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Backstepping sliding mode control for combined spacecraft with nonlinear disturbance observer

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Abstract—**To attenuate the effects of inertia uncertainty and external disturbance of combined spacecraft on attitude control accuracy and stability, a composite control law by combining nonlinear disturbance observer (NDO) and backstepping sliding mode control is proposed. In this paper, the nonlinear disturbance observer (NDO) is added to observe the unknown inertia and external disturbance, and asymptotic stability of the backstepping sliding mode control with NDO is demonstrated. The performance of the proposed attitude control for combined spacecraft is discussed, and the simulation results demonstrate the effectiveness and feasibility of the proposed controller.**

Keywords—combined spacecraft; nonlinear disturbance observer; backstepping; sliding mode

I. INTRODUCTION

On-orbit service such as on-orbit maintenance, fuel adding, catching non-cooperative target and so on has become one of the most important fields in space development. The uncertainties of a combined spacecraft including momentum of inertia uncertainty and external disturbance for the combined spacecraft composed of chaser spacecraft and non-cooperative target becomes a non-ignorable issue in on-orbit servicing task accordingly. However, due to the uncertainty of non-cooperative target, the mass parameter of the combined spacecraft with rigid connection turns to be unknown. In view of the uncertainty above together with external disturbance, the robustness and stability of the system will be reduced.

Among the previous researching findings, generally speaking, there are two main methods to solve the problem of system uncertainties. The first one is using adaptive estimation method to estimate the upper bound of disturbance, in which the uncertainties always need to be extracted linearly to be a vector. This method also exists some conservations due to the utilizing of the upper bound[1][2]. The second one is the observer method which includes linear disturbance observer (LDO) and nonlinear disturbance observer (NDO). As an effective disturbance attenuation strategy, the NDO, which can estimate unknown disturbance and effectively compensate for them through feedforward, has attracted the attention of many researchers. In [3], a NDO method is developed in combination with SMC control law for a fin stabilizer system. In [4], an adaptive sliding mode controller has been applied to the attitude control for near space vehicles with the help of NDO. In [5], the NDO is used in a fight control system for a hypersonic gliding vehicle. Above all, the NDO has extensive applicability in disturbance attenuation and is easy to combine with other control methods. Inspired by the advantages of NDO, it has been introduced in the design of rigid connection combined spacecraft attitude controller.

The stability and robustness is a primary consideration for the design of attitude controller. Backstepping method is able to ensure better stability due to the iterative derivatives of virtual controller (recursive approach), and so it has been widely used in the fields of aeronautics and astronautics. In [6], an adaptive backstepping controller is used to solve attitude tracking of a rigid spacecraft without considered mass parameter uncertainty. In [7], the authors investigate the attitude control problem for a rigid spacecraft. The approach is based on backstepping and non-linear damping method, but the coefficient of damped term is constant which may result in large control moment. As what the advantages of backstepping method show in the researches above, in the paper, backstpping is adopted to make sure the stability of control law. Sliding mode control is categorized as a variable structure control, which has excellent robustness, and disturbance rejection characteristics[8]. Sliding mode is an effective control technique, and many researchers have utilized its robust property to control a variety of spacecraft[9][10][11]. So in the paper, sliding mode control is adopted to make sure the robustness of the control law .

On the basis of the researches in above papers, the backstepping sliding mode method combined with NDO is proposed to stabilize the rigid connection combined spacecraft attitude. The NDO, which can estimate unknown disturbances caused by inertia uncertainty and external disturbance, is designed for feedforward compensation. Then the backstepping sliding mode is used to control the attitude of rigid connection combined spacecraft precisely. Stability of the closed-loop system is analyzed using the Lyapunov method. The simulation results show that the control strategy is effective to stabilize rigid connection combined spacecraft with large inertia uncertainty, and the attitude control system has high precision and high stability.

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II. PROBLEM FORMULATION

Since the combined spacecraft in this paper is fully constrained, and the connection between the spacecraft and the non-cooperative target is rigid. So the rigid connection combined spacecraft can be seen as a rigid one. The attitude kinematics of combined spacecraft can be expressed by

$$\dot{\boldsymbol{q}} = \frac{1}{2}\boldsymbol{E}(\boldsymbol{q})\boldsymbol{\omega} \tag{1}$$

$\boldsymbol{q} = [q_0, \boldsymbol{q}_v^{\mathrm{T}}]^{\mathrm{T}} = [q_0, q_1, q_2, q_3]^{\mathrm{T}}, \boldsymbol{q}_v = [q_1, q_2, q_3]^{\mathrm{T}}$ is the unit quaternion that satisfies the constraint $\|\boldsymbol{q}\| = 1$, and represents the body frame attitude orientation with respect to the inertial frame. $\boldsymbol{\omega}(\boldsymbol{\omega} \in \mathbb{R}^{3\times1})$ is the body frame angular velocity with respect to inertial frame. $\boldsymbol{E}(\boldsymbol{q})$ is defined as $\boldsymbol{E}(\boldsymbol{q}) = \begin{bmatrix} -\boldsymbol{q}_v^{\mathrm{T}} \\ [\boldsymbol{q}_v^{\times}] + q_0\boldsymbol{I}_3 \end{bmatrix}$, $\boldsymbol{I}_3$ is the 3×3 identity matrix, $\boldsymbol{q}_v^{\times}$ is a skew-symmetric matrix.

