 Cars Served = 0 Total Time in Queue = 0
Initialization	**Next Arrival**	**Next Departure**	Mean Waiting Time = N/A
	Car 1 at t=2	-	Utilization = N/A

Clock	Server	Queue	Statistics
00:02			Cars Entered = 1 Cars Served = 0 Total Time in Queue = 0
Event	**Next Arrival**	**Next Departure**	Mean Waiting Time = 0
Car 1 Arrival	Car 2 at t=10	Car 1 at t=13	Utilization = 0

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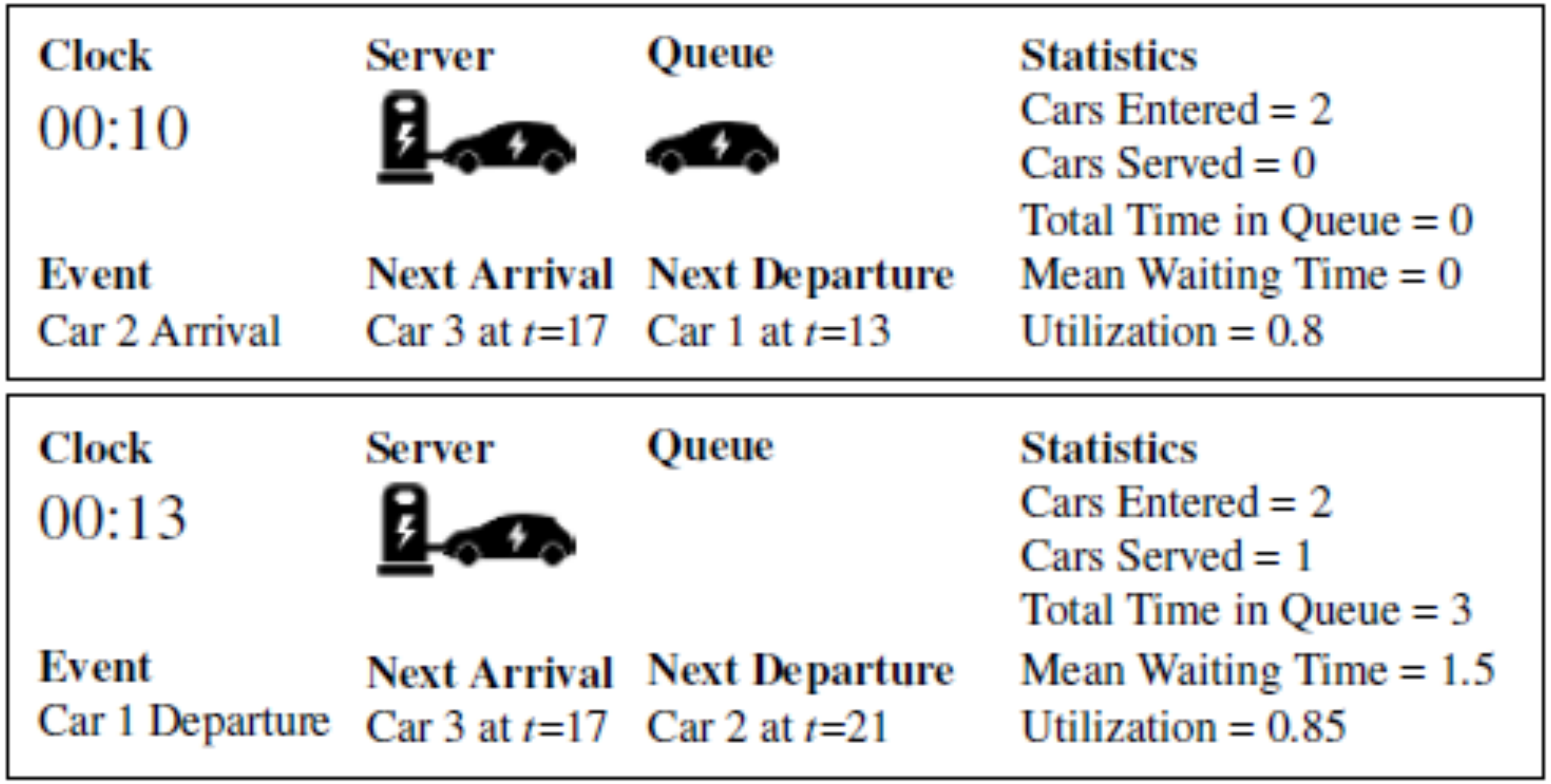
Discrete-event Simulation Example



