

# Exercise 1 - Theory

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## 1 Properties of Rotation Matrices

### 1.1 Showing that $U^T = U^{-1}$ holds for *general* rotation matrix $U \in \mathbb{R}^3$

Generally note that  $UU^{-1} = I$  subject to  $U^{-1} = U^T$ , as provided by  $U$  being rotation matrix and as such orthogonal, yields  $UU^T = I$ .

For given column-vectors  $c_1, c_2, c_3$  of  $U$  we observe

$$\langle c_i, c_j \rangle = \delta_{ij}$$

which in combination with

$$U^T = (c_1, c_2, c_3)^T = (c_1^T, c_2^T, c_3^T)$$

results in

$$UU^T = (\langle c_i, c_j \rangle)_{1 \leq i, j \leq 3} = I$$

Motivated by [1][2][3].

### 1.2 Geometric interpretation of determinant of 3x3 matrix

The determinant of a matrix  $A \in \mathbb{R}^3$  has the geometric intuition of a scaling factor for a given (unit) volume within the cartesian (standard) coordinate system and how its volume changes when applying the matrix transform. [4]

As such, with  $\det(A) = 1$  the matrix does not change the size of any given volume within its space, which intuitively makes sense for a rotation matrices as its only task is to rotate - not scale - within any given coordinate system.

## 2 Transformation Chain

Normally, the transformation mapping from the world coordinate system to pixel coordinate system is.

$$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = K[I_3|O_3] \begin{bmatrix} R & T \\ 0_3^T & 1 \end{bmatrix} \begin{bmatrix} X_s \\ Y_s \\ Z_s \\ 1 \end{bmatrix}$$

Where  $x_{pix}$  and  $y_{pix}$  are  $u'/w'$  and  $v'/w'$  and  $[X_s \ Y_s \ Z_s \ 1]^T$  are the coordinates of a 3D point in the world coordinate space

Homogeneous Coordinates is good. It enables us to use Matrix Multiplication on all transformations. To represent 3d coordinate points you need to have 4 elements in the vector.

$$[x, y, z, w] = [x/w, y/w, z/w] \quad (1)$$

However, when  $w$  in (1) is 0. There is no such 3D vector on that. We call points in this form vanishing point.[5]

## 2.1 Chain Step

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ 1 \end{bmatrix} = \begin{bmatrix} R & T \\ 0_3^T & 1 \end{bmatrix} \begin{bmatrix} X_s \\ Y_s \\ Z_s \\ 1 \end{bmatrix}$$

multiplying  $\begin{bmatrix} R & T \\ 0_3^T & 1 \end{bmatrix}$  to the world coordinates  $[X_s, Y_s, Z_s, 1]^T$  gives a camera coordinates  $[x_s, y_s, z_s, 1]^T$

$$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \begin{bmatrix} a_x & s & x_0 & 0 \\ 0 & a_y & y_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_s \\ y_s \\ z_s \\ 1 \end{bmatrix}$$

By multiplying the camera coordinate points with calibration matrix, the result is in the image plane coordinates. After that we can do  $u'/w'$  and  $v'/w'$  to get the pixel coordinates

## 2.2 Intrinsic parameters

$K$  is a camera matrix, or a matrix of intrinsic parameters  $K = \begin{bmatrix} a_x & s & x_0 \\ 0 & a_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$  Where

- $a_x$  and  $a_y$  are focal lengths in pixel
- $x_0$  and  $y_0$  are the image center
- $s$  the skew coefficient between the x,y axis which is 0 in most cases [6][7]

$K$  can transform image plane to pixels and  $[I_3 \mid O_3] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$  can be use for perspective projection which can be combined to  $K$  matrix and facilitate the matrix multiplication

## 2.3 Extrinsic parameters

$$extrinsicParams = \begin{bmatrix} R & T \\ 0_3^T & 1 \end{bmatrix}$$

This extrinsic parameters is used to explain the camera motion along the scene. It translates world-coordinates of a point  $[X, Y, Z, 1]$  to a camera coordinate system

$$R = \begin{bmatrix} I_i & J_i & K_i \\ I_j & J_j & K_j \\ I_k & J_k & K_k \end{bmatrix} T = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}$$

$R$  is a rotational matrix such that it helps rotating the point from world to camera.  $T$  is a translational metric to translate the point from world to camera. Motivated from [8]

## References

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