The dynamic model of rigid connection combined spacecraft can be found from Euler's moment equation as

$$\boldsymbol{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}^{\times}\boldsymbol{J}\boldsymbol{\omega} = \boldsymbol{u} + \boldsymbol{d} \tag{2}$$

$\boldsymbol{J} \in \mathbb{R}^{3\times3}$ is the total inertia matrix of rigid connection combined spacecraft, $\boldsymbol{u} \in \mathbb{R}^{3\times1}$ is the total control torque, and $\boldsymbol{d} \in \mathbb{R}^{3\times1}$ is the total external disturbance.

The relative attitude error between the reference frame and the desired reference frame is used in this paper for attitude control. The desired attitude quaternion is $\boldsymbol{q}_d = [q_{0d}, \boldsymbol{q}_{vd}^{\mathrm{T}}]^{\mathrm{T}} = [q_{0d}, q_{1d}, q_{2d}, q_{3d}]^{\mathrm{T}}$, where $\boldsymbol{q}_{vd} = [q_{1d}, q_{2d}, q_{3d}]^{\mathrm{T}}$. The error attitude quaternion is $\boldsymbol{q}_e = [q_{0e}, \boldsymbol{q}_{ve}^{\mathrm{T}}]^{\mathrm{T}} = [q_{0e}, q_{1e}, q_{2e}, q_{3e}]^{\mathrm{T}}$, where $\boldsymbol{q}_{ve} = [q_{1e}, q_{2e}, q_{3e}]^{\mathrm{T}}$. The relationship between $\boldsymbol{q}_d$ and $\boldsymbol{q}_e$ is $q_{0e} = \boldsymbol{q}_{vd}^{T}\boldsymbol{q}_v + q_0 q_{0d}$, $\boldsymbol{q}_{ve} = q_{0d}\boldsymbol{q}_v - \boldsymbol{q}_{vd}^{\times}\boldsymbol{q}_v - q_0\boldsymbol{q}_{vd}$.

The desired frame angular velocity is $\boldsymbol{\omega}_d \in \mathbb{R}^{3\times1}$, which is set to 0 for the attitude stabilization control

$$\boldsymbol{\omega}_e = \boldsymbol{\omega} - \boldsymbol{\omega}_d = \boldsymbol{\omega} \tag{3}$$

Then, the attitude and the dynamics equations can be written as

$$\dot{\boldsymbol{q}}_e = \frac{1}{2}\boldsymbol{E}(\boldsymbol{q}_e)\boldsymbol{\omega}_e \tag{4}$$

$$\boldsymbol{J}\dot{\boldsymbol{\omega}}_e = -\boldsymbol{\omega}_e^{\times}\boldsymbol{J}\boldsymbol{\omega}_e + \boldsymbol{u} + \boldsymbol{d} \tag{5}$$

The rigid connection combined spacecraft has large inertia uncertainty. In this paper, we assume that the inertia matrix is $\boldsymbol{J} = \boldsymbol{J}_0 + \Delta\boldsymbol{J}$, where $\boldsymbol{J}_0$ denotes the known constant matrix. $\Delta\boldsymbol{J}$ denotes the uncertainty. The dynamics equation can be written as

$$(\boldsymbol{J}_0 + \Delta\boldsymbol{J})\dot{\boldsymbol{\omega}}_e = -\boldsymbol{\omega}_e^{\times}(\boldsymbol{J}_0 + \Delta\boldsymbol{J})\boldsymbol{\omega}_e + \boldsymbol{u} + \boldsymbol{d} \tag{6}$$

Then, (6) can be written as (7) through simple algebraic transformations

$$\dot{\boldsymbol{\omega}}_e = \boldsymbol{F} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{D} \tag{7}$$

Where

$$\boldsymbol{F} = \boldsymbol{J}_0^{-1}(-\boldsymbol{\omega}_e^{\times}\boldsymbol{J}_0\boldsymbol{\omega}_e) \tag{8}$$

$$\boldsymbol{D} = \boldsymbol{J}_0^{-1}(-\boldsymbol{\omega}_e^{\times}\Delta\boldsymbol{J}\boldsymbol{\omega}_e - \Delta\boldsymbol{J}\dot{\boldsymbol{\omega}}_e + \boldsymbol{d}) \tag{9}$$

Assumption 1: The desired attitude frame angular velocity $\boldsymbol{\omega}_d$ and its first time derivative $\dot{\boldsymbol{\omega}}_d$ are bounded.

Assumption 2: External disturbance torque $\boldsymbol{d}$ is bounded.