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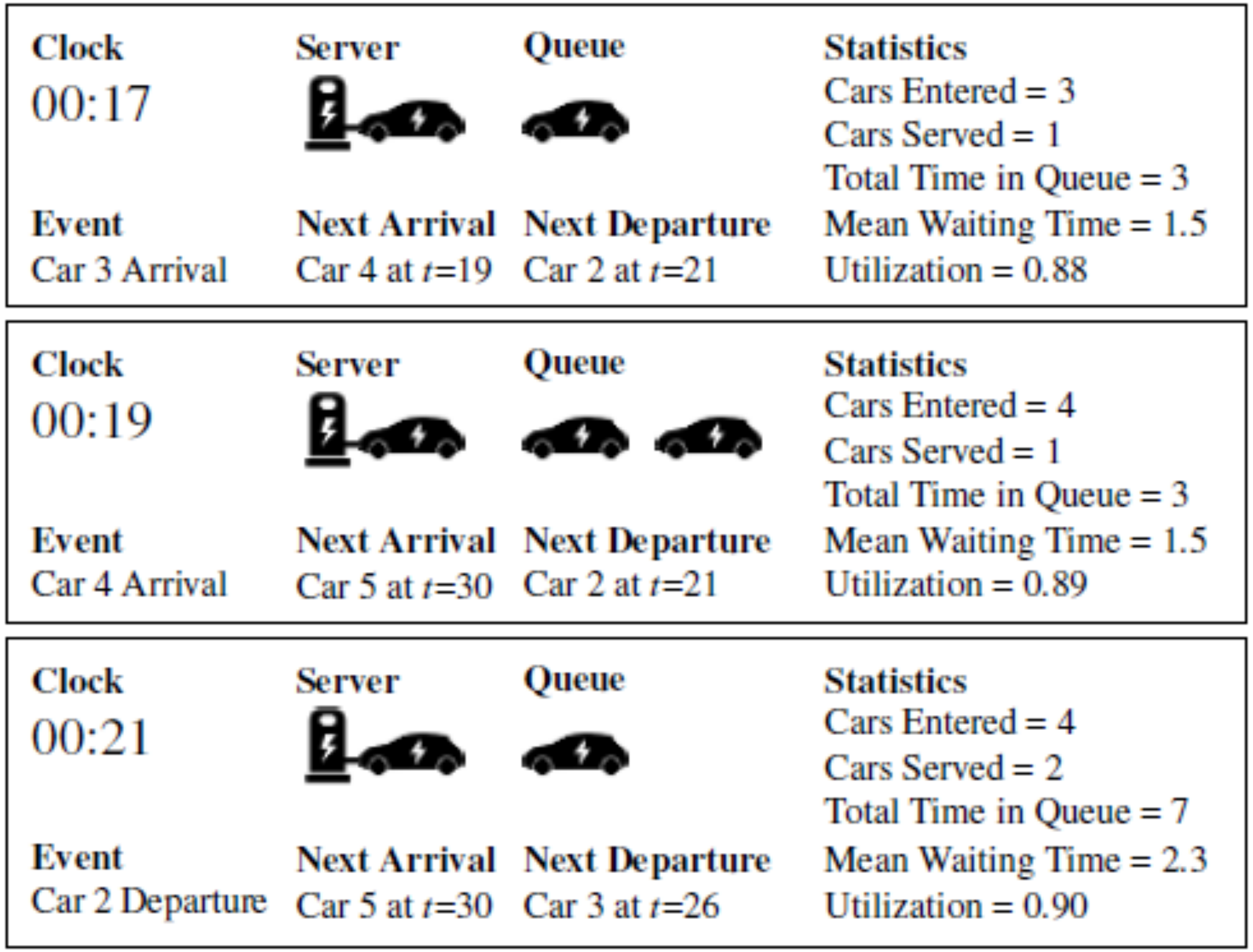
Discrete-event Simulation Example

