Lab schedule
Learn

ECE105 - Introduction

Sep 8/2017

Practice:

$V_1 = 23 m/s \quad \vec{a} = -9.8 m/s^2 \quad V_2 = 0$

$d = V_2^2 - V_1^2 / 2a = 27.0 m$

$V_f^2 - V_0^2 = 2\vec{a}\Delta\vec{y}$

$0 - (23)^2 = 2|\vec{a}||\Delta\vec{y}| \cos 180$ -1 this is where the - sign comes from (a is ~~not~~ -9.8)

$\Delta y = \dfrac{-23^2}{-2(9.8)}$

Definitions:

$\vec{v}_{avg} = \dfrac{\Delta\vec{x}}{\Delta t}$ } $\lim_{\Delta t \to 0} \vec{v}_{avg} = \lim_{\Delta t \to 0} \dfrac{\Delta\vec{x}}{\Delta t} =$ inst. $\vec{v}$ } $\dfrac{d\vec{x}}{dt}$

$\vec{a}_{avg} = \dfrac{\Delta\vec{v}}{\Delta t}$ } $\lim_{\Delta t \to 0} \vec{a}_{avg} = \lim_{\Delta t \to 0} \dfrac{\Delta\vec{v}}{\Delta t} =$ inst. $\vec{a}$ } $\dfrac{d\vec{v}}{dt}$

$\Delta\vec{x} = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t$

$\hookrightarrow \vec{x}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{x}_0$ (just made it Δd)

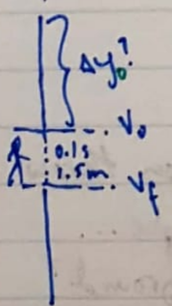
} assuming constant $\vec{a}$

$x(t) = t^3$
$v(t) = 3t^2$
$a = 6t$

} $x(t) = \int v\,dt + C$
$= t^3 + C$ ← or x_0
$v(t) = \int a\,dt + v_0$ ← need const for initial values

Example:

You are sitting in your room. Stone dropped from top of building, you see it from window for 0.1s. From what height above ⊞ was dropped if ⊞ 1.5m



$v_1 = 0 \quad a = +9.8$

$\Delta\vec{y} = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t$

$1.5 = \frac{1}{2}(+9.8)(0.1)^2 + \vec{v}_0(0.1)$

$\vec{v}_0 = 14.51 m/s$

$v_f^2 - v_0^2 = 2(9.8)\Delta y$

$14.51^2 - 0 = 19.6\Delta y$

$\Delta y =$

Scanned by CamScanner

Lecture 2 - Relative Motion

Sep 11/2017

Example:

1) You throw 2 balls simultaneously. one ↑ 30 m/s one ↓ 15 m/s. Distance between them 3 seconds later?

Ball 1 Δd: $\frac{1}{2}\vec{a}t^2 + V_0 t$
$= -\frac{1}{2}(9.8)(3)^2 + (30)(3)$
$= 45.9\,m$

Ball 2 Δd: $\frac{1}{2}\vec{a}t^2 + V_0 t$
$= -\frac{1}{2}\vec{a}t^2 + V_0 t$
$= -\frac{1}{2}(9.8)(3)^2 + (-15)(3) = -89.1$

$|\Delta d_2 - \Delta d_1| = 135\,m$

Best way $\Rightarrow$ $\vec{V}_{12} = 45$ $\vec{V}_{12} \times t = \Delta d = 135$

2) You let go of a ball and 0.5 seconds later you throw another straight down at 9.8 m/s. $\vec{V}_{12}$ after 3 seconds?

ball 1 is going 4.9 m/s

4.9 m/s, no matter what t is.

Relative Motion:

$\vec{V}_{12} = \vec{V}_1 - \vec{V}_2$

$\Delta\vec{x}_{12} = \Delta\vec{x}_1 - \Delta\vec{x}_2$

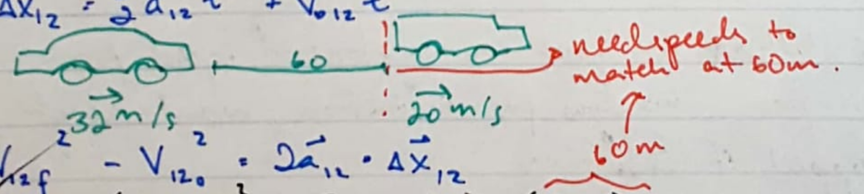
$\vec{a}_{12} = \vec{a}_1 - \vec{a}_2$

Example:

1) You are driving at 32 m/s, 60 m behind a truck moving at 20 m/s. Truck brakes at 4 m/s². What must be your $\vec{a}$ for no collision?

- Can't assume truck stops before you hit it.

$\Delta\vec{x}_{12} = \frac{1}{2}\vec{a}_{12}t^2 + V_{0\,12}t$



match $\cancel{V_{12f}^2} - V_{12_0}^2 = 2\vec{a}_{12} \cdot \Delta\vec{x}_{12}$

$-(v_{1_0} - v_{2_0})^2 = 2(\vec{a}_1 - \vec{a}_2)\cdot(\vec{x}_2 - \vec{x}_1)\cdot(-1)$

$-(12)^2 = -2(\vec{a}_1 - 4)(60)$

Find $\vec{a}_1$

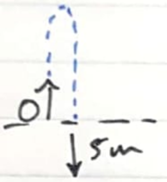
2) You are in an elevator moving up at 6 m/s (open roof), you throw an apple straight up at 25 m/s (rel. to you). Max height of apple...
a) rel to you?   b) rel to original height above ground?

Projectile Motion:

Sep 13, 2017

Example:

1) $v = 20\,m/s$ ↓5m $v_f = ?$



$\Delta d_v = -5m$

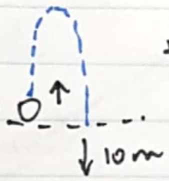
$\Delta d_v = \frac{v_f^2 - v_o^2}{2a}$

$-5 = \frac{v_f^2 - 20^2}{-2(9.8)}$

$v_f = 22.3\,m/s$ (down)

$\left\{ v_f^2 - v_o^2 = 2|\vec{a}||\Delta\vec{d}|(\underbrace{\cos\theta}_{+1}) \right.$ with $|\vec{a}||\Delta\vec{d}|$ +ve

2) $v = 30\,m/s$ ↓10m $t = ?$

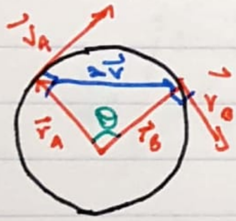


$\Delta d_v = \frac{1}{2}\vec{a}t^2 + \vec{v}_o t$

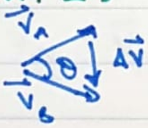
$-10 = \frac{1}{2}(-9.8)t^2 + (30)t$

$t =$

Circular Motion:



Sim. Δs



$\Delta\vec{v} \perp \Delta\vec{r}$ + // $\Delta\vec{a}$

$a_r = \frac{v^2}{r}$

$s = \theta r$

$v = \omega r$

$a = \alpha r$

Example:

1) An object moves around a vertical circle starting at $\theta = 0$, $t = 0$, its speed is $10\,m/s$ 2 seconds later. What is the total $\vec{a}$ at $t = 2$?

Not constant $\vec{a}$. $\nwarrow \vec{a}_t$ $\swarrow \vec{a}_r$ *Forgot to give r = 7m

Tangential $\vec{a}_t$: Use kin. equations to find $\vec{a}_t = 5\,m/s^2$

$\left\{ \vec{a}_c + \vec{a}_t = \vec{a} \right.$

↑ perpendicular.

$\vec{a}_c = \frac{v^2}{r} = \frac{100}{7}$

$|\vec{a}| = \sqrt{a_t^2 + a_c^2}$

$= \sqrt{25 + (\frac{100}{7})^2}$

Albert Wasef

ECE 105 Lab Introduction

Sep 14, 2017

Review guidlines on learn.

Read lab manual before lab

Submit on learn dropbox

- ↳ prelab1, data1, report1, lab1
- ↳ Same file renamed.

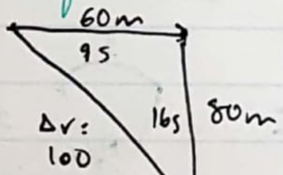
Lab 1 on 28th

Error Analysis - read on learn.

Example:

Sep 15, 2017

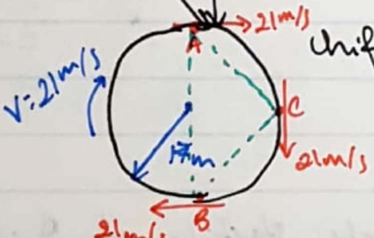
1)



$\vec{V}_{avg} = \frac{100}{25} = 4\,m/s$

$V_{avg}\text{ (speed)} = \frac{140}{25} = 5.6\,m/s$

2)



Uniform.

Find: $\vec{V}_{avg_{AB}} = ?$ $\vec{a}_{avg_{AB}} = ?$

$\vec{V}_{avg_{AC}} = ?$ $\vec{a}_{avg_{AC}} = ?$

$V_{avg}\text{(speed)} = ?$

$\vec{A} \to \vec{B}$:

$\vec{V}_{avg} = \frac{2(17)}{(\pi r/21)}$ ← $\Delta d/\Delta t$, so $\Delta t = (\Delta d)(\vec{v})$ $= \frac{34}{(\pi r)(1/21)}(-\hat{j})$

$\vec{A} \to \vec{C}$:

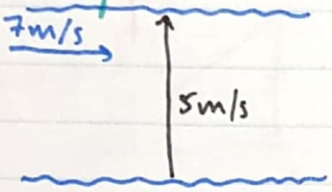
$\vec{V}_{avg} = \left[\frac{(\sqrt{2})(17)}{(\pi r/(2)(21))}\right]\cos 45^\circ\,\hat{i} + \left[\frac{(\sqrt{2})(17)}{(\pi r/(2)(21))}\right]\sin 45^\circ\,\hat{j}$

$\vec{a}_{avg}\ \vec{A} \to \vec{B} = \frac{42(-\hat{i})}{(\pi r/(21))}$ $\xrightarrow{v_A}$ $\xleftarrow{v_B}$ $\therefore \vec{a} \leftarrow$

$\vec{a}_{avg}\ \vec{A} \to \vec{C}$: v_A, v_C, Δv, $\vec{a}_{avg}$ $= \frac{\Delta \vec{V}}{\Delta t} = \frac{\vec{V}_c - \vec{V}_A}{\Delta t} = \frac{}{(\pi r/(2)(21))}$

Example..?