$\boldsymbol{x} = [\boldsymbol{x}_1, \boldsymbol{x}_2]^{\mathrm{T}} = [\boldsymbol{q}_e, \boldsymbol{\omega}_e]^{\mathrm{T}}$ are taken as state variables. Then, the attitude and the dynamics equation can be written as

$$\begin{cases} \dot{\boldsymbol{x}}_1 = \frac{1}{2}(q_{0e}\boldsymbol{I}_3 + \boldsymbol{x}_1^{\times})\boldsymbol{x}_2 = \boldsymbol{G}(\boldsymbol{x})\boldsymbol{x}_2 \\ \dot{\boldsymbol{x}}_2 = \boldsymbol{F}(\boldsymbol{x}) + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{D} \end{cases} \tag{10}$$

The control aim is to propose a control law to guarantee that $\lim_{t\to\infty}\boldsymbol{x}_1(t) = 0$ and $\lim_{t\to\infty}\boldsymbol{x}_2(t) = 0$.

III. ADAPTIVE CONTROL LAW BASED ON ESTIMATION OF UCERTAINTIES

In this section, an adaptive method is adopted to deal with the uncertainties, which is also used to contrast with the method proposed in next section. In this method, the state variables can be defined as

$$\begin{cases} \boldsymbol{z}_1 = \boldsymbol{x}_1 \\ \boldsymbol{z}_2 = \boldsymbol{x}_2 - \boldsymbol{\omega}_r \end{cases} \tag{11}$$

$\boldsymbol{x}_2$ is the virtual control input; $\boldsymbol{\omega}_r$ is the stabilization function which is defined as

$$\boldsymbol{\omega}_r = -\alpha\boldsymbol{x}_1 \tag{12}$$

From (11), we can get

$$\dot{\boldsymbol{z}}_2 = \dot{\boldsymbol{x}}_2 - \dot{\boldsymbol{\omega}}_r \tag{13}$$

Then, (13) can be written as (14) through $\boldsymbol{J}$ premultiplication

$$\boldsymbol{J}\dot{\boldsymbol{z}}_2 = -\boldsymbol{\omega}^{\times}\boldsymbol{J}\boldsymbol{\omega} + a\boldsymbol{J}\dot{\boldsymbol{x}}_1 + \boldsymbol{u} + \boldsymbol{d} = \boldsymbol{Y}\boldsymbol{\eta} + \boldsymbol{u} \tag{14}$$

Where $\boldsymbol{\eta} = [J_{11} \quad J_{22} \quad J_{33} \quad J_{12} \quad J_{13} \quad J_{23} \quad d_1 \quad d_2 \quad d_2]^{\mathrm{T}}$ denotes the uncertainties.

$\boldsymbol{Y}$ is the corresponding coefficient matrix

$$
\boldsymbol{Y}=\begin{bmatrix} a\dot{q}_{e1} & \omega_2\omega_3 & -\omega_2\omega_3 & a\dot{q}_{e2}+\omega_1\omega_3 & a\dot{q}_{e3}-\omega_1\omega_2 & \omega_3^2-\omega_2^2 & 1 & 0 & 0 \\ -\omega_1\omega_3 & a\dot{q}_{e2} & \omega_1\omega_3 & a\dot{q}_{e1}-\omega_2\omega_3 & \omega_1^2-\omega_3^2 & a\dot{q}_{e3}+\omega_1\omega_2 & 0 & 1 & 0 \\ \omega_1\omega_2 & -\omega_1\omega_2 & a\dot{q}_{e3} & \omega_2^2-\omega_1^2 & a\dot{q}_{e1}+\omega_2\omega_3 & a\dot{q}_{e2}-\omega_1\omega_3 & 0 & 0 & 1 \end{bmatrix} \tag{15}
$$

Theorem 1: Considering the plant (4) and (5), the closed-loop system is uniformly ultimately bounded stability (UUB) under the control law (16) and adaptive law (17). Where k_1 and k_2 are positive constants, $\boldsymbol{P}$ is the estimation matrix. $\hat{\boldsymbol{\eta}}$ is the estimation of uncertain.

$$
\dot{\hat{\boldsymbol{\eta}}}=\boldsymbol{P}^{-1}\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{z}_2 \tag{16}
$$

$$
\boldsymbol{u}_1=-k_1\boldsymbol{z}_1-k_2\boldsymbol{z}_2-\boldsymbol{Y}\hat{\boldsymbol{\eta}} \tag{17}
$$

The proof of Theorem 1 is similar to the one in [7]. And the control law (17) is a relatively straightforward method. The stability and robustness of it is not very desirable.

Remark: The estimate of inertia uncertainties matrix is not always symmetric. Meanwhile, the estimate value is not the true value, because when z_2 comes to zero, the $\hat{\boldsymbol{\eta}}$ is unchanged.

IV. Backtepping Sliding Mode Control Design with Nolinear Disturbance Observer

The control law in above section is not an exact method. In order to achieve a better control characteristic, an attitude control based on NDO is proposed in this section. First, a NDO is adopted to estimate the merged disturbance precisely. Second, an adaptive backstepping sliding mode control law is applied based on the estimation value.

