LQR control for a quadrotor using unit quaternions: Modeling and simulation

Conference Paper · March 2013

DOI: 10.1109/CONIELECOMP.2013.6525781

4 authors, including:

Rogerio Enriquez-Caldera
National Institute of Astrophysics, Optics and Electronics

Sergio Camacho
Centro Regional de Enseñanza de Ciencia y Tecnología del Espacio para América Lat…

Jose S. Guichard
National Institute of Astrophysics, Optics and Electronics

LQR Control for a Quadrotor using Unit Quaternions: Modeling and Simulation

Elias Reyes-Valeria, Rogerio Enriquez-Caldera,
Electronics Department
INAOE
Puebla, Mexico
rogerio@inaoep.mx

Sergio Camacho-Lara and Jose Guichard
Astrophysics Department
CRECTEALC/INAOE
Puebla, Mexico
jguichard@inaoep.mx

***Abstract*— Nowadays, quadrotors have become a popular UAV research platform because full control can be achieved through speed variations in each and every one of its four rotors. Here, a non-linear dynamic model based on quaternions for attitude is presented as well as its corresponding LQR Gain Scheduling Control. All considerations for the quadrotor movements are described through their state variables. Modeling is carried out through the Newton-Euler formalism. Finally, the control system is simulated and the results shown in a novel and direct unit quaternion. Thus, a successful trajectory and attitude control of a quadrotor is achieved.**

Keywords*—*Quadrotor, UAV, LQR, Unit Quaternion

I. Introduction

Interest is growing around Unmanned Aerial Vehicles (UAV's) due to their industrial and military uses. Aerial photography, mapping, surveillance, search and rescue are among of many intelligence duties that can be carried out without risking the life of an operator.

Many research communities continue to develop a great variety of controllers for UAV. Nevertheless, in most of practical control implementations, position control is done by a remote operator that obtains visual feedback information using an on board camera. Meanwhile, attitude control is automatically performed by a local control coupled to the aircraft. However, it is also common for the Control to use rotation matrices based on *Tait-Bryan* angles whose main flaw is the existence of certain critical points with repercussion in the lost of degrees of freedom. The alternative to such a problem is to make the attitude control using a unit quaternion.

These kind of aircrafts make use of four motors m_i to produce the required forces for corresponding maneuver on the UAV as depicted in Fig. 1. Thus, the angular movement around the *x*-axis, known as *roll* ϕ, is produced by the resulting torque of m_2 and m_4. Similarly, the resulting torque of m_1 and m_3 produce an angular movement around the *y*-axis, known as *pitch* θ. The movement around *z*-axis, known as *yaw* ψ, is due to the torque around *z*-axis when incrementing the speed of motors along *x*-axis while lowering speed of motors along *y*-axis if the principal uplifting force is kept constant. Such movements can be understood from Fig. 1.

This article is organized as follows: Section II briefly describes quaternions and all mathematical expressions used in this paper. Section III, introduces the necessary quaternion Kinematics. Section IV, describes the dynamic model for the quadrator under study taking into account the proper considerations. In Section V, the quadrator model is linearized with the purpose to later use Linear Control. Section VI, develops the corresponding LQR Gain Scheduling Control. Section VII, shows the simulated results for the designed system which was implemented using *Simulink* for the observer and feedback. Finally, in Section VIII, conclusions are given as well as future recommendations.

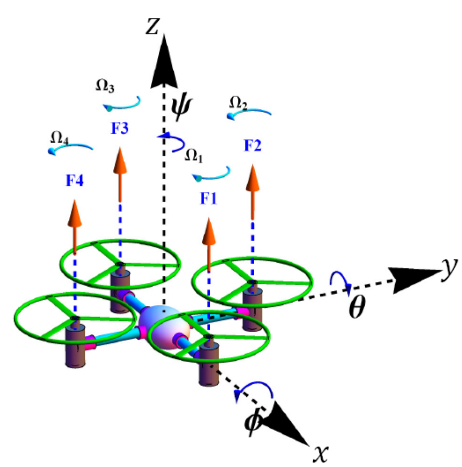

Figure 1. Quadrotor: Forces and movements

II. QUATERNIONS

In Aeronautics, the attitude of a rigid body, with respect to an inertial frame of reference, can be described through the *Tait-Bryan* angles. Recently, attitude has been described using a set of four parameters known as Quaternion [1]. A quaternion $\boldsymbol{q}=(q_0 \quad \boldsymbol{q}_{1:3})$ is formed by a scalar q_0 and a vector part $\boldsymbol{q}_{1:3}$, which can be mathematically grouped [2] as a single vector $\boldsymbol{q}$ of the type

$$\boldsymbol{q}=\begin{pmatrix} q_0 & q_1 & q_2 & q_3 \end{pmatrix}=\begin{pmatrix} q_0 \\ \boldsymbol{q}_{1:3} \end{pmatrix}.$$