17-21



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Discrete-event Simulation Example

26-30

Clock	Server	Queue	Statistics
00:26			Cars Entered = 4 Cars Served = 3 Total Time in Queue = 14
Event	**Next Arrival**	**Next Departure**	Mean Waiting Time = 3.5
Car 3 Departure	Car 5 at t=30	Car 4 at t=34	Utilization = 0.92

Clock	Server	Queue	Statistics
00:30			Cars Entered = 5 Cars Served = 3 Total Time in Queue = 14
Event	**Next Arrival**	**Next Departure**	Mean Waiting Time = 3.5
Car 5 Arriving	Car 6 at t=33	Car 4 at t=34	Utilization = 0.93

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33-42

Discrete-event Simulation Example

Clock	**Server**	**Queue**	**Statistics**
00:33			Cars Entered = 6
			Cars Served = 3
			Total Time in Queue = 14
Event	**Next Arrival**	**Next Departure**	Mean Waiting Time = 3.5
Car 6 Arrival	Car 7 at t=48	Car 4 at t=34	Utilization = 0.94

Clock	**Server**	**Queue**	**Statistics**
00:34			Cars Entered = 6
			Cars Served = 4
			Total Time in Queue = 18
Event	**Next Arrival**	**Next Departure**	Mean Waiting Time = 3.6
Car 4 Departure	Car 7 at t=48	Car 5 at t=42	Utilization = 0.94

Clock	**Server**	**Queue**	**Statistics**
00:42			Cars Entered = 6
			Cars Served = 5
			Total Time in Queue = 18
Event	**Next Arrival**	**Next Departure**	Mean Waiting Time = 3.6
Car 5 Departure	Car 7 at t=48	Car 6 at t=47	Utilization = 0.95

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47-48

Discrete-event Simulation Example

Clock	**Server**	**Queue**	**Statistics**
00:47			Cars Entered = 6 Cars Served = 6 Total Time in Queue = 27
Event	**Next Arrival**	**Next Departure**	Mean Waiting Time = 4.5
Car 6 Departure	Car 7 at t=48	-	Utilization = 0.96

Clock	**Server**	**Queue**	**Statistics**
00:48			Cars Entered = 7 Cars Served = 6 Total Time in Queue = 27
Event	**Next Arrival**	**Next Departure**	Mean Waiting Time = 4.5
Car 7 Arrival	Car 8 at t=57	Car 7 at t=58	Utilization = 0.94