A. Nonlinear Disturbance Observer Design

To estimate the disturbances of system (7), we construct the NDO as follow[12]

$$
\begin{cases} \hat{\boldsymbol{D}}=z+\boldsymbol{p}(\boldsymbol{x}) \\ \dot{z}=-\boldsymbol{L}(\boldsymbol{x})z+\boldsymbol{L}(\boldsymbol{x})[-\boldsymbol{p}(\boldsymbol{x})-\boldsymbol{F}(\boldsymbol{x})-\boldsymbol{Bu}] \end{cases} \tag{18}
$$

Assumption 3： The merged disturbances $\boldsymbol{D}$ are slow varying and bounded. Therefore, $\dot{\boldsymbol{D}}=0$ is reasonable.

Where $\hat{\boldsymbol{D}}$ is the estimation of the merged disturbance, $\boldsymbol{p}(\boldsymbol{x})$ is the nonlinear function need to be designed; $\boldsymbol{L}(\boldsymbol{x})$ is the nonlinear observer gain function, which satisfies $\boldsymbol{L}(\boldsymbol{x})\dot{\boldsymbol{x}}_2=\mathrm{d}\boldsymbol{p}(\boldsymbol{x})/\mathrm{d}t$, $\tilde{\boldsymbol{D}}$ is observer error with $\tilde{\boldsymbol{D}}=\boldsymbol{D}-\hat{\boldsymbol{D}}$.

Theorem 2: Considering the dynamic system (7) and NDO (18), when $\boldsymbol{L}(\boldsymbol{x})$ is a positive definite matrix, the NDO can track the disturbance $\boldsymbol{D}$, and the estimation error $\tilde{\boldsymbol{D}}$ converges to the origin.

Proof: The candidate Lyapunov function is selected as

$$
V_d=\frac{1}{2}\tilde{\boldsymbol{D}}^{\mathrm{T}}\tilde{\boldsymbol{D}} \tag{19}
$$

The first time derivative of (19) is

$$
\dot{V}_d=\tilde{\boldsymbol{D}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{D}}}=\tilde{\boldsymbol{D}}^{\mathrm{T}}[-\dot{z}-\dot{\boldsymbol{p}}(\boldsymbol{x})] \tag{20}
$$

By substituting (18) into (20), we obtain that

$$
\begin{aligned} \dot{V}_d=\tilde{\boldsymbol{D}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{D}}}=\tilde{\boldsymbol{D}}^{\mathrm{T}}[&\boldsymbol{L}(\boldsymbol{x})z+ \\ &\boldsymbol{L}(\boldsymbol{x})[\boldsymbol{p}(\boldsymbol{x})+\boldsymbol{F}(\boldsymbol{x})+\boldsymbol{Bu}]-\boldsymbol{L}(\boldsymbol{x})\dot{\boldsymbol{x}}_2] \end{aligned} \tag{21}
$$

By substituting (10) into (21), we obtain that

$$
\dot{V}_d=-\tilde{\boldsymbol{D}}^{\mathrm{T}}\boldsymbol{L}(\boldsymbol{x})\tilde{\boldsymbol{D}} \tag{22}
$$

If $\boldsymbol{L}(\boldsymbol{x})$ is a positive definite matrix, then $\dot{V}_d<0$. It means that the NDO can track the disturbance $\boldsymbol{D}$, and $\boldsymbol{D}(t)=e^{-\boldsymbol{L}(\boldsymbol{x})(t-t_0)}\boldsymbol{D}(0)$, $\boldsymbol{D}(0)$ is the initial value of $\boldsymbol{D}$.

B. Adaptive Backstepping Sliding Mode Control Law

Backstepping method usually starts from the minimum order of the system. In every step of design, the virtual control is introduced into system for satisfying the corresponding subsystem sliding conditions. By this way, the final control law is achieved.

The system error is defined as

$$
\begin{cases} \boldsymbol{e}_1=\boldsymbol{x}_1-\boldsymbol{y}_d \\ \boldsymbol{e}_2=\boldsymbol{x}_2-\boldsymbol{\alpha}_1(\boldsymbol{x}) \end{cases} \tag{23}
$$

The first time derivative of (23) is

$$
\begin{cases} \dot{\boldsymbol{e}}_1=\dot{\boldsymbol{x}}_1-\dot{\boldsymbol{y}}_d \\ \dot{\boldsymbol{e}}_2=\dot{\boldsymbol{x}}_2-\dot{\boldsymbol{\alpha}}_1(\boldsymbol{x}) \end{cases} \tag{24}
$$

According to the backstepping method, the virtual control law is defined as:

$$
\boldsymbol{\alpha}_1(\boldsymbol{x})=\boldsymbol{G}^{-1}(-k_3\boldsymbol{e}_1+\dot{\boldsymbol{y}}_d) \tag{25}
$$

