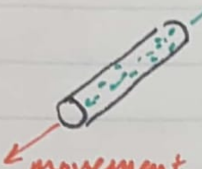
ECE 140

Current:

Jan 4, 2018

Moving charges.



movement of + charges

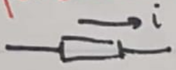
movement of - charges

$$\boxed{i_{(t)} = \frac{dq_{(t)}}{dt}}$$ ← charge in coulomb (C)

high school way $\left\{ i = \frac{Q}{t} \right.$ in APs

often need to assume direction.

Reference Direction:



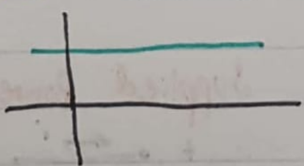
if $i$ +, flows →
else, flows ←

Charge:

$$q_{(t)} = q(t_0) + \int_{t_0}^{t} i(\tau)\,d\tau$$

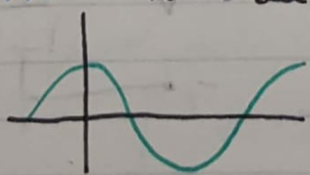
Types of Current:

1) DC - constant wrt time.



$i = 5A$

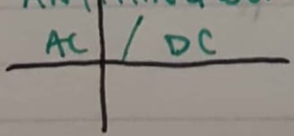
2) AC - sinusoidal



$i = 2\sin(5t)$

3) Time Varying:

ANYTHING BUT AC / DC



$i = 5e^{-2t}$

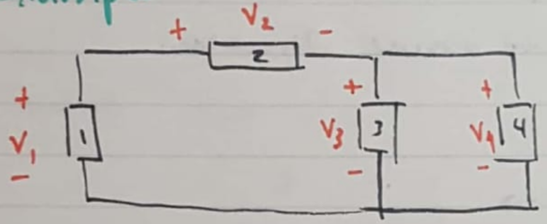
**Voltage:**

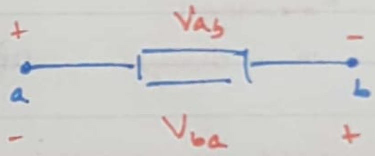
$V(t) = \dfrac{dW(t)}{dq}$ ← Energy (J), ← charge in Coulombs (C)

$\left(\dfrac{E}{Q}\right.$ from high school$\left.\right)$

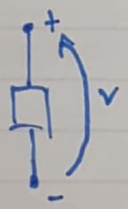
AKA emf, PD

**Example:**




$V_{ab} = -V_{ba}$




Also sometimes used. ←

**Power:**

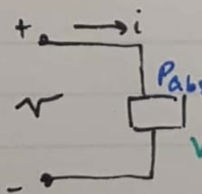
Jan 8, 2018

$p(t) = \dfrac{dw}{dt}$ ← J $= \dfrac{dw}{dq} \cdot \dfrac{dq}{dt} = v \cdot i$

↑ watts

**Absorbed Power:**

Current + → − of v



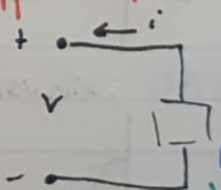

$P_{abs} = v \cdot i$

V drop

$P_{abs} = -P_{sup}$

**Supplied Power:**

Current − → + of v




$P_{sup} = v \cdot i$

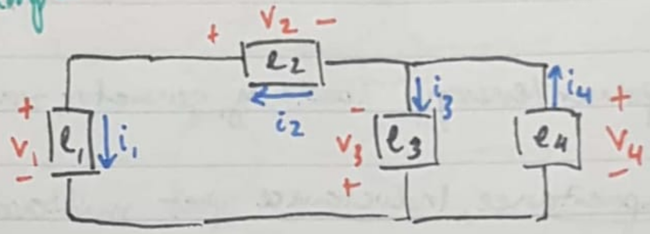
V rise

Passive sign convention ↑

$+ \rightarrow i \rightarrow -$

If $P_{abs} > 0$, actually absorbed. $< 0$ supplied.

Example:




$i_1 = i_2 = 5A$ $V_1 = 8V$
$i_3 = -2A$ $V_2 = -2V$ $V_4 = 10V$
$i_4 = 3A$ $V_3 = -10V$

a) Power for $e_1$? Not specified? Assume abs.
$P_{abs} = V_1 i_1 = 40W$ +40, so actually abs.

b) $P_{abs}$ for $e_2$?
$P_{abs} = -V_2 i_2 = 10W$ +10, so actually abs.

c) $P_{sup}$ for $e_3$?
$P_{sup} = V_3 i_3 = 20W$ +20, so actually sup.

4) $P_{abs}$ for $e_4$?
$P_{abs} = -V_4 i_4 = -30W$ −30, so actually sup.

Conserved

Conservation of Power/Energy:

$\sum P_{abs} = \sum P_{sup}$ $\sum P = 0$

Energy:

$w = w(t_2) - w(t_1) = \int_{t_1}^{t_2} p(\tau) d\tau$ } number

Assume $t_1 = -\infty$, $w(t_1) = 0$.

$w(t) = \int_{-\infty}^{t} p(\tau) d\tau$ } function of t.

## Circuit Elements:

Jan 9, 2018

1) Active:

↳ Generates power/energy (battery, generator, transistor)

2) Passive:

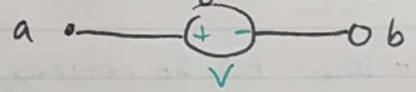
↳ Resistance, Capacitance, Inductance post midterm.

↳ Absorbs / stores energy.

I, V independent of each other

3) Independent Voltage Source:

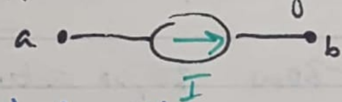
↳ Specific v. regardless of i. (Not constant, but independent)




$V_{ab} = V$, no matter $i$

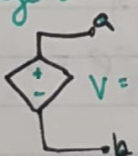
4) Independent Current Source:

↳ Specific current regardless of v. ( " " )




5) Dependent Sources:

a) Voltage-controlled Voltage Source:

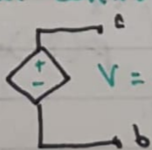



$V = \alpha V_x$ $\alpha$ = constant.

control V.

$V_x$ somewhere in rest of circuit.

b) Current-controlled Voltage Source:

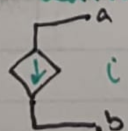



$V = r \cdot i_x$

control current

Same deal

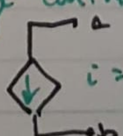
c) Current-controlled Current Source:




$i = \beta \cdot i_x$

Same

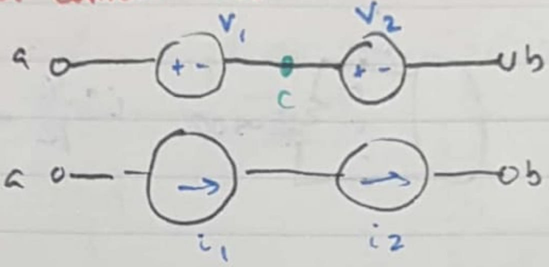
d) Voltage-controlled Current Source:




$i = g \cdot V_x$

Same

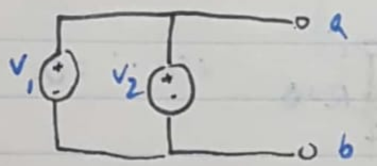
## Series Connection:



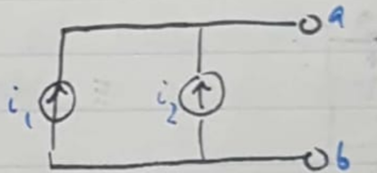

Only $v_1, v_2$ connected at c.

$i_1$ should $= i_2$ (else invalid)

## Parallel connection:



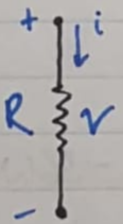

$v_1$ should $= v_2$
(else invalid)




valid

"Parallel" - connected btwn same two nodes (a, b).