Such description uses Euler's Method to obtain all four parameters [3]. Hamilton introduced quaternions for the first time in 1844 to describe rotations through a quadratic homogeneous function given by the following matrix [4]:

$$A(\boldsymbol{q})=\left(q_0^2-\boldsymbol{q}_{1:3}^T\boldsymbol{q}_{1:3}\right)I_{3\times3}+2\boldsymbol{q}_{1:3}\boldsymbol{q}_{1:3}^T-2q_0[\boldsymbol{q}_{1:3}\times]$$

where $I_{3\times3}$ is the identity matrix, and $\left[\boldsymbol{q}_{1:3}\times\right]$ is the skew-symmetric cross product matrix function given by:

$$[\boldsymbol{q}_{1:3}\times]=\begin{pmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{pmatrix}.$$

A geometric interpretation of a quaternion is made ease if for a given angle of rotation around an arbitrary direction, an eigenvector $\boldsymbol{e}=\begin{pmatrix} \boldsymbol{e}_1 & \boldsymbol{e}_2 & \boldsymbol{e}_3 \end{pmatrix}^T$ and its corresponding eigenvalue is found [5]. This eigenvector is in fact the Euler axis of rotation along which the quaternion is defined:

$$\boldsymbol{q}=\begin{pmatrix} cos\left(\dfrac{\alpha}{2}\right) \\ \boldsymbol{e}\ \ sin\left(\dfrac{\alpha}{2}\right) \end{pmatrix}.$$

Furthermore, the conjugate, the norm and the inverse of a quaternion $\boldsymbol{q}$ are respectively defined as follows:

$$\bar{\boldsymbol{q}}=\begin{pmatrix} q_0 \\ -\boldsymbol{q}_{1:3} \end{pmatrix}$$

$$\|\boldsymbol{q}\|=\sqrt{q_0{}^2+q_1{}^2+q_2{}^2+q_3{}^2}$$

$$\boldsymbol{q}^{-1}=\frac{\bar{\boldsymbol{q}}}{\|\boldsymbol{q}\|}$$

A unit quaternion is such that the condition $\|q_0+\boldsymbol{q}_{1:3}\|=1$ is fulfilled. Under such condition, the conjugate and the inverse quaternion are $\bar{\boldsymbol{q}}=\boldsymbol{q}^{-1}=\begin{pmatrix} q_0 & -\boldsymbol{q}_{1:3} \end{pmatrix}^T$ which in fact is only valid for just unit quaternions and do not hold for any other more general quaternions.

The only binary operation used in this paper is the quaternion multiplication between $\boldsymbol{q}$ and $\boldsymbol{p}$ which is denoted by the operator $\otimes$ and defined as:

$$\boldsymbol{q}\otimes\boldsymbol{p}=\begin{pmatrix} p_0 & -\boldsymbol{p}_{1:3}^T \\ \boldsymbol{p}_{1:3} & p_0\boldsymbol{I}_{3\times3}+[\boldsymbol{p}_{1:3}\times] \end{pmatrix}\begin{pmatrix} q_0 \\ \boldsymbol{q}_{1:3} \end{pmatrix}.$$

Since $\boldsymbol{q}\otimes\boldsymbol{p}\neq\boldsymbol{p}\otimes\boldsymbol{q}$, quaternion multiplication is not commutative, It is associative though [6] [7].

The identity quaternion $(1,\mathbf{0})$ complies with $\boldsymbol{q}\otimes\boldsymbol{q}^{-1}=\boldsymbol{q}^{-1}\otimes\boldsymbol{q}=(1,\mathbf{0})$.