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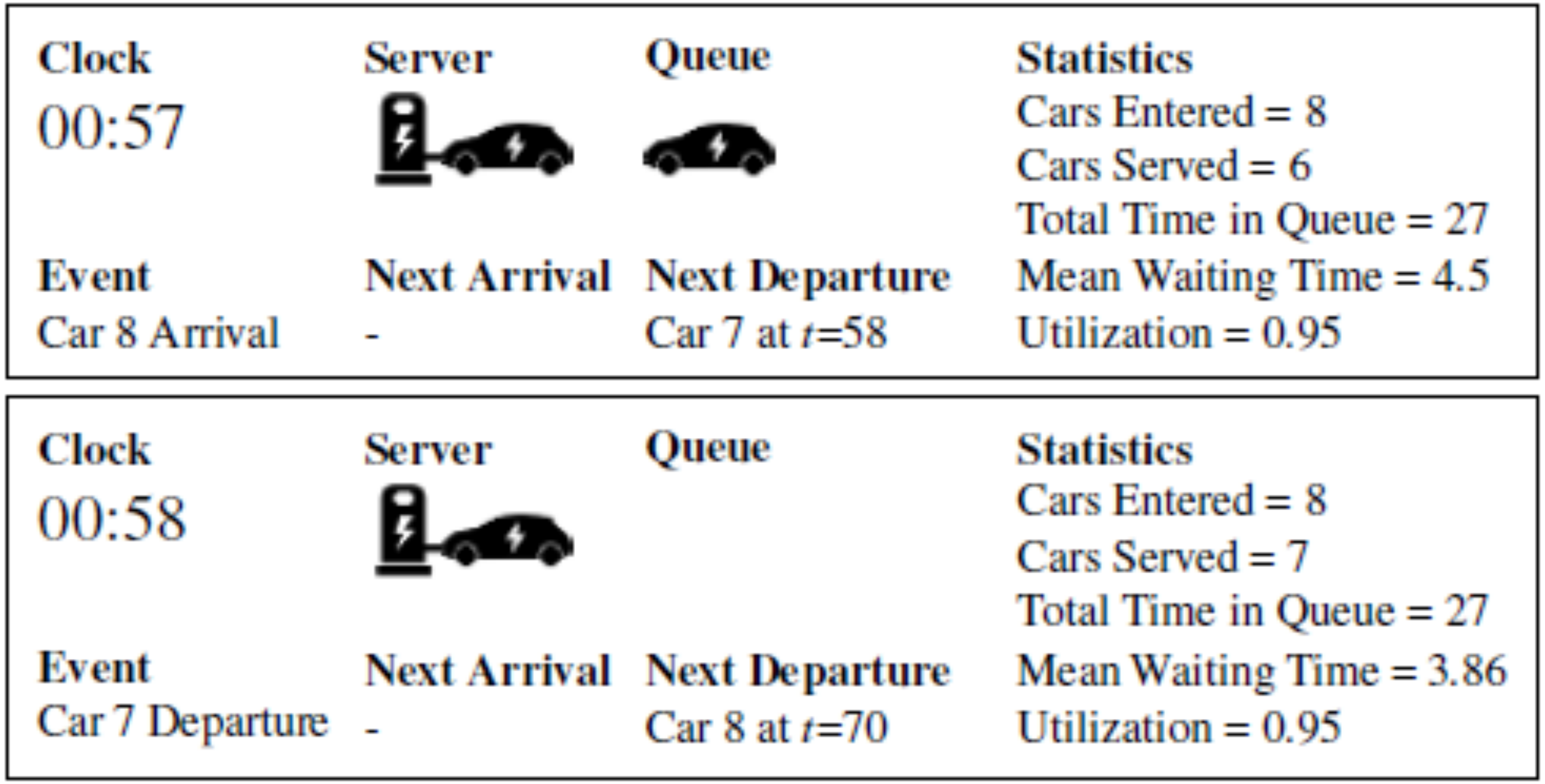
Discrete-event Simulation Example



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Discrete-event Simulation Example

Mean Waiting Time

The basic statistics for waiting time are the mean (average) waiting time in Equation 1.3 and sample variance in Equation 1.4.

$$\overline{W} = \frac{1}{n}\sum_{i=1}^{n} W_i \tag{1.3}$$

$$Var_W = \frac{1}{n-1}\sum_{i=1}^{n} (W_i - \overline{W})^2 \tag{1.4}$$

The mean waiting time for the first hour of operation (seven cars completing) is

Mean Waiting Time ($\overline{W}$) = (0 + 3 + 4 + 7 + 4 + 9 + 0) / 7
= 3.86 minutes.

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Discrete-event Simulation Example

Queue Waiting Time

The mean and variance of the queue length are in Equations 1.5 and 1.6 where T is the total simulation time and t_i is the length of time between the $(i-1)^{\text{th}}$ and i^{th} events. A graph of queue length is shown in Figure 1.1 and continued in Figure 1.2 (which can also be derived from Table 1.2).

$$\overline{L} = \frac{1}{T}\sum_{i=1}^{n} L_i t_i \qquad (1.5)$$

$$Var_L = \frac{1}{T}\sum_{i=1}^{n} (L_i - \overline{L})^2 t_i \qquad (1.6)$$

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Discrete-event Simulation Example