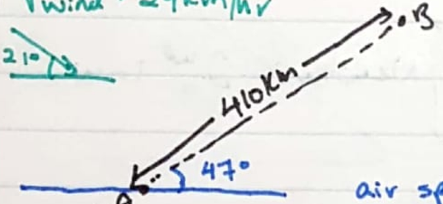
1)



Must aim straight across to cross in fastest time.

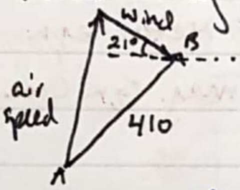
Bearing: where you aim
Heading: where you end up.

2) $V_{wind} = 29$ Km/hr

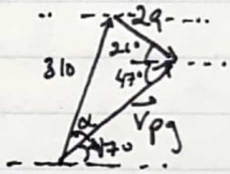


air speed = 310km/hr

a) How long does trip take?



$\vec{V}_{pg} = \vec{V}_{pa} + \vec{V}_{ag}$



$\frac{\sin 68}{310} = \frac{\sin \alpha}{29}$

$\alpha = 4.98°$

$\frac{\sin 107°}{\vec{V}_{pg}} = \frac{\sin 68}{310}$

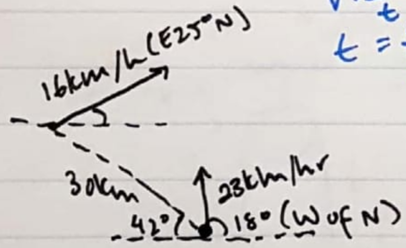
$\vec{V}_{pg} = 319.7$ m/s

$V = \frac{d}{t}$

$t = \frac{d}{V} = \frac{410}{319.7} = 1.28$ hr

bearing: 51.98°

3)



what is closest distance of approach?

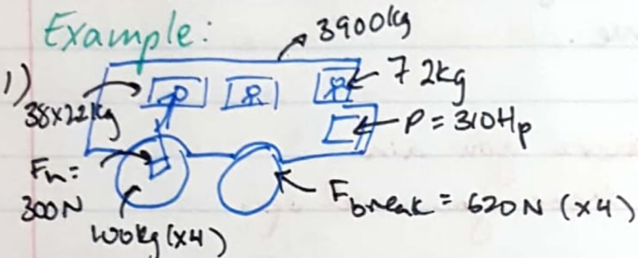
Find relative V rel to time. Derive for rel. Δd.

Sep 18, 2017

Forces:

Example:

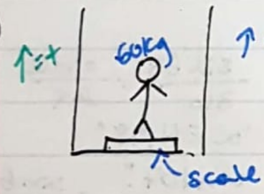
1)



F_{net} = wheel?

$F_{net} = ma$
$= (100)(3.2)$
$= 320\,N$ } Newton's 2^{nd} law

2)



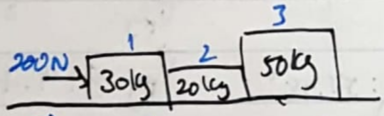
$\sum \vec{F}_y = m\vec{a}$

$\vec{N} + m\vec{g} = m\vec{a}$ ← vector eqn

$N - mg = ma$ ← mag. eqn (cos θ makes −)

$N = 1008\,N$

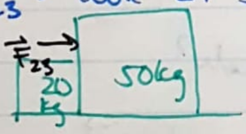
3)



$\sum F = ma = 200$

$a = 2\,m/s^2$

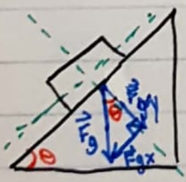
$\vec{F}_{23}$ = look at 3 in isolation



$\sum F = ma$
$= (2)(50)$
$= 100\,N$

Sep 20, 2017

*Gr12 physics? axis in direction of $\vec{a}$

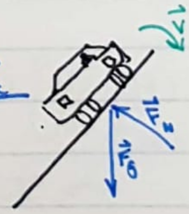


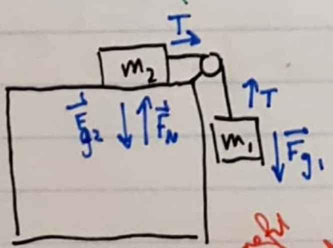
$\vec{F}_{gy} = -\vec{F}_N = \vec{F}_g \cos\theta$

Not a law!

$\vec{F}_{gx} = \vec{F}_g \sin\theta$

$\vec{F}_c$





*be careful w/ a direction

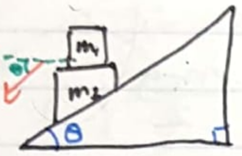
$\vec{F}_N = \vec{F}_{g2}$

$\sum \vec{F}_x = ma$

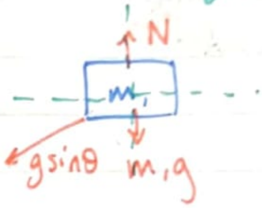
$T = m_2 a$ or $m_1 a = F_{g1} - T$ (depends on $\vec{a}$)

$a = \dfrac{m_1 g}{m_1 + m_2}$ $\quad T =$

Find the $\vec{N}$ between m_1 and m_2.



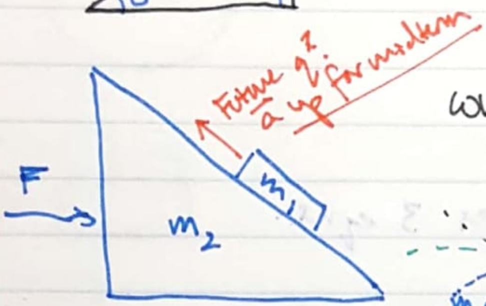
break up $\vec{a}$ into components



$\vec{N} + m_1\vec{g} = m\vec{a}_y$

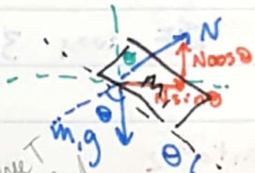
$N = m_1 g - m_1 g \sin^2\theta$

$= m_1 g \cos^2\theta$



What does F = ? so m_1 does not slide on m_2

$F = (m_1 + m_2)a$



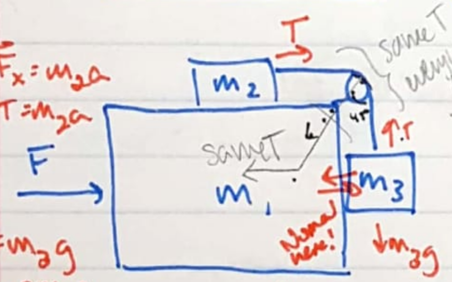
$N = \cancel{mg\cos\theta}$ more $\vec{F}_{applied}$

$\Sigma F_y = m\vec{a}_y$ $\quad$ $\Sigma F_x = m\vec{a}_x = 0$

$\boxed{N\sin\theta = m_1 a}$ $\quad$ $\boxed{N\cos\theta = m_1 g}$ $\quad$ $\boxed{a = g\tan\theta}$

$\Sigma \vec{F}_x = m_2 a$

$T = m_2 a$



1) What force do you have to push m_1 w/ so m_3 does not slide on m_1?

2) You let system go. Find $\vec{a}$ of m_1 ←.

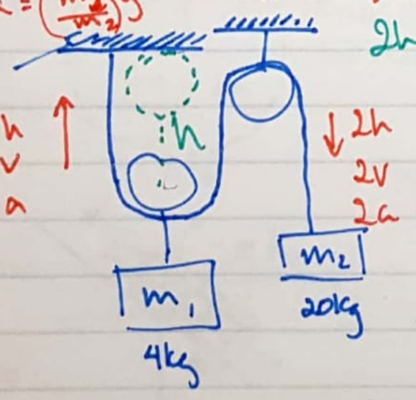
m_1 feels tension ← since pulley is part of mass.

$T = m_3 g$

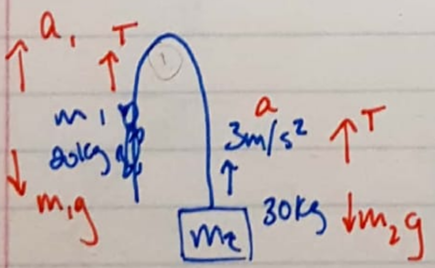
$m_3 g = m_2 a$

$a = \left(\frac{m_3}{m_2}\right)g$ $\quad$ $\therefore F = (m_1 + m_2 + m_3)\left(\frac{m_3}{m_2}\right)g$

$2h$ slack avail, so $m_2 \downarrow 2h$.



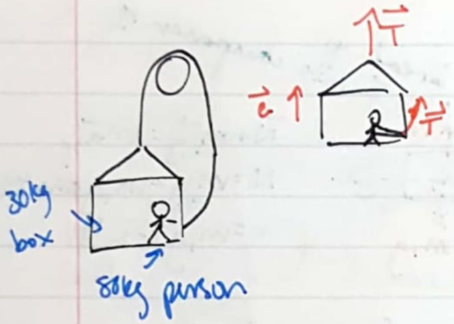
☆ Review GR notes ☆



$\Sigma \vec{F}_{y,m_2} = m_2\vec{a}_y = \vec{T} + m_2\vec{g} = m_2\vec{a}_2$

$\Sigma F_{y,m_1} = m_1\vec{a}_y = \vec{T} + m_1\vec{g} = m_1 a_2$

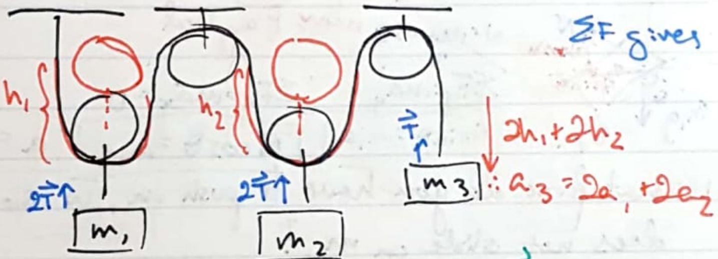
$a_1 = \frac{m_2 g - m_1 g + m_2 a}{m_1}$



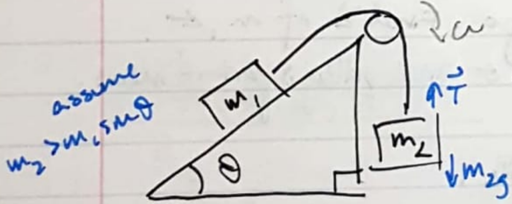
$$\Sigma \vec{F}_y = m\vec{a}_y$$

$$2\vec{T} + m_T\vec{g} = m_T\vec{a}$$

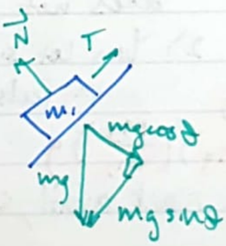
$$T = \frac{m_T g + m_T a}{2} = \frac{m_T g}{2}$$

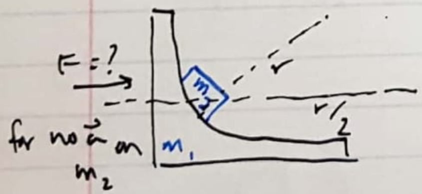


ΣF gives 3 eqns.