III. QUATERNION KINEMTICS

Let $\boldsymbol{q}(t)$ be a quaternion changing at the time $t+\Delta t$

$$\boldsymbol{q}(t+\Delta t)=\left(\cos\left(\frac{\Delta\alpha}{2}\right)\boldsymbol{I}+\sin\left(\frac{\Delta\alpha}{2}\right)\begin{pmatrix} 0 & \boldsymbol{e}_3 & -\boldsymbol{e}_2 & \boldsymbol{e}_1 \\ -\boldsymbol{e}_3 & 0 & \boldsymbol{e}_1 & \boldsymbol{e}_1 \\ \boldsymbol{e}_2 & -\boldsymbol{e}_1 & 0 & \boldsymbol{e}_3 \\ -\boldsymbol{e}_1 & -\boldsymbol{e}_2 & -\boldsymbol{e}_3 & 0 \end{pmatrix}\right)\boldsymbol{q}(t)$$

with $\boldsymbol{q}=\left(cos\left(\dfrac{\alpha}{2}\right) \quad \boldsymbol{e}\ \ sen\left(\dfrac{\alpha}{2}\right)\right)^{\boldsymbol{T}}$ and $\Delta\alpha=\omega\Delta t$ as in [8].

When Δt can be consider small, the corresponding small angle approximation will give:

$$cos\left(\frac{\Delta\alpha}{2}\right)\approx1 \quad \text{and} \quad sin\left(\frac{\alpha}{2}\right)\approx\frac{1}{2}\omega\Delta t$$

which leads to $\boldsymbol{q}(t+\Delta t)=\left(1+\dfrac{1}{2}\boldsymbol{\Omega}\Delta t\right)\boldsymbol{q}(t)$ .Thus, the kinematic quaternion behaviour is:

$$\dot{\boldsymbol{q}}=\lim_{\Delta t\to0}\frac{\boldsymbol{q}(t+\Delta t)-\boldsymbol{q}(t)}{\Delta t}=\frac{1}{2}\boldsymbol{\Omega}\boldsymbol{q}$$

with $\boldsymbol{\Omega}$ is the skew-symmetric matrix

$$\boldsymbol{\Omega}=\begin{pmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{pmatrix}.$$

Finally, after some mathematical arrangement, kinematic equations of movement are obtained in terms of quaternions:

$$\dot{\boldsymbol{q}} = \frac{1}{2}\begin{pmatrix} [\boldsymbol{q}_{1:3}\times] + q_0 \boldsymbol{I}_{3\times3} \\ -\boldsymbol{q}_{1:3}^T \end{pmatrix}\boldsymbol{\omega} = \frac{1}{2}\boldsymbol{q}\otimes\begin{pmatrix} 0 \\ \boldsymbol{\omega} \end{pmatrix}$$

IV. Dynamic System Model

Let us consider the quadrotor as being a rigid body under external forces applied to its center of mass, the dynamic equation referred to the body coordinate system under the Newton-Euler formulation is [9]:

$$\begin{pmatrix} m\boldsymbol{I}_{3\times3} & 0 \\ 0 & \boldsymbol{J}_{3\times3} \end{pmatrix}\begin{pmatrix} \dot{\boldsymbol{V}} \\ \dot{\boldsymbol{\omega}} \end{pmatrix} + \begin{pmatrix} \boldsymbol{\omega}\times m\boldsymbol{V} \\ \boldsymbol{\omega}\times \boldsymbol{J}_{3\times3}\boldsymbol{\omega} \end{pmatrix} = \begin{pmatrix} \boldsymbol{F}+\boldsymbol{F_d} \\ \boldsymbol{\tau}+\boldsymbol{\tau_d} \end{pmatrix}$$

where $\boldsymbol{J}_{3\times3}$ is the inertia tensor, $\boldsymbol{V}$ is the translation velocity, $\boldsymbol{\omega}$ is the angular velocity and m is the mass associated to the quadrotor, $\boldsymbol{F}+\boldsymbol{F}_d$ and $\boldsymbol{\tau}+\boldsymbol{\tau_d}$ are the total external forces and torques referred to the body centre of mass. It is assumed that the quadrotor is fully symmetric and therefore the inertia tensor is given by:

$$\boldsymbol{J}_{3\times3} = \begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{pmatrix}.$$