By substituting (25) into (24), we obtain that:

$$
\begin{cases} \dot{\boldsymbol{e}}_1=\boldsymbol{G}[\boldsymbol{e}_2+\boldsymbol{G}^{-1}(-k_3\boldsymbol{e}_1+\dot{\boldsymbol{y}}_d)]-\dot{\boldsymbol{y}}_d=\boldsymbol{G}\boldsymbol{e}_2-k_3\boldsymbol{e}_1 \\ \dot{\boldsymbol{e}}_2=\boldsymbol{F}+\boldsymbol{Bu}+\boldsymbol{D}-\dot{\boldsymbol{\alpha}}_1(\boldsymbol{x}) \end{cases} \tag{26}
$$

To improve the robustness of the attitude control law, the sliding mode control is adopted. And the sliding surface is selected as follow, where $c>0$

$$
\boldsymbol{s}=c\boldsymbol{e}_1+\boldsymbol{e}_2 \tag{27}
$$

The error of observer is bounded and $\|\tilde{\boldsymbol{D}}\|=\|\boldsymbol{D}-\hat{\boldsymbol{D}}\|\le\beta$, $\hat{\beta}$ is the estimate of β , we select the adaptive law as

$$
\dot{\hat{\beta}}=\lambda|\boldsymbol{s}| \tag{28}
$$

Theorem 3: For the system (10), control law (29) and NDO (18), when $h>0$, the closed-loop system is asymptotic stability.

$$\boldsymbol{u}_2 = \boldsymbol{B}^{-1}[-\boldsymbol{F}(\boldsymbol{x})+\dot{\boldsymbol{\alpha}}_1(\boldsymbol{x})-h\boldsymbol{s}-\hat{\beta}\,\mathrm{sgn}(\boldsymbol{s})-\hat{\boldsymbol{D}} \\ -c(\boldsymbol{G}\boldsymbol{e}_2-k_3\boldsymbol{e}_1)] \tag{29}$$

Proof: Consider a candidate Lyapunov function V as

$$V=\frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\boldsymbol{s}+\frac{1}{2\lambda}\tilde{\beta}^2 \tag{30}$$

The first time derivative of V is

$$\dot{V}=\boldsymbol{s}^{\mathrm{T}}(c\dot{\boldsymbol{e}}_1+\dot{\boldsymbol{e}}_2)-\frac{1}{\lambda}\tilde{\beta}\dot{\hat{\beta}} \tag{31}$$

And substituting (29) into (31), we can obtain

$$\begin{aligned}\dot{V}&=\boldsymbol{s}^{\mathrm{T}}[c(\boldsymbol{G}\boldsymbol{e}_2-k_3\boldsymbol{e}_1)+\boldsymbol{F}(\boldsymbol{x})+\boldsymbol{B}\boldsymbol{u}+\boldsymbol{D}-\dot{\boldsymbol{\alpha}}_1(\boldsymbol{x})]-\frac{1}{\lambda}\tilde{\beta}\dot{\hat{\beta}}\\ &=\boldsymbol{s}^{\mathrm{T}}[c(\boldsymbol{G}\boldsymbol{e}_2-k_3\boldsymbol{e}_1)+\boldsymbol{F}(\boldsymbol{x})+[-\boldsymbol{F}(\boldsymbol{x})+\dot{\boldsymbol{\alpha}}_1(\boldsymbol{x})-h\boldsymbol{s}\\ &-\hat{\beta}\,\mathrm{sgn}(\boldsymbol{s})-\hat{\boldsymbol{D}}-c(\boldsymbol{G}\boldsymbol{e}_2-k_3\boldsymbol{e}_1)]+\boldsymbol{D}-\dot{\boldsymbol{\alpha}}_1(\boldsymbol{x})]-\frac{1}{\lambda}\tilde{\beta}\dot{\hat{\beta}}\end{aligned} \tag{32}$$

Substituting adaptive law (28) into (32), we can obtain

$$\begin{aligned}\dot{V}&\le|\boldsymbol{s}|\beta-h\boldsymbol{s}^2-|\boldsymbol{s}|\hat{\beta}-|\boldsymbol{s}|\tilde{\beta}\\ &\le -h\boldsymbol{s}^2\end{aligned} \tag{33}$$

When the following condition $h>0$ is satisfied, $\dot{V}\le 0$.

V. Simulation Results

In this section, the effectiveness of the proposed controller is verified through numerical simulations. And for better illustration, the simulation of control law (17) is also be conducted.