$a = ?$

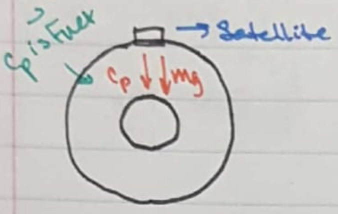


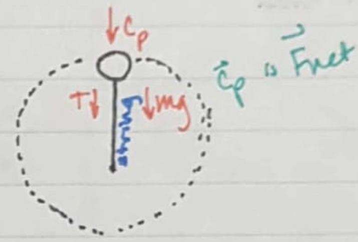


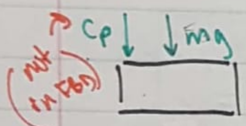
$(m_1+m_2)g\tan\theta$

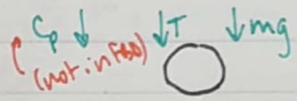
Satellite Motion:

Sep 25, 2017

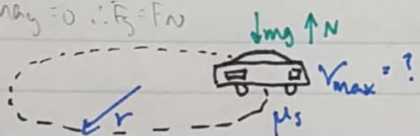








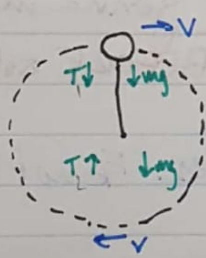
$\sum \vec F_y = m\vec a_y = 0 \therefore F_g = F_N$



$\sum \vec F_r = \frac{mv^2}{r} = \mu_s \vec F_N = \mu_s F_g = \mu_s m g$

$\frac{mv^2}{r} = \mu_s m g$

$v = \sqrt{\mu_s r g}$



vertical ○.
Find ΔT from top and bottom.

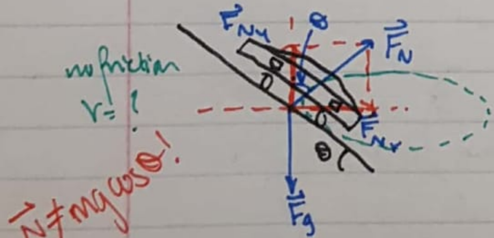
Top: $\sum \vec F_r = \frac{mv^2}{r} = T_1 + mg$ $\quad T_1 = \frac{mv^2}{r} - mg$

Bottom: $\sum \vec F_r = \frac{mv^2}{r} = T_2 - mg$ $\quad T_2 = \frac{mv^2}{r} + mg$

$\Delta T = T_1 - T_2$

$= \frac{mv^2}{r} - mg - \left(\frac{mv^2}{r} + mg\right)$

$= -2mg$



$\sum \vec F_y = \vec F_{N_y} + \vec F_g$

$\therefore F_{N_y} = F_g = mg$

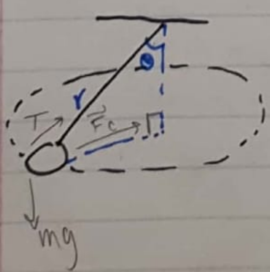
$F_N \cos\theta = mg$

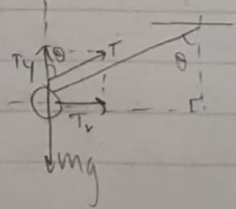
$\sum \vec F_r = m\vec a_r$

$\vec F_{N_r} = m\frac{v^2}{r}$

$F_N \sin\theta = \frac{mv^2}{r}$

$\tan\theta = \frac{v^2}{rg}$





$\sum \vec F_y = 0$

$\vec T_y + m\vec g = 0$, so $T\cos\theta = mg$ (2)

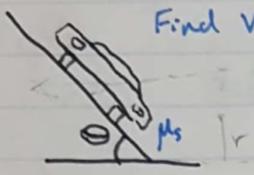
$\sum \vec F_r = m\vec a_r$

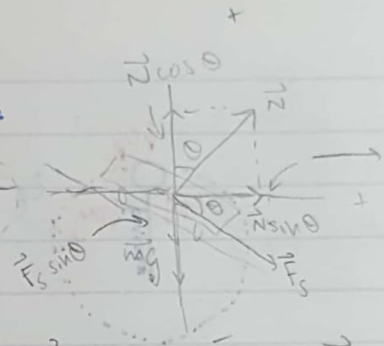
$\vec T_r = m\frac{v^2}{r}$

(1) $T\sin\theta = \frac{mv^2}{r}$

$\tan\theta = \frac{v^2}{rg}$

Find V_{max}





$\vec{F}_s = \vec{F}_{s\,max} = \mu_s F_N$

$\sum \vec{F}_x = m\vec{a}_x = m\frac{v^2}{r}$

$\vec{F_N}\sin\theta + \vec{F}_s\cos\theta = \frac{mv^2}{r}$

$\vec{F_N}\sin\theta + \mu_s \vec{F}_N\cos\theta = \frac{mv^2}{r}$

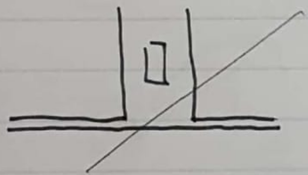
$\div$ $\underline{F_N\cos\theta - \mu_s F_N\sin\theta = mg}$

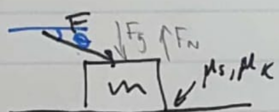
$\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} = \frac{v^2}{rg}$

$\sum \vec{F}_y = m\vec{a}_y = 0$

$\vec{F}_N\cos\theta + \vec{F}_s\sin\theta + \vec{F}_g = 0$

$F_N\cos\theta - \mu_s F_N\sin\theta = mg$





*Don't assume this object is gonna move! Need to establish this BEFORE solving.

$f_{s\,max} = F_N\mu_s = (mg + F\sin\theta)\mu_s$

$F\cos\theta \geq f_{s\,max}$ ∴ object moves. F_k not F_s

$\sum \vec{F}_x = m\vec{a}_x = \vec{F}\cos\theta + \vec{F}_K$

$= \vec{F}\cos\theta + \vec{F}_N\mu_K$

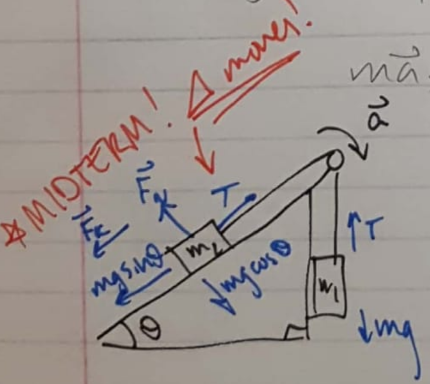
$m\vec{a}_x = F\cos\theta - F_N\mu_K$

$\sum \vec{F}_y = m\vec{a}_y = 0 = \vec{F}\sin\theta + \vec{F}_N + \vec{F}_g$

$\vec{F_N} = \vec{F}\sin\theta + mg$

$m\vec{a}_x = F\cos\theta - (\vec{F}\sin\theta + mg)\mu_K$

$= F\cos\theta - F\sin\theta\,\mu_K - mg\mu_K$

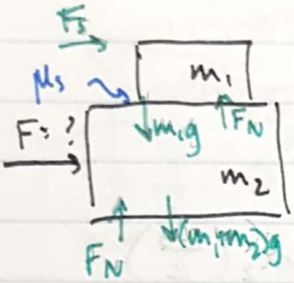


$T - m_2 g\sin\theta - f_K = m_2 a$

$f_K = F_N\mu_K = mg\cos\theta\,\mu_K$

$T - m_1 g = m_1 a$

Sep 29, 2017



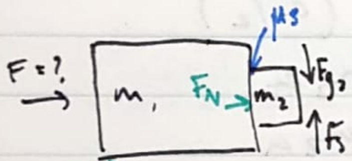
$F_{max} = ?$ So m_1 doesn't slide on m_2

$F_s = ma$

$F_N \mu_s = ma$

$mg\mu_s = ma$

$a = g\mu_s \quad \therefore \quad F = (m_1+m_2)g\mu_s$



$F_{min} = ?$ so m_2 doesn't move ↑↓ ?

$\Sigma \vec{F}_x = m_2 a$

$F_N = m_2 a$

$F_s = F_{g_2}$

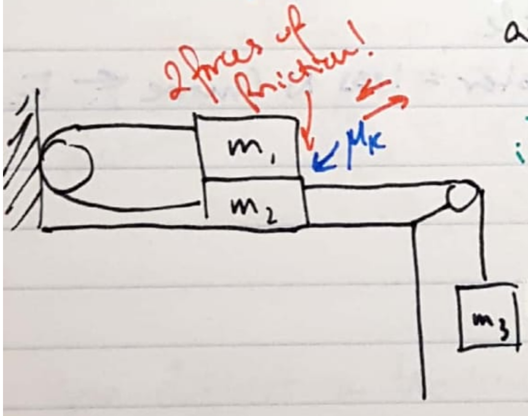
$F_N \mu_s = F_{g_2} = m_2 g$

$m_2 a \mu_s = m_2 g$

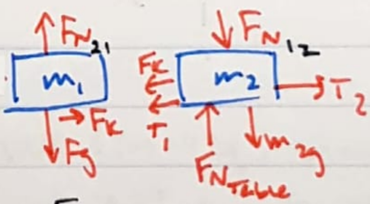
$a = g/\mu_s$

$F = (m_1+m_2)a$

$= \dfrac{g(m_1+m_2)}{\mu_s}$



$i)\ m_3 g - 2m_1 g\mu_K = (m_1+m_2+m_3)a$



$F_{N_{12}} = F_{N_{21}} = m_1 g$

$m_1)\ \Sigma \vec{F}_y = 0 \Rightarrow \vec{F}_{N_{21}} = m_1 g$

$\Sigma \vec{F}_x = m_1 \vec{a}_x$

$\vec{T}_1 + \vec{f}_K = m_1 \vec{a}$

$T - m_1 g \mu_K = m_1 a$ (1)

$m_2)\ \Sigma \vec{F}_x = m_2 \vec{a}_x$

$\vec{T}_2 + \vec{T}_1 + \vec{F}_K = m_2 \vec{a}$

$T_2 - T_1 - F_K = m_2 a$

$T_2 - T_1 - m_1 g \mu_K = m_2 a$

$m_3)\ \Sigma \vec{F}_y = m_3 \vec{a}_y$

$\vec{T}_2 + m_3 \vec{g} = m_3 \vec{a}_y$

$m_3 g - T_2 = m_3 a$ (3)

Now, add all 3 eqns to get i).

