University of Science and Technology
Zewail City

CIE 425 Information Theory And Coding

Project 3 Report

Ibrahim Hanafy 202200518
Khaled Mahmoud 202200282
Mai Ibrahim 202200504

Instructor
Dr. Samy Soliman

Teaching Assistants
Eng. Heba Selim
Eng. Aya Shehata

Contents

1 Introduction

This report presents the design, implementation, and analysis of a Bit Error Rate (BER) simulation tool for wireless communication systems. The primary objective of this project is to investigate the performance of Binary Phase Shift Keying (BPSK) modulation over different channel conditions and to evaluate the effectiveness of Channel Coding in mitigating the effects of fading.

We simulate transmission over two specific channel models:

- **AWGN Channel:** An ideal channel with Additive White Gaussian Noise.
- **Rayleigh Fading Channel:** A realistic wireless channel model that accounts for multipath fading.

To improve system reliability over the Rayleigh channel, we implement and compare several Forward Error Correction (FEC) schemes, specifically Repetition Codes and Hamming Block Codes.

2 System Model

2.1 Modulation Scheme

The system utilizes Binary Phase Shift Keying (BPSK). The mapping from binary data to transmit symbols is defined as:

$$s_i = \begin{cases} +1 & \text{if bit is 1} \\ -1 & \text{if bit is 0} \end{cases}$$

2.2 Channel Models

2.2.1 AWGN Channel

In the Additive White Gaussian Noise (AWGN) channel, the received signal y is the sum of the transmitted signal x and noise n:

$$y = x + n$$

where n is a Gaussian random variable with zero mean and variance $\sigma^2 = \frac{N_0}{2}$.

2.2.2 Rayleigh Fading Channel

The Rayleigh fading channel models the effect of a signal taking multiple paths to the receiver. The received signal is given by:

$$y = hx + n$$

where h is the fading coefficient modeled as a complex Gaussian random variable (magnitude follows a Rayleigh distribution). The simulator assumes perfect Channel State Information (CSI) at the receiver and performs Zero-Forcing equalization:

$$\hat{x} = \frac{y}{h} = x + \frac{n}{h}$$

3 Channel Coding Implementation

3.1 Repetition Codes

We implemented repetition codes where every information bit is repeated n times. The receiver uses a "majority vote" hard decision rule.

- **Rate 1/3:** $n = 3$.
- **Rate 1/5:** $n = 5$.

3.2 Hamming Codes

Hamming codes are a class of linear block codes that can correct single-bit errors. We implemented two variants.

3.2.1 Hamming (7,4)

For the (7,4) code, 4 data bits are encoded into 7 transmit bits.

- **Generator Matrix (G):** Used for encoding ($c = mG$).
- **Parity Check Matrix (H):** Used for decoding ($S = rH^T$).

The matrices used in our implementation are:

$$G_{(7,4)} = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

3.2.2 Hamming (15,11)

The (15,11) code encodes 11 data bits into 15 transmit bits. The parity check matrix H is a 4×15 matrix containing all non-zero 4-bit vectors as columns. The decoder calculates the syndrome and matches it to a column in H to identify and correct the error position.

4 User Interface

The simulation tool was developed using Python (Flask) for the backend and HTML/JS for the frontend. The interface allows users to configure the SNR range, select the channel model, and choose specific coding parameters.

