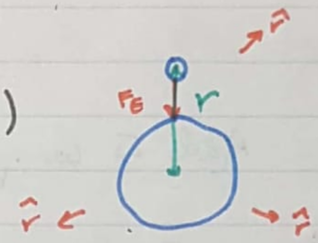
ECE106

## Force and Field:

$\vec{F}_E = \frac{G m_E m}{r^2}(-\hat{r})$

$= m\vec{g} \leftarrow$ field




$\vec{g} = \frac{G m_E}{r^2}(-\hat{r})$

$\vec{F}_{net} = m\vec{g}_{net}$

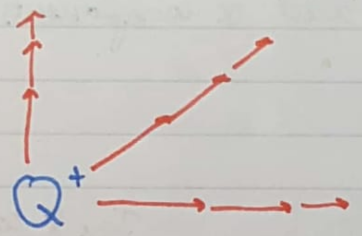
Jan 3, 2018

## Electrostatics:


$\vec{F}_{Qq} = \frac{KQq}{r^2}(\hat{r})$

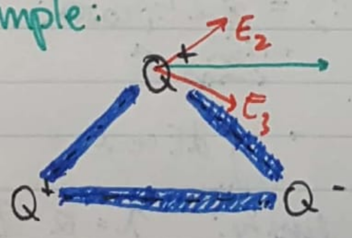
$= \vec{E}_Q q$ where $\vec{E}_Q = \frac{KQ}{r^2}\hat{r}$

$= q\vec{E}$ in general.




## Example:

1) $\vec{E}$? $F_{net}$?



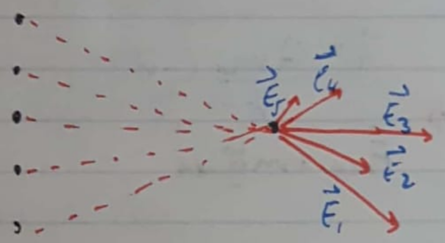

$|\vec{E}_{net}| = (E_2 + E_3)\cos 60$
$= 2E\cos 60$
$= E$

$E_{net}^2 = E_2^2 + E_3^2 + 2E_2E_3\cos\theta$

w/ $\theta$ is tail to tail.

## Point charge Model:

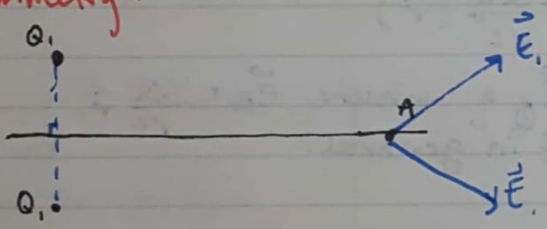
Jan 5, 2018




Add w/ components

$$E = \frac{kQ}{r^2} = \frac{Q}{4\pi\varepsilon_0 r^2} \quad \left(k = \frac{1}{4\pi\varepsilon_0}\right)$$

## Symmetry:

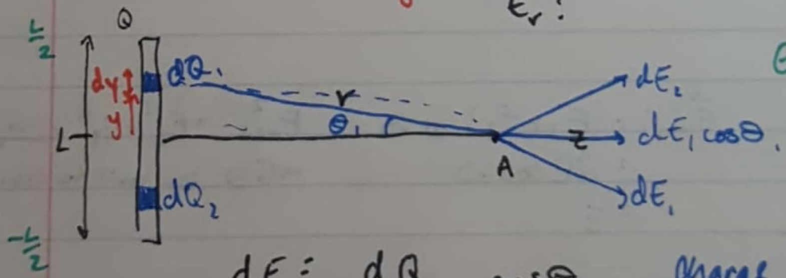



just add x components

## Continuous Charges:

Jan 8, 2018

$\vec{E}_r$ ?




$$E_r \cong \sum dE_i \cos\theta_i = \lim_{i\to\infty} = \int_{line} dE\cos\theta$$

$$dE_z = \frac{dQ}{4\pi\varepsilon_0 r^2}\cos\theta$$

charge density: $\lambda = \frac{Q}{L} \Big\} dQ = \lambda dL$

$$= \frac{\lambda\, d_y \cos\theta}{4\pi\varepsilon_0 r^2}$$

$$\therefore E_z = \int_{line} dE_z = \int_{-L/2}^{L/2} \frac{\lambda\, d_y \cos\theta}{4\pi\varepsilon_0 r^2} = \int_{-L/2}^{L/2} \frac{\lambda\, d_y}{4\pi\varepsilon_0 r^2}\left(\frac{z}{r}\right)$$

$$= \int_{-L/2}^{L/2} \frac{\lambda\, d_y\, z}{4\pi\varepsilon_0 (y^2+z^2)^{3/2}}$$

$\Big\}$ Evaluate w/ tan sub.
$y = z\tan\alpha$

$y = z\tan\alpha$
$dy = z\sec^2\alpha\, d\alpha$

$(y^2+z^2)^{3/2} = (z^2(\tan^2\alpha+1))^{3/2}$
$= (z^2\sec^2\alpha)^{3/2}$
$= z^3\sec^3\alpha$

$$E_z = \int_{line} \frac{\lambda z^2 \sec^2\alpha\, d\alpha}{4\pi\varepsilon_0 z^3 \sec^3\alpha}$$

$\left(P_e = \lambda = \frac{Q}{L} = \text{Charge Density}\right)$

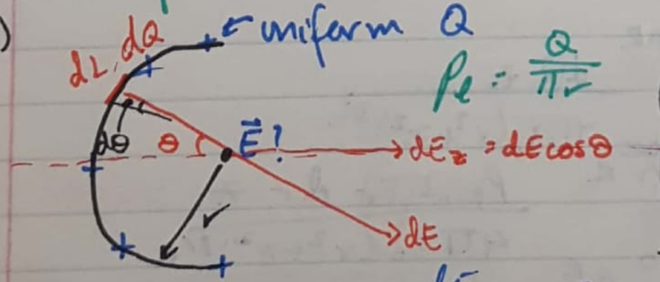
$$= \frac{\lambda}{4\pi\varepsilon_0 z}\int \frac{d\alpha}{\sec\alpha}$$
$$= \frac{\lambda}{4\pi\varepsilon_0 z} \sin\alpha \Big|_{line}$$
$$= \frac{\lambda}{4\pi\varepsilon_0 z}\left(\frac{y}{\sqrt{y^2+z^2}}\right)\Bigg|_{-L/2}^{L/2}$$
$$= \frac{\lambda L}{4\pi\varepsilon_0 z\sqrt{\frac{L^2}{4}+z^2}}$$

$\lim_{L>>z} = 2$ for $\frac{L}{\sqrt{\frac{L^2}{4}+z^2}}$

$$\approx \frac{\lambda}{2\pi\varepsilon_0 z}$$

Jan 10, 2018

Example:

1) 


$$dE_z = \frac{dQ}{4\pi\varepsilon_0 r^2}\cos\theta$$
$$dQ = P_e\, dL = P_e r\, d\theta$$
sub back in.

$$dE_z = \frac{P_e r}{4\pi r^2 \varepsilon_0}\cos\theta\, d\theta$$
$$E_z = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\lambda}{4\pi\varepsilon_0 r}\cos\theta\, d\theta$$
$$= \frac{\lambda}{2\pi\varepsilon_0 r}$$

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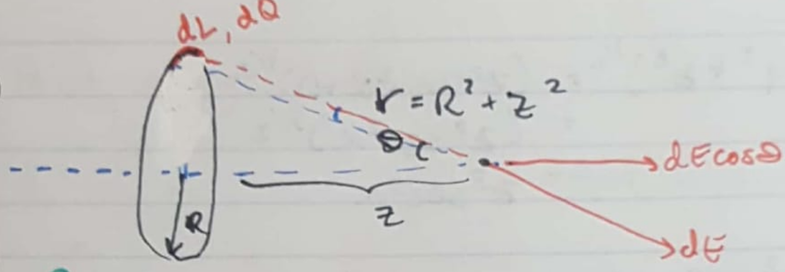
2)




$\rho_l = \frac{Q}{2\pi R}$

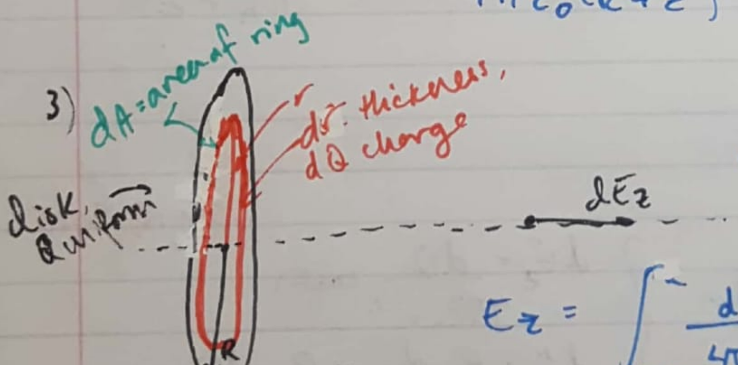
$$dE_z = \frac{dQ}{4\pi\varepsilon_0 r^2}\cos\theta$$

$$= \frac{dQ \; z}{4\pi\varepsilon_0 r^3} = \frac{dQ \; z}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}}$$

$$E_z = \int_{ring} \frac{z\,dQ}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}} = \left(\int_{ring} dQ\right)\frac{z}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}}$$

$$= \frac{Qz}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}}$$

3)




Jan 15, 2018

$$dE_z = \frac{dQ\,z}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}$$

$$E_z = \int_{disk} \frac{dQ\,z}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}$$

$\rho_s = \frac{Q}{\pi r^2}$

$$= \int_0^R \frac{\rho_s 2\pi r\,dr\,z}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}$$

$dQ = \rho_s (\text{Area})$
$= \rho_s (2\pi r\,dr)$

$$= \frac{\rho_s z}{2\varepsilon_0}\int_0^R \frac{r\,dr}{(r^2+z^2)^{3/2}}$$

u-sub $u = r^2 + z^2$

$$\rightarrow = \frac{-2z\rho_s}{4\varepsilon_0}\left(\frac{1}{\sqrt{r^2+z^2}}\right)\Bigg|_0^R$$

$$= \frac{z\rho_s}{2\varepsilon_0}\left[\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}}\right]$$

$$\lim_{R\to\infty} E_z = \frac{\rho_s}{2\varepsilon_0}$$ } distance doesn't matter

