Exercise 1 - Theory

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1 Properties of Rotation Matrices

1.1 Showing that $U^T = U^{-1}$ holds for *general* rotation matrix $U \in \mathbb{R}^3$

Generally note that $UU^{-1} = I$ subject to $U^{-1} = U^T$, as provided by U being rotation matrix and as such orthogonal, yields $UU^T = I$.

For given column-vectors c_1, c_2, c_3 of U we observe

$$< c_i, c_j > = \delta_{ij}$$

which in combination with

$$U^T = (c_1, c_2, c_3)^T = (c_1^T, c_2^T, c_2^T)$$

results in

$$UU^T = (< c_i, c_j >)_{1 \leq i,j \leq 3} = I$$

Motivated by [1][2][3].

1.2 Geometric interpretation of determinant of 3x3 matrix

The determinant of a matrix $A \in \mathbb{R}^3$ has the geometric intition of a scaling factor for a given (unit) volume within the cartesian (standard) coordinate system and how its volume changes when applying the matrix transform. [4]

As such, with $det(A) = 1$ the matrix does not change the size of any given volume within its space, which intuitively makes sense for a rotation matrices as its only task is to rotate - not scale - within any given coordinate system.

2 Transformation Chain

[...]

References

[1] matrix transpose * itself = identity. [Online]. Available: https://math.stackexchange.com/questions/768098/matrix-transpose-itself-identity

[2] Why is inverse of orthogonal matrix is its transpose. [Online]. Available: https://math.stackexchange.com/questions/1097422/why-is-inverse-of-orthogonal-matrix-is-its-transpose

[3] Orthogonale matrix. [Online]. Available: https://de.wikipedia.org/wiki/Orthogonale_Matrix

[4] The determinant | essence of linear algebra, chapter 5. [Online]. Available: https://www.youtube.com/watch?v=Ip3X9LOh2dk