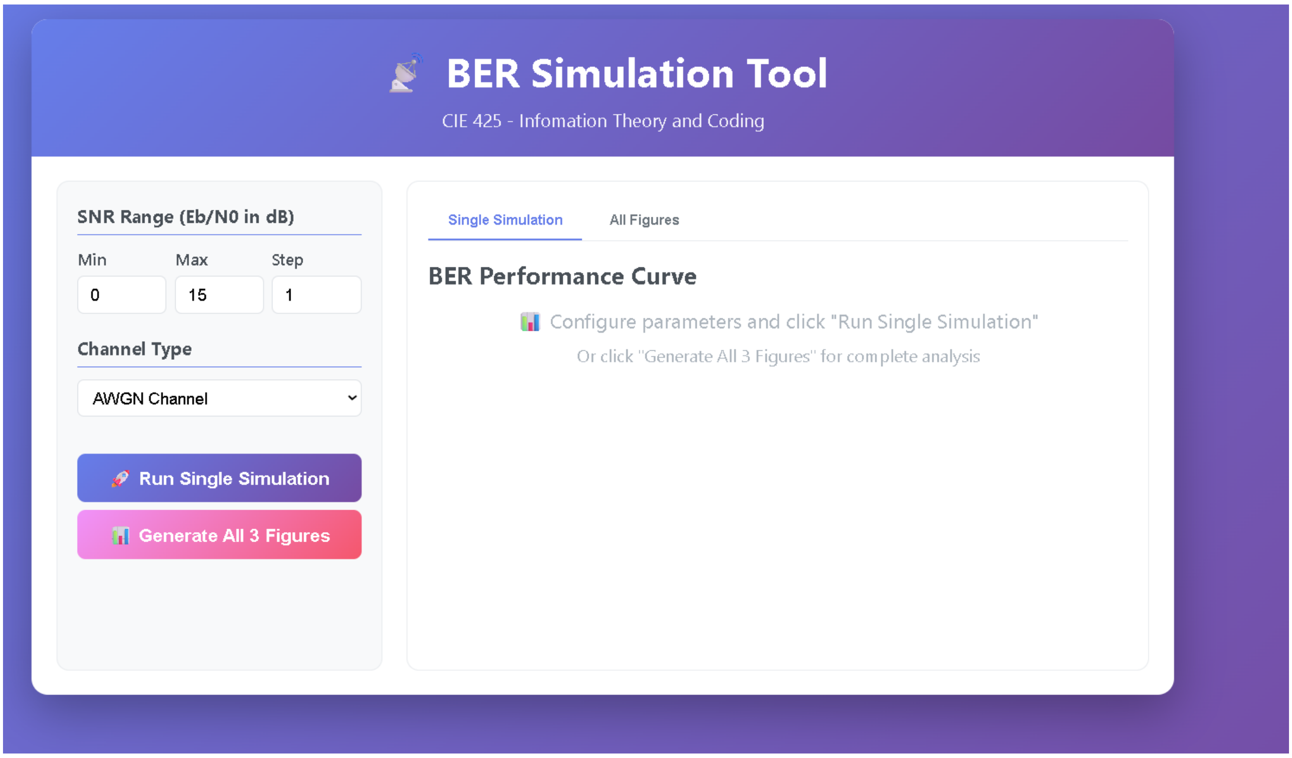

Figure 1: Screenshot of the developed BER Simulator GUI.

5 Simulation Results and Analysis

The simulation evaluates the BER across a range of SNR values (E_b/N_0 from 0 dB to 15 dB).