## Summary of Field Calculations:

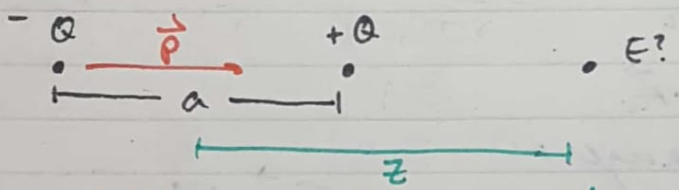
Jan 17, 2018

Point charge: $\dfrac{Q}{4\pi\varepsilon_0 r^2}$

Infinite Line: $\dfrac{\rho_\ell}{2\pi\varepsilon_0 r}$

Infinite Sheet: $\dfrac{\rho_s}{2\varepsilon_0}$

Field due to Dipole:




$$E = \frac{kQ}{(z-\frac{a}{2})^2} - \frac{kQ}{(z+\frac{a}{2})^2}$$

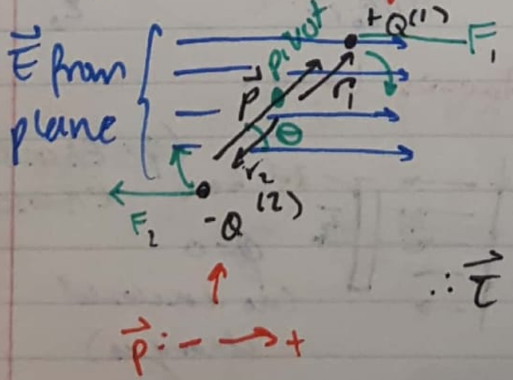
$$= \frac{2azQ}{4\pi\varepsilon_0\left[z^2 - \frac{a^2}{4}\right]^2}$$

$$\lim_{z \gg a} E_d = \frac{Qa}{2\pi\varepsilon_0 z^3}$$

$P = Qa$, so $\lim\limits_{z \gg a} \vec{E}_d = \dfrac{\vec{P}}{2\pi\varepsilon_0 z^3}$ ← Electric Dipole moment

↳ points in $\vec{E}_d$ direction, ⟶ (+ charge closer, so)

## Torque on a Dipole:




$\vec{p}$: − ⟶ +

$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2$$

$$= \tau_1 + \tau_2$$

$$= r_1 \sin\theta F + r_2 \sin\theta F$$

$r = \frac{a}{2}$, so...

$$= aF\sin\theta$$

$$= pE\sin\theta$$

$$\therefore \vec{\tau} = \vec{p} \times \vec{E}$$

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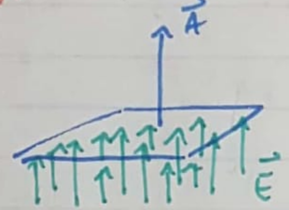
# Gauss' Law:

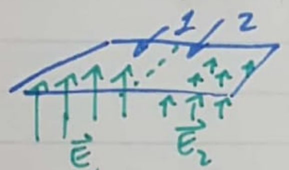
## Flux:




$$\boxed{\varphi = \vec{E}\cdot\vec{A} = |E||A|\cos\theta}$$

If uneven $E$:

Add up ind. flux

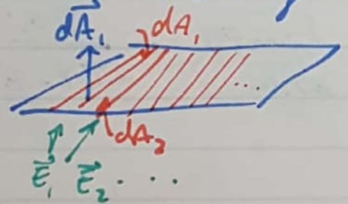



$\vec{A} = \vec{A}_1 + \vec{A}_2$

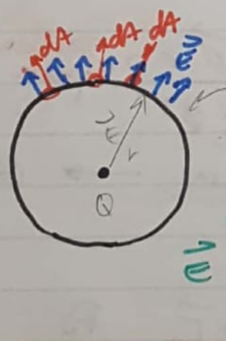
$\varphi = \vec{E}_1\cdot\vec{A}_1 + \vec{E}_2\cdot\vec{A}_2$

$$\varphi = \sum_i \vec{E}_i\cdot\vec{A}_i$$

Continuous changes:




$$\varphi \approx \sum_i \vec{E}_i\cdot d\vec{A}_i = \int_{area} \vec{E}\cdot d\vec{A}$$




$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}$

$|\vec{A}| = 4\pi r^2$

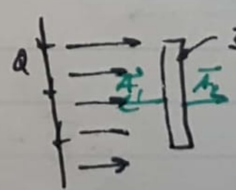
$\theta = 0$ and $\vec{E}$ same everywhere

$$\varphi = \oint \vec{E}\cdot d\vec{A} = |\vec{E}_1||d\vec{A}_1| + |\vec{E}_2||d\vec{A}_2| + \dots = |\vec{E}|(dA_1 + dA_2 + \dots) = |\vec{E}|\cdot|A| = \frac{Q}{4\pi\varepsilon_0 r^2}\cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\boxed{\varphi_{\substack{\text{enclosed}\\ \text{surface}\\ (3D)}} = \frac{Q_{enc}}{\varepsilon_0}}$$ Gauss's Law.


$\varphi = \frac{5Q}{\varepsilon_0}$

Area vector always points away from encl.




$(\varphi = EA\cos 180 + EA\cos 0)$

$\varphi = 0$ (no enclosed charges)

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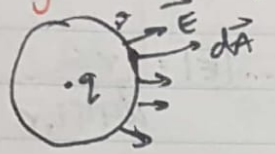
## Using GL to find $\vec{E}$ from P.C:

Jan 19, 2018



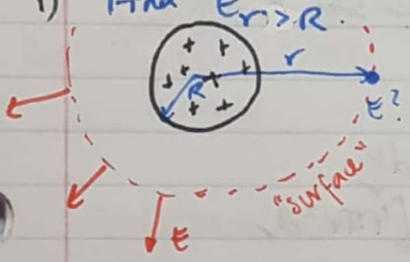
$$\varphi_e = \frac{Q_{enc}}{\varepsilon_0} = \oint \vec{E}\cdot d\vec{A}$$

$$= \oint |\vec{E}||d\vec{A}|\cos\theta \leftarrow \theta = 0 \text{ e'where}$$

$$= \oint |\vec{E}||d\vec{A}| = |\vec{E}|\oint |d\vec{A}| = |\vec{E}||\vec{A}| \text{ etc.}$$

### Example:

1) Find $E_{r>R}$.




$$\varphi = \frac{Q_{enc}}{\varepsilon_0}$$

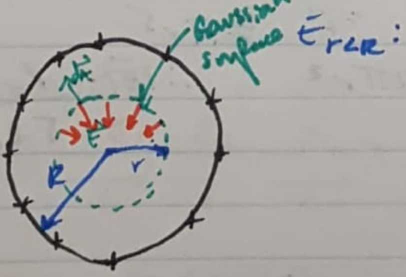
$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$ } Becomes same as pt. charge.

$$E 4\pi r^2 = \frac{Q}{\varepsilon_0}$$ ← Same as charged shell

Note: PC, sphere, shell, w/ uniform dist. behave same (r>R)

### Field inside ($r \leq R$):

Uniform shell:




$E_{r<R}$: $\varphi = \frac{Q_{enc}}{\varepsilon_0}$

$$\oint \vec{E}\cdot d\vec{A} \rightarrow \frac{Q_{enc}}{\varepsilon_0} = 0. = -|\vec{E}|\oint dA = 0$$

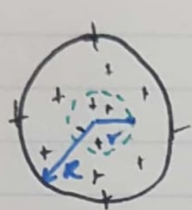
$\therefore \vec{E}_r = 0$ ← only if symmetric

Non-uniform shell: $\varphi = 0$, but $\vec{E} \neq 0$!!!

← $\vec{E}$ can't be 0, no symmetry.

↳ No sym? Can't param. $\vec{E}$, aka bring it out of integral.

uniform sphere:



$$\phi_e = \frac{Q_{enc}}{\varepsilon_0} = \frac{r^3 Q}{R^3 \varepsilon_0}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{r^3 Q}{R^3 \varepsilon_0} \quad \text{etc...} \quad |\vec{E}| = \frac{rQ}{R^3 4\pi\varepsilon_0}$$

$$Q_{enc} = \rho_v \left(\tfrac{4}{3}\pi r^3\right), \quad \rho_v = \frac{Q}{\frac{4}{3}\pi R^3} \quad \text{(sphere)}$$

(Gauss)

## More Charge Density:

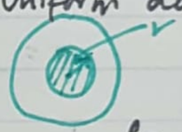
Jan 22, 2018

$$Q_{line} = \int_{line} dQ = \int_{line} \rho_\ell \, dL$$

$$Q_{plane} = \int_{plane} dQ = \int_{plane} \rho_s \, ds$$

$$Q_{sphere} = \int_{sphere} dQ = \int_{sphere} \rho_v \, dV$$

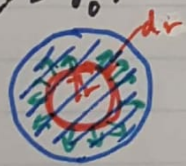
Uniform density $\rho_v$



$$Q_{(r)} = \int \rho_v \, dV = \rho_v \int_0^r 4\pi r^2 \, dr = \rho_v \frac{4}{3}\pi r^3$$

## Example:

1) $\rho_v = \rho_0 r^2$ $\quad E_{r<R}$?



$$\phi_e = \frac{Q_{enc}}{\varepsilon_0}$$

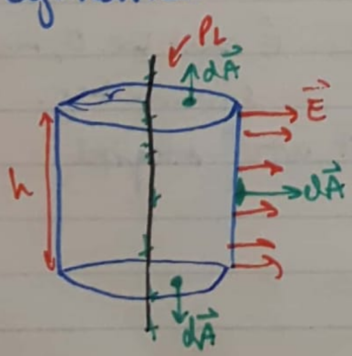
$$Q_{enc} = \int_0^r \rho_0 r^2 \cdot 4\pi r^2 \, dr = \rho_0 \int 4\pi r^4 \, dr = \rho_0 \frac{4}{5}\pi r^5$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{\rho_0 4\pi r^5}{5\varepsilon_0} = E \, 4\pi r^2 \quad \therefore |\vec{E}| = \frac{\rho_0 r^3}{5\varepsilon_0}$$