The above two simulation conditions are the same except the external disturbance. And the initial conditions of angle and angular velocity are selected as $\begin{bmatrix}10^\circ & -10^\circ & 15^\circ\end{bmatrix}^{\mathrm{T}}$ and $\begin{bmatrix}0.02 & 0.02 & 0.02\end{bmatrix}^{\mathrm{T}}$ rad/s, respectively. The initial target angle is set to $\begin{bmatrix}0^\circ & 0^\circ & 0^\circ\end{bmatrix}^{\mathrm{T}}$ and the desired angular velocity is $\begin{bmatrix}0 & 0 & 0\end{bmatrix}^{\mathrm{T}}$ rad/s, the nominal and parameter uncertainty of rigid connection combined spacecraft inertia matrix are selected as

$$\boldsymbol{J}_0=\begin{bmatrix}1349.616 & 6.563 & -13.321\\ 6.563 & 1240.404 & 5.244\\ -13.321 & 5.244 & 724.423\end{bmatrix}\ \mathrm{N\cdot m^2}$$

$$\Delta\boldsymbol{J}=\begin{bmatrix}500 & 0 & 0\\ 0 & 600 & 0\\ 0 & 0 & 300\end{bmatrix}\ \mathrm{N\cdot m^2}$$

The external disturbance of control law (17) and control law (29) are $\boldsymbol{d}_1(t)$ and $\boldsymbol{d}_2(t)$, respectively.

$$\boldsymbol{d}_1(t)=\begin{bmatrix}-1 & 2 & -3\end{bmatrix}^{\mathrm{T}}\times10^{-3}\,\mathrm{N\cdot m}$$

$$\boldsymbol{d}_2(t)=\begin{bmatrix}1\sin(0.1t) & 2\sin(0.2t) & 3\sin(0.3t)\end{bmatrix}^{\mathrm{T}}\times10^{-3}\,\mathrm{N\cdot m}$$

The parameters of the control law (17) are set to $\boldsymbol{P}=10^{-2}\cdot\mathrm{diag}\{0.01\ 0.001\ 0.01\ 0.1\ 0.1\ 0.1\ 100\ 100\ 100\}$ $k_1=185,\ k_2=150$ and $\alpha=0.2$.

The parameters of the proposed control law (29) are set to $L(x_1,x_2)=a=2$, $k_3=0.1$, $c=0.1$, $\lambda=0.001$ and $h=0.2$。

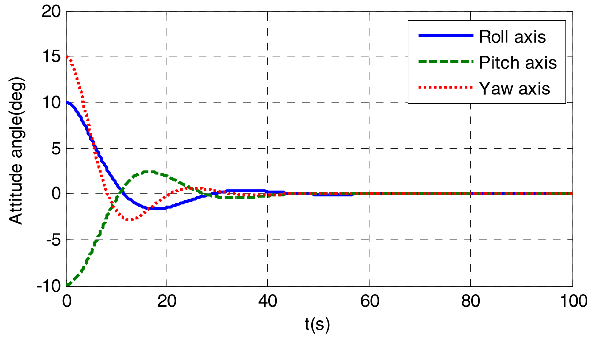

Fig. 1. Attitude angle of control law $\boldsymbol{u}_1$

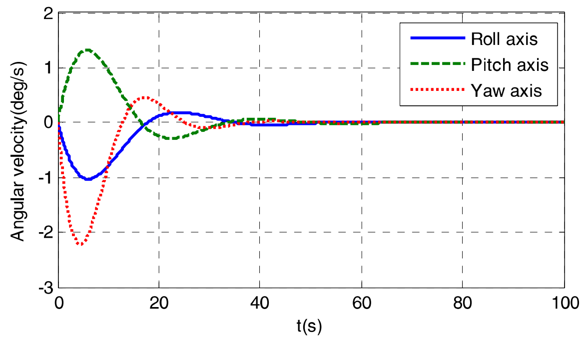

Fig. 2. Angular velocity of control law $\boldsymbol{u}_1$

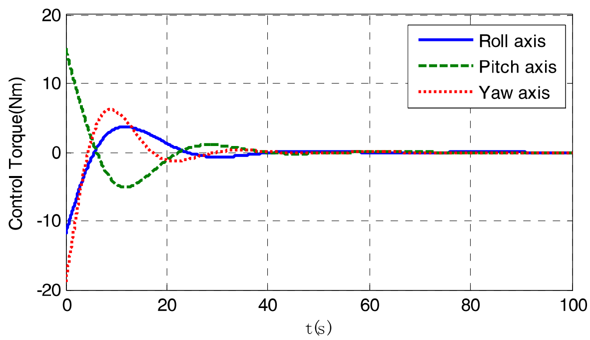

Fig. 3. The control input $\boldsymbol{u}_1$.