## Resistance and Ohm's Law:

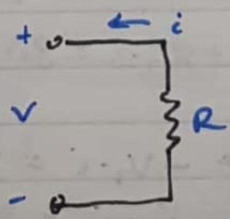



$\boxed{v = R \cdot i}$ $R \geq 0$.

$\hookrightarrow$ $i = \frac{1}{R} \cdot v$

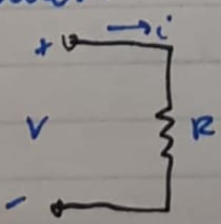
$= G \cdot v$ where $G$ = conductance in Siemons (S or $\Omega^{-1}$)

Note:




$v = -R \cdot i$ for this
(watch for current direction)

## Power:

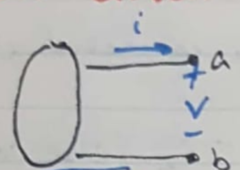



$$\left.\begin{aligned} P_{abs} &= v \cdot i \\ &= R \cdot i^2 \\ &= \frac{v^2}{R} \end{aligned}\right\} \geq 0 \text{ b/c of } {}^2$$

resistors always absorb power.

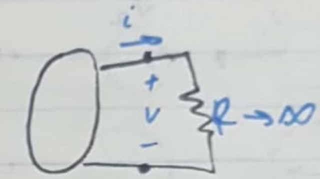
Open Circuit:

Jan 10, 2018



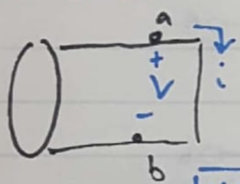

$\boxed{i=0}$

≡



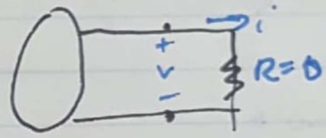

$i = \frac{V}{R} \to 0.$

Short circuit:




$\boxed{V=0}$

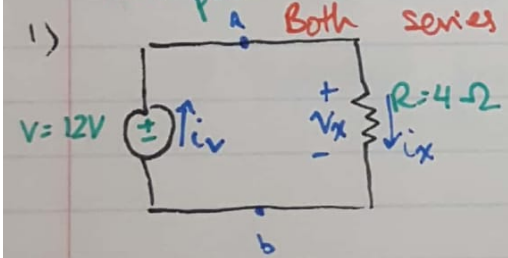
≡




$V = R \cdot i = 0.$

Example:

1) Both series + parallel



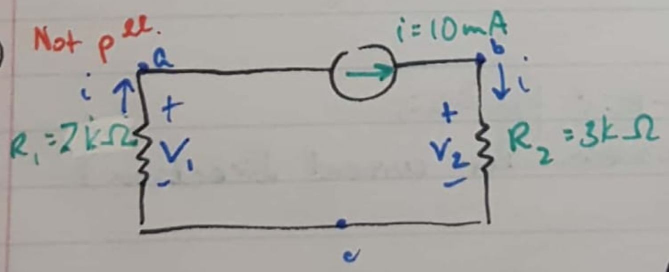

$V_x = 12V$ (−12V if $\overset{-}{}\ \overset{+}{}$)

$i_x = \frac{V_x}{R} = 3A$

$i_V = i_x = 3A.$

$P_{R,abs} = 12 \cdot 3 = 36W$ and $P_{v,abs} = -V \cdot i = -36W.$

2) Not $p^{ll}$.




$V_1 = -R_1 \cdot i = -20V$

$V_2 = R_2 \cdot i = 30V$

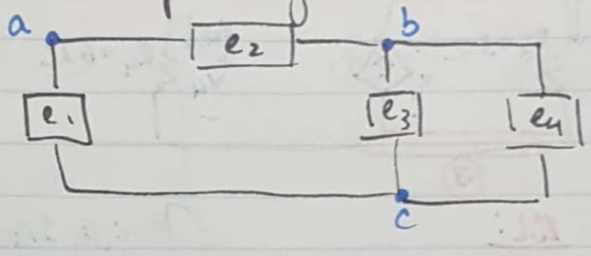
$P_{R_1 abs} = -V_1 \cdot i = 200mW$

$P_{R_2 abs} = V_2 \cdot i = 300mW$

## Kirchhoff's Current Law (KCL):

Jan 17, 2018

A node: pt or junction where 2+ elements connected.

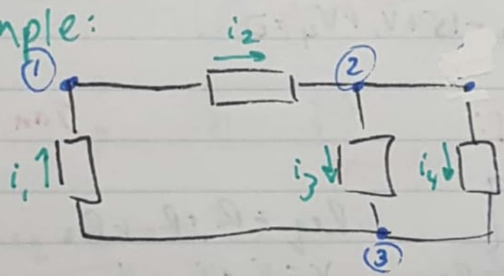



KCL: Sum of all currents at any node = 0.

$\boxed{\sum_n i_n = 0}$ i enter = i leaving node.

Example:

1)




1) $i_1 = i_2$
2) $i_2 = i_3 + i_4$
3) $i_3 + i_4 = i_1$

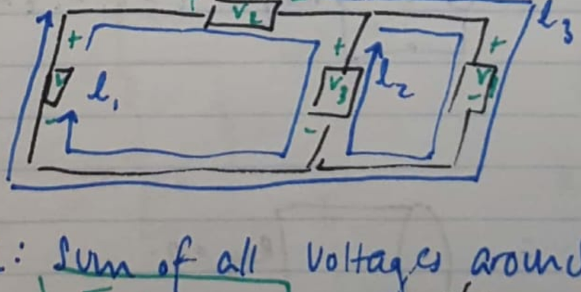
Note linear independence/dependence

N nodes give N-1 independent KCL eqn.

## Kirchhoff's Voltage Law (KVL):

A loop: closed path where no element is encountered more than once.

mesh: loop w/o any loops inside it.



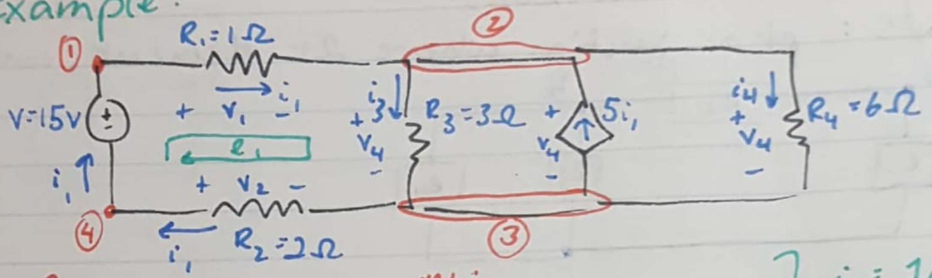

1) $V_1 = V_2 + V_3$
2) $V_3 = V_4$
3) $V_2 + V_4 = V_1$

KVL: Sum of all voltages around any loop is 0.

$\boxed{\sum_n V_n = 0}$ $V_{drops} = V_{rises}$

N meshes give N independent KVL eqn.