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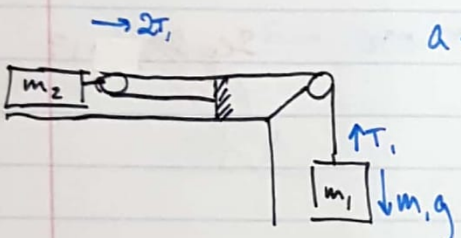


$a = ?$

$\sum \vec{F}_{m_1 y} = m_1 \vec{a}_y$

$m_1 g - T = m_1 \vec{a}_y$ (1)

$\sum \vec{F}_{m_2 x} = m_2 \vec{a}_x = m_2 \left(\frac{\vec{a}_y}{2}\right)$

$2T = m_2 \left(\frac{\vec{a}_y}{2}\right)$ (2)

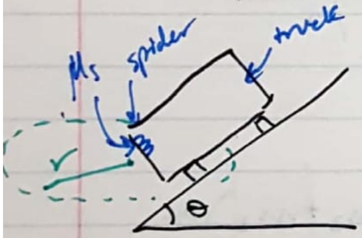
$\frac{4m_1 g}{4m_1 + m_2} = a$

$2(m_1 g - m_1 \vec{a}) = m_2 (\vec{a}/2)$

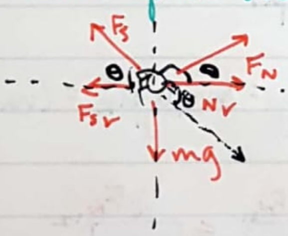
$4(m_1)(g - a) = m_2 a$

$4m_1 g - 4m_1 a = m_2 a$

$4m_1 g = m_2 a + 4m_1 a$

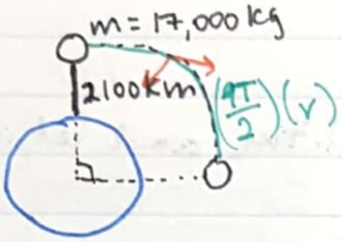


Vmax so spider doesn't slide



Faster = less N (more ← F_c)

Oct 2, 2017.



Find work done by gravity.

$W_T = \Delta K_e = 0$!

or... $W_T = \vec{F} \cdot \Delta\vec{d} = |\vec{F}||\vec{d}|\cos\theta \leftarrow \cos 90° = 0.$

These are ⊥

Total work on object = Change in object's kinetic energy

You hurl a cooper mini ($m = 690$ kg) at 6 m/s @ Mansour. He stops over a distance of 3m then hurls it back @ same $\vec{v}$. W?

$W_T = \Delta K_e = 0$, again.

F, d stay the same: $\begin{cases} W_{F_1} = \vec{F}\cdot\vec{d} = |\vec{F}||\vec{d}|\cos 180° \leftarrow -1 & \text{when you stop} \\ W_{F_2} = \vec{F}\cdot\vec{d} = |\vec{F}||\vec{d}|\cos 0° \leftarrow +1 & \text{when you release} \end{cases}$

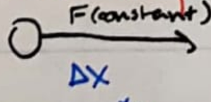
$\Delta W_T = W_{F_1} + W_{F_2}$ cancel out.

You deflect a hockey puck ($m = 42$g) moving at 32 m/s by 30 degrees. You hit it at 20° to its original direction so the final speed is same as initial. W?

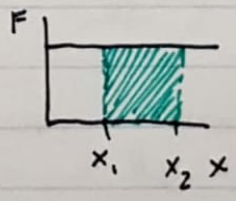
∴ No change in kinetic energy

$W_T = \Delta E_k = 0$, again.

Variable Forces in 1D:

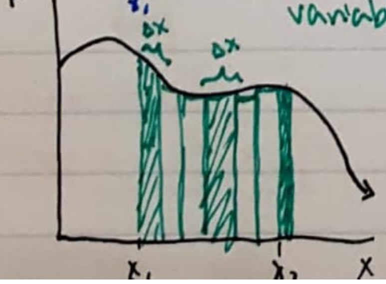


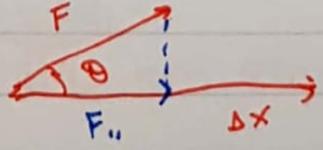
$W = |\vec{F}||\vec{d}|\cos 0 = |\vec{F}||\vec{d}|$



W = area under curve.

$\int_{x_1}^{x_2} \vec{F}\cdot d\vec{x}$





$W = |\vec{F}||\Delta\vec{x}|\cos\theta$
$= \vec{F}_{\parallel}\Delta x$

Example:

1) You lift a heavy ball from ground ↑ 1.7m and let it go at 7m/s. (12kg)

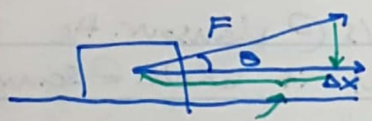
$W_T = \Delta K$

cosθ changes sign

$W_{you} + W_g = \Delta K$ ← actually, final - initial

$W_{you} = \frac{1}{2}mv^2 + m\vec{g}\cdot\vec{h}$

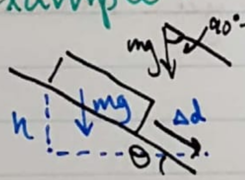
Work in 2D:



$F_{\parallel}$, or $F\cos\theta$.

Example:

1)



$W_g = ?$

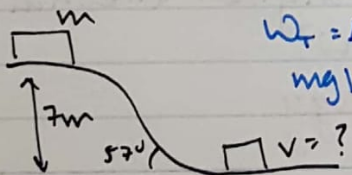
$W_g = m\vec{g}\cdot\Delta\vec{s}$

$= mg\Delta s(\sin\theta)$

$= mgh$

moral of the story: work done by gravity? all that matters is vertical displacement!

2)

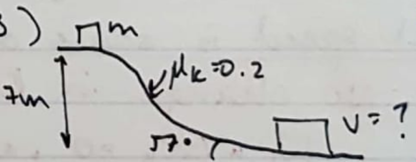


$W_T = \Delta K_e$

$mgh = \frac{1}{2}mv^2$

$v = \sqrt{2gh}$

3)



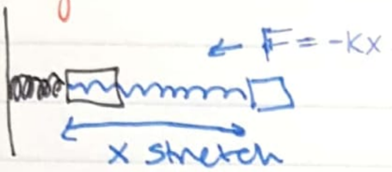
$W_T = \Delta K_e$

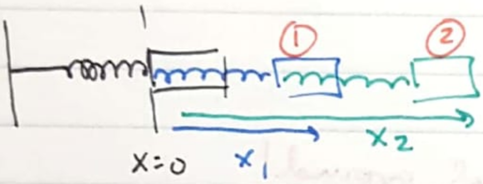
$W_g + W_f = \Delta K_e$

$mgh + (-mg\cos\theta\,\mu_k)\left(\frac{h}{\sin 57°}\right) = \frac{1}{2}mv^2$

Springs

Oct 4, 2017

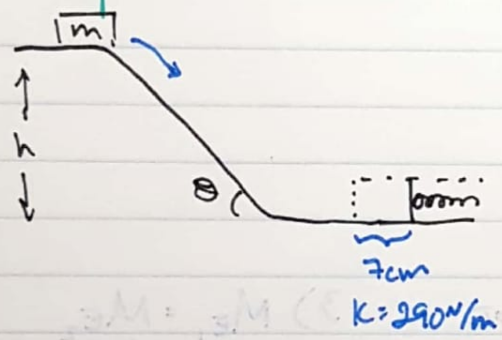




$$W_s = \int_{x_1}^{x_2} -kx\,dx \;\boxed{= \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2}$$

Example:

1)



Block slides and hits already compressed spring. Max comp. of string?

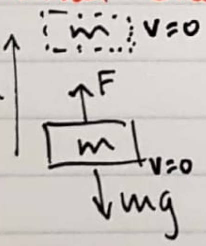
$W_T = \cancel{\Delta K_e}^{\,0}$ $\quad W_T = W_g + W_s = 0$

$mgh + \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2 = 0$

$(12)(9.8)(0.9) = \frac{1}{2}(290)(0.07)^2 = \frac{1}{2}(290)x_2^2$

Potential Energy:

A)



$W_F = \vec{F}\cdot\vec{h} = mgh$

$W_g = m\vec{g}\cdot\vec{h} = -mgh.$

$\Delta P_E = -W_g$. ← this applies for any cons. force.

↳ $= \Delta U_g$.

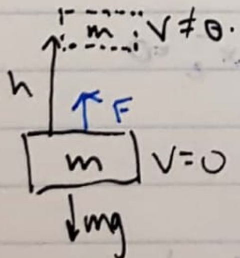
→ Conservative Forces:

↳ Can get ΔE back.

1. ↳ $\boxed{F_F}$ is not conservative (can't get heat back).
2. ↳ $\boxed{F_{app}}$ by humans

}☆

B)



$W_T = \Delta K_e$

$W_F + W_g = \Delta K_e \rightarrow W_{noncons.} + W_{cons.} = \Delta K_e$

$W_F - \Delta U_g = \Delta K_e \qquad W_{nc} - U_c = \Delta K_e$

$W_F = \underbrace{\Delta K_e + \Delta U_g}_{= \Delta M_E}$. mech. energy

$$\boxed{W_{non.c} = \Delta M_E}$$

$$\boxed{\begin{aligned} W_T &= \Delta K \\ W_C &= -\Delta U_c \\ W_{NC} &= \Delta M_E \end{aligned}}$$

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$W_{nc} = \Delta M_E$ VERY IMPORTANT

If $W_{nc} = 0$

↳ $\Delta M_E = 0$, so

1) $\Delta K + \Delta U = 0$

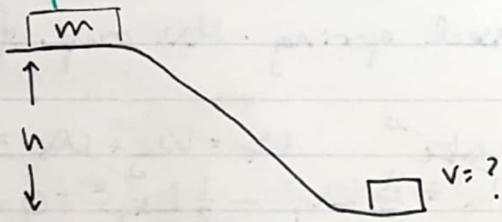
2) $M_{E_1} = M_{E_2} = M_{E_3} \ldots$

So...

$$\boxed{\begin{aligned}\Delta U_s &= -W_s \\ &= \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2\end{aligned}}$$

switched around!