The state vector can be written as $(\boldsymbol{p} \quad \mathbf{v} \quad \boldsymbol{q} \quad \boldsymbol{\omega})^T$ where $\boldsymbol{p} = (x \quad y \quad z)^T$, $\mathbf{v} = (U \quad V \quad W)^T$ represent position and velocity with respect to the specific inertial frame of reference and $\boldsymbol{q} = (q_0 \quad q_1 \quad q_2 \quad q_3)^T$, $\boldsymbol{\omega} = (P \quad Q \quad R)^T$ are the attitude and angular velocity with respect to the body frame of reference. Thus, the dynamic equations for the quadrotor become:

$$\dot{\boldsymbol{p}} = \boldsymbol{q}\otimes\mathbf{v}\otimes\overline{\boldsymbol{q}},$$

$$m\dot{\mathbf{v}} = -mg\boldsymbol{z}_b + \boldsymbol{q}\otimes\begin{pmatrix} 0 \\ 0 \\ U_1 \end{pmatrix}\otimes\overline{\boldsymbol{q}} - \boldsymbol{\omega}\times m\mathbf{v} + \Gamma_{\boldsymbol{p}},$$

$$\dot{\boldsymbol{q}} = \frac{1}{2}\boldsymbol{q}\otimes\begin{pmatrix} 0 \\ \boldsymbol{\omega} \end{pmatrix}$$

and $$\boldsymbol{J}\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega}\times\boldsymbol{J}\boldsymbol{\omega} - \sum_{i=1}^{4} J_R\left(\boldsymbol{\omega}\times\boldsymbol{z_b}\right)\bullet\Omega_i + \boldsymbol{\tau}_U + \Gamma_{\boldsymbol{\omega}}.$$

where, g is the gravitational force, $\boldsymbol{z}_b$ is the force along the z-axis, $U_1 = b\sum_{i=1}^{4} F_i$ is the total lift generated by the four rotors (b is the lifting coefficient), $\Gamma_{\boldsymbol{p}} = (\Gamma_x \quad \Gamma_y \quad \Gamma_z)^T$ are the forces, J_R is the tensor of inertia for each rotor around its own axis [10]. $\boldsymbol{\tau}_U$ is the necessary control torque to obtain the desired attitude, $\Gamma_{\boldsymbol{\omega}} = (\Gamma_P \quad \Gamma_Q \quad \Gamma_R)^T$ are the aerodynamic moments on the quadrotor which are given by $\Gamma_i = \frac{1}{2}\rho_{air}C_i v^2$ (ρ_{air} is the air density, v is the quadrotor's velocity respect to the air and C_i are aerodynamic coefficients).

In terms of U_1 and $\boldsymbol{\tau}_U$, the control signals produced by the rotors are:

$$\boldsymbol{U} = \begin{pmatrix} U_1 \\ \boldsymbol{\tau}_U \end{pmatrix} = \begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix} = \begin{pmatrix} b & b & b & b \\ 0 & lb & 0 & -lb \\ -lb & 0 & lb & 0 \\ d & -d & d & -d \end{pmatrix}\begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{pmatrix}$$

where l and d are the distance to the center of mass and the drag of each rotor respectively (chosen to be equal for each rotor) and $(F_1 \quad F_2 \quad F_3 \quad F_4)^T = (\Omega_1^2 \quad \Omega_2^2 \quad \Omega_3^2 \quad \Omega_4^2)^T$ with Ω_i^2 the square magnitude of the angular velocity for the *i-th* rotor.

V. Linear Model

The selected operating point for the quadrotor is such that its 3D position is stationary and represented by $\overline{\overline{\mathbf{v}}} = (0 \quad 0 \quad 0)^T$, $\overline{\overline{\boldsymbol{\omega}}} = (0 \quad 0 \quad 0)^T$ and therefore $\overline{\overline{\boldsymbol{q}}} = (1 \quad 0 \quad 0 \quad 0)^T$.

Now, at that operating point quadrotor rotations can be written using the Hamilton quaternion rotation matrix as:

$$A(\boldsymbol{q})\big|_{\boldsymbol{q}=\overline{\overline{\boldsymbol{q}}}} = A\left(\overline{\overline{\boldsymbol{q}}}\right) = \boldsymbol{I}_{3\times3} - 2[\boldsymbol{q}_{1:3}\times].$$