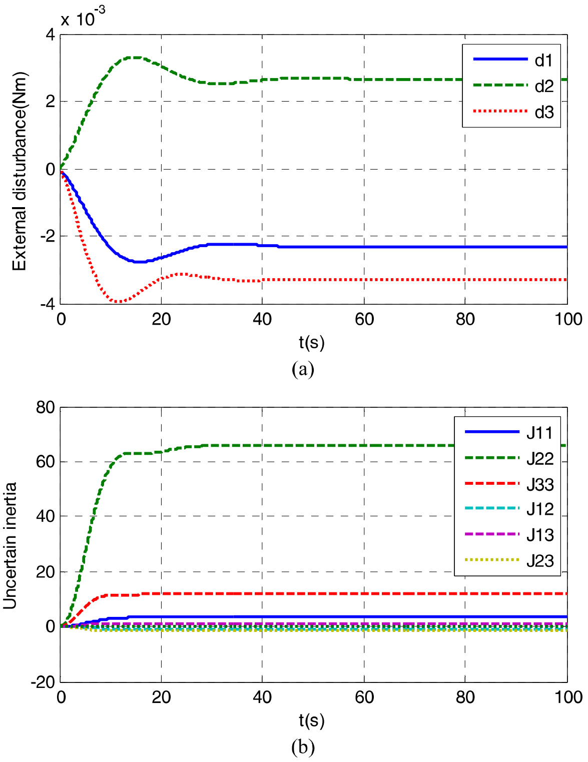

Fig. 4. Estimation of external disturbance and unkown inertia

The attitude states are depicted in Fig. 1, which shows that the designed attitude states achieve stable after 45 s. The angular velocity states are shown in Fig. 2. The control inputs are shown in Fig. 3. The estimation of external disturbance and unknown inertia are shown in Fig. 4. These results illustrated that the control law (17) has some effects on the control of rigid connection combined spacecraft.

The attitude states, angular velocity states and control inputs of rigid connection combined spacecraft with the proposed control law (29) are shown in Fig. 5, Fig. 6 and Fig. 7, respectively. The performance of NDO is illustrated in Fig. 8, which shows that the disturbance observer can effectively estimate the total disturbances.

According to Figs. 1 and 5, we can see that the stable times of control law (17) and control law (29) are the same. According to Figs. 2 and 6, it shows that the maximum angular velocity state of control law (17) is larger than the ones of control law (29). Additional according to Figs. 3 and 7, it exhibits that the control inputs of control law (17) are larger than the ones of control law (29) until the attitude states become stable. A big control input will be required for control law (17). According to Figs. 4 and 8, it shows that the NDO can effectively estimate the total disturbances rather than the unknown inertia. Meanwhile the NDO has better traceability. All above demonstrate the effectiveness of the control law (29).

These comparisons show the validity of the conclusion of **Theorem 3** that the system states converge to the origin in spite of the inertia uncertainty and external disturbance.

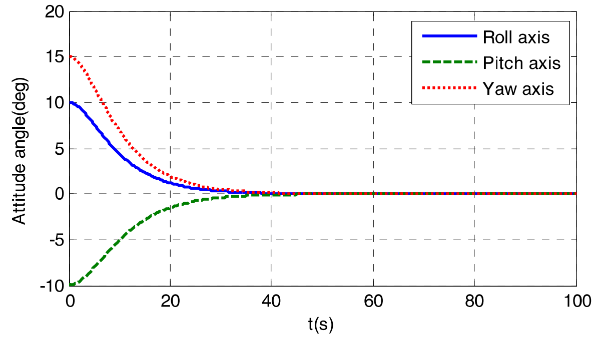

Fig. 5. Attitude angle of control law $\boldsymbol{u}_2$

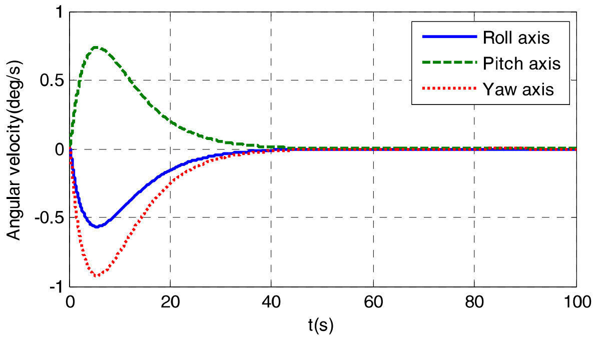

Fig. 6. Angular velocity of control law $\boldsymbol{u}_2$

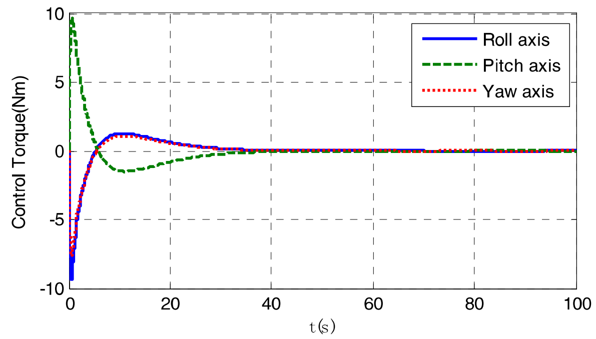

Fig. 7. The control input $\boldsymbol{u}_2$.

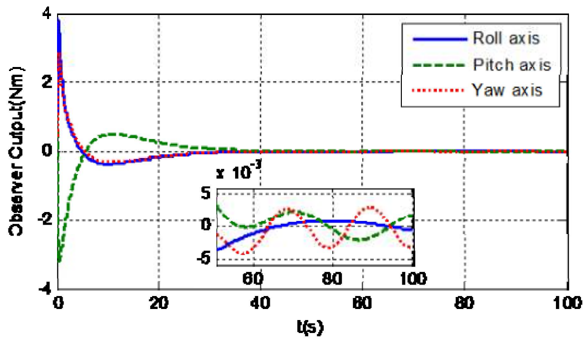

Fig. 8. Estimation of disturbances via NDO

Furthermore, some comparative simulations are also carried out to illustrate the effectiveness of NDO. From Fig. 9 and Fig. 10, we can see the backstepping sliding mode control with NDO has better high steady accuracy.