Example:




**OL:**

$V_1 = R_1 i_1$

$V_2 = -R_2 i_1$

$V_4 = R_3 i_3$

$V_4 = R_4 i_4$

**KCL:** Only need node 2

$i_1 + 5i_1 = i_3 + i_4$

**KVL:** Only need $\ell_1$

$-V_2 - 15 + V_1 + V_4 = 0$

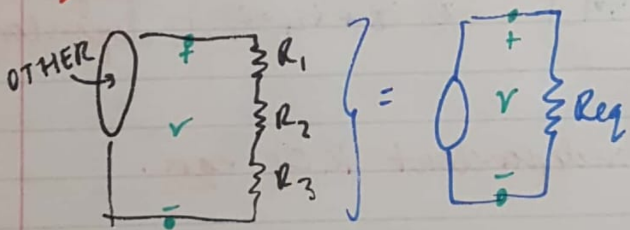
$i_1 = 1A \quad V_1 = 1V$

$i_4 = 2A \quad V_2 = -2V$

$i_3 = 4A \quad V_4 = 12V$

Jan 15, 2018

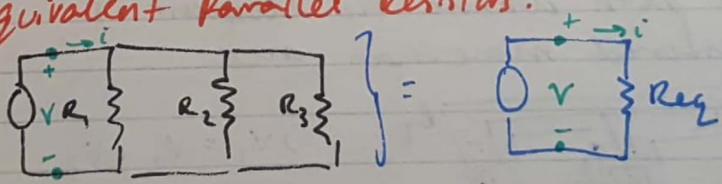
**Equivalent Series Resistors:**




$R_{eq} = R_1 + R_2 + R_3$

$V = V_1 + V_2 + V_3$

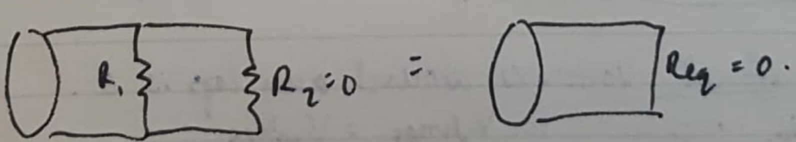
**Equivalent Parallel Resistors:**




$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

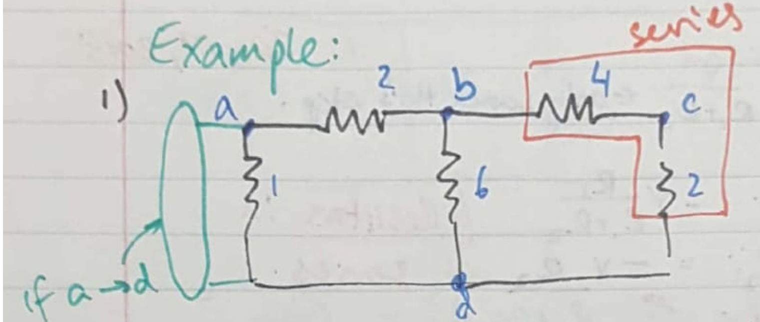
$i = i_1 + i_2 + i_3$

Note:



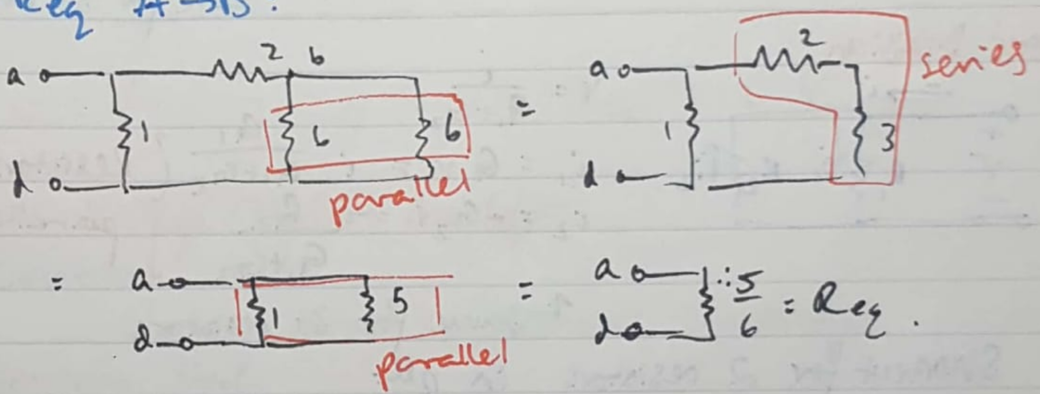
Example:

1)

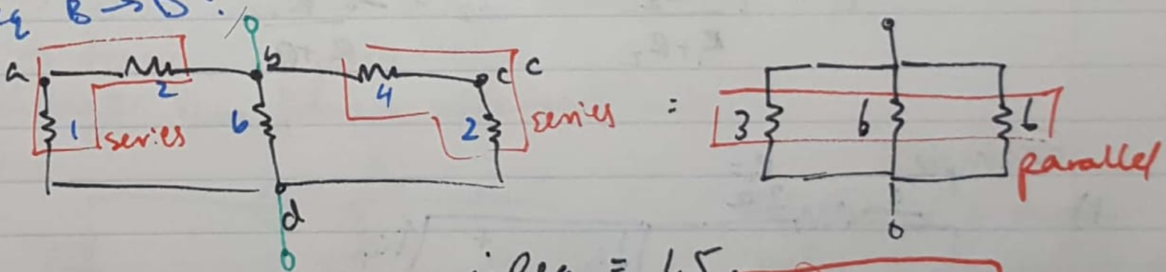



Should draw rest of circuit on left to show 1,2 not in series.
→ depends on nodes you want (a,d)

a) $R_{eq}$ A→D?

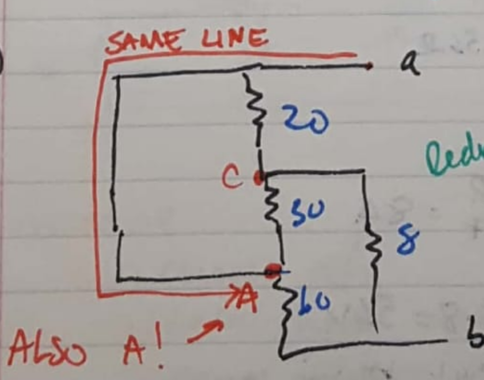



b) $R_{eq}$ B→D?

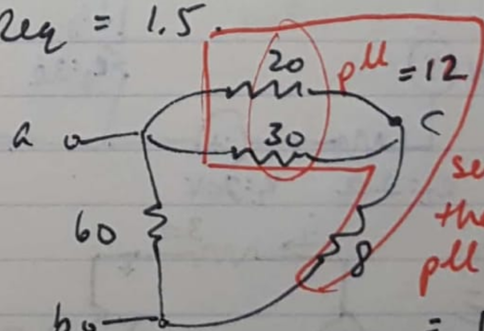



∴ $R_{eq}$ = 1.5.

2)




Redraw:




series, = 20
then
pll w 60
= 15 = $R_{eq}$.

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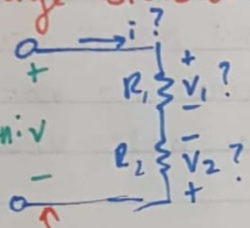
## Voltage Division:

Given: v




$i = \frac{v}{R_1+R_2}$ ← only sames this step.

$v_1 = R_1 i = v\frac{R_1}{R_1+R_2}$

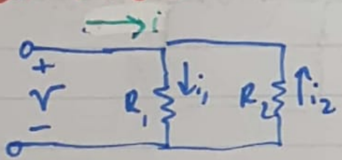
$v_2 = -R_2 i = -v\frac{R_2}{R_1+R_2}$

based on +V−

Resistors in series.

↑ same for 3+ resistors

## Current Division:




$v = \frac{i}{G_1+G_2}$

$i_1 = G_1 v = i\frac{G_1}{G_1+G_2}$

$i_2 = -G_2 v = -i\frac{G_2}{G_1+G_2}$

Resistors in parallel

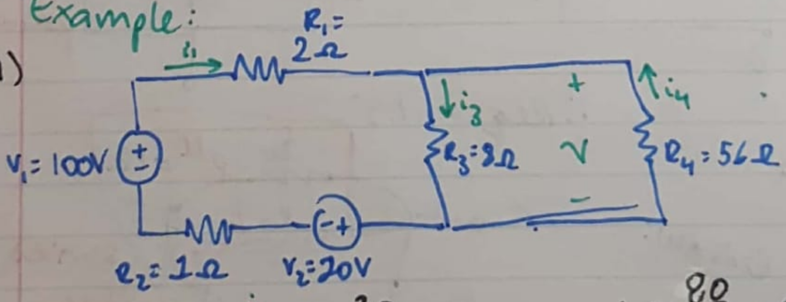
↑ Same for 3+ resistors

Shortcut for 2 resistors in ∥:

$i_1 = i\frac{R_2}{R_1+R_2}$ $\quad i_2 = -i\frac{R_1}{R_1+R_2}$

## Example:

1)

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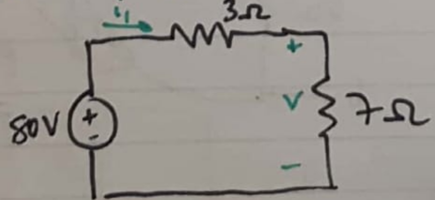



$i_1 = \frac{80}{3+7} = 8A$

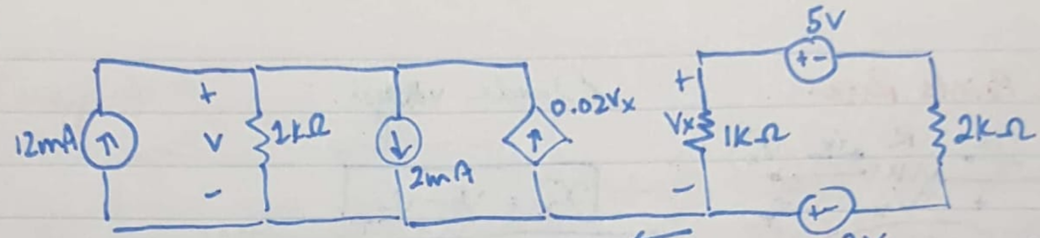
$V = 7 \cdot 8 = 56V$

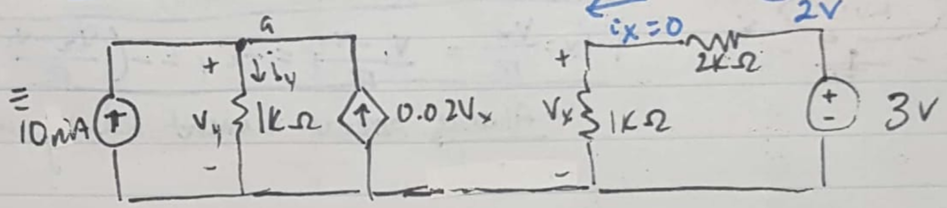
Now, just Ohm's law.

Or, CD: $i_3 = i_1 \cdot \frac{R_4}{R_3+R_4}$ $\quad i_4 = -i_1 \cdot \frac{R_3}{R_3+R_4}$

Scanned by CamScanner

2)




By VD: $V_x = 3 \cdot \frac{1}{1+2} = 1V$ Then KCL $V_y = 30V$

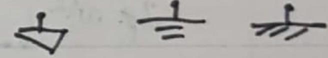
Node-Voltage (Nodal) Analysis:

Essential Node:

→ Node where 3+ elements connected.

Reference Node:

→ One of essential nodes chosen arb.



→ Assumed 0 potential.

Node Voltage: $V_{drop}$ from a node → ref. node.

Example:

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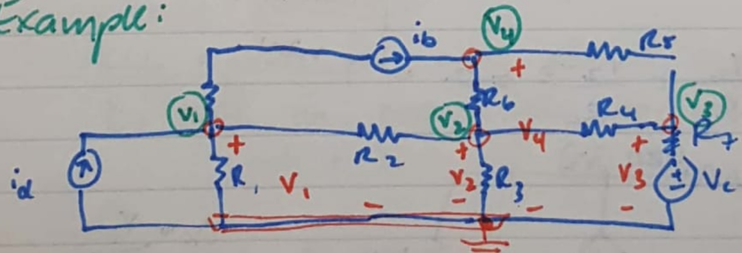
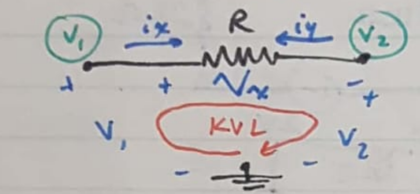



# of node V's @ essential nodes = # of ind. eqns. (4)

Procedure:

1) choose ref.
2) Label node voltages
3) Assign currents to branches (optional)
4) KCL at each non-ref essential node.
5). Ohm's law + KVL to get all I in terms of Voltages.

## Three Points Rule:




@node voltages.

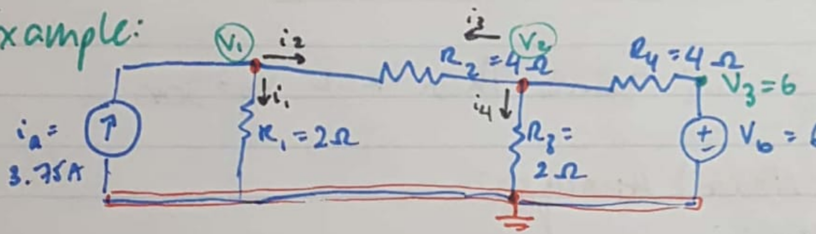
$$\boxed{v_x = V_1 - V_2}$$

$$\boxed{i_x = \frac{v_x}{R} = \frac{V_1 - V_2}{R} = -i_y = -\left(\frac{V_2 - V_1}{R}\right)}$$

↳ Always assume current leaving node

## Example:

1)




KCL 1: $3.75 = i_1 + i_2$

$i_1 + i_2 = -3.75$ ← enter = −ve, leave = +ve

3pt rule: $\dfrac{V_1}{2} + \dfrac{V_1 - V_2}{4} = 3.75$

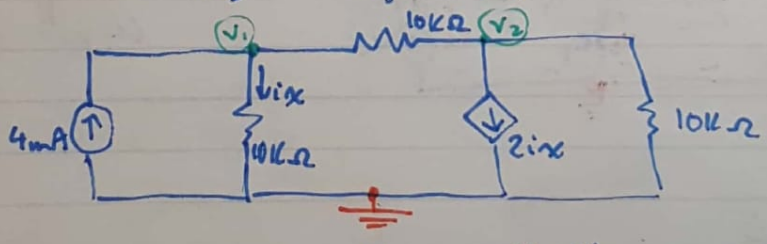
$0.75V_1 - 0.25V_2 = 3.75$

KCL2:

$\dfrac{V_2 - V_1}{R_2} + \dfrac{V_2}{R_3} + \dfrac{V_2 - 6}{R_4} = 0.$

can solve now.

2)




$i_x = \dfrac{V_1}{10}$

KCL 1: $-4mA + \dfrac{V_1}{10k} + \dfrac{V_1 - V_2}{10k} = 0$

$\frac{2}{10}V_1 - \frac{1}{10}V_2 = 4$

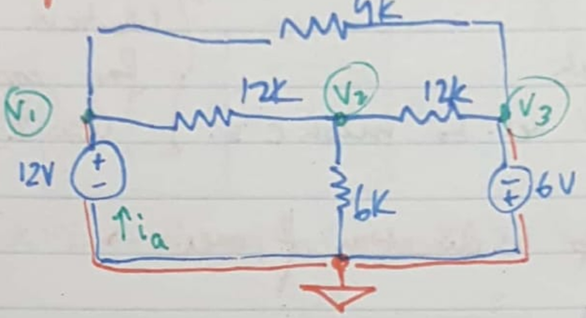
KCL2: $\dfrac{V_2 - V_1}{10} + 2i_x + \dfrac{V_2}{10} = 0$

Solve.
$V_1 = 16V$
$V_2 = -8V$

## Exceptions:

Jan 22, 2018

1)




can't write current w/o resistance

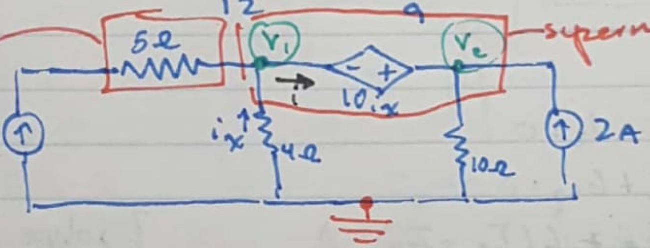
$V_1 = 12V$ $V_3 = -6V$ and KCL for 2

$V_2 = 1.6V$

To get $i_a$, KCL $v_1$:

$$i_a = \frac{V_1 - V_2}{12} + \frac{V_1 - V_3}{9} = 2.875 \text{ mA}.$$

2)




supernode

$i_x = \frac{-V_1}{4}$

has no effect on $V_1, V_2$, etc...