## Symmetries:

(Spherical for PC)

cylindrical:




$$\phi_e = \cancel{\phi_{top}} + \cancel{\phi_{bottom}} + \phi_{walls} \quad \cos 90 = 0$$

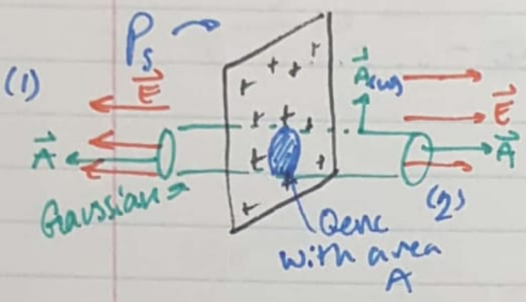
$$|\vec{E}| \int_{walls} |d\vec{A}| = \frac{Q_{enc}}{\varepsilon_0}$$

$$|\vec{E}| \, 2\pi r h = \frac{\rho_L \cdot h}{\varepsilon_0}$$

$$E = \frac{\rho_L}{2\pi\varepsilon_0 r}$$

Jan 24, 2018

(1)




$$\varphi_1 + \varphi_2 + \varphi_w = \frac{Q_{enc}}{\varepsilon_0}$$

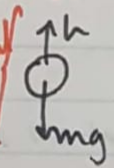
$$\vec{E}\cdot\vec{A} + \vec{E}\cdot\vec{A} = Q_{enc}/\varepsilon_0$$

$$2EA = \frac{Q_{enc}}{\varepsilon_0}$$

$$E = \frac{\rho_s \cdot A}{2\varepsilon_0 \cdot A} = \frac{\rho_s}{2\varepsilon_0}$$

## Potential:

Gravitational

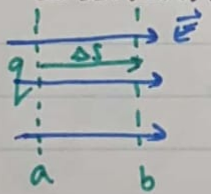



$\frac{\Delta U}{m} = gh = \Delta V_g$ (change in gravitational potential)

$*\Delta U_g = -W_g$, always. $W_{force} = \Delta U \Leftrightarrow \Delta K = 0$.

## Generalization:

$\Delta U_{conservative} = -W_{conservative}$.




$\Delta U_e = -W_{F_e} = -\vec{F}_e\cdot\Delta\vec{s}$ *only if F const (plane)

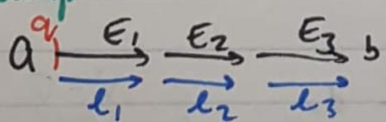
$$= -\int_a^b q\vec{E}\cdot d\vec{\ell}$$

$$\Delta V = -\int_a^b \vec{E}\cdot d\vec{\ell}$$

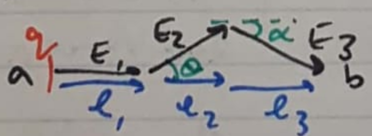
## Examples:

| | $W_F$? | $\Delta U$? | $\Delta V$? |
|---|---|---|---|
| | $q(E_1\ell_1 + E_2\ell_2 + E_3\ell_3)$ | $-q(E_1\ell_1 + E_2\ell_2 + E_3\ell_3)$ | $-E_1\ell_1 - E_2\ell_2 - E_3\ell_3$ |

1)

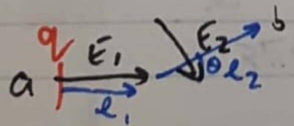



2)




$\Delta V = -E_1\ell_1\cos\theta - E_2\ell_2\cos\theta - E_3\ell_3\cos\alpha$

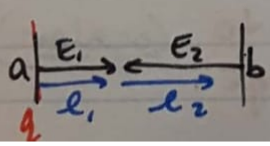
3)



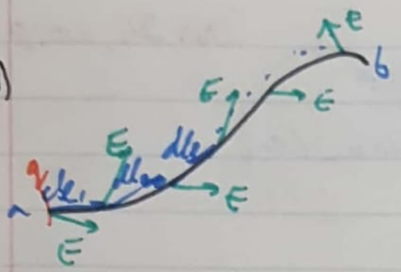

$\Delta U = -qE_1\ell_1 - qE_2\ell_2\cos\theta$  $\theta > 90$, $\cos\theta = -ve$

$\Delta V = -E_1\ell_1 - E_2\ell_2\cos\theta$

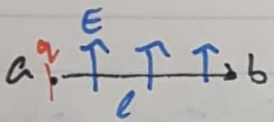
3.5)



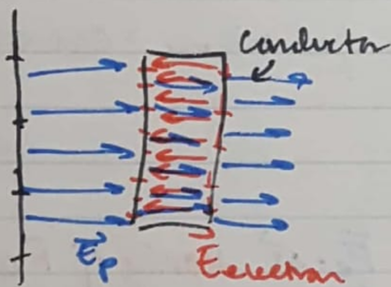

$\Delta V = -E_1\ell_1 + E_2\ell_2$ (gain potential

4) 


$$\Delta V = -E_1 \cdot dl_1 - E_2 \cdot dl_2 - \ldots\ldots$$
$$= -\int_a^b \vec{E} \cdot d\vec{l}$$

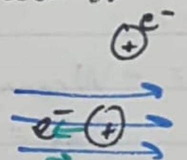
5) 


$\Delta V = \Delta U = 0.$ $(\cos 90)$

6) 

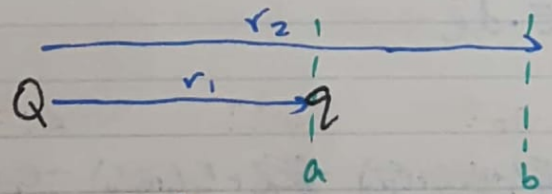

$E_{inside} = 0$ (free electrons make field)

insulator



$E_{induced} < E_p$

$E_{inside} = 0$, so $\Delta V = 0$
everywhere in a conductor.

7) 


$$W_r = \int q\vec{E}_a \cdot \vec{dr} = q\int_{r_1}^{r_2} \frac{Q}{4\pi\varepsilon_0 r^2}\,dr$$

$V = 0$ at $r = \infty$
$Q>0$: $V = \infty$ at $r \to 0$
$Q<0$: $V = -\infty$ at $r = 0$

$$\Delta U = -q\int_{r_1}^{r_2} \frac{Q}{4\pi\varepsilon_0 r^2}\,dr$$

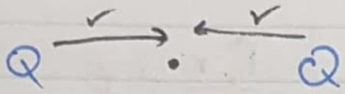
$$\Delta V = -\int_{r_1}^{r_2} \frac{Q}{4\pi\varepsilon_0 r^2}\,dr = \frac{Q}{4\pi\varepsilon_0 r_2} - \frac{Q}{4\pi\varepsilon_0 r_1}$$

$V_Q(r) = \dfrac{Q}{4\pi\varepsilon_0 r}$ } potential due to point charge. (scalar)

$U_Q(r) = \dfrac{Qq}{4\pi\varepsilon_0 r}$

Jan 29, 2018

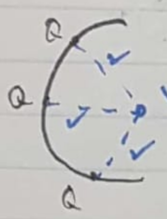
1)




$V = V_1 + V_2$

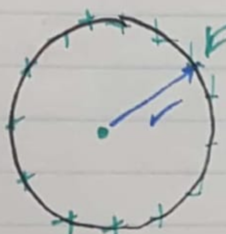
$V = \frac{2Q}{4\pi\varepsilon_0 r}$ (but $|\vec{E}| = 0$)

2)




$V = \frac{3Q}{4\pi\varepsilon_0 r}$ Total charge

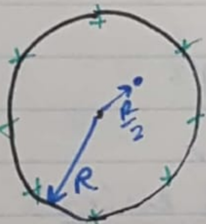
3)




$V_{(0)} = \frac{Q}{4\pi\varepsilon_0 r}$ Total charge.

4)

shell $r < R$




$V_{\frac{R}{2}} = ?$

$\Delta V = -\int_a^b \vec{E} \cdot d\vec{l}$

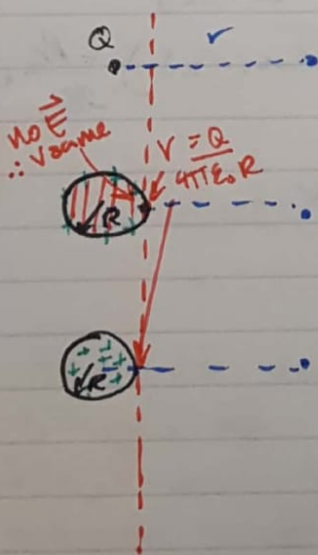
$\vec{E}_{inside} = 0$ (symt Gauss)

$\therefore \Delta V = 0$, so

V same as center $\left(\frac{Q}{4\pi\varepsilon_0 r}\right)$

5)

$r > R$

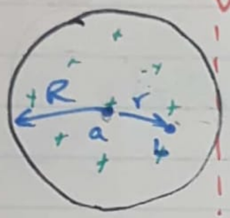



$E = \frac{Q}{4\pi\varepsilon_0 r^2}$ $V = \frac{Q}{4\pi\varepsilon_0 r}$

$E = \frac{Q}{4\pi\varepsilon_0 r^2}$ $V = \frac{Q}{4\pi\varepsilon_0 r}$

$E = \frac{Q}{4\pi\varepsilon_0 r^2}$ $V = \frac{Q}{4\pi\varepsilon_0 r}$

6)




$V = \frac{Q}{4\pi\varepsilon_0 R}$ ← use this as comp. at surface.

$$\Delta V = -\int_0^r \vec{E} \cdot d\vec{r} \qquad \frac{\rho_v r}{3\varepsilon_0} = E$$

