# Exercise 2 - Theory

Abdelaziz, Ibrahim Somkiadcharoen, Robroo Berg, Oliver

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## 1 Theory

### 1.1 Homography Definition

In case of $P^2$ (2D Projection plane) we have homogeneous coordinates as $[x_1, x_2, x_3]$ and the $H$ transformation matrix is of size $3 \times 3$ as $\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$ where 8 of them are independent ratios(DOF) and another one is the gain.

Using the same logic as above, you can get a point from $P^n$ (n-dimensional Projection Space) as $[x_1, x_2, ..., x_n, x_{n+1}]$ and the $H$ transformation matrix is of size $(n+1) \times (n+1)$. Thus, $(n+1)^2 - 1$ DOF. Motivated From [1][2]

### 1.2 Line preservation

Given that a point $x = [x1, x2, x3]$ is a point in 2D Projection plane which is also on a line $l$, and all the points are on $l$ which gives $l^T x_i = 0$. We can derive

$$l^T x_i = 0 = l^T H^{-1} H x_i \tag{1}$$

From (1) we get that the points $x' = Hx_i$ that is transformed lie on the line $l' = l^T H^{-1}$ In other words, we can perceived from the equation that points are transformed by $x'$ and line is transformed by $l'$

Highly Motivated by [3] [4]

## 2 Implementation

You may find the implementation code inside the `main.py` file. To run the code, call `python main.py`.

### 2.1 Relative rotation estimation from homography

To 4. : $H_2$ needs correction, as it not strictly conforms to the properties of a rotation matrix, namely $MM^T$ does not exactly match the identity matrix and its determinant is not strictly equal to 1. This is most likely due to measuring errors.

### 2.2 Camera pose estimation from homography

To 2. "meaning of $t = -RC$": Motivated by [5], because the chessboard lies within the xy-plane, it holds

$$[K|0_3] \begin{bmatrix} R & -RC \\ 0_3^T & 1 \end{bmatrix} (XYZ1)^T \tag{2}$$

being subject to $Z = 0$ corresponds to

$$K[r_1 r_2 t](XY1)^T \tag{3}$$

where $t$ now responds to the initial $-RC$. We translate the virtual camera to the real camera.

To 3. "3rd element of $t$ being negative": With the translation being negative, this means the camera resides within the positive X-Y-Z quadrant of the world coordinate system, such that we need to do negative translation to move the camera to the world coordinate systems origin.

# References


[1] Homogeneous matrix has eight independent ratios of matrix elements? [Online]. Available: https://stackoverflow.com/questions/9534453/homogeneous-matrix-has-eight-independent-ratios-of-matrix-elements

[2] Dynamic pn to pn alignment. [Online]. Available: https://www.cs.tau.ac.il/~wolf/papers/dyn-alignment.pdf

[3] Projective geometry. [Online]. Available: http://www.umiacs.umd.edu/~ramani/cmsc828d/ProjectiveGeometry.pdf

[4] Projective geometry, camera models and calibration. [Online]. Available: http://www.cse.iitd.ernet.in/~suban/vision/geometry.pdf

[5] Camera calibration. [Online]. Available: http://www.epixea.com/research/multi-view-coding-thesisse9.html