Thus, the dynamic equations for the quadrator can be linearized using Taylor's Theorem giving:

$$\dot{\boldsymbol{p}} = \mathbf{v},$$

$$\dot{\mathbf{v}} = \begin{pmatrix} -2q_2 g & 2q_1 g & \frac{U_1}{m} \end{pmatrix}^{\boldsymbol{T}},$$

$$\dot{\boldsymbol{q}} = \frac{1}{2}\begin{pmatrix} 0 \\ \boldsymbol{\omega} \end{pmatrix}$$

and $$\dot{\boldsymbol{\omega}} = \left(I_{xx}^{-1}U_2 \quad I_{yy}^{-1}U_3 \quad I_{zz}^{-1}U_4\right)^{\boldsymbol{T}}.$$

VI. LQR CONTROLLER

Let the controller be of the form $u(t) = -K\left(Xt - X_{ref}\right)$ where the feedback gain matrix K is found when minimizing the cost function

$$J = \int_0^\infty \left[(Xt - X_{ref})^T Q (Xt - X_{ref}) + u(t)^T R u(t) \right] dt$$

with Q a semi-positive definite matrix and R a positive definite matrix and due to implementation convenience, the state vector X is formed by the state variables

$$\left(x_1 \quad x_3 \quad x_5\right)^T = \boldsymbol{q}_{1:3} \, ,$$

$$\left(x_2 \quad x_4 \quad x_6\right)^T = \left(\dot{x}_1 \quad \dot{x}_3 \quad \dot{x}_5\right)^T ,$$

$$\left(x_7 \quad x_9 \quad x_{11}\right)^T = \left(x \quad y \quad z\right)^T ,$$

$$\left(x_8 \quad x_{10} \quad x_{12}\right)^T = \left(\dot{x}_7 \quad \dot{x}_9 \quad \dot{x}_{11}\right)^T ,$$

and X_{ref} is the corresponding reference matrix.

Fig. 2 shows how all variables are related and having the following meaning: the error in the attitude is obtained by means of the quaternion reference $\boldsymbol{q}_{ref}$ as $\hat{\boldsymbol{q}} = \boldsymbol{q}_{ref} \otimes \bar{\boldsymbol{q}}$ and the corresponding position error $\hat{\boldsymbol{p}} = \boldsymbol{p} - \boldsymbol{p}_{ref}$.

Since the unit quaternion is used, the real part q_0 is directly obtained and just the vector part $\boldsymbol{q}_{1:3}$ is used for feedback.

GAIN SCHEDULING

The main idea of controlling the quadrotor is to define a specific trajectory for the quadrotor to follow. The trajectory and the quadrotor start at different points and then when the quatrotor is closing to the predefined path, a tracking of the theoretical trajectory takes place.

Therefore, the gain matrix K was calculated for two different situations: i) when the quadrotor is far from the reference and ii) when the quadrotor is already tracking. Thus, there are two different matrices:

$$KG = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 31623 & 365 \\ 14 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 2 & 0 & 0 \\ 0 & 0 & 14 & 1 & 0 & 0 & -3 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$KP = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 316.2 & 316.7 \\ 10.8 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1.5 & 0 & 0 \\ 0 & 0 & 10.9 & 1 & 0 & 0 & -1 & -1.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Scheduling Gain LQR Control

displays the design of the gain scheduling LQR control used.

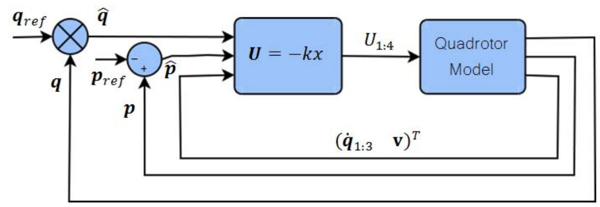

Figure 2. LQR Quadrotor Control using quaternions

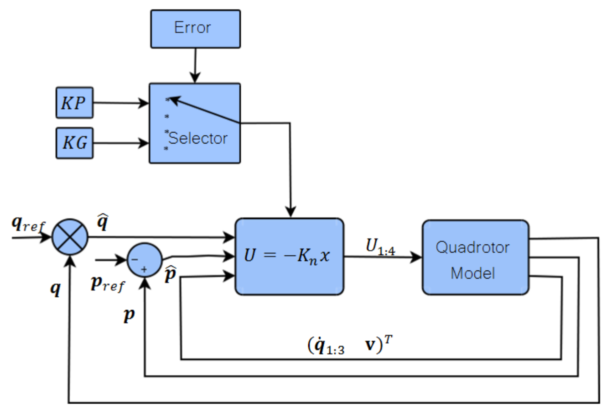

Figure 3. Scheduling Gain LQR Control

VII. SIMULATION AND RESULTS

Simulations were carried out using Simulink with the values shown in Table I [11].