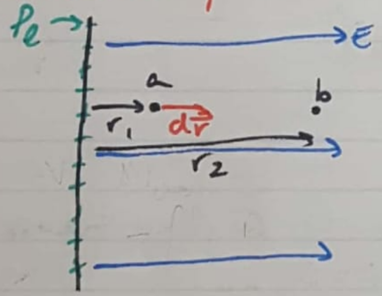
$$= -\int_0^r \frac{rQ}{R^3 4\pi\varepsilon_0}\, dr \qquad = -\int \frac{\rho_v r \cdot \hat{r}}{3\varepsilon_0} \cdot d\vec{r}$$

$$= -\left[\frac{r^2 Q}{R^3 8\pi\varepsilon_0}\right]_0^r \qquad = -\frac{\rho_v r^2}{6\varepsilon_0}\Big|_0^r$$

$$= -\frac{r^2 Q}{R^3 8\pi\varepsilon_0} \qquad = \frac{-\rho_v r^2}{6\varepsilon_0}$$

## Potential from a Line:

Jan 31, 2018




$$\Delta V = -\int_{r_1}^{r_2} \frac{\rho_\ell \hat{r}}{2\pi\varepsilon_0 r} \cdot d\vec{r}$$

$$= -\int_{r_1}^{r_2} \frac{\rho_\ell\, dr}{2\pi\varepsilon_0 r}$$

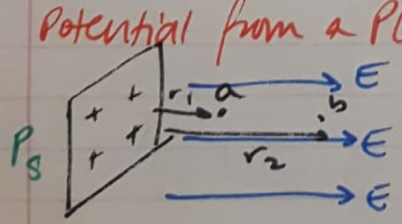
$$= -\left[\frac{\rho_\ell}{2\pi\varepsilon_0} \ln(r)\right]_{r_1}^{r_2}$$

$$= \frac{\rho_\ell \ln(r_1)}{2\pi\varepsilon_0} - \frac{\rho_\ell \ln(r_2)}{2\pi\varepsilon_0}$$

$$\Delta V = \frac{\rho_\ell}{2\pi\varepsilon_0} \ln\left(\frac{r_1}{r_2}\right)$$

$$= \frac{\rho_\ell}{2\pi\varepsilon_0}\left(\ln(r_1) - \ln(r_2)\right) = \frac{\rho_\ell}{2\pi\varepsilon_0} \ln\left(\frac{r_1}{r_2}\right)$$

## Potential from a Plane:




$$\Delta V = -\int_{r_1}^{r_2} \frac{\rho_s}{2\pi\varepsilon_0}\, dr$$

$$= \frac{-\rho_s}{2\pi\varepsilon_0} \Delta r$$

$$\Delta V = \frac{-\rho_s}{2\pi\varepsilon_0} \Delta r$$

# Conductors in Electrostatic Equilibrium:

Conductors - free electrons.

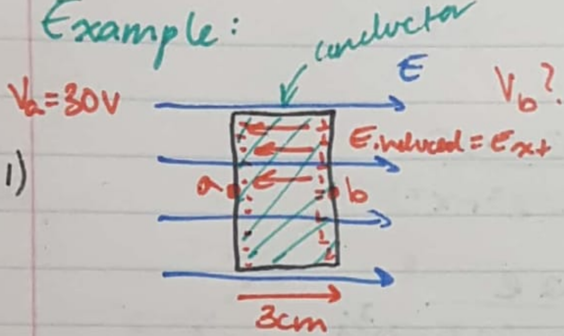
→ Some are not as "bound" to nucleus.

↳ Will move in presence of field (current) ← external field.

↳ To be in eq, field inside conductor must = 0. always eq!

Example:

1)




$\vec{E}_{inside} = 0$, so $\Delta V = 0$

$\therefore V_b = V_a = 30V$

2)

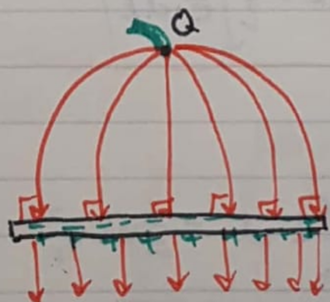

charge Q in conducting sphere.

$V_{\frac{R}{2}}$? $E_{inside}$ must $= 0$.

↳ By Gauss's Law, charge must be at edge for $E = 0$. ($Q_{enc} = 0$)

→ $V_{r=0} = \frac{Q}{4\pi\epsilon_0 R}$. $\Delta V = 0 = \vec{E}$ so $V_{\frac{R}{2}} = \frac{Q}{4\pi\epsilon_0 R}$

3)



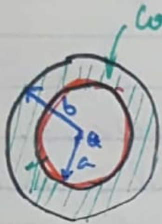

Must intersect at 90°

so $E_{inside} = 0$ by shielding.

1) $\vec{E} = 0$ inside conductor
2) charge on surface
3) $E$ just outside $= \rho_{local} / \epsilon_0$
4) $\vec{E}$ normal to surface
5) Same V everywhere
6) Pointy areas have less charge, more density, more field. O-o

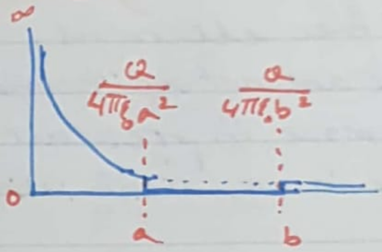
Scanned by CamScanner

4)



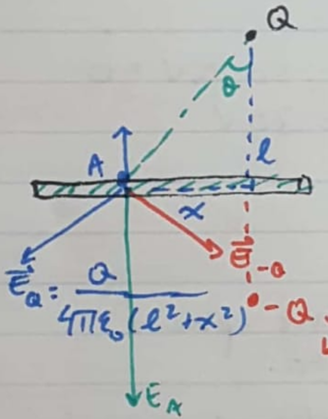

○ electrons w/ $-Q$ charge

$|\vec{E}|$ :




Same as P.C but 0 gap.

5) $E_A$ ?

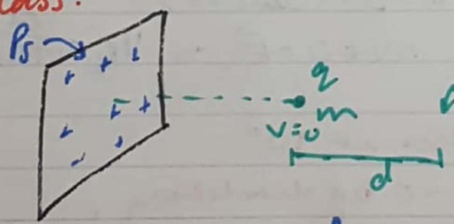



$$E_A = \frac{2Q\cos\theta}{4\pi\varepsilon_0(l^2+x^2)}$$

$$= \frac{2Ql}{4\pi\varepsilon_0(l^2+x^2)^{3/2}}$$

Mass:

Feb 2, 2018.




$$\Delta K = -\Delta U$$

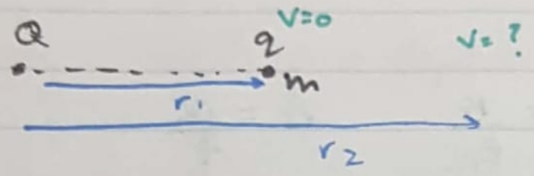
$$\boxed{\Delta k = q\int_a^b \vec{E}\cdot d\vec{l}}$$

$$\Delta U = -q\int \vec{E}_{plane}\, d\vec{l}$$

$$\Delta K = q\int \frac{\rho_s}{2\varepsilon_0} dl = \frac{q\rho_s}{2\varepsilon_0} d = \frac{1}{2}mv_2^2$$

$$v = \sqrt{\frac{q\rho_s}{m\varepsilon_0}}$$

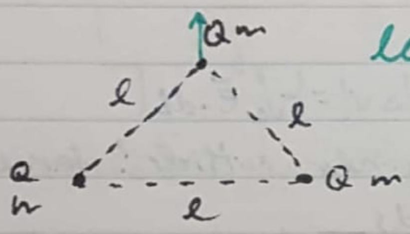
1)




$$\Delta K = -\Delta U = q\int_{r_1}^{r_2} \frac{Q}{4\pi\varepsilon_0 r^2}\,dr$$

$$= \frac{Qq}{4\pi\varepsilon_0 r_1} - \frac{Qq}{4\pi\varepsilon_0 r_2} = \frac{1}{2}mv_2^2 - 0.$$

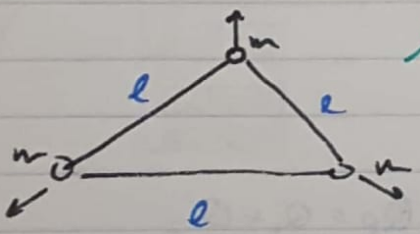
2)




let one charge go. Speed @ $\infty$?

$$\Delta K = -\Delta U = -(U_f - U_i)$$
$$\frac{1}{2}mv^2 = -\left(\frac{Qq}{4\pi\varepsilon_0 l} - \frac{3Qq}{4\pi\varepsilon_0 l}\right)$$
whole system.

3)



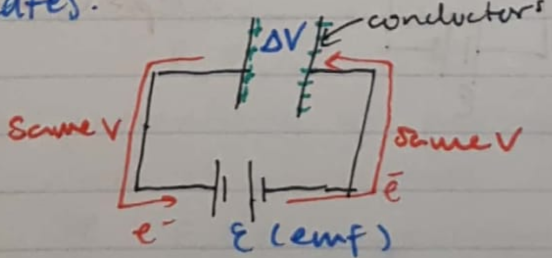

let all three charges go. Speed at $\infty$?

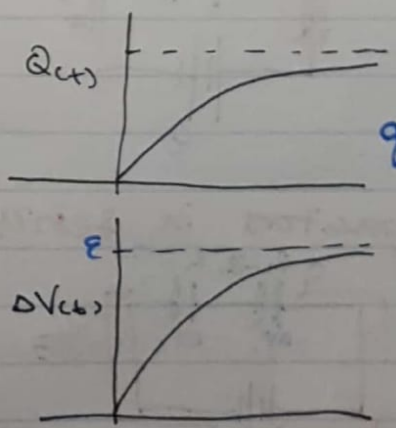
$$\Delta K = -\Delta U_s = -\left(0 - \frac{3Qq}{4\pi\varepsilon_0 l}\right) = 3\left(\frac{1}{2}mv^2\right)$$

Capacitance:

Feb 5, 2018

$\parallel$ plates:



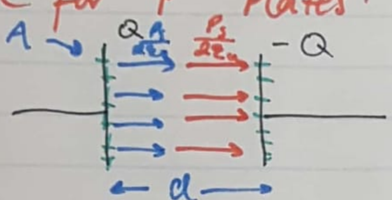




$$q_{(t)} = C\Delta V_{(t)} = C\varepsilon$$

Battery will pump $e^-$ from plate to plate until equillibrium.