Example:

1) 

1) $W_T = \Delta K$

$W_g = \Delta K$

$mgh = \frac{1}{2}mv^2$

2) $W_{nc} = \Delta M_e = 0$

$\Delta M_E = 0$

$\Delta K + \Delta U = 0$

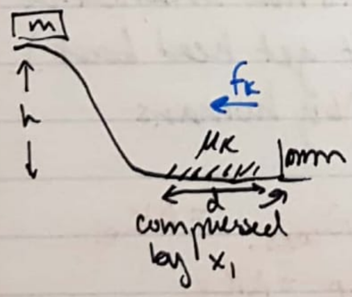
$\Delta K = -\Delta U$

$\Delta K = -(-mgh)$

$\frac{1}{2}mv^2 = mgh$

3) $M_{E_1} = M_{E_2}$

$mgh = \frac{1}{2}mv^2$

2) 

1) $W_T = \Delta K_e$

$W_g + W_F + W_s = \Delta K_e$

$mgh - mg\mu_k d + \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2 = 0$

2) $W_{nc} = \Delta M_e$

$-mg\mu_k d = \Delta K_e + \Delta U$

$-mg\mu_k d = -mgh + \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$

Momentum:

Oct 6, 2017.

$\vec{p} = m\vec{v}$

$$\frac{dp}{dt} = \frac{d(m\vec{v})}{dt} = \vec{v}\frac{dm}{dt} + m\frac{d\vec{v}}{dt} = \vec{F}_{net}$$

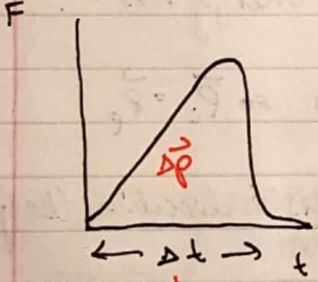
If m is constant, this = 0, so $F_{net} = ma$.

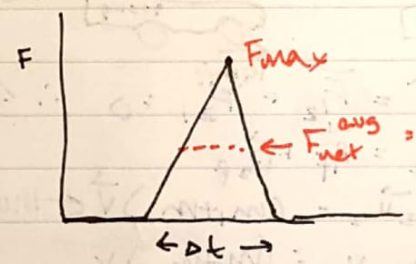
$dp = \vec{F}dt$

$\Delta\vec{p} = \int_{t_1}^{t_2} \vec{F}dt = \vec{F}_{net}^{avg}\Delta t$.

$\vec{F}_{net}^{avg} = \frac{\Delta\vec{p}}{\Delta t}$ $\Big\}$ $\Delta\vec{p}$:

Impulse is Δmomentum

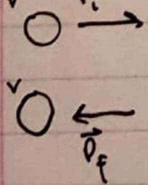




$\Delta\vec{p} = \vec{I} = \int_{T_1}^{T_2} \vec{F}\cdot dt = \vec{F}_{avg}\Delta t$

Example:

1) You throw a ball which strikes a wall horizontally. The impact lasts 70ms, m = 33g. Vin and Vout = 42 m/s. Find Favg of ball on wall.



$\vec{p}_f \quad \vec{p}_i$

$\Delta\vec{p}$

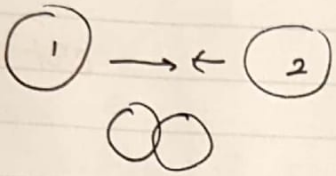
$\vec{F}_{avg} = \frac{\Delta\vec{p}}{\Delta t}$

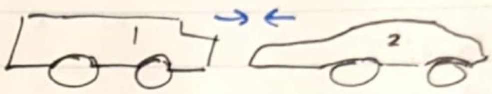
Wall→ball

$$= \frac{\vec{P}_f - \vec{P}_i}{\Delta t} = \frac{mv - (-mv)}{\Delta t} = \frac{2mv}{\Delta t}$$

$\vec{F}_{b\to w} = -\vec{F}_{w\to b}$

2)





on m_1, $\frac{\vec{F}_{21}}{m_1} = \vec{a}_1$, m_2, $\frac{\vec{F}_{12}}{m_2} = \vec{a}_2$

$F^{net}_{system} = \vec{F}_{12} + \vec{F}_{21} = 0$.

If $F^{net}_{system} = 0$, $\Delta \vec{P}_s = 0$.

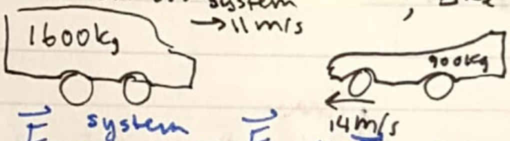
Collisions:

Inelastic collisions:

→ momentum$_{system}$ conserved, ΔKE not. ← entire system!

"complete inelastic" → cars become a clump

Ex:



Inelastic Energy loss = ?

$\vec{F}^{system}_{net} = \vec{F}_{12} + \vec{F}_{21} = 0$ ∴ $\Delta \vec{P}_s = 0$ so $\vec{P}_i = \vec{P}_f$

$\vec{P}_{i_1} + \vec{P}_{i_2} = \vec{P}_{1f} + \vec{P}_{2f}$

$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1+m_2)\vec{v}$ ← must guess $\vec{v}$ direction (keep +)

$m_1 v_1 - m_2 v_1 = (m_1+m_2) v$

$v = 2 m/s$

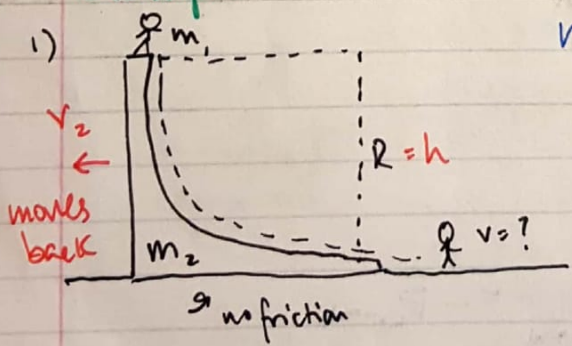
$E_{loss} = KE_i - KE_f$

$= \left(\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v^2\right) - \left(\frac{1}{2}(m_1+m_2)v^2\right)$

$= 180{,}000 J$

Example:

1)



$mgh = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$m_1 v_1 = m_2 v_2$

If moves:

$\vec{P}_{Tx_1} = \vec{P}_{Tx_2}$

$0 = m_1\vec{v}_1 + m_2\vec{v}_2$

$m_1 v_1 = m_2 v_2$

$m_1 g R = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$m_1 g R = \frac{1}{2} m_1 \left(\frac{m_2}{m_1}\right)^2 v_2^2 + \frac{1}{2} m_2 v_2^2$

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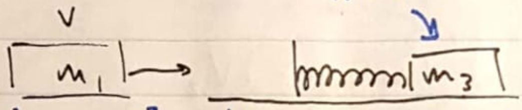
2) moves →



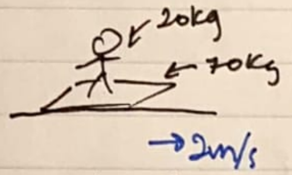
max comp?
↳ when speeds are equal
↳ both move as 1 object

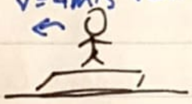
$\frac{1}{2}m_1v_1^2 = \frac{1}{2}(m_1+m_2)v^2 + \frac{1}{2}kx_{max}^2$

energy eqn.

$m_1v_1 = (m_1+m_2)v \leftarrow \vec{P}_{T_1} = \vec{P}_{T_2}$

3)





Final sled speed?

$\vec{P}_{T_1} = \vec{P}_{T_2}$

$(90)(2) = (x+4)(20) + (70)(x)$

$x = 2.9\,m/s$

assume ←

OR

$m_T\vec{v} = m_k\vec{v}_k + m_s\vec{v}_s$

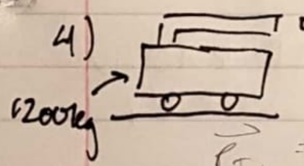
$m_Tv = -m_kv_k + m_sv_s$

$\vec{v}_{ks} = \vec{v}_k - \vec{v}_s$ → +

$-4 = -v_k - x$ sub

$4 - x = v_k$

4)



V = 500 m/s rel. to cannon
recoil speed?

$\vec{P}_{T_1} = \vec{P}_{T_2}$

$(1240)(0) = (1200)(x) + (40)(500)$

$x = -16.67\,m/s$

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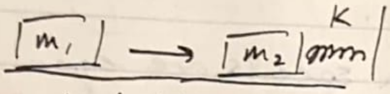
ECE Midterm Review Session — Oct 13, 2017.



inelastic. x_{max}?

$\vec{p}_{T_1} = \vec{p}_{T_2}$

$m_1 v_1 = (m_1 + m_2) V_2$

$\frac{1}{2}(m_1 + m_2) V_2^2 = \frac{1}{2} k x^2$

Energy not conserved during collisions (after, yes).

$h = l - l\cos\theta$

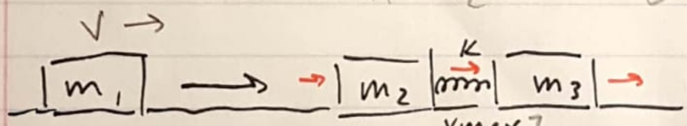
$m_1 v_1 = (m_1 + m_2) V_2$

$V_2 = \left(\frac{m_1 V_1}{m_1 + m_2}\right)$

$\frac{1}{2}(m_1+m_2) v_2^2 = (m_1+m_2) g \Delta h$

$v_2^2 = 2g(l - l\cos\theta)$

$\left(\frac{m_1 v_1}{m_1 + m_2}\right)^2 = 2g(l - l\cos\theta)$ All known but V_1



Inelastic

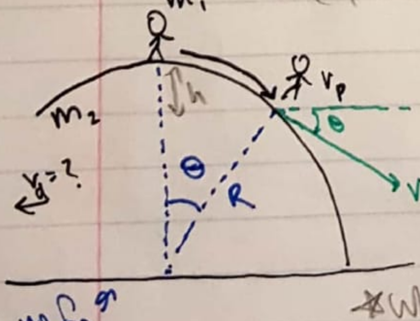
xmax?