KCL 1: $-4 + \frac{V_1}{4} + i = 0$

KCL 2: $-i + \frac{V_2}{10} - 2 = 0$

$V_2 - V_1 = 10\left(\frac{-V_1}{4}\right)$. Now solve.

## Supernode Shortcut:

KCL applies for any closed surface.

Do KCL on supernode to avoid using $i$.

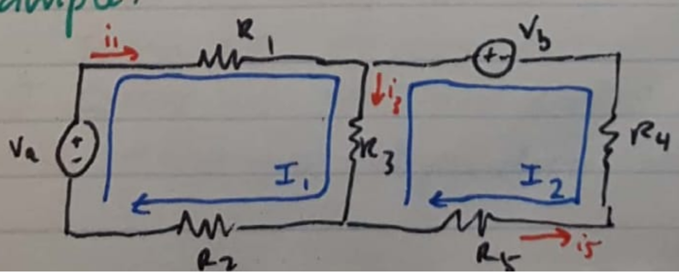
$$-4 + \frac{V_1}{4} + \frac{V_2}{10} - 2 = 0.$$

## Mesh Current Analysis:

Jan 23, 2018

Mesh current - Imaginary current, one direction + value.

### Example:




$I_1 I_2$ mesh currents

branch currents

$i_1 = I_1$ $i_3 = I_1 - I_2$ $i_5 = -I_2$

Procedure:

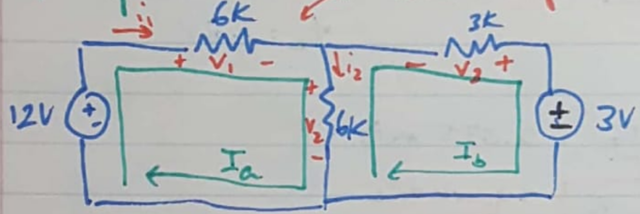
1) Draw + label mesh c's
2) KVL around each mesh
3) O'L and KCL to get V rel. to mesh c's.

} Watch for control (ex 2)

Example:

Assume Vdrop in direction of mesh →.

1)




KVL $I_a$: $-12 + V_1 + V_2 = 0$

$-12 + 6i_1 + 6i_2 = 0$

$-12 + 6I_a + 6(I_a - I_b) = 0$

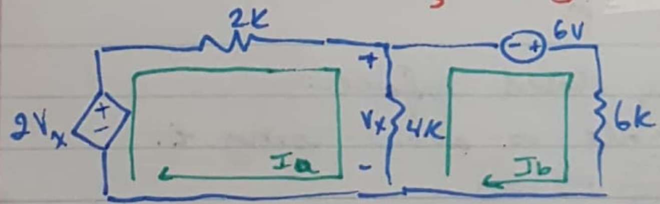
KVL $I_b$ (short form): $6(I_b - I_a) + 3I_b = -3$

} solve for $I_a = \frac{5}{4}$ mA, $I_b = \frac{1}{2}$ mA

Can solve e'thang $\bar{w}$ $I_b, I_a$ (ex: $i_2 = I_a - I_b$)

$\Rightarrow V_3 = -3I_b$

2)




$V_x = 4(I_a - I_b)$

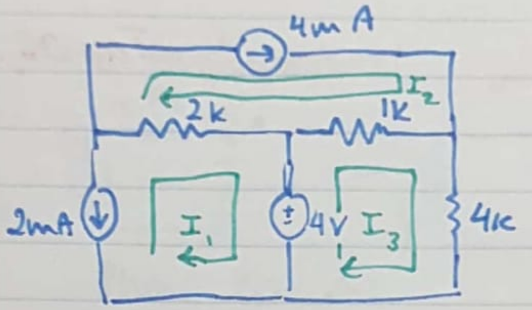
KVL $I_a$: $-2V_x + 2I_a + V_x = 0$

KVL $I_b$: $4(I_b - I_a) - 6 + 6I_b = 0$.

} solve for $I_a = 6$ mA, $I_b = 3$ mA

Jan 24, 2018

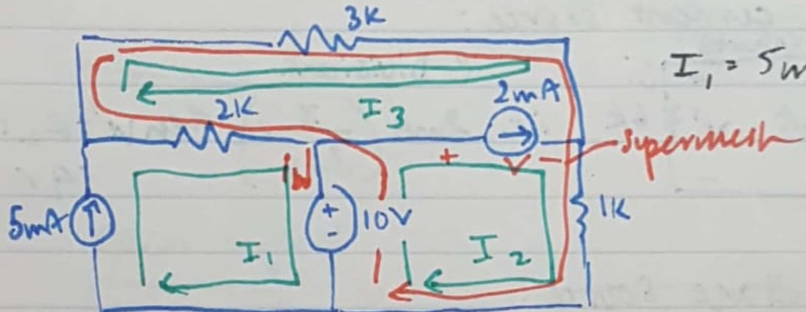
3)




★ $I_1 = -2mA$
$I_2 = 4mA$ } Don't need KVL for these.

KVL 3: $-4 + (I_3 - I_2) + 4I_3$

4)




$I_1 = 5mA$

KVL 2: $-10 + V + I_2 = 0$
KVL 3: $3I_3 - V + 2(I_3 - I_1) = 0$
$I_2 - I_3 = 2$

} $I_2 = 5mA$
$I_3 = 3mA$

Supermesh Shortcut:

KVL: $-10 + 2(I_3 - I_1) + 3I_3 + I_2 = 0$ (KVL 2 + KVL 3)

$i_b = I_1 - I_2 = 0 \Rightarrow$ removing this branch makes no dif.

Principle of Superposition:

Current/Voltage in a linear circuit can be found as the alg. sum of the indv. contributions of each indep. source acting alone.

Deactivating Indep. Sources:

V source = 0. ⇒ Short circuit.

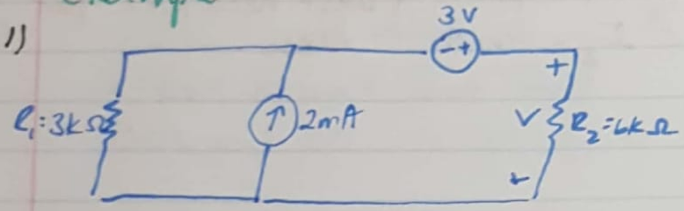
I source = 0. ⇒ Open circuit.

★ Not for dependent sources!!!!!

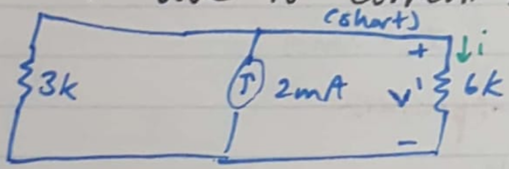
Jan 26, 2018

Example:

1)

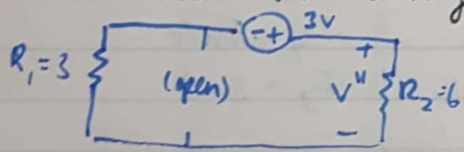



$V'$ due to current source:




C Division: $i = 2mA\left(\frac{3}{3+6}\right)$ then $V' = R_2 i$
$= 4V$

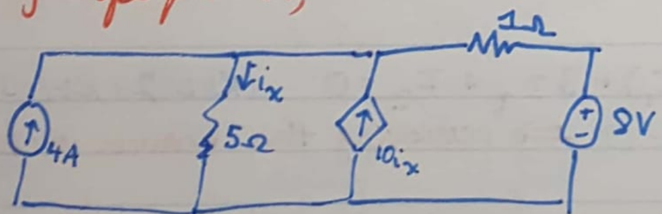
$V''$ due to voltage source:



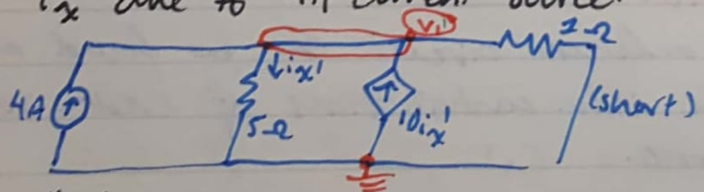

V division: $V'' = \frac{(3)(R_2)}{(R_1+R_2)} = 2V$

By superposition, $V = V' + V'' = 6V$

2)



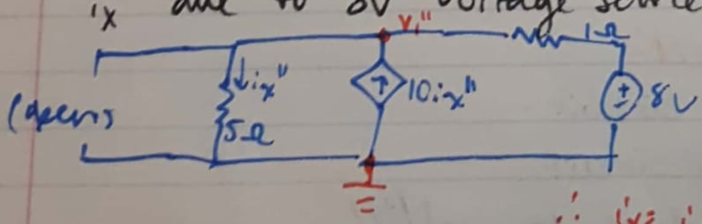

$i_x'$ due to 4A current source:




KCL $V_1'$: $-4 + \frac{V_1'}{5} - 10 \cdot \frac{V_1'}{5} + V_1' = 0$

$V_1' = -5V$, $i_x' = -1A$

$i_x''$ due to 8V voltage source:

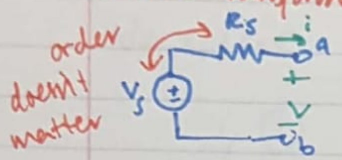



KCL $V_1''$: $\frac{V_1''}{5} - 10 i_x'' + \frac{V_1'' - 8}{1} = 0$

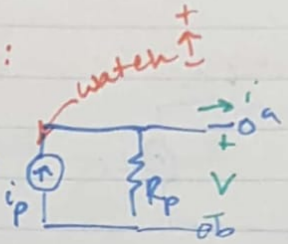
$V_1'' = -10V$, $i_x'' = -2A$

$\therefore i_x = i_x' + i_x'' = -3A$ by superposition.

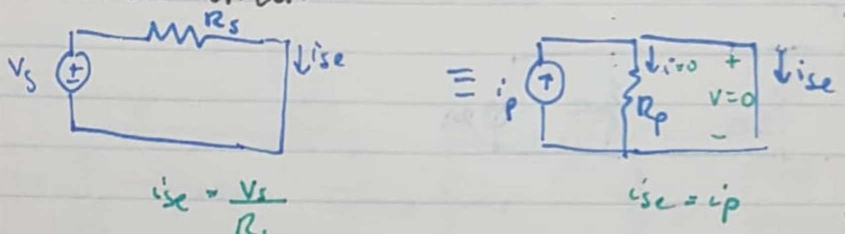
## Source Transformation:




$\equiv$




## Short Circuit:

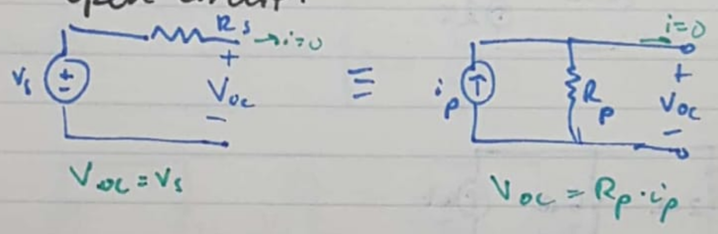



$i_{sc} = \frac{V_s}{R_s}$

$i_{sc} = i_p$

$\therefore\ i_p = \frac{V_s}{R_s}$

## Open circuit:




$V_{oc} = V_s$

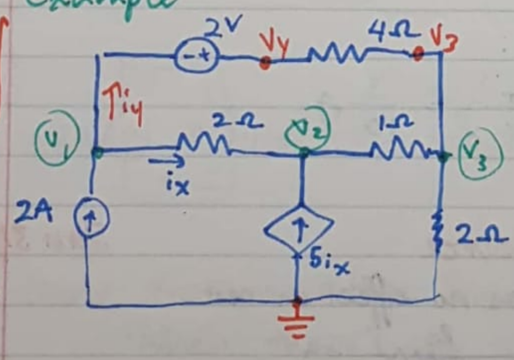
$V_{oc} = R_p \cdot i_p$

$\therefore\ R_p = R_s$

unrelated

just practice

## Example:



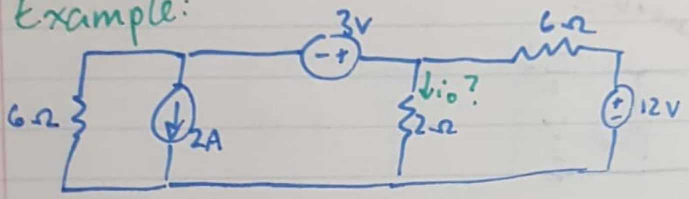

$V_y = 2 + V_1$ $\therefore\ i_x = \frac{V_1 - V_2}{2}$

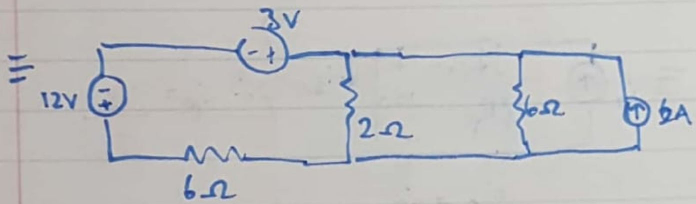
KCL1: $-2 + \frac{V_1 - V_2}{2} + \frac{V_1 + 2 - V_3}{4} = 0$

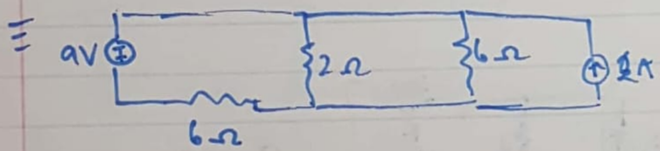
KCL2: $-5i_x + V_2 - V_3 - i_x = 0$

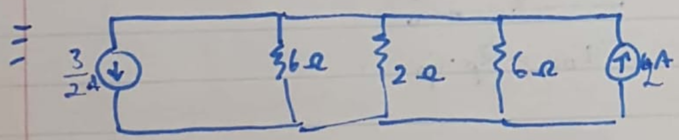
KCL3: $\frac{V_3}{2} + V_3 - V_2 + \frac{V_3 - V_1 - 2}{4} = 0$

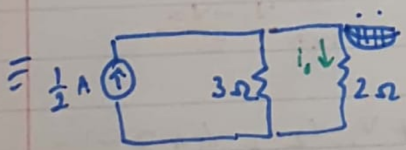
Example:



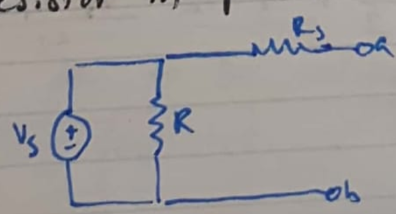

c' Division:

$$i_o = \frac{1}{2}\left(\frac{3}{5}\right) = \frac{3}{10}\,A$$

## Resister in $p^{ll}$ $\bar{w}$ Voltage source

Jan 31, 2018

Resistor in $p^{ll}$ $\bar{w}$ V source has no effect on rest.

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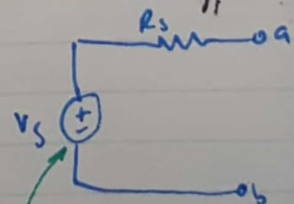


★ has dif current, but rest of circut has no dif.