TABLE I. ACTUAL MODEL PARAMETER VALUES

Variable	*Description*	*Value*
g	Gravity force	$9.8 \frac{m}{s^2}$
m	Quadrotor mass	0.52 kg
J_R	Single rotor moment of inertia	$8.66^{-7} \frac{rad}{s}$
$\lVert\Omega_i\rVert$	i-th rotor angular velocity range	$0 - 278 \frac{rad}{s}$
I_{xx}	Quadrotor *x-axis moment of inertia*	$6.228^{-2} kgm^2$
I_{yy}	Quadrotor *y-axis moment of inertia*	$6.225^{-2} kgm^2$
I_{zz}	Quadrotor *z-axis moment of inertia*	$1.121^{-2} kgm^2$
l	Rotor-mass centre length	0.235 m
b	Lift coefficient	$3.13^{-5} Ns^2$
d	Drag coefficient	$7.5^{-7} Nms^2$

A first test with a step input was run using *KP* gain for the LQR control of Fig. 2 for $(x \quad y \quad z)^T = (5 \quad 0 \quad 0)^T$. The step response is shown in Fig. 4.

It is clear that exists a constant error during tracking. Meanwhile, Fig. 5 shows the same test for the LQR control but this time using the gain *KG* and it can be easily seen that the error presents oscillation.

A second test was run using the scheduling gain control and the step response is shown in Fig. 6.

At this moment is important to shift from the classical graphic representation for the step response to the unit quaternion behaviour because It is the way the whole model was developed Fig. 7.

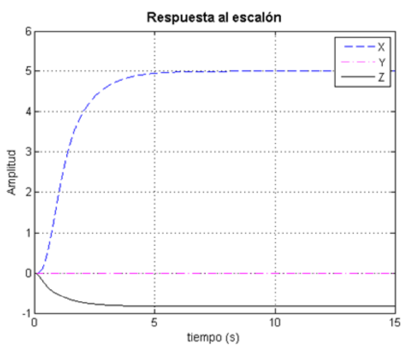

Figure 4. *KP* gain step response

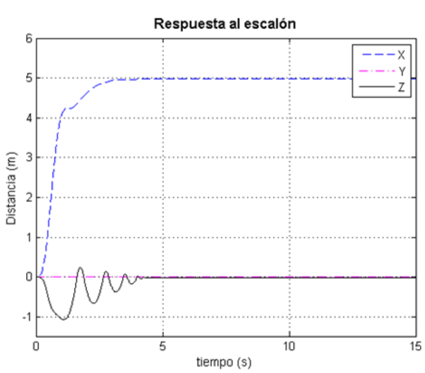

Figure 5. KG gain step response

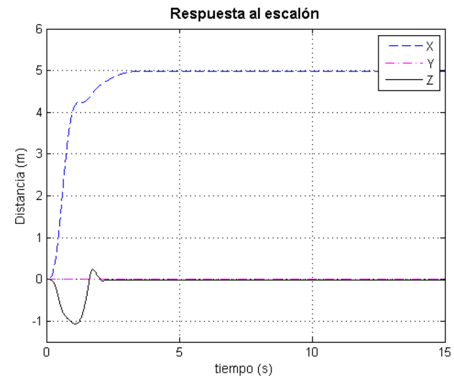

Figure 6. Scheduling Gain step response

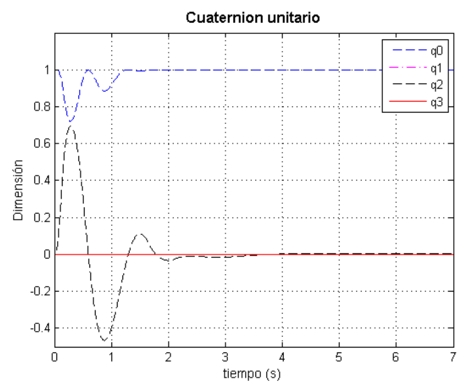

Figure 7. Unit quaternion step under Scheduling Gain

TRAJECTORY TRACKING

Parametric equations were used to program different test trajectories and Gain Scheduling LQR Controls were applied to the quadrotor to follow such trajectories. The whole set of responses are shown from Fig. 8 to Fig.14.