$\vec{P}_{T_1} = P_{T_2}$

$m_1 v_1 = (m_1 + m_2) V_2$ and at max amp, same V, so $m_1 v_1 = (m_1 + m_2 + m_3) V_3$

Energy now:

$\frac{1}{2}(m_1 + m_2) v_2^2 = \frac{1}{2}(m_1 + m_2 + m_3) V_3^2 + \frac{1}{2} k x_{max}^2$



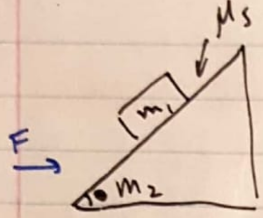
no f_f

$h = R - R\cos\theta$ $\quad m g(R - R\cos\theta) = \frac{1}{2} m_1 v_p^2 + \frac{1}{2} m_d V_d^2$

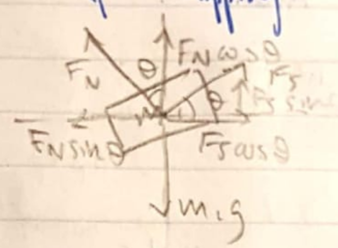
$\vec{P}_{x_i} = \vec{P}_{xf}$

$0 = m_1 v_p \cos\theta - m_d V_d$ ← solve

*when does he fly off?



Fmax before slipping?



$F = (m_1 + m_2)a$

$\sum \vec{F}_x = m_1 a$

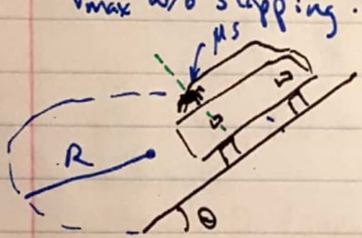
① $\mu_s F_N \cos\theta - F_N \sin\theta = m_1 a$

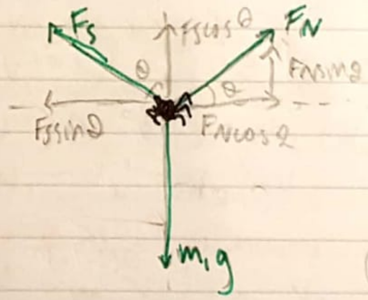
$\sum F_y = 0$

② $F_N\cos\theta + \mu_s F_N \sin\theta = m_1 g$

$$①/② = \frac{\mu_s \cos\theta - \sin\theta}{\cos\theta + \mu_s \sin\theta} = \frac{a}{g}$$

Vmax w/o slipping.





$\sum \vec{F}_x = \frac{mv^2}{r}$

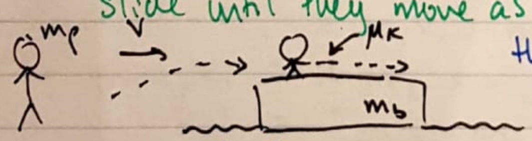
① $\mu_s F_N \sin\theta - F_N\cos\theta = \frac{mv^2}{r}$

$\sum \vec{F}_y = 0 \quad \mu_s F_N \cos\theta = F_N \sin\theta + m_1 g$

② $\mu_s F_N \cos\theta + F_N \sin\theta = m_1 g$

$$①/② \quad \frac{\mu_s \sin\theta - \cos\theta}{\mu_s \cos\theta + \sin\theta} = \frac{v^2}{rg}$$

slide until they move as one



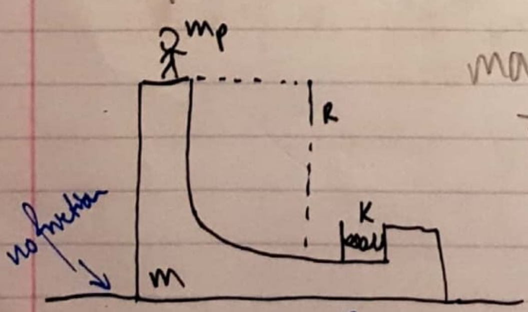
How far slide on block? rel to block

$\vec{p}_{T_1} = \vec{p}_{T_2}$

$m_p v_1 = (m_p + m_b) v_2$ move as one

no f

$\frac{1}{2} m_p v_1^2 = \frac{1}{2}(m_p + m_b) v_2^2 + m_p g \mu_K \cdot d$



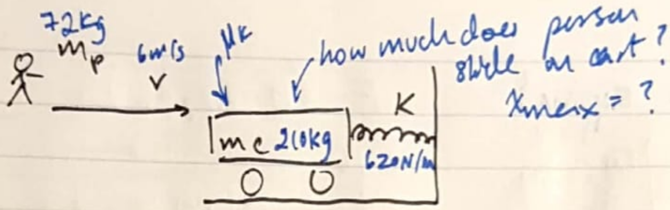
Xmax?

max comp when speeds are equal!

$\vec{p}_{T_1} = 0 = \vec{p}_{T_2}$

$0 = (m_1 + m_2) v_2$ so $v_2 = 0$.

$\therefore m g R = \frac{1}{2} k x^2$ since objects stop moving.



$\vec{p}_{T_1} = \vec{p}_{T_2}$

$m_p v_p = (m_p + m_c) v_c$

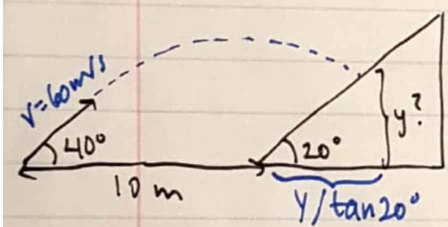
After collision ↳ $\frac{1}{2}(m_c + m_p) v_c^2 = \frac{1}{2} k x_{max}^2$

Solve for x_{max}

$\frac{1}{2} m_p v_p^2 = m g \mu_k d + \frac{1}{2} k x_{max}^2$

Solve for d.

$\frac{1}{2}(m_c + m_p) v_c^2 + m_p g \mu_k d = \frac{1}{2} m_p v_p^2$



$d_x = v \cos\theta t$

$10 + \frac{y}{\tan 20°} = v \cos\theta t$

$d_y = v \sin\theta t - \frac{1}{2} a t^2$

$y = v \sin\theta t - \frac{1}{2} g t^2$

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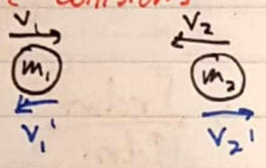
Elastic Collisions Oct 25, 2017.



Energy and momentum conserved.

momentum $\longrightarrow +$

1) Conserve momentum:

① $m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$ *even if 0, don't cancel yet.

2) Conserve energy:

② $\frac{1}{2}m_1\vec{v}_1^{\,2} + \frac{1}{2}m_2\vec{v}_2^{\,2} = \frac{1}{2}m_1\vec{v}_1'^{\,2} + \frac{1}{2}m_2\vec{v}_2'^{\,2}$ now they bounce off

→ Group by mass, choose + momentum, and assume unknown directions

① $m_1(v_1 + v_1') = m_2(v_2 + v_2')$
③ $\frac{1}{2}(m_1)(v_1^2 - v_1'^2) = \frac{1}{2}m_2(v_2^2 - v_2'^2)$ } Group by mass

sub: $m_1(v_1 - v_1')(v_1 + v_1') = m_2(v_2 - v_2')(v_2 + v_2')$

divide by ①: $v_1 - v_1' = v_2' - v_2$

Now: 2 linear eqns, 2 unknowns:

① $m_1v_1 + m_1v_1' = m_2v_2' + m_2v_2$
② $v_1 - v_1' = v_2' - v_2$

Want: v_2'.

② $m_1v_1 - m_1v_1' = m_1v_2' - m_1v_2$ $(\times m_1)$

$-$② $+$①: $2m_1v_1 = m_2v_2' + m_2v_2 + m_1v_2' - m_1v_2$

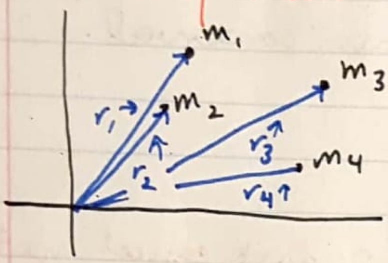
$$v_2' = \frac{2m_1v_1 - m_2v_2 + m_1v_2}{m_1 + m_2}$$

Now find v_1' NOT by subbing in v_2'. Do same as above but $\times m_2$.

−ve answer? opposite assumption.

★ → If $v_1' = -5$? WRONG ASSUMPTION!

Centre of Mass:

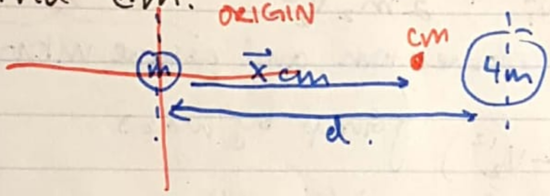


$$\vec{r}_{cm} = \frac{\sum_i m_i \vec{r_i}}{\sum_i m_i} \rightarrow \frac{\int r\,dm}{\int dm}$$

$$x: \vec{x}_{cm} = \frac{m_1 \vec{x}_1 + m_2 \vec{x}_2 \ldots}{m_1 + m_2 \ldots} \quad y, z \text{ etc.}$$

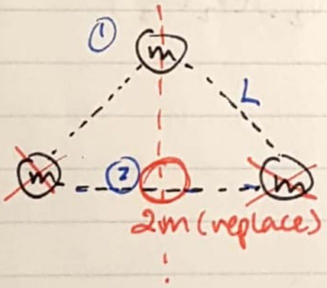
Example:

1) Find c.m.



$$\vec{x}_{cm} = \frac{m(0) + 4m(d)}{5m} = \frac{4}{5}d.$$

2) eq. Δ.
30°



$$\vec{y}_{cm} = \frac{m_1 \vec{y}_1 + m_2 \vec{y}_1}{m_1 + m_2} = \frac{m\left(\frac{\sqrt{3}}{2}\right)l + 2m(0)}{3m} = \frac{\sqrt{3}}{6} l.$$

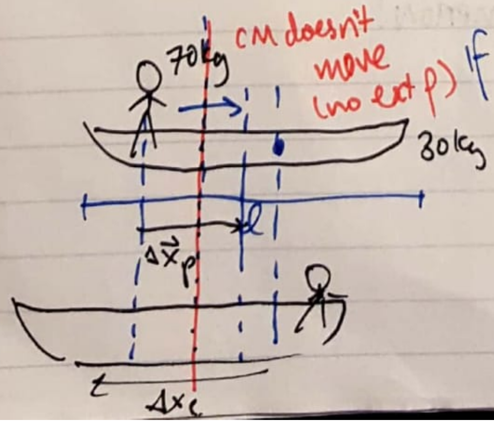
$m_T \vec{r}_{cm} = m_1 \vec{r}_1 + m_2 \vec{r}_2 + \ldots$

$m_T \vec{V}_{cm} = m_1 \vec{v}_1 + m_1 \vec{v}_2 + \ldots$ CM momentum

$m_T \vec{a}_{cm} = m_1 \vec{a}_1 + m_2 \vec{a}_2 + \ldots$ CM acceleration

$= \vec{F}_{net_1} + \vec{F}_{net_2} + \ldots$ EXTERNAL forces only.