5.1 AWGN vs. Rayleigh (Uncoded)

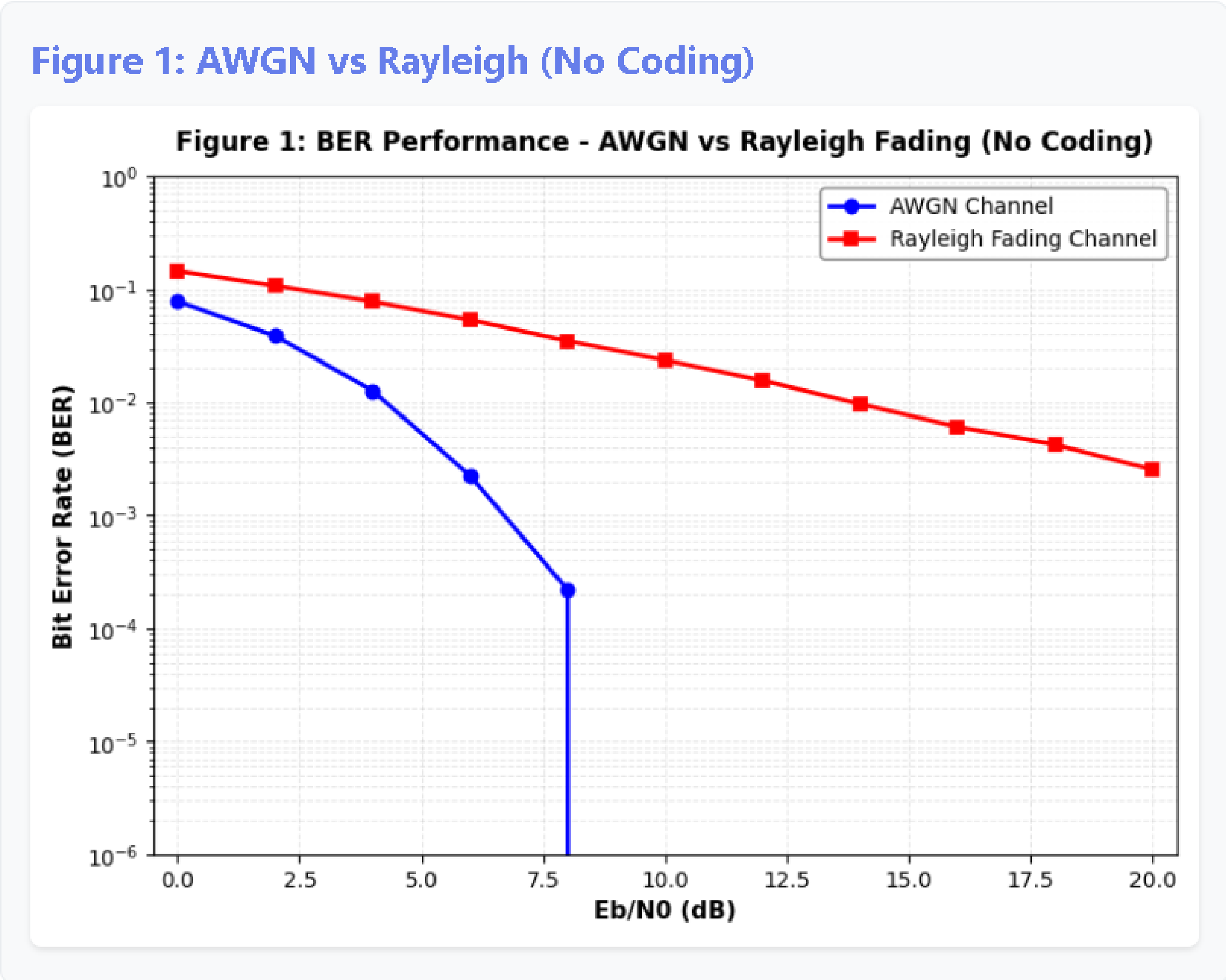

Figure 2: BER Performance: AWGN vs. Rayleigh Fading (Uncoded).

Comments: The figure illustrates the severe impact of fading on system performance.

- The **AWGN curve** drops exponentially (the infamous waterfall shape), indicating reliable performance improvement with increasing transmit power, although of course it is still noise added to the signal but it is just additive.
- The **Rayleigh curve** drops linearly on the log-log scale. This is due to the probability of deep fades, where the signal power drops significantly below the noise level, causing burst errors that are hard to overcome simply by increasing average power as the term is always multiplcated by the signal.
- For the same E_b/N_0 it will AWGN will result in about 1000 less erroneous bits or for a specific bit error rate AWGN will always outperform and will need less E_b/N_0

5.2 Comparison of Coding Schemes

Figure 2: Channel Coding Cases (4 Cases)