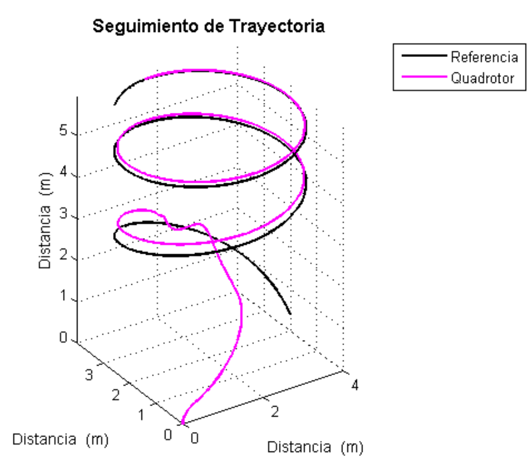

Figure 8. Trajectory tracking in 3D

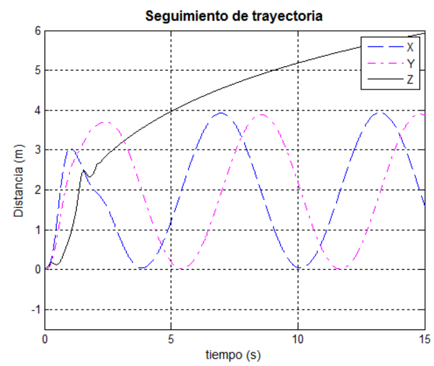

Figure 9. Trajectory tracking in 2D

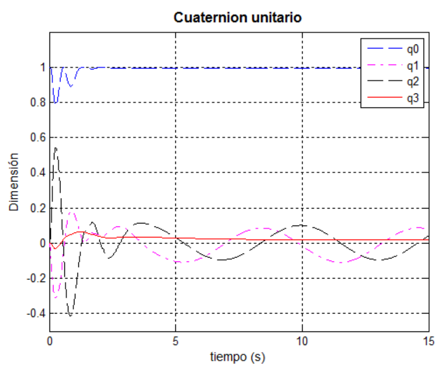

Figure 10. Unit quaternion for trajectory of Fig 8.