Average Queue Length

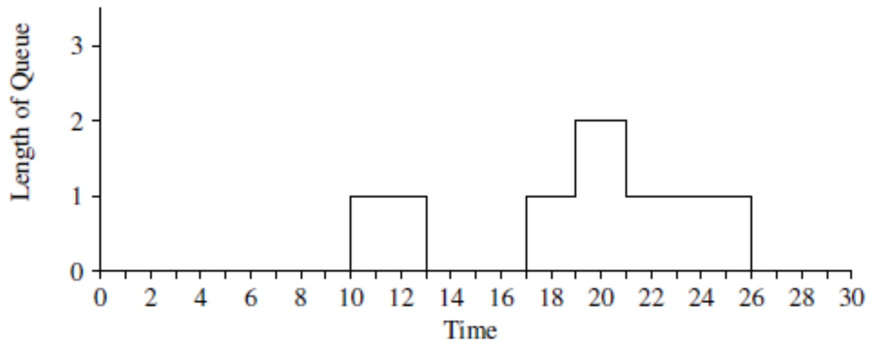

FIGURE 1.1: Length of Queue (0–30 Minutes)

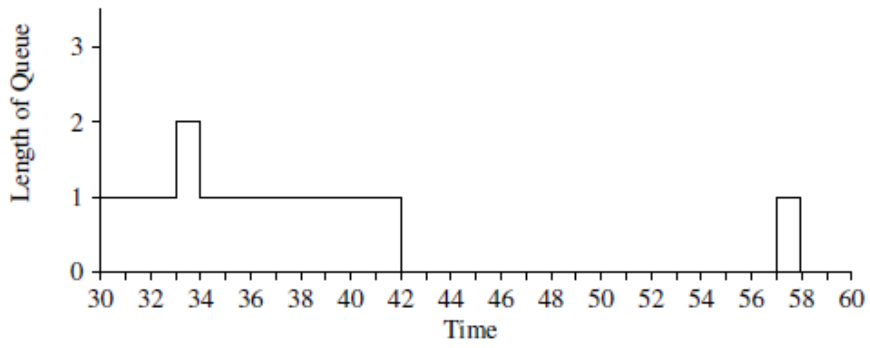

FIGURE 1.2: Length of Queue (30–60 Minutes)

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Discrete-event Simulation Example

Average Queue Length

$$
\begin{aligned}
\text{Average queue length } (\bar{L}) &= [(10-0)0+(13-10)1 \\
&\quad +(17-13)0+(19-17)1+(21-19)2+(26-21)1 \\
&\quad +(30-26)0+(33-30)1+(34-33)2+(42-34)1 \\
&\quad +(57-42)0+(58-57)1+(60-58)0]/60 \\
&= [0+3+0+2+4+5+0+3+2+8+0+1+0]/60 \\
&= .47 \text{ cars}
\end{aligned}
$$

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Discrete-event Simulation Example

Resource Utilization

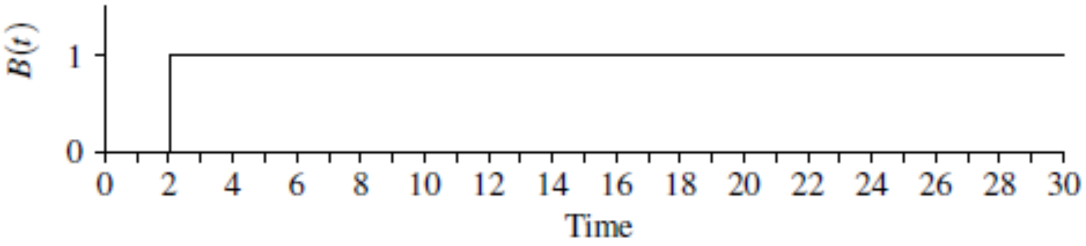


FIGURE 1.3: Resource Utilization (0–30 Minutes)

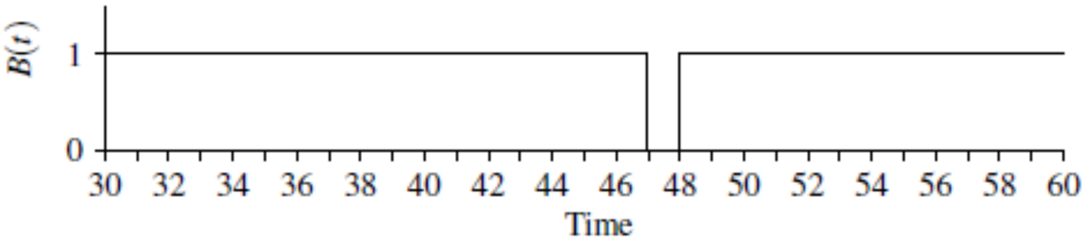


FIGURE 1.4: Resource Utilization (30–60 Minutes)