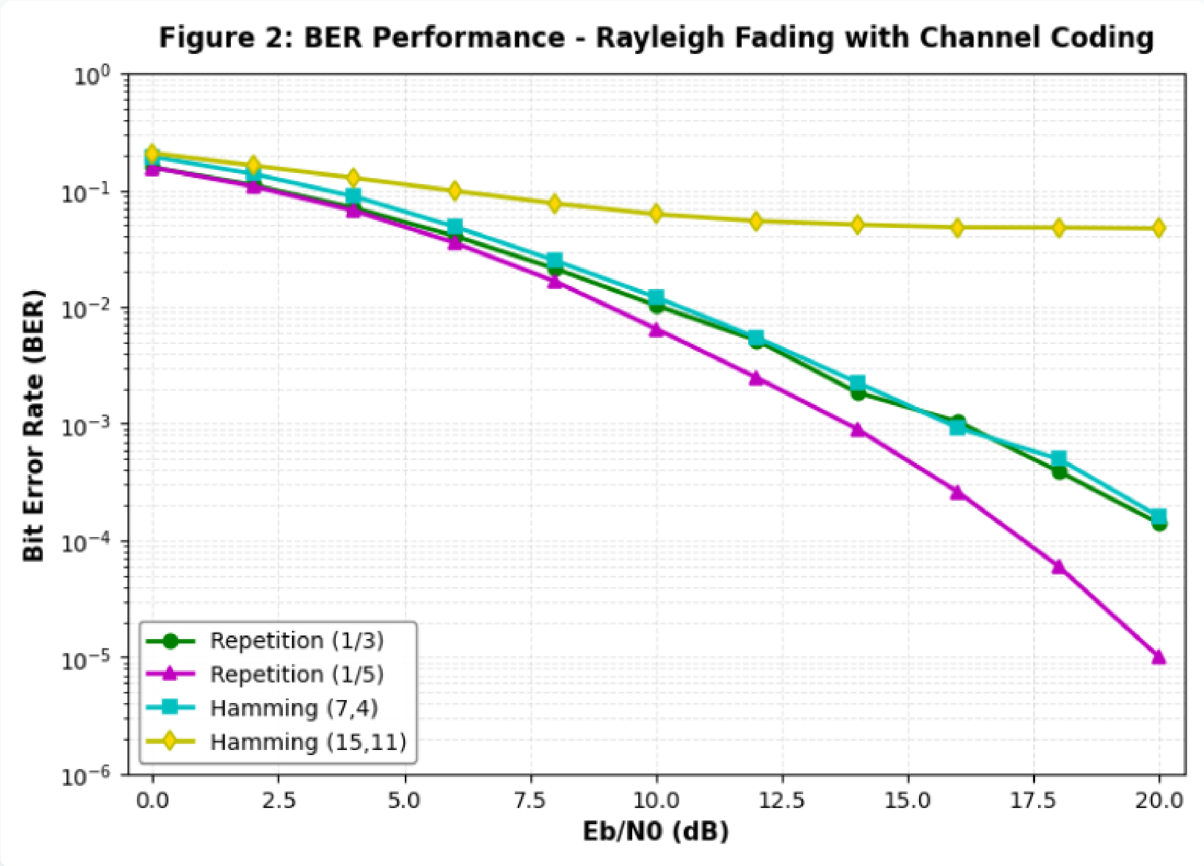

Figure 3: BER Performance of Different Channel Coding Schemes over Rayleigh Channel.

Commentary: This figure compares the raw performance of the four implemented codes over the Rayleigh channel.

- **Repetition Code (1/5) Performance:** This code achieves the lowest raw BER among all tested schemes. This is because it introduces the highest diversity order (5 repetitions). The decoder can tolerate up to 2 bit errors in a 5-bit block, making it highly resilient to deep channel fades compared to the 1/3 repetition code (tolerates 1 error) or Hamming codes (tolerates 1 error).
- **Hamming Codes ((7,4), (15,11)):** While they do not provide as much raw reliability as the 1/5 repetition code, they are significantly more spectrally efficient, offering rates of 0.57 and 0.73 respectively, compared to the 0.2 rate of the 1/5 repetition code.