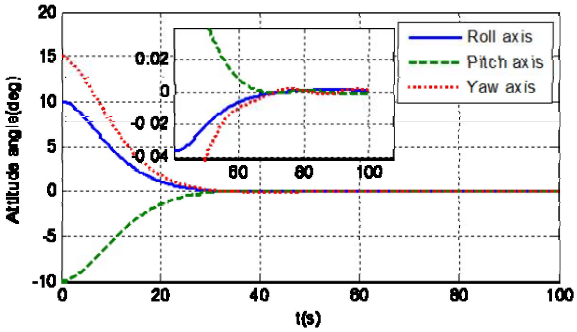

Fig. 9. Combined spacecraft with backstepping sliding mode control

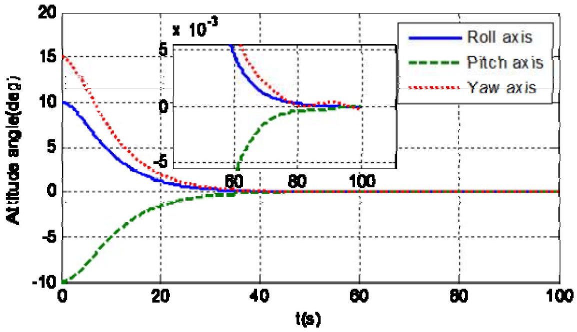

Fig. 10. Combined spacecraft with backstepping sliding mode control + NDO

VI. CONLUSIONS

In this paper, we considered the attitude control problem for a rigid connection combined spacecraft with inertia uncertainty and external disturbance. The NDO with the backstepping sliding mode method is proposed. The NDO is applied to estimate the external disturbance and the parametric uncertainty of the inertia matrix then compensate for them through feedforward. The backstepping sliding mode controller can achieve a high performance on attitude control in the presence of various disturbances. Simulation results showed that the composite controller can improve robust dynamic performance and attitude control accuracy.

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REFERENCES

[1] G. Chen and F. L. Lewis, Distributed adaptive tracking control for synchronization of unknown networked Lagrangian systems. Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on, 2011. 41(3): pp. 805-816.

[2] Z. Chen and J. Huang, Attitude tracking and disturbance rejection of rigid spacecraft by adaptive control. IEEE Transactions on Automatic Control, 2009. 54(3): pp. 600.

[3] Y. T. Zhang, W. R. Shi and M. B. Qiu, Sliding backstepping control for fin stabilizer with nonlinear disturbance observer. Control and Decision, 2010. 25(8): pp. 1255-1260.

[4] J Yu, M Chen, C S Jiang, Adaptive sliding mode control for nonlinear uncertain systems based on disturbance observer. Control Theory & Applications, 2014. 8: pp. 001.

[5] C. S. Qian, C. Y. Sun, Y. Q. Huang, et al, Design of flight control system for a hypersonic gliding vehicle based on nonlinear disturbance observer. In: Proceedings of the 10th IEEE International Conference on Control and Automation. Hangzhou: IEEE press. 2014:pp. 1573-1577.

[6] Q. Hu, B. Li, X. Huo, et al, Spacecraft attitude tracking control under actuator magnitude deviation and misalignment. Aerospace Science and Technology, 2013. 28(1): pp. 266-280.

[7] B. Song, C. J. Li, and G. F. Ma, Robust adaptive controller design for spacecraft during attitude maneuver. Journal of Astronautics, 2008. 29(1): pp. 121-125.

[8] C. Pukdeboon, A.S. Zinober, M.-W. Thein, Quasi-continuous higher order sliding-mode controllers for spacecraft-attitude-tracking maneuvers. Industrial Electronics, IEEE Transactions on, 2010. 57(4): pp. 1436-1444.

[9] C. Zhong, Y. Guo, Z. Yu, et al, Finite-time attitude control for flexible spacecraft with unknown bounded disturbance. Transactions of the Institute of Measurement and Control, 2016. 38(2): pp. 240-249.

[10] P. M. Tiwari, S. Janardhanan, and M. un Nabi, Rigid Spacecraft Attitude Control Using Adaptive Non-singular Fast Terminal Sliding Mode. Journal of Control, Automation and Electrical Systems, 2015. 26(2): pp. 115-124.

[11] H. Gui, and G. Vukovich, Adaptive integral sliding mode control for spacecraft attitude tracking with actuator uncertainty. Journal of the Franklin Institute, 2015. 352(12): pp. 5832-5852.

[12] W. H. Chen, D. J Ballance, P. J Gawthrop, et al, A nonlinear disturbance observer for robotic manipulators. Industrial Electronics, IEEE Transactions on, 2000. 47(4): pp. 932-938.