4. Systems, Models & Simulation

Discrete-event Simulation Example

Resource Utilization

The resource utilization of the charging server for the simple example would be calculated as:

$$\begin{aligned}\text{Utilization } u(n) &= [(2-0)0+(47-2)1 \\ &\quad +(48-47)0+(60-48)1]/60 \\ &= [0+45+0+12]/60 \\ &= .95.\end{aligned}$$

Simulation Methodology

- A methodology is simply a series of steps to follow
- A general methodology for solving problems can be stated as follows:
  1. Define the problem.
  2. Establish measures of performance for evaluation.
  3. Generate alternative solutions.
  4. Rank alternative solutions.
  5. Evaluate and iterate during process.
  6. Execute and evaluate the solution.

Simulation Methodology

- This methodology can be referred to by using the first letter of each step.
- The DEGREE methodology for problem solving represents a series of steps that can be used during the problem-solving process.

    1. **Define the problem:** The first step helps to ensure that you are solving the right problem.

    2. **Establish measures of performance for evaluation:** The second step helps to ensure that you are solving the problem for the right reason, that is, your metrics must be coherent with your problem.

Simulation Methodology

3. 4. Generate alternative solutions & Rank alternative solutions:

Steps 3 and 4 ensure that the analyst looks at and evaluates multiple solutions to the problem. In other words, these steps help to ensure that you develop the right solution to the problem.

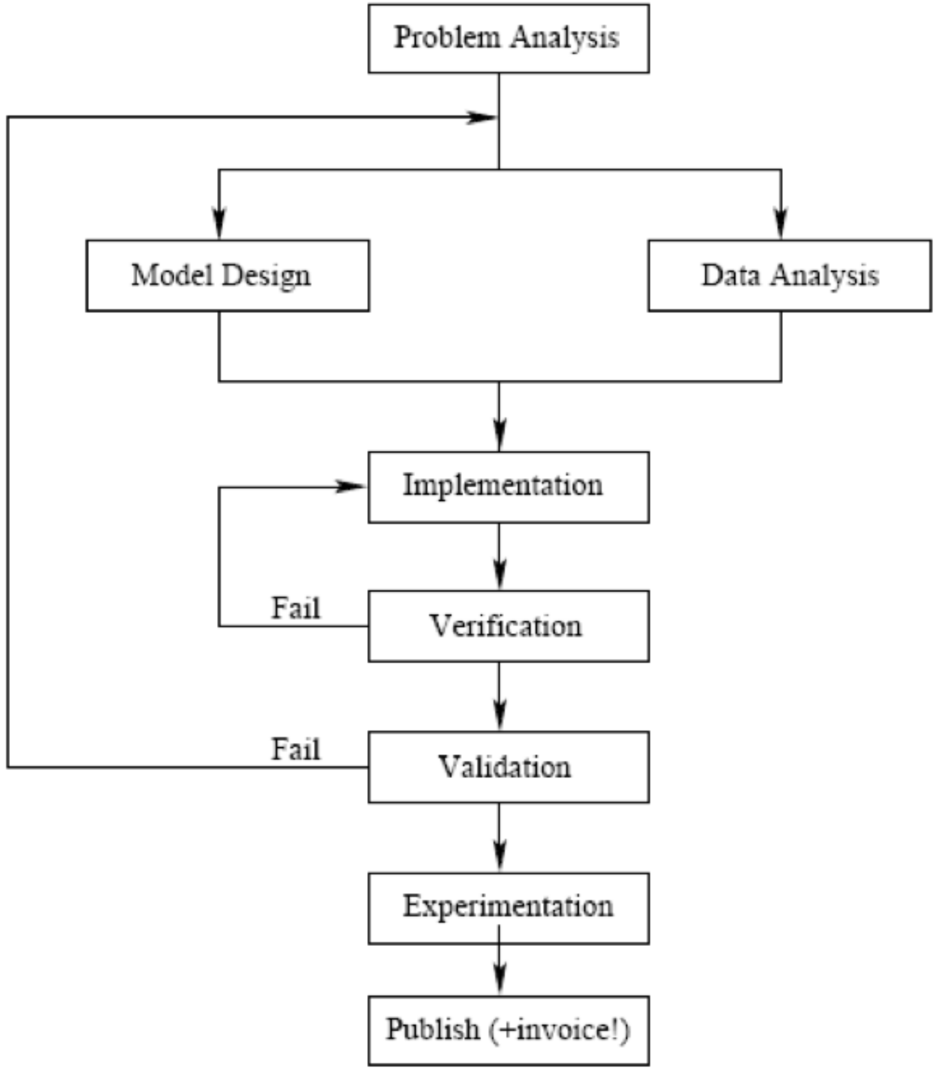
5. Evaluate and iterate during process: In step 5, the analyst evaluates how the process is proceeding and allows for iteration.