5.3 Coded vs. Uncoded (Fair Comparison)

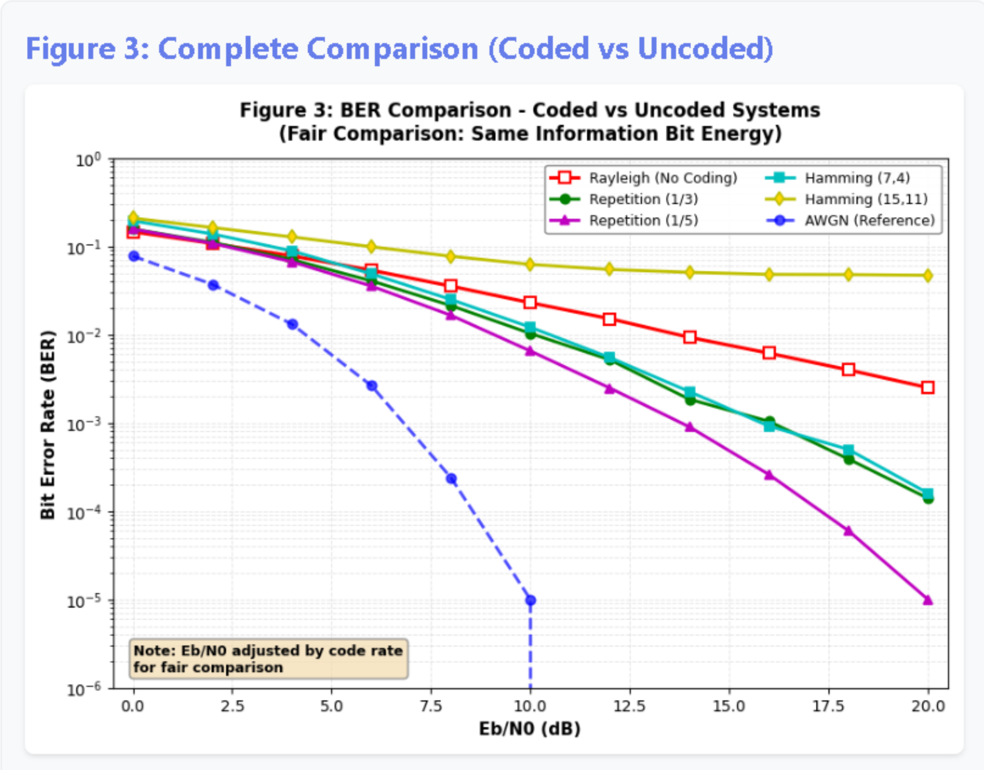

Figure 4: Fair Comparison: Coded vs. Uncoded Systems (Normalized E_b).

Comments: This is the most critical analysis, as it accounts for the energy penalty ($10 \log_{10} R$) incurred by adding redundant bits.

- **Low SNR Region (< 5-8 dB):** The coded systems actually perform *worse* or nearly the same than the uncoded system. This is because the energy per bit is split among many coded bits ($E_c < E_b$). In deep noise, the channel introduces more errors than the decoder can correct.
- **High SNR Region (> 10 dB):** The "Coding Gain" becomes visible. The error correction capability overcomes the energy penalty. The BER for the Hamming (7,4) code, for example, drops much faster than the uncoded Rayleigh curve.
- **Conclusion:** Channel coding is highly effective for high-reliability requirements (low BER), provided the SNR is sufficient to operate above the coding threshold.Also, we have to keep in mind that all of these are always worse than the AWGN if UnCoded!! but unfortunately Raw AWGN Isn't always Practical.

6 Conclusion

This project demonstrated that wireless channels like Rayleigh fading pose significant challenges to reliable communication. While the **Repetition (1/5)** code yielded the lowest raw Bit Error Rate due to its high diversity order, it suffers from extremely low spectral efficiency (Rate = 0.2). In contrast, the **Hamming (7,4)** and **(15,11)** codes provide a more practical balance, offering significant coding gain at high SNR levels while maintaining much higher data rates. This confirms that for practical communication standards where bandwidth is a constraint, block codes like Hamming are often preferred over simple repetition schemes despite the latter's raw robustness.