All conductors, so $\Delta V$ across wire $= 0$.

## C for $\parallel^{\text{el}}$ Plates:

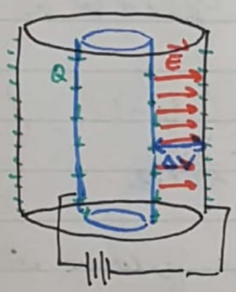



$\vec{E} = \frac{\rho_s}{\varepsilon_0}$ $\quad \rho_s = \frac{Q}{A}$

$|\Delta V| = \left|-\int_a^b \vec{E}\cdot d\vec{l}\right| = \left|\frac{-\rho_s}{\varepsilon_0} d\right| = Ed.$

$$c = \frac{Q}{\Delta V} = \frac{Q}{\frac{\rho_s}{\varepsilon_0} d} = \frac{Q\,\varepsilon_0}{\rho_s d} = \frac{A^2 \varepsilon_0}{A d} = \boxed{\frac{A\,\varepsilon_0}{d}}$$

## Cylindrical:




$Q = c\,\Delta V$

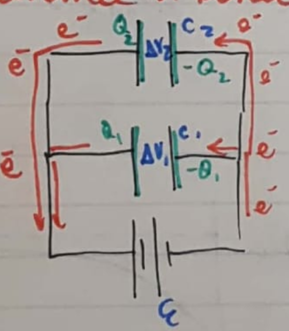
$Q = \int \rho_s\, ds \quad |\Delta V| = \left|-\int_a^b \vec{E}\cdot d\vec{l}\right|$

Field from cylinder outside? Same as line.

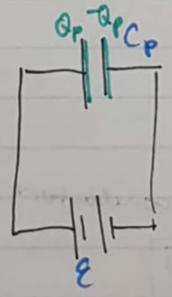
$$c = \frac{\int \rho_s\, ds}{\left|-\int (\vec{E}\cdot d\vec{l})\right|}$$

## Capacitance in Parallel:

$\Delta V_1 = \Delta V_2 = \mathcal{E}$




$\equiv$




$Q_p = Q_1 + Q_2$

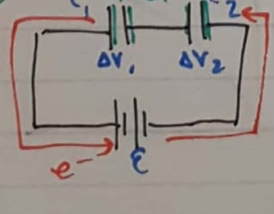
$\Delta V_p = \mathcal{E} = \Delta V_1 = \Delta V_2$

So, $C_p \Delta V_p = C_1 \Delta V_1 + C_2 \Delta V_2$

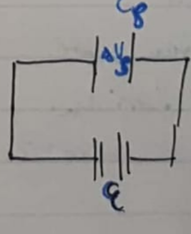
$C_p = C_1 + C_2$

## Capacitors in Series:

$-Q_1$ because G. shielding




$\equiv$




$\Delta V_s = \Delta V_1 + \Delta V_2 = \mathcal{E}$

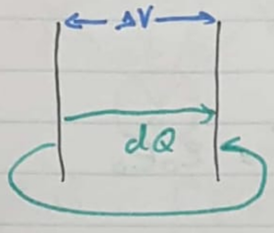
$\Delta V_s = \frac{q_1}{C_1} + \frac{q_2}{C_2} = \frac{Q_s}{C_s}$

But $Q_s = Q_2 = Q_1$

$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$

## Energy Stored in a Capacitor:

Feb 7, 2018



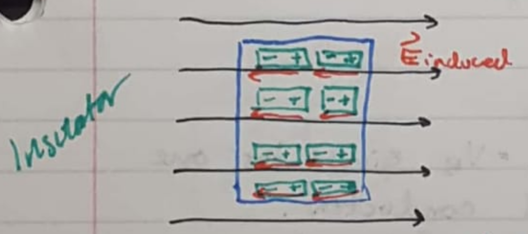

$dU = \Delta V dQ$

energy needed to bring $dq$ across $\Delta V$.

$$U \cong \sum du = \sum \Delta V dQ = \int_0^Q \Delta V dQ = \int_0^Q \frac{q}{C} dQ = \frac{Q^2}{2C}$$

$$\boxed{U = \frac{Q^2}{2C} = \frac{1}{2} C \Delta V^2 = \frac{1}{2} Q \Delta V}$$

## Dielectrics:




$$\vec{E} = \vec{E}_0 + \vec{E}_{ind}$$
$$= E_0 - E_{ind}.$$
$$= \frac{E_0}{\varepsilon_r} = \frac{E_0}{K}$$

rel. permativity / dielectric constant

Applies when $\vec{E}$ is perpendicular to boundary.

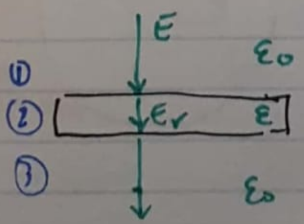
## Gauss's Law for Dielectrics:

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{enc}}{\varepsilon_0 \varepsilon_r} = \frac{Q_{enc}}{\varepsilon}$$

$\varepsilon_0$: permativity of free space

$\varepsilon_r$: relative permativity (depends on material)

$\varepsilon = \varepsilon_0 \varepsilon_r$: permativity




$E_r$(inside dielectric) $= \dfrac{E}{\varepsilon_r}$ ← >1

E is not continuous across boundary.

$\vec{D} = \epsilon \vec{E}$ is continuous

1) $E = E_0$ $\qquad D = \varepsilon_0 E_0$

2) $E = E_r = \frac{E_0}{\varepsilon_r}$ $\qquad D = (\varepsilon_r \varepsilon_0)\frac{E_0}{\varepsilon_r} = \varepsilon_0 E_0$

3) $E = E_0$ $\qquad D = \varepsilon_0 E_0$

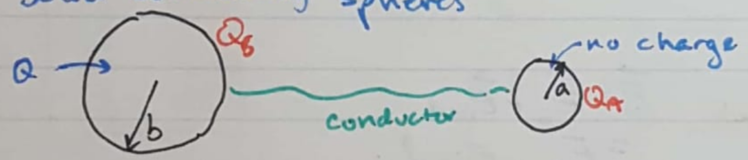
So, Gauss' Law for Dielectrics:

$$\oint \vec{D} \cdot d\vec{s} = Q_{enc}.$$

Midterm Review:

1) Solid conducting spheres

$V_{a/2}$ on small sphere

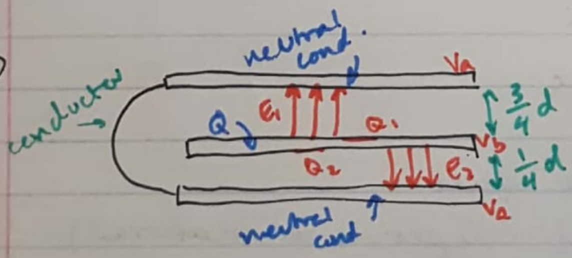



$V_B = \frac{Q_B}{4\pi\varepsilon_0 b}$ $\qquad V_A = \frac{Q_a}{4\pi\varepsilon_0 a}$ $\Big\}$ At eq, $V_A = V_B$ since its one overall conductor.

$\hookrightarrow \frac{Q_B}{b} = \frac{Q_a}{a}$ $\qquad Q_a + Q_B = Q \Big\{ Q_A = \frac{Q}{(1+\frac{b}{a})} \Rightarrow V_A =$ ____

All over A.

2)




$Q_1 + Q_2 = Q$ $\qquad E_1 = \frac{\rho_{s1}}{\varepsilon_0}$ $\qquad E_2 = \frac{\rho_{s2}}{\varepsilon_0}$

$\Delta V_1 = \Delta V_2$

$$\left(\frac{\rho_{s1}}{\varepsilon_0}\right)\left(\frac{3d}{4}\right) = \frac{\rho_{s2}}{\varepsilon_0}\left(\frac{d}{4}\right) \Rightarrow \rho_{s1} = \frac{\rho_{s2}}{3}$$

$$Q_1 = \frac{Q_2}{3} \Rightarrow Q_2 = \frac{3Q}{4}$$

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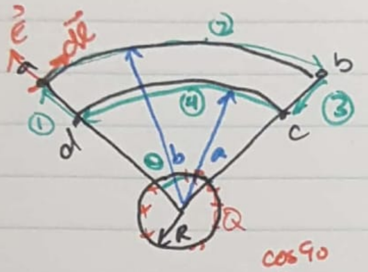
3)




$\oint \vec{E} d\vec{l} = 0.$

2, 4 are equipotential, no $|\vec{E}|$ change ($\cos 90$)
1, 3 cancel.

$$\oint \vec{E}\cdot d\vec{l} = \cancel{\int_a^b \vec{E}\cdot d\vec{l}}^{\cos 90} + \int_b^c \vec{E}\cdot d\vec{l} + \cancel{\int_c^d \vec{E}\cdot d\vec{l}}^{\cos 90} + \int_d^a \vec{E}\, d\vec{l}$$

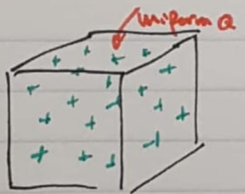
$$= 0 \text{ since } \int_d^a \vec{E}\cdot d\vec{l} = \frac{Q}{4\pi\varepsilon_0 d} - \frac{Q}{4\pi\varepsilon_0 a} = -\int_b^c \vec{E}\, d\vec{l}$$

4)


$\varphi_e$ through one face?

$\varphi_e = \frac{Q_{enc}}{6\varepsilon_0}$ (6 faces)

6)

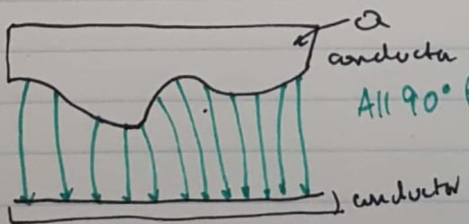



$\cdot\ E$?