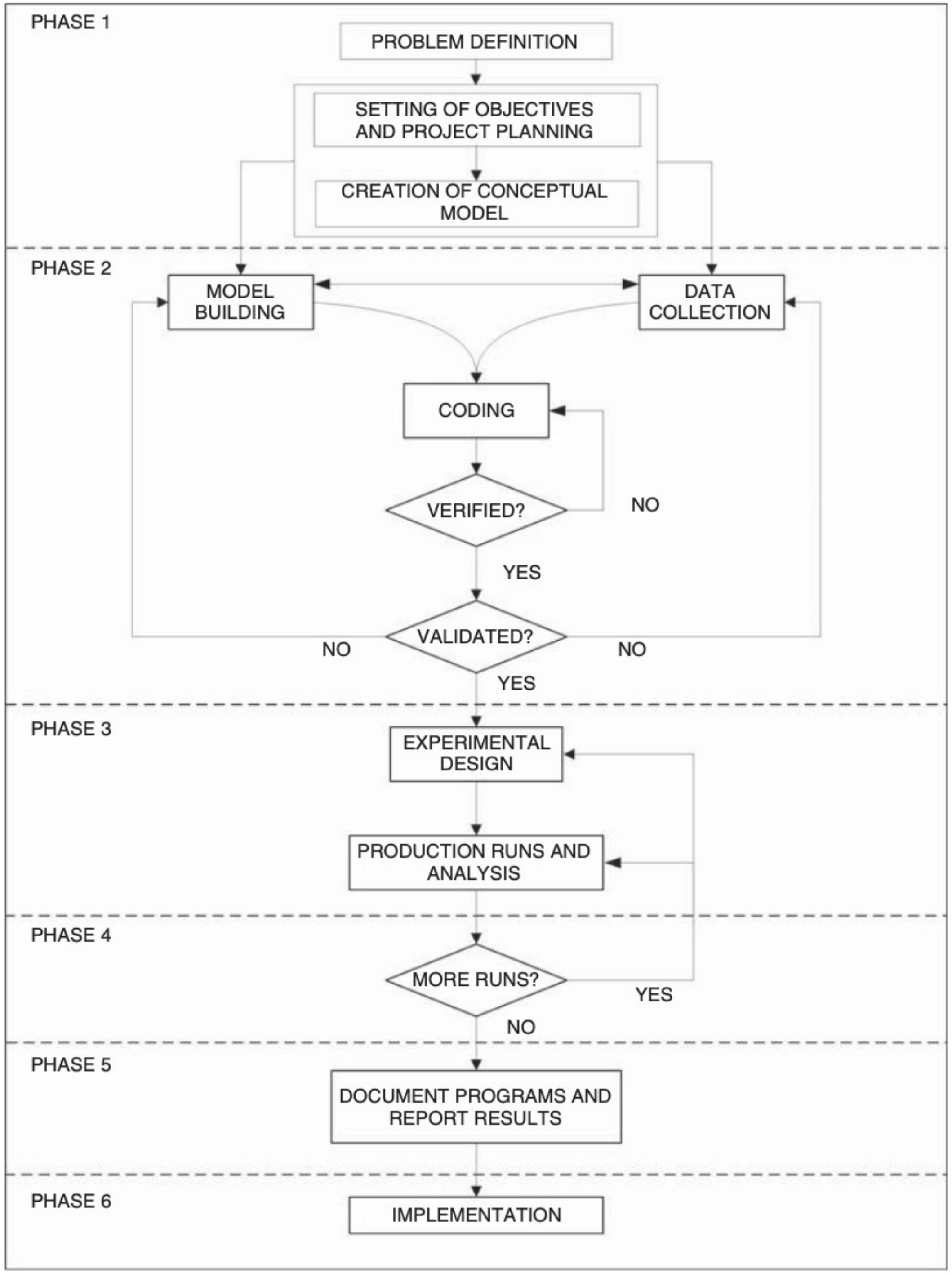
6. Execute and evaluate the solution: Step 6 indicates that if you have the opportunity, you should execute the solution by implementing the decisions. Finally, you should always follow up to ensure that the projected benefits of the solution were obtained.