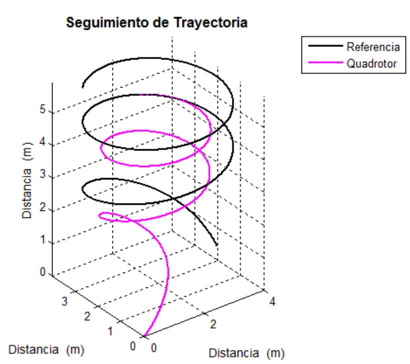

Figure 11. 3D Trajectory traking under *KP*

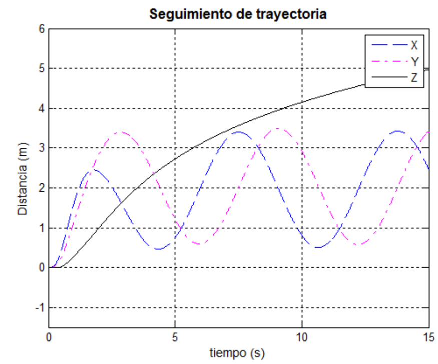

Figure 12. 2D trajectory tracking under *KP*

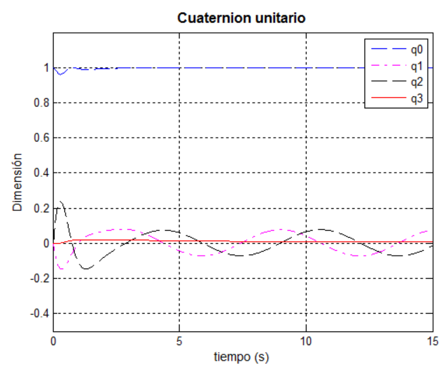

Figure 13. Unit quaternion for trajectory Fig.11

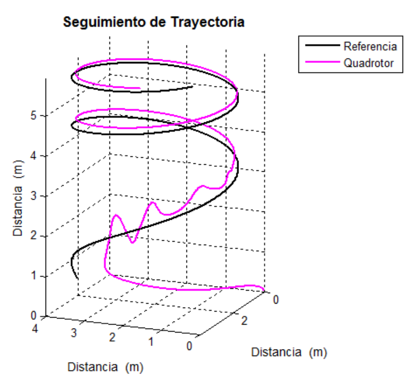

Figure 14. 3D trajectory tracking under *KG* gain

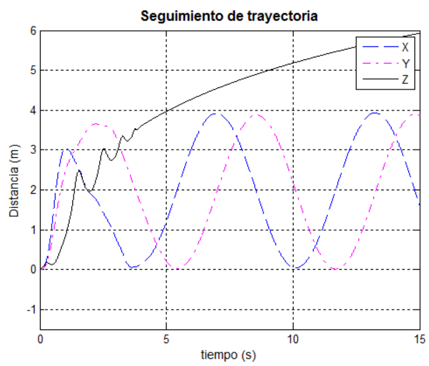

Figure 15. 2D trajectory tracking under *KG*

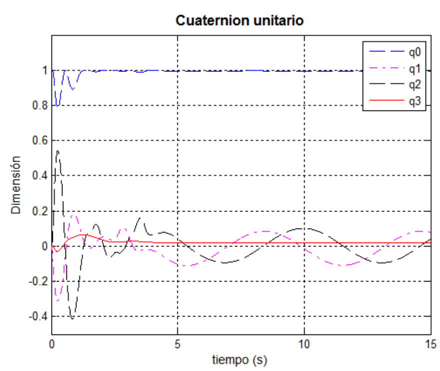

Figure 16. Unit quaternion for trajectory of Fig. 14

I. CONCLUSION

A Gain Scheduling LQR Control for a quadrotor was introduced. A unit quaternion approach was used to control the quadrotor's attitude.

The control system proposed was tested using simulation with real parameters.

The presented results show that quaternions can effectively substitute *Tait-Bryan* angles for attitude control.

It is worth to notice that for large gains in the controller the unit quaternion grows indefinitely.

REFERENCES

[1] G. Vianna Raffo, MODELADO Y CONTROL DE UN HELICÓPTERO QUADROTOR, Sevilla, España: Universidad de Sevilla, 2007.

[2] J. Diebel, *Reppresenting Attitude: Euler Angles, Unit Quaternions, and Rotation Vectors,* Stanford, California: Stanford University, 20 October 2006.

[3] I. D. Allerton, Principles of Fligth Simulation, United Kingdom: John Wiley & Sons, 2009, pp. 122-124.

[4] F. Landis Markley, *ATTITUDE ESTIMATION OR QUATERNION ESTIMATION?,* N. G. S. F. Center, Ed., Greenbelt: Guidance, Navigation and Control Systems Engineering Branch, 2005.

[5] R. H. Bishop, THE MECHATRONIC HANDBOOK, Austin, Texas: CRC PRESS, 2002.

[6] A. Tayebi, «Unit Quaternion observer based attitude stabilization of a rigid spacecraft without velocity measurement,» *Proceedings of the 45th IEEE Conference on Desicion & Control,* vol. 06, nº 1-4244-0171-2, pp. 1557-1561, December 13-15, 2006.

[7] E. Stingu y F. Lewis, «Design ans Implementation of a Structured Fligth Controller for a 6DoF Quadrotor Using Quaternions,» *17th Mediterranean Conference on Control & Automation,* vol. 09, nº 978-1-4244-4685-8, pp. 1233-1238, June 24-26, 2009.

[8] K. L. Makovec, A Nonlinear Magnetic Controller for Three-Axis Stability of Nanosatellites, Blackburg, Virginia: Virginia Politechnic Institute and State University, July 23, 2001

[9] S. Bouabdallah, DESIGN AND CONTROL OF QUADROTORS WITH APPLICATION TO AUTONOMOUS FLYING, Lausanne: ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LASUSANNE, 23 FÉVRIER 2007.

[10] C. M. Esparza Martinez de Luco, M. J. Pérez-Ilzarbe Torralba y J. Elso Torralba, MODELADO, DISEÑO, CONSTRUCCIÓN Y CONTROL BORROSO DE UN HELICÓPTERO QUADROTOR, Pamplona: ESCUELA TÉCNICA SIPERIOR DE INGENIEROS INDUSTRIALES Y DE TELECOMUNICACIONES, 24 de Febrero de 2012.

[11] «Draganfly Innovations Inc.;,» Draganfly Innovations Inc.;, 2012. [En línea]. Available: www.rctoys.com. [Último acceso: 1 Noviembre 2012].