Gauss's Law doesn't work ($E$ not $\perp$)

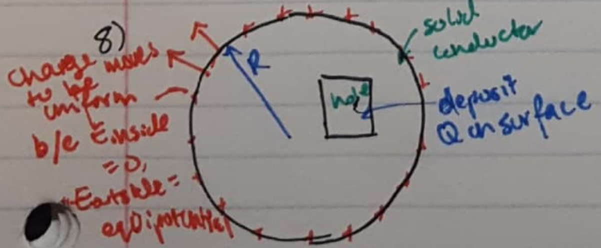
(would triple integral)

7)




All 90° for eq potential surface

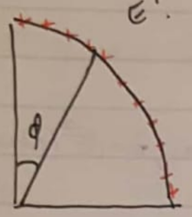
8)




a) $E_{in\ cube}$? b) $V_{center\ sphere}$? c) $V_{cube}$?

Field 0 inside, so a) = 0

b) = c) = $\frac{Q}{4\pi\varepsilon_0 R}$

9) 

$E$?

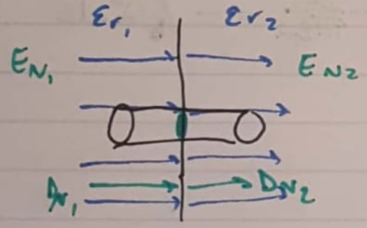
$P = P_0 \cos\varphi$

from Quiz.

## Boundary Conditions: Normal

Feb 9, 2018

$\vec{E}$ not continuous across boundary. $D_N$ is. N (normal)

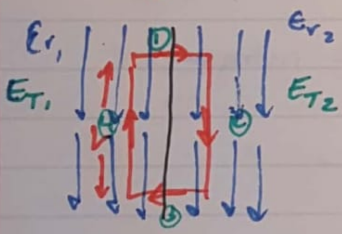


$\oint \vec{D} \cdot d\vec{s} = Q_{enc}$

$-D_{N_1} A_1 + D_{N_2} A_2 = Q_{enc}$

$\boxed{\text{If } Q_{enc} = 0, \text{ then } D_{N_1} = D_{N_2} \Rightarrow E_{N_1} \neq E_{N_2}}$

## Boundary Conditions: Tangential.



$E_{T_1} = E_{T_2}$

$\oint \vec{E} \cdot d\vec{l} = 0$.

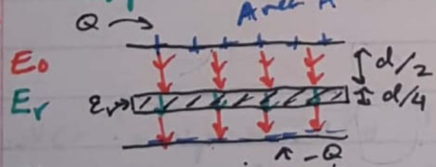
$① \cancel{\int \vec{E} \cdot d\vec{l}}^{\,0} + ② \int \vec{E} \cdot d\vec{l} + ③ \cancel{\int \vec{E} \cdot d\vec{l}}^{\,0} + ④ \int \vec{E} \cdot d\vec{l} = 0$

$E_{T_2} \cdot L - E_{T_1} L = 0$

$\therefore E_{T_1} = E_{T_2}$

## Example:

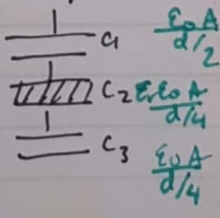
FINAL



charge capacitor, then insert dielectric.

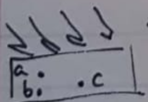
$\Delta V_{slab} = E_r \frac{d}{4} = \frac{E_0}{\varepsilon_r} \frac{d}{4} = \frac{\rho_s}{\varepsilon_r \varepsilon_0} \frac{d}{4} = \frac{Qd}{4A\varepsilon_r \varepsilon_0}$

$C_{equiv} = \frac{Q}{\Delta V}$ OR (where $\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3$)



combine in series.

Quiz?



split E into $E_T$, $E_N$ in and outside slab.

parameterize $V_{ab}$, $V_{ac}$

$\varepsilon_1 E_{N_1} = \varepsilon_2 E_{N_2}$

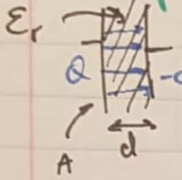
$E_{T_1} = E_{T_2}$

FINAL MATERIAL!

Example:

Feb 28, 2018

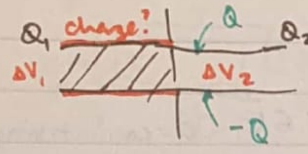
1) $\varepsilon_r$ $C_r$?




$C_r = \frac{Q}{\Delta V}$ $\quad |\Delta V| = Ed = \frac{E_0}{\varepsilon_r} d$

$\therefore C_r = \varepsilon_r \frac{Q}{E_0 d} = \varepsilon_r \frac{\rho_s A}{\frac{\rho_s}{\varepsilon_0} d} = \varepsilon_r \frac{\varepsilon_0 A}{d} = \boxed{\varepsilon_r C}$

2) Insert dielectric, how much charge on left?




$Q_1 + Q_2 = Q$

$\Delta V_1 = E_1 d = \Delta V_2 = E_2 d$

$E_1 d = E_2 d$

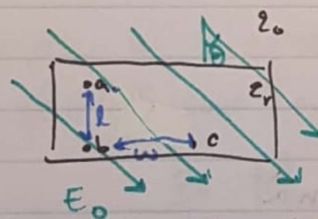
$\frac{\rho_{s1}}{\varepsilon_0 \varepsilon_r} \cancel{d} = \frac{\rho_{s2}}{\varepsilon_0} \cancel{d}$

$\rho_{s1} = \varepsilon_r \rho_{s2}$

$\rightsquigarrow Q_1 = \varepsilon_r Q_2$

$Q_1 = \frac{Q}{1 + \frac{1}{\varepsilon_r}}$

3)




$\Delta V_{ba} = -\int_a^b \vec{E} \cdot d\vec{l} = -\vec{E}_r \cdot \vec{l} = -E_{rN}\, l = -\frac{E_0}{\varepsilon_r} \cos\theta\, l$

$\Delta V_{cb} = -E_{T_r} w = -E_0 \sin\theta\, w$

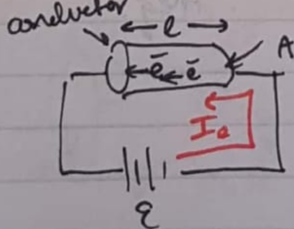
$\Delta V_{ca} = \Delta V_{ba} + \Delta V_{cb}$

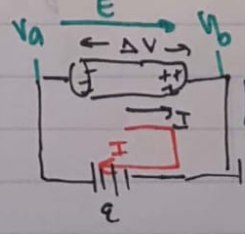
plug in here

$\vec{D}_{n1} = D_{n2}$ $\quad E_N = E_0 \cos\theta \neq E_{N_r}$ } Boundary conditions

$E_T = E_0 \sin\theta = E_{T_r}$

4)




$\boxed{\Delta V = IR}$

$= El$

Mar 2, 2018

$V_a > V_b$, so $\vec{E} \longrightarrow$

pushes $e^- \longleftarrow$, $p^+ \longrightarrow$

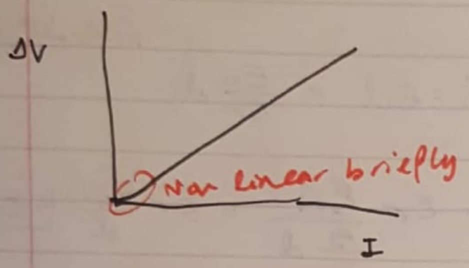
$e^-$ movement

$p^+$ movement

Drift speed: $V_d$

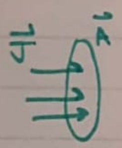
Mean free path.

* Not in eq! ∴ ΔV not 0.

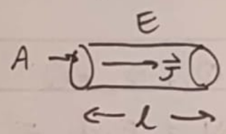



$$I = \frac{Q}{t} = \frac{dQ}{dt}.$$

## Current Density:




$$\boxed{I = \vec{J} \cdot \vec{A}}$$



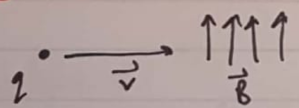

$\vec{J} = \sigma \vec{E}$ ($\sigma$ = conductivity)


$$I = \int \vec{J} \cdot d\vec{A}$$

$\sigma = \frac{1}{\rho}$ $\rho$ = resistivity.

$EL = IR \Rightarrow R = \frac{\rho l}{A}$ ← Uniform E, I

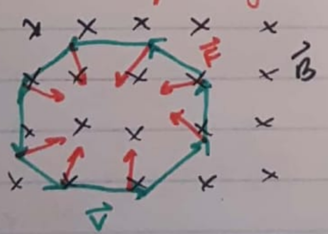
## Magnetism:




$F = q\vec{v} \times \vec{B}$

→ q +, F ⊙ . q −, F ⊗

## Cyclotron Frequency:




$$q\vec{v} \times \vec{B} = m\vec{a}$$

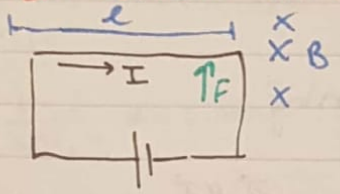
$$qvB = \frac{mv^2}{r} \quad (\sin 90)$$

$$qB = \frac{mv}{r} \quad . \text{ Freq?}$$

$$v = \frac{2\pi r}{T}. \Rightarrow \boxed{\frac{qB}{2\pi m} = f}$$

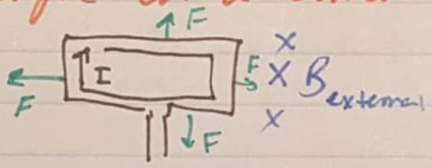
Scanned by CamScanner

## Force on Current in wire:

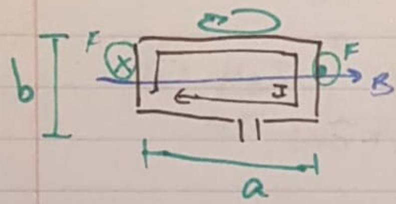



$$\vec{F} = q\vec{v} \times \vec{B}$$
$$= q\frac{\vec{l}}{t} \times \vec{B}$$
$$= I\vec{l} \times \vec{B}$$

## Torque on a current Loop:




No torque.