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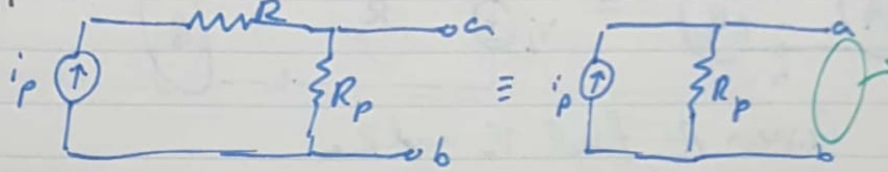
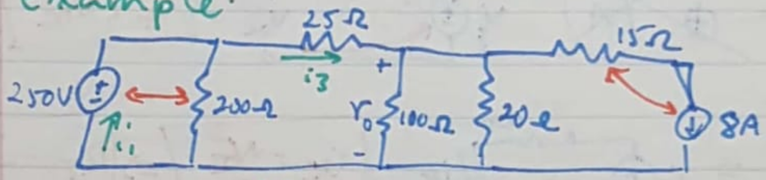
## Resistor in Series with Current Source:

No effect

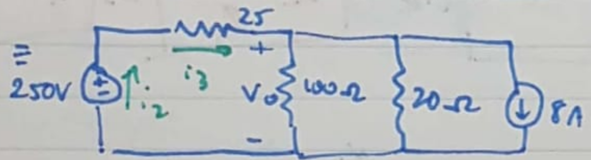



*only rest of circuit feels no dif!

### Example:

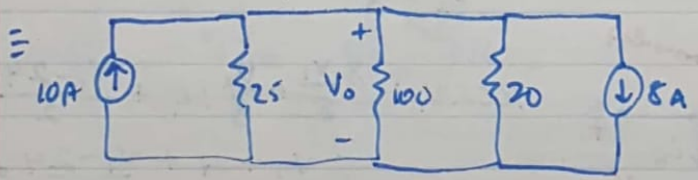



Find $V_o$ w̄ source transformation.



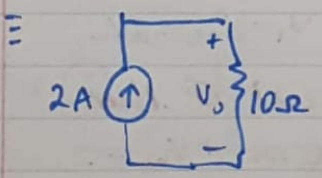

Now, source transformation.

$\boxed{i_1 \neq i_2}$ !!!




$$i_3 = \frac{250-20}{25}$$

$$i_1 = i_3 + \frac{250}{200}$$

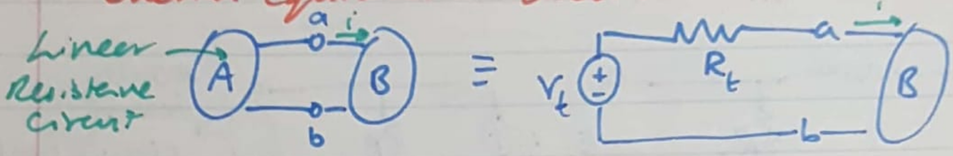



$\therefore V_0 = 20V$

## Source Transformation can't always be used:

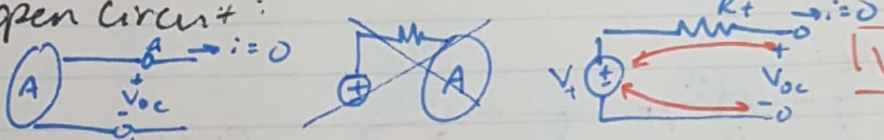
→ When there's no $V_{source}$ in series w̄ R or $I_{source}$ in $p^{ll}$ w̄ R.

# Thevenin Equivalent circuits:



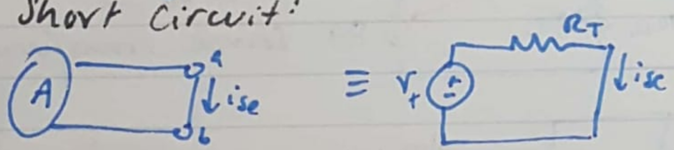

Given A, find $V_t$ and $R_t$

open circuit:




$\boxed{V_t = V_{oc}}$

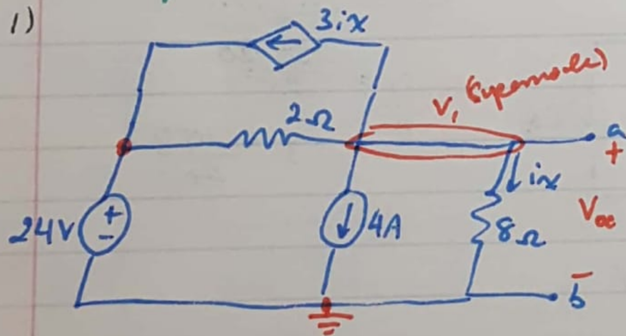
Short circuit:




$\boxed{i_{sc} = \frac{V_t}{R_t}} \Rightarrow \boxed{R_T = \frac{V_{oc}}{i_{sc}}}$

## Example:

1)




$i_x = \frac{V_1}{8}$.

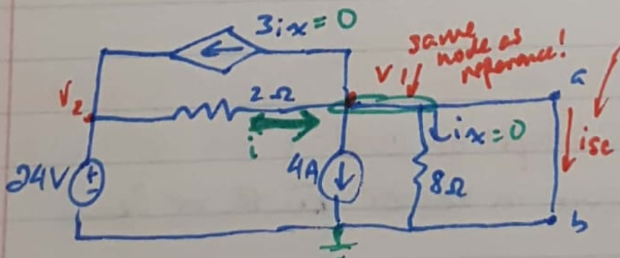
KCL $V_1$: $3\frac{V_1}{8} + \frac{V_1}{8} + 4 + \frac{V_1 - 24}{2} = 0$

$\frac{1}{2}V_1 + 4 + \frac{V_1 - 24}{2} = 0$

$V_1 = V_{oc} = 8V$.

Redraw circuit $\bar{c}$ short.

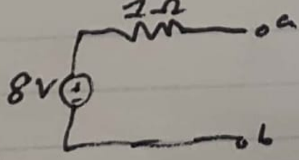
direction based on $+V_{oc}-$.




$i = \frac{V_2 - V_1}{2}$ But $V_1 = 0$ (short). $= 12A$

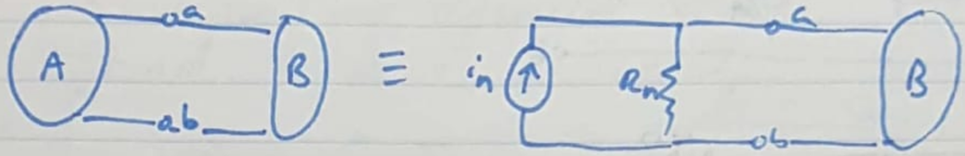
KCL ①: $i_{sc} = i - 4 - 0 - 0$. $i_{sc} = 8A$.

Now, $R_T = 1\Omega$.

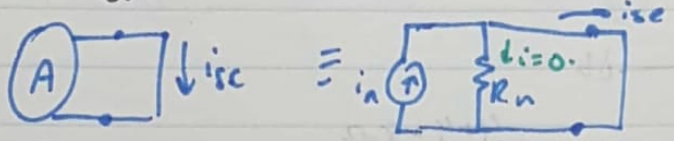


# Norton Equivalent Circuit:

Feb 2, 2018.

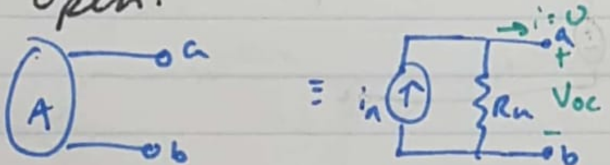



Short:




$$\boxed{i_n = i_{sc}}$$

Open:

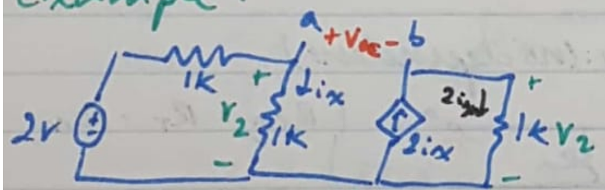



$V_{oc} = R_n \cdot i_{sc}$

$$\boxed{R_n = \frac{V_{oc}}{i_{sc}}}$$

Same as thev.

