Exercise 2 - Theory

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1 Theory

1.1 Homography Definition

In case of P^2 (2D Projection plane) we have homogeneous coordinates as $[x_1, x_2, x_3]$ and the H transformation matrix is of size 3×3 as $\begin{bmatrix} h_1 1 & h12 & h13 \\ h_2 1 & h22 & h23 \\ h_3 1 & h32 & h33 \end{bmatrix}$ where 8 of them are independent ratios(DOF) and another one is the gain.

Using the same logic as above, you can get a point from P^n (n-dimensional Projection Space) as $[x_1, x_2, ...x_n + 1]$ and the H transformation matrix is of size $(n+1) \times (n+1)$. Thus, $(n+1)^2 - 1$ DOF. Motivated From [1][2]

1.2 Line preservation

Given that a point $x = [x1, x2, x3]$ is a point in 2D Projection plane which is also on a line l, and all the points are on l which gives $l^T x_i = 0$. We can derive

$$l^T x_i = 0 = l^T H^{-1} H x_i \tag{1}$$

From (1) we get that the points $x' = Hx_i$ that is transformed lie on the line $l' = l^T H^{-1}$ In other words, we can perceived from the equation that points are transformed by x' and line is transformed by l'

Highly Motivated by [3] [4]

References

[1] Homogeneous matrix has eight independent ratios of matrix elements? [Online]. Available: https://stackoverflow.com/questions/9534453/homogeneous-matrix-has-eight-independent-ratios-of-matrix-elements

[2] Dynamic pn to pn alignment. [Online]. Available: https://www.cs.tau.ac.il/~wolf/papers/dyn-alignment.pdf

[3] Projective geometry. [Online]. Available: http://www.umiacs.umd.edu/~ramani/cmsc828d/ProjectiveGeometry.pdf

[4] Projective geometry, camera models and calibration. [Online]. Available: http://www.cse.iitd.ernet.in/~suban/vision/geometry.pdf