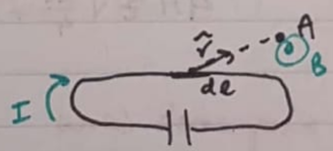

$$\Sigma\vec{\tau} = \vec{r}\times\vec{F} + \vec{r}\times\vec{F}$$
$$= 2\left(\frac{a}{2}\right)F = aF$$
$$\vec{\tau} = aF = a(IbB) = IAB \text{ (area)}.$$
$$= \underbrace{I\vec{A}}\times\vec{B} = \vec{\mu}\times\vec{B}$$

dipole moment: $= \vec{\mu}$.

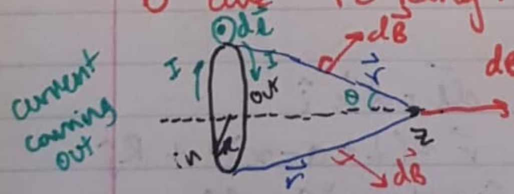
## Sources of Magnetic Fields:

Mar 7, 2018

Biot Savart Law: $\vec{B}_A = \int \frac{\mu_0 I d\vec{l} \times \hat{r}}{4\pi r^2}$




## $\vec{B}$ due to Ring:

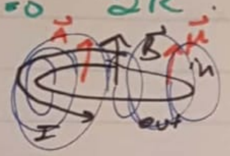



$$dB_z = dB\sin\theta = \frac{\mu_0 I}{4\pi r^2}|d\vec{l}||\hat{r}|\sin 90° \sin\theta$$
$$= \frac{\mu_0 I dl}{4\pi r^2}\sin\theta$$
$$= \frac{\mu_0 I}{4\pi r^2}dl\frac{R}{\sqrt{R^2+z^2}}$$

$$B = \int_{ring} dB_z = \frac{\mu_0 I R}{4\pi(R^2+z^2)^{3/2}}\int_{ring} dl = \boxed{\frac{\mu_0 I 2\pi R^2}{4\pi(R^2+z^2)^{3/2}}}$$

So $B_{ring} = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}$

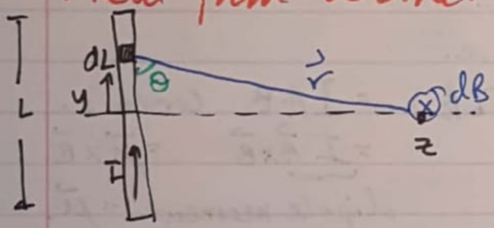
$\lim_{R \gg z / z=0} B_z = \frac{\mu_0 I}{2R}$ center of ring.




curl $\bar{\omega}$ I. $\vec{B} \parallel \vec{A}$

$\lim_{z \gg R} B_z = \frac{\mu_0 \vec{\mu}}{2\pi z^3}$

## Field from a Line:




$dB = \frac{\mu_0 I}{4\pi r^2} \vec{dl} \times \hat{r}$

$= \frac{\mu_0 I}{4\pi r^2} |dl||r| \sin\theta$

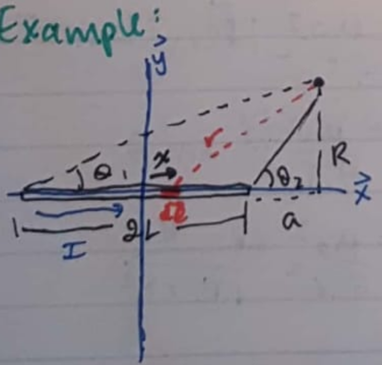
$= \frac{\mu_0 I}{4\pi r^2} \frac{z}{r} dl = \frac{\mu_0 I \; z \; dy}{4\pi (y^2+z^2)^{3/2}}$

$B = \int_{line} dB$

$= \frac{\mu_0 I}{4\pi} \int_{-\frac{L}{2}}^{L/2} \frac{z\,dy}{(y^2+z^2)^{3/2}}$ $\quad y = z\tan\alpha \Rightarrow B = \frac{\mu_0 I \cdot L}{4\pi z \sqrt{\frac{L^2}{4} + z^2}}$

$\lim_{L \gg z} B = \frac{\mu_0 I}{2\pi z}$ } Same as ampere's law

## Example:




$d\vec{B} = \frac{\mu_0 I}{4\pi r^2} d\vec{l} \times \hat{r}$

$d\vec{l} = d\vec{x}\,\hat{i}$

$\vec{r} = (L+a-x)\hat{i} + R\hat{j}$

$\hat{r} = \frac{(L+a-x)\hat{i} + R\hat{j}}{\sqrt{(L+a-x)^2 + R^2}} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$$d\vec{B} = \frac{\mu_0 I}{4\pi r^2}\, dx\,\hat{i} \times (\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$

$$= \frac{\mu_0 I}{4\pi r^2}\, dx \sin\theta\,\hat{k} \Rightarrow \vec{B} = \int_{line} dB = \int_{line} \frac{\mu_0 I}{4\pi r^2}\, d_x \sin\theta\,\hat{k}$$

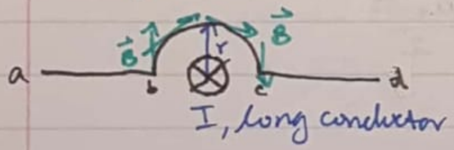
Parameterize in terms of $\theta$:

$r = \frac{R}{\sin\theta} = R\csc\theta$. Also, $\frac{R}{L+a-x} = \tan\theta$

$$\frac{R}{\tan\theta} = L+a-x \Rightarrow dx = -R\csc^2\theta$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_{line} \frac{-R\csc^2\theta}{R^2\csc^2\theta} \sin\theta = \frac{-\mu_0 I}{4\pi R} \int_{\theta_1}^{\theta_2} \sin\theta$$

**Ampere's Law:** Mar 14, 2018



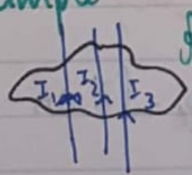

$$\int_a^d \vec{B}\cdot d\vec{l} = \int_b^c \vec{B}\cdot d\vec{l} = \int_b^c B\,dl$$
$$= B\int_b^c dl$$
$$= B\pi r.$$
$$= \frac{\mu_0 I}{2\pi r}\cdot \pi r = \frac{\mu_0 I}{2}$$

Over whole O, $\boxed{\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}}$

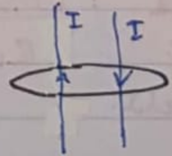
Any closed loop. Applies everywhere.

Can only use w/ $\vec{B}$sym

Example:

1) 


$\oint \vec{B}\cdot d\vec{l}$
$= \mu_0(I_1 + I_2 + I_3)$

2) 


$\oint \vec{B}\cdot d\vec{l} = 0.$

But $B \neq 0$!!! O not correct.

**Amperian Surfaces:**

B, dl must be pll or ⊥. Circular surfaces.

↳ B constant over line.

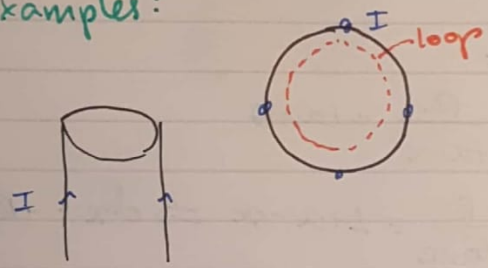
→ Need infinite line of current to gaurentee no other interference

## Field inside Ideal solenoid:

$\oint \vec{B}\cdot d\vec{l} = Bl = \mu_0 \cdot NI$ . $B = \mu_0 n I$ where $n$ = loops/unit of length

## Examples:

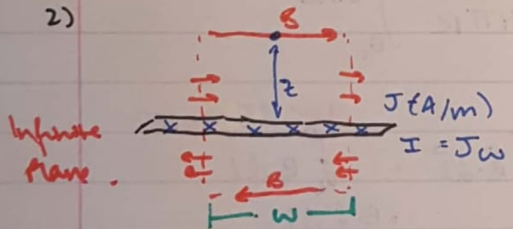
1)




$\oint \vec{B}\cdot d\vec{l} = 0$

B around loop is symmetrical, so comes out of integral.
∴ B also = 0

2)

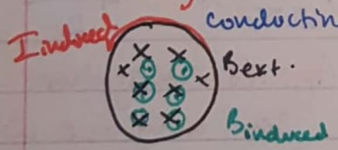



$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$

$Bw + 0 + Bw = \mu_0 Jw$.

$B = \frac{\mu_0 J}{2}$

## Faraday's Law:




If $\vec{B}_{ext}$ ↑, $\vec{B}_{internal}$ will increase in direction of $\vec{B}_{ext}$ to oppose ↑ flux.

EMF$_{ind}$ from $I_{ind}$.

$\left|\frac{d\Phi_B}{dt}\right| = |\mathcal{E}_{ind}| = I_{ind} \cdot R_{loop}$

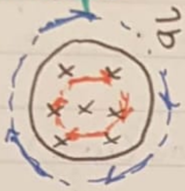
R.O.C. of flux due to ext. field.