3)

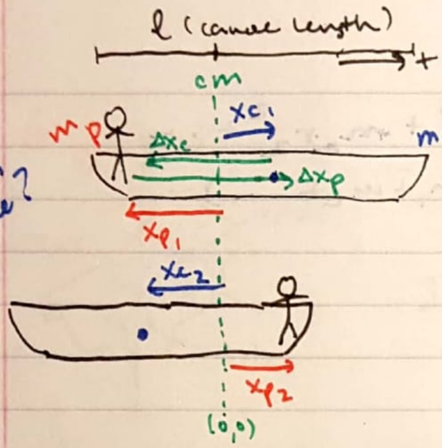


If no ext forces, CM doesn't move!

$m_p \Delta x_p = m_c \Delta x_c$.

1)

How far does canoe move?



both 0.

$m_T \vec{x}_{cm_1} = m_p \vec{x}_{p_1} + m_c \vec{x}_{c_1}$ Oct 25, 2017

$m_T \vec{x}_{cm_2} = m_p \vec{x}_{p_2} + m_c \vec{x}_{c_2}$

$0 = m_p \Delta\vec{x}_p + m_c \Delta\vec{x}_c$

can get from momentum b/c mom. conserved.

↳ Internal forces only.

$m_1\vec{v}_1 + m_2\vec{v}_2 = 0 \quad \vec{p}_i = \vec{p}_f = 0.$

$\Delta\vec{x}_{pc} = \Delta\vec{x}_p - \Delta\vec{x}_c \qquad m_p \Delta x_p - m_c \Delta x_c = 0 \; (\rightarrow +)$ ①

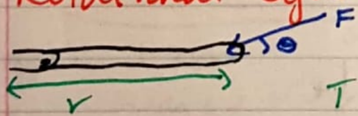
$l = \Delta x_p - (-\Delta x_c)$

② $= \Delta x_p + \Delta x_c$

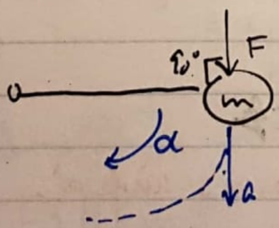
$\Delta x_p = \dfrac{m_c \Delta x_c}{m_p} \rightarrow$ sub② $l = \dfrac{m_c \Delta x_c}{m_p} + \Delta x_c$

$\Delta x_c = \dfrac{l}{1 + \frac{m_c}{m_p}}$

Rotational Dynamics: Torque



$T = rF\sin\theta$



Always specify pivot as subscript of T

$F = ma$

$T = |r||F| = rma$

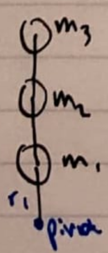
$T = rm\alpha r$

$T = mr^2\alpha$

r×F but sin 90 = 1

inertia: ability to resist angular acc. linear? m.

relationship btwn F and a. T is a F.

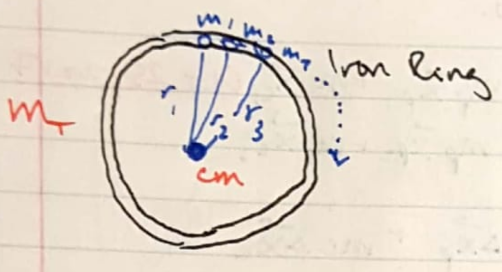


$T_{net} = T_1 + T_2 + T_3$

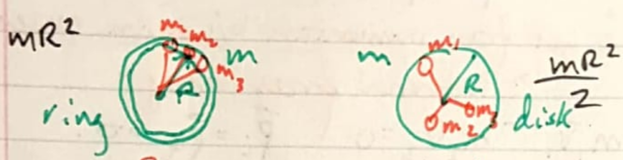
$= (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2)\alpha$

$= I_{total}\,\alpha$

Total Inertia $= m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$

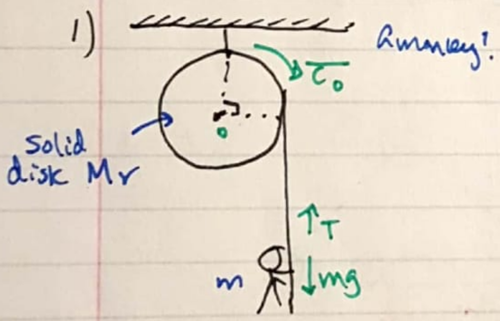


$I_{cm} = m_1 r_1^2 + m_2 r_2^2 + \dots$
$= r^2(m_1 + m_2 + \dots)$
$= m_T r^2$



Same mass, but disk has much less inertia, since mass is spread so some has smaller r.
Disk has smaller r's. Ring has max r for every m_i.

1)

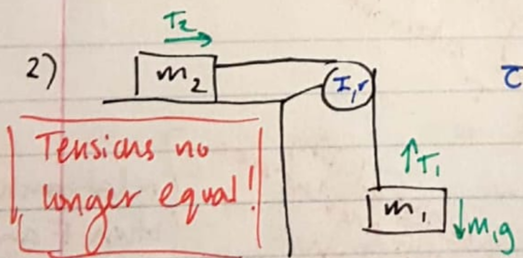


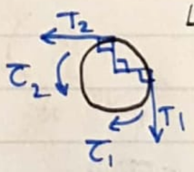
Oct 30, 2017

$\Sigma\vec{\tau}_0 = \vec{I}_0\alpha$
$rT = \frac{Mr^2}{2}\alpha$
$\frac{2T}{M} = \frac{a}{r}$

$mg - T = ma$, so
$a = \frac{mg}{\left(m + \frac{M}{2}\right)}$

2)

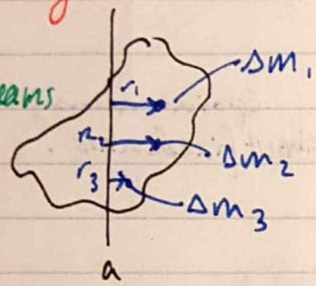




Let ⟲+ →+ ↓+
$\Sigma\vec{\tau}_0 = \vec{I}_0\alpha$
$\tau_1 - \tau_2 = I\alpha$
$rT_1 - rT_2 = I\frac{a}{r}$
Sub T's rel. to m's and a's.

Nov 1, 2017

In general:



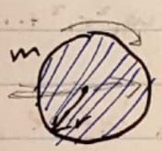
★ Δm means small value.

$I_a = ?$

$I_o \cong \sum_i \Delta m_i r_i^2$ ← the same.

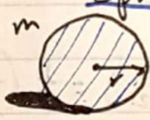
↳ lim becomes integral. $\int_{V_{objet}} r^2 dm$

Can use integration to show:



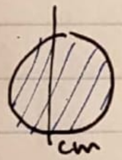
disk

$\boxed{I_{cm} = \frac{mr^2}{2}}$



sphere

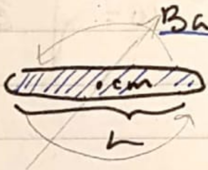
$\boxed{I_{cm} = \frac{2}{5} mr^2}$



spherical shell

$\boxed{I_{cm} = \frac{2}{3} mr^2}$

(hollow sphere)

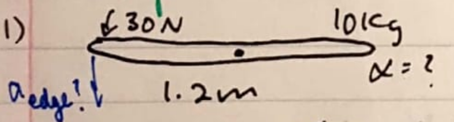


Bar

m

$\boxed{I_{cm} = \frac{1}{12} mL^2}$

Example:

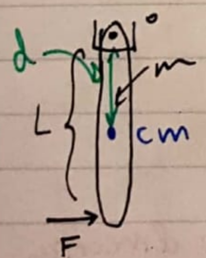
1) 

$\sum \vec{\tau}_o = \vec{I}_o \alpha$

$(r)(F) = (\frac{1}{12})(mL^2)\alpha \qquad \alpha = \frac{6F}{mL}$

$a_{edge} = \alpha r$

$= \frac{6F(\frac{L}{2})}{mL} = \frac{3F}{m}$ } makes sense, since bar ≠ point mass

2) 

$\tau_o = I_o \alpha$

$LF = (I_{cm} + md^2)\alpha$

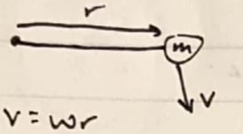
$LF = \left(\frac{mL^2}{3}\right)\alpha$

etc.