Example:



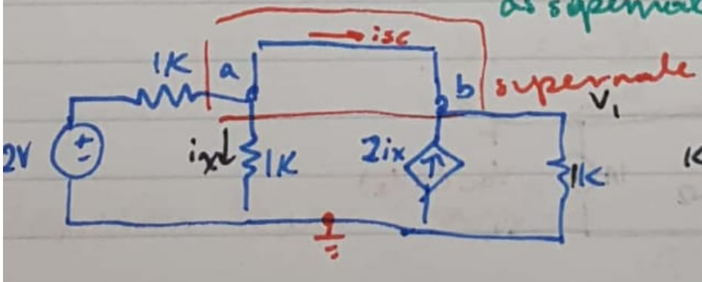

$\vec{i} = 0$ (take right half as supernode)

$V_1 = \frac{(2)(1)}{(2)} = 1V$ (V division)

$i_x = \frac{1V}{1K} = 1mA$.

$V_2 = (1k)(2i_x) = 2V$.

$\therefore V_{oc} = V_1 - V_2 = -1V$

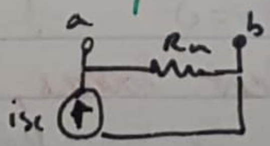



$i_x = \frac{V_1}{1} = V_1$

KCL $V_1$: $\frac{V_1 - 2}{1} + \frac{V_1}{1} - 2\frac{V_1}{1} + \frac{V_1}{1} = 0$

$V_1 = 2V$.

KCL: $i_{sc} = \frac{2 - V_1}{1} - \frac{V_1}{1} = -2mA$. $\Rightarrow R_n = \frac{-1}{-2mA} = 0.5k\Omega$

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Midterm:

→ up to source transformation

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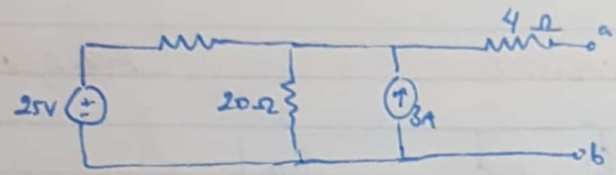
Feb 4, 2018

2)

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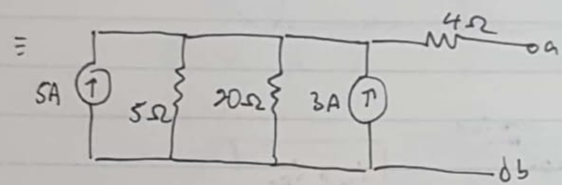


Source Transformation

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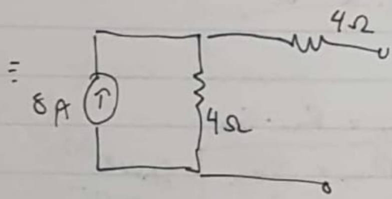


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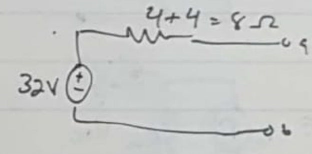
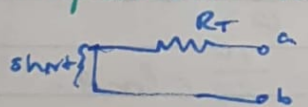
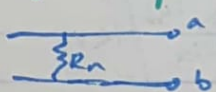



Finding $R_T$ or $R_N$ with $R_{eq}$:

→ Independent sources only. (no dependent).

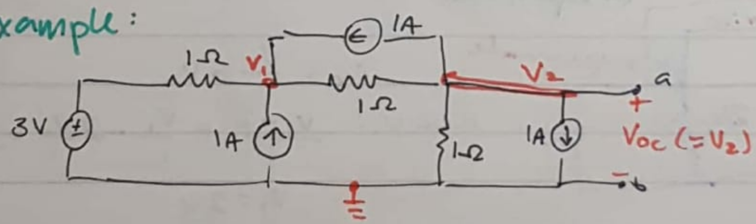



$\Big\}$ $R_{eq} = R_T = R_N$

Example:

1)

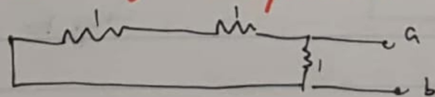



KCL 1: $-1-1+\frac{V_1-3}{1}+\frac{V_1-V_2}{1}=0$

KCL 2: $1+1+\frac{V_2}{1}+\frac{V_2-V_1}{1}=0$

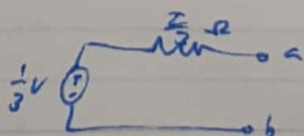
$\Big\}$ $V_{oc} = V_2 = \frac{1}{3}V$.

Since only independent sources, $R_T = R_{eq}$. Deactivate sources.




$R_{eq} = \frac{2}{3}\Omega$

∴ T. equiv:

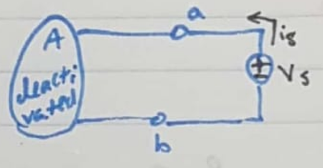


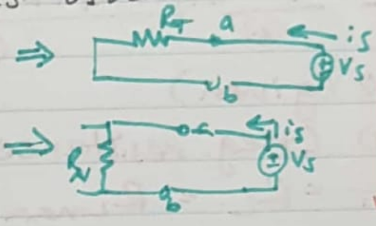
## Finding $R_T$ or $R_N$ with External Sources:

↳ Independent/dependent sources.

↳ Find $V_T$ or $i_N$ as usual.

Can also do ⊕ $i_s$, then find $V_s$



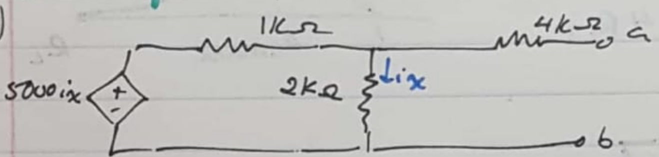




$$\boxed{R_T = \frac{V_s}{i_s}}$$

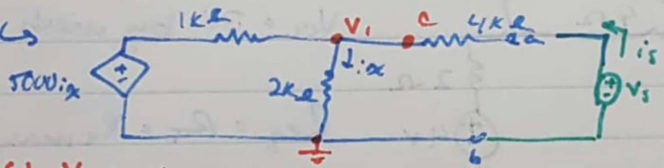
$$\boxed{R_N = \frac{V_s}{i_s}}$$

### Example:

1)




$V_{oc}$, $i_{sc} = 0$ (no ind. sources)



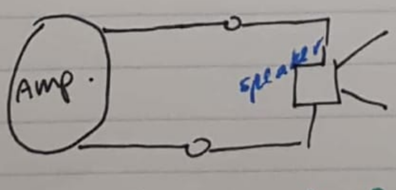

$R_T$' c→B = ~~1~~kΩ ö

KCL $V_1$: $\frac{V_1 - 5000 i_x}{1k} + \frac{V_1}{2k} + \frac{V_1 - V_s}{4} = 0$ { Sub $i_x = \frac{V_1}{2k}$

↳ $V_1 = -\frac{V_s}{3}$, $i_s = \frac{V_s - V_1}{4}$ ⇒ $\boxed{\frac{V_s}{i_s} = 3k\Omega = R_T}$

## Maximum Power Transfer:

Feb 6, 2018

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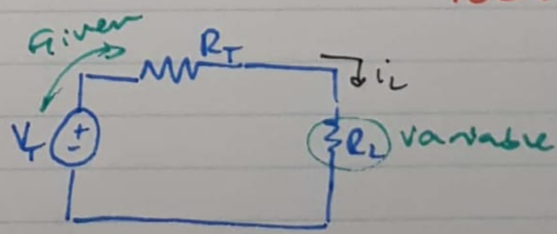


Given A, find $R_L$ for max power

$\boxed{i_L = \frac{V_T}{R_T + R_L}}$   $\boxed{P_L = R_L \left(\frac{V_T}{R_T + R_L}\right)^2}$ ← $R_L$ variable, derive wrt $R_L$.