Simulation Methodology

- When applying DEGREE to a problem that may require simulation, the general DEGREE approach needs to be modified to explicitly consider how simulation will interact with the overall problem-solving process.

Simulation Methodology




PHASE 1
PROBLEM DEFINITION
SETTING OF OBJECTIVES AND PROJECT PLANNING
CREATION OF CONCEPTUAL MODEL
PHASE 2
MODEL BUILDING
DATA COLLECTION
CODING
VERIFIED?
NO
YES
VALIDATED?
NO
NO
YES
PHASE 3
EXPERIMENTAL DESIGN
PRODUCTION RUNS AND ANALYSIS
PHASE 4
MORE RUNS?
YES
NO
PHASE 5
DOCUMENT PROGRAMS AND REPORT RESULTS
PHASE 6
IMPLEMENTATION

Simulation Methodology

1. Problem formulation
   a) Define the problem
   b) Define the system
   c) Establish performance metrics
   d) Build conceptual model
   e) Document model assumptions
2. Simulation model building
   a) Model translation
   b) Input data modeling
   c) Verification
   d) Validation

Simulation Methodology

3. Experimental design and analysis
   a) Preliminary runs
   b) Final experiments
   c) Analysis of results

4. Evaluate and iterate
   a) Documentation
   b) Model manual
   c) User manual Implementation

5. Implementation

Simulation Methodology

The first phase, problem formulation, captures the essence of the first two steps in the DEGREE process. The second phase, model building, captures the essence of step 3 of the DEGREE process.

A problem starts with a perceived need. These activities are useful in developing an appreciation for and an understanding of what needs to be solved. The basic output of the problem definition activity is a problem definition statement. A problem definition statement is a narrative discussion of the problem.

Simulation Methodology

- The problem formulation phase of the study consists of five primary activities:

1. Defining the problem.
2. Defining the system.
3. Establishing performance metrics.
4. Building conceptual models.
5. Documenting modeling assumptions.

Simulation Methodology

- The general goals of a simulation study often include the following:

• *Comparison*. To compare system alternatives and their performance measures across various factors (decision variables) with respect to some objectives.

•*Optimization*. This is a special case of comparison in which you try to find the system configuration that optimizes performance subject to constraints.

• *Prediction*. To predict the behavior of the system at some future point in time.

• *Investigation*. To learn about and gain insight into the behavior of the system, given various inputs.

Simulation Methodology

Within the context of a simulation project, this process includes the following:

Input Data Preparation. Input data is analyzed to determine the nature of the data

*ModelTranslation.*Description of the procedure for coding the model,including timing and general procedures and the translation of the conceptual models

Verification. Verification of the computer simulation model is performed to determine whether or not the program performs as intended

Validation. Validation of the simulation model is performed to determine whether or not the simulation model adequately represents the real system

Simulation Methodology

After you are confident that your model has been verified and validated to suit your purposes, you can begin to use the model to perform experiments that investigate the goals and objectives of the project.

If you are satisfied that the simulation has achieved your objectives, then you should document and implement the recommended solutions. If not, you can iterate as necessary and determine if any additional data, models, experimentation, or analysis is needed to achieve your modeling objectives.