$\Phi_B = \int \vec{B}\cdot d\vec{A}$

If uniform, $\Phi_B = BA\cos\theta$

Example:

1) ($\vec{J}$ uniform)




$$\oint \vec{B}\cdot d\vec{l} = \mu_0 J \cdot \pi r^2$$

$$B \cdot 2\cancel{\pi} \cancel{r} = \mu_0 J \cancel{\pi} r^2$$

$$B = \frac{\mu_0 J r}{2} \quad (r<R)$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 J \pi R^2$$

$$B = \frac{\mu_0 I}{2\pi r}$$

2) Current density for r-independent $\vec{B}$?

$$B \cdot 2\pi r = \mu_0 \int \vec{J}\cdot d\vec{A}$$

$$= \mu_0 \int J \cdot 2\pi r\, dr$$

$$\therefore J = \frac{C}{R} \leftarrow \text{constant}$$

3) $J_{(r)} = J_0 r$. Find $B_{r<R,\, r>R}$

Mar 16, 2018

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \int \vec{J}\cdot d\vec{A}$$

$$B \cdot \cancel{2\pi r} = \mu_0 \int_0^r J_0 \cdot \cancel{2\pi} r^2\, dr$$

$$B = \frac{\mu_0 J_0 r^2}{3} \Big\} r<R$$

$$\int^R \vec{J}\cdot d\vec{A}$$

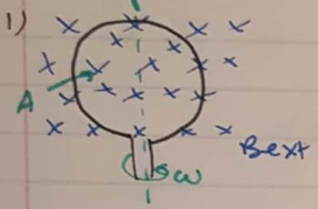
$$B = \frac{\mu_0 I}{2\pi \underline{r}_{\text{still}}} \Big\} r>R \quad (r>R)$$

Lenz's Law:

Emf induced such that $I_{induced}$ will make $B_{induced}$ resist flux.

$$\varepsilon_{ind} = \frac{-d\varphi_B}{dt}$$

Example:  Mar 19, 2018

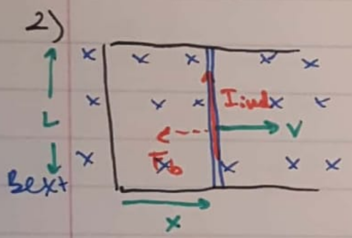
1) 


Spins at ω. Find $\mathcal{E}_{ind}$. B const.

$$\varphi_B = \int \vec{B}\cdot d\vec{A} = BA\cos\theta = BA\cos(\omega t)$$

$$\mathcal{E}_{ind} = -\frac{d\varphi_B}{dt} = BA\omega\sin(\omega t)$$

Initial $B_{ind}$ ⊗ to resist lower flux. $I_{ind}$ ↻

2) 


Conducting frame and bar. Grab bar + move right. F to pull? Resistance R, const V.

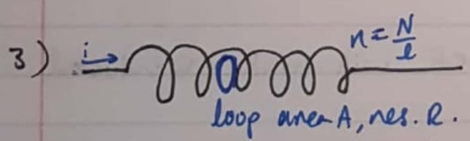
Flux ↑ ⇒ $B_{ind}$ ⊙, $I_{ind}$ ↺

$$\varphi_B = BA = BLx. \quad \therefore |\mathcal{E}_{ind}| = |I_{ind}R| = BL\frac{dx}{dt} = BLV.$$

$$\therefore I = \frac{BLV}{R} \Rightarrow F = I\vec{l}\times\vec{B} = IlB = \frac{B^2L^2v}{R}$$

$P_{loss}$?

$$P_{loss} = I^2R = \frac{B^2L^2v^2}{R} \qquad P_{in} = \frac{W}{t} = \frac{F\cdot d}{t} = Fv = \frac{B^2L^2v^2}{R}$$

3) 

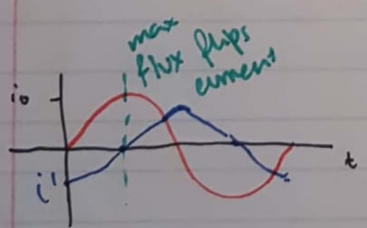

$i = i_0\sin(\omega t)$

$B = \mu_0 n I = \mu_0 n i_0 \sin(\omega t)$  Mar 21, 2018

$$\varphi_B = \int \vec{B}\cdot d\vec{A} = \mu_0 n i_0 \sin(\omega t)(A)$$

$$\left|\frac{d\varphi_B}{dt}\right| = |\mu_0 n i_0 \omega A\cos(\omega t)| = |\mathcal{E}_{ind}| = |I_{ind}\cdot R|$$

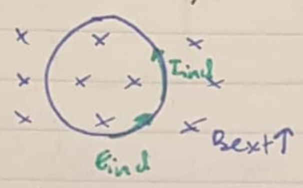
$$\therefore I_{ind} = -\frac{\mu_0 n i_0 \omega A\cos(\omega t)}{R}$$



Mar 23, 2018

## Induced Fields:

$\mathcal{E}$ and $I$ pll.




$$\oint \vec{E}_{ind} \cdot d\vec{l}$$

$$\mathcal{E}_{ind} = E_{ind} \cdot 2\pi r$$

The same 2 points have a potential difference.

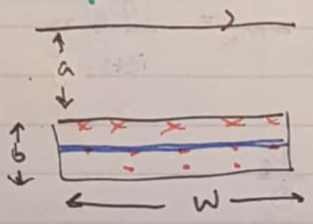
$\therefore$ Not conservative force.

$$B_{(t)} \xLeftrightarrow{\text{causes}} E_{(t)}$$

## Electrostatic Fields:

$$\Delta V_{a \to a} = \oint \vec{E} \cdot d\vec{l} = 0 \quad \text{(conservative force)}$$

## Example:

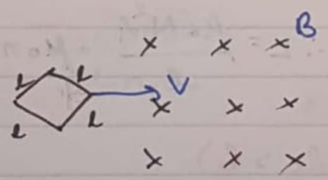



$$\Phi_B = B \cancel{\times} A \qquad d\Phi = B\,dA$$

$$= B w\,dx$$

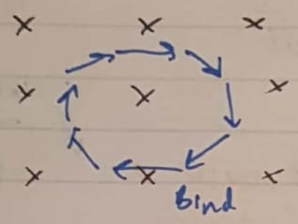
$$\therefore \Phi = \int_a^{a+b} B w\,dx = \int \frac{\mu_0 I}{2\pi x} w\,dx$$

Quiz




$\frac{d\Phi}{dt}$? parameterize area in terms of V.

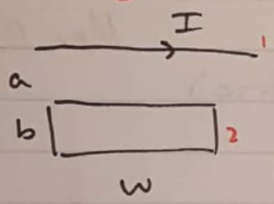
JCM: Mar 28, 2018




$\mathcal{E}_{(t)} \uparrow$

Not needed.

Inductance:




$$\varphi = \frac{\mu_0 i(t)}{2\pi} w \ln\left(\frac{b+a}{a}\right)$$

$$\mathcal{E}_{ind} = \frac{-\mu_0 w}{2\pi} \ln\left(\frac{b+a}{a}\right) \frac{di(t)}{dt}$$

$$\left|\frac{\mathcal{E}_{ind}}{\left(\frac{di(t)}{dt}\right)}\right| = \text{Mutual Inductance} = M_{21} = \frac{\varphi_{B(2)}}{i(t)_{(1)}}$$

Assume I →

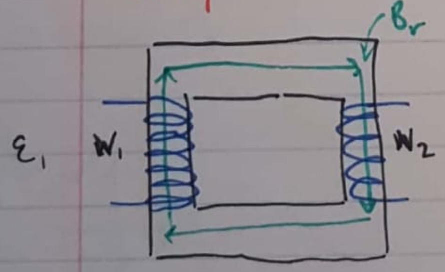
$L$ : Self inductance $= \frac{\varphi_{B_1}}{I_1}$

$\ell, N, A$

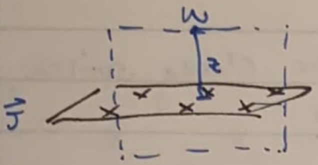
$$\left.\begin{aligned} \varphi_{loop} &= B \cdot A = \mu_0 n I A = \mu_0 \frac{N}{\ell} I A \\ \varphi_{all\ loops} &= N \cdot \varphi_{loop} = \frac{\mu_0 N^2 I A}{\ell} \end{aligned}\right\} \therefore L = \frac{\mu_0 N^2 A}{\ell} = \mu_0 n^2 \ell A$$

If iron placed, $B_r = \mu_r B_0$ ($B_{inside} > B$)

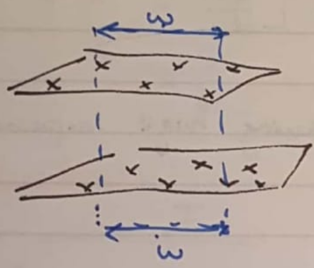
Transformers:



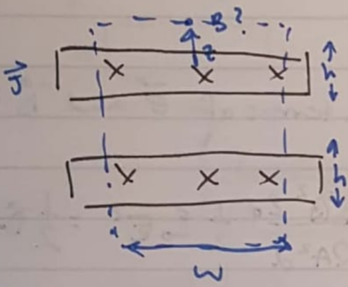

$$\left.\begin{aligned} \varphi_2 &= N_2 \varphi_{loop} \\ \varphi_1 &= N_1 \varphi_{loop} \end{aligned}\right\} \therefore \mathcal{E}_2 = \frac{N_2}{N_1} \mathcal{E}_1$$

1) 

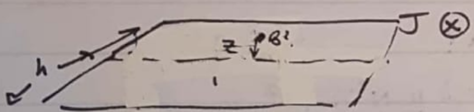

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc} = \mu_0 \cdot J \cdot w$$
$$\oint \vec{B}\cdot d\vec{\ell} = 2Bw = \mu_0 J w$$
$$B = \frac{\mu_0 J}{2}$$

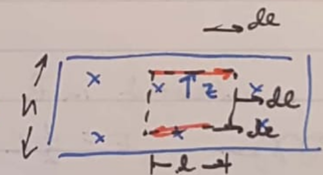
2) 


$$2Bw = \mu_0 J \cdot w \cdot 2$$
$$B = \frac{\mu_0 J}{1} = \mu_0 J$$

3. 


$$\cancel{2}B\cancel{w} = \mu_0 J \cdot h \cdot \cancel{w} \cdot \cancel{2}$$
$$B = \mu_0 J h$$

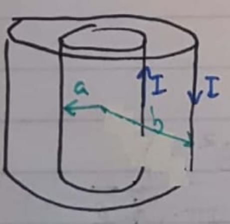
4. 

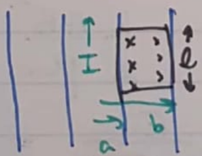

$\Rightarrow$ 


$$2B\ell = \mu_0 J \ell z$$
$$B = \mu_0 J z$$

## Inductance Continued: Apr 2, 2018

Inductance/unit length?




$$L = \frac{\Phi}{I} = \frac{\int_a^b \frac{\mu_0 I}{2\pi x}\,\ell\, dx}{I} = \frac{\mu_0 \cancel{I}}{2\pi}\frac{\ell(\ln(\frac{b}{a}))}{\cancel{I}}$$

$$\frac{L}{\ell} = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right)$$