Parralel Axis Theorem: $\boxed{I_a = I_{cm} + md^2}$

↳ Two axis must be ∥

Rotational Energy:



$v = \omega r$

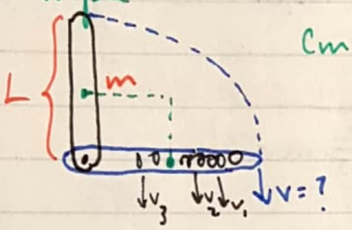
$K = \frac{1}{2} m v^2$

$= \frac{1}{2} m \omega^2 r^2$

$\boxed{= \frac{1}{2} I \omega^2}$ Rotational KE about fixed axis. (point mass)

Example:

1)



Cm fell by $\frac{L}{2}$. $mg\left(\frac{L}{2}\right) = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \dots$

$= \frac{1}{2} m (\omega_1^2 r_1^2 + \omega_1^2 r_1^2 + \dots)$

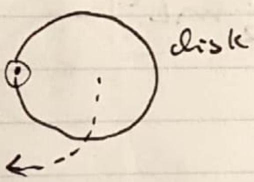
$= \frac{1}{2} I \omega^2$

$mg\left(\frac{L}{2}\right) = \frac{1}{2}\left(\frac{1}{3} m L^2\right)\omega^2$ from before

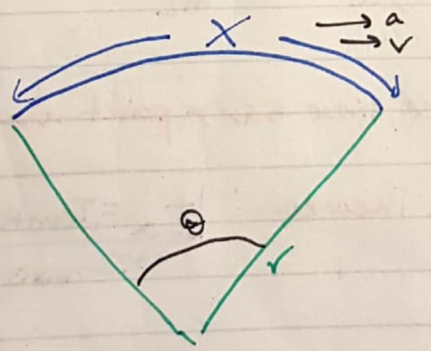
$\omega = \sqrt{\frac{3g}{L}}$ $\quad V = \omega L$

$V = \sqrt{3gL}$

2)



3)



$\Delta x = \vec{v}_1 t + \frac{1}{2}\vec{a} t^2 / r$

$\Delta \theta = \omega t + \frac{1}{2} \alpha t^2$

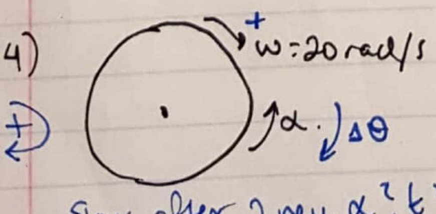
$\vec{V}_2 = \vec{V}_1 + \vec{a} t / r$

$\omega_2 = \omega_1 + \alpha t$

$V_f^2 - V_i^2 = 2\vec{a} \cdot \Delta \vec{d} / r^2$

$\omega_f^2 - \omega_i^2 = 2\vec{\alpha} \cdot \Delta \theta$

4)



Stops after 2 rev. α? t?

$\omega_f^2 - \omega_i^2 = 2 \vec{\alpha} \cdot \Delta \theta$ opp direction

$-400 = 2(\alpha)(4\pi)(-1)$

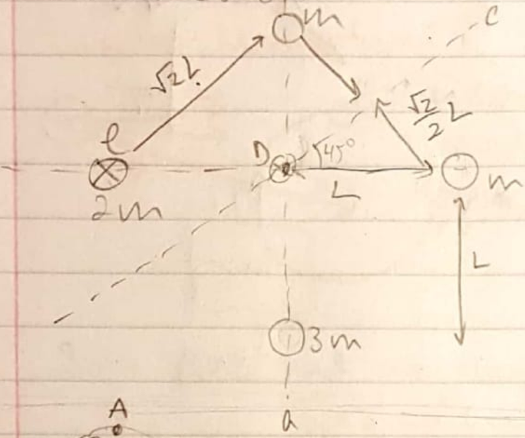
$\alpha =$

$t = \frac{\vec{\omega}_2 - \vec{\omega}_1}{\vec{\alpha}}$ need sign convention

$= \frac{\omega_1}{\alpha}$

Review Session

Nov 1, 2017



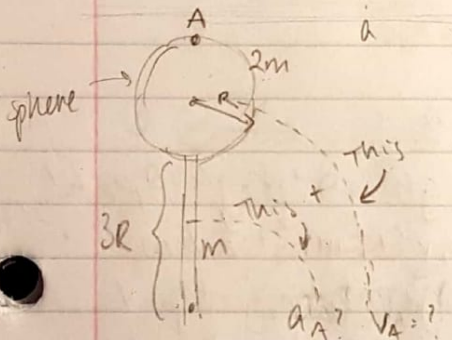
$I_a: mL^2 + 2mL^2 = 3mL^2$

$I_c: \frac{1}{2}mL^2 + \frac{1}{2}mL^2 + \frac{1}{2}3mL^2 = \frac{1}{2}L^2 3m$

$* \left(\frac{\sqrt{2}}{2}L\right)^2 = \left(\frac{1}{2}\right)L^2$

$I_d: mL^2 + mL^2 + 2mL^2 + 3mL^2$

$I_e: m(\sqrt{2}L)^2 + m(2L)^2 + 3m(\sqrt{2}L)^2 + 0$



$\Delta K = -(\Delta U)$ Can also do c of mass approach

$2mg(4r) + mg(1.5R) = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$

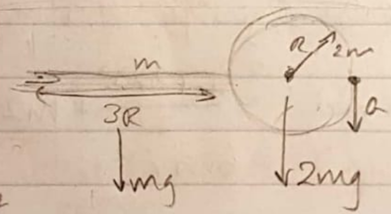
$9.5mgr = \frac{1}{2}\left(\frac{1}{3}m(3r)^2\right)\omega^2 + \frac{1}{2}\left(\frac{2}{5}mr^2 + 32mr^2\right)\omega^2$

$* \hookrightarrow I_{sphere} = I_{cm} + md^2 = \frac{2}{5}mr^2 + 2m(4r)^2$

$9.5gr = \frac{3}{2}r^2\omega^2 + \frac{82}{5}r^2\omega^2 = \frac{179}{10}r^2\omega^2$

$\omega^2 = \frac{95}{179}\frac{g}{R}$ $V_A = \omega 5r$

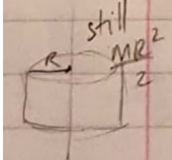
$I_o = \left\{\frac{64}{5}mr^2 + 3mr^2\right\}$
$I_1 + I_2$

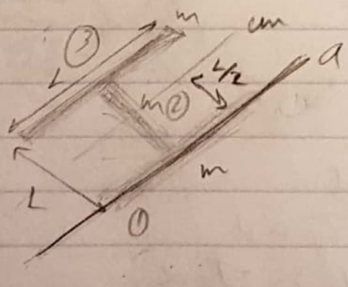


$a = \alpha 5r$ $\sum \tau_o = I_o\alpha$

$\tau_1 + \tau_2 = I_o\alpha$

$1.5Rmg + 4R(2mg) = I_o mR^2\alpha$





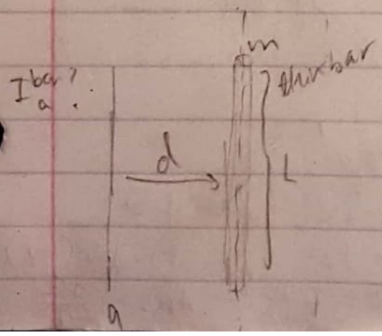
Thin bars

$I_a = I_1 + I_2 + I_3$

$= md_1^2 + md_2^2 + md^2$

$= \frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2 + md^2$

$= \frac{1}{12}mL^2 + \frac{1}{4}mL^2 + mL^2 = \frac{4}{3}mL^2$



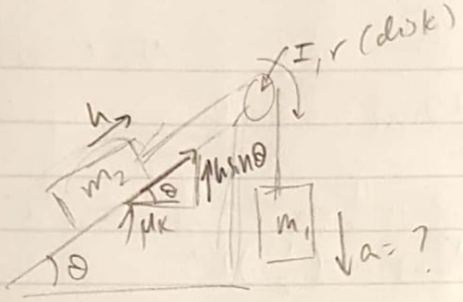
$I_a = I_{cm} + md^2$

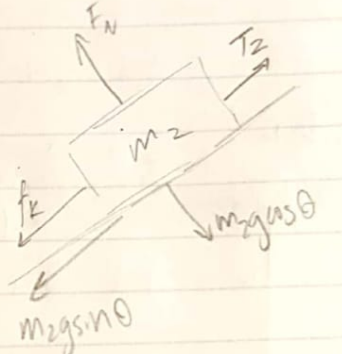
$= \frac{mr^2}{2} + md^2$

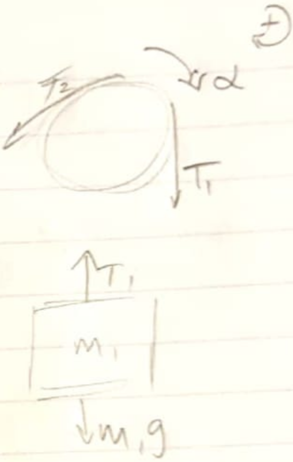
$= md^2$

or... $I_T = m_1d^2 + m_2d^2 \ldots$

$= md^2$







$m_1 : \sum \vec{F_y} = m_1 \vec{a}$

① $m_1 g - T_1 = m_1 a$

$m_2 : \sum \vec{F_x} = m_2 \vec{a_x}$

② $T_2 - m_2 g\sin\theta - m_2 g\cos\theta\mu_K = m_2 a$

Pulley: $\sum \vec{\tau_0} = \vec{I_0}\alpha$

$rT_1 - rT_2 = \frac{mr^2}{2}\left(\frac{a}{r}\right)$

③ $T_1 - T_2 = (ma)/2$

$m_2 : \sum F_y = 0.$

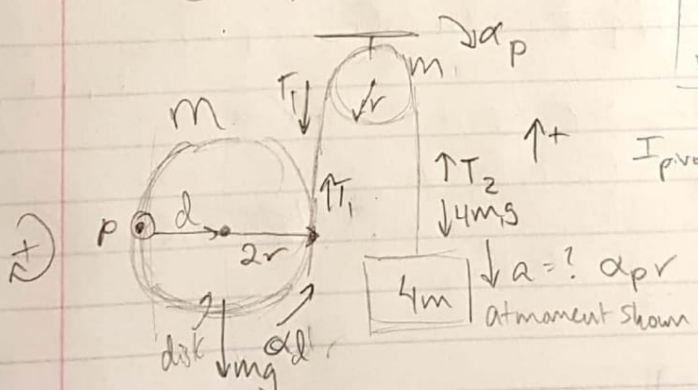
$F_N = m_2 g\cos\theta$

*One line solution:

$m_1 g h = m_2 g h\sin\theta + m_2 g\cos\theta\mu_K h + \frac{1}{2}m_1 v^2 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$\frac{d}{dt} : m_1 g v = m_2 g v\sin\theta + m_2 g\cos\theta\mu_K v + m_1 va + m_2 va + I\omega\alpha$

$m_1 g = m_2 g\sin\theta + m_2 g\cos\theta\mu_K + m_1 a + m_2 a + \frac{I}{r^2}$



$I_{pivot} = I_{cm} + md^2 = \frac{m(2r)^2}{2} + m(2r)^2$
$= 6mr^2$

$4m : 4mg - T_2 = 4ma$

Pulley: $\sum \vec{\tau_0} = \vec{I_0}\alpha_p$

$\vec{\tau_{T_1}} + \vec{\tau_{T_2}} = I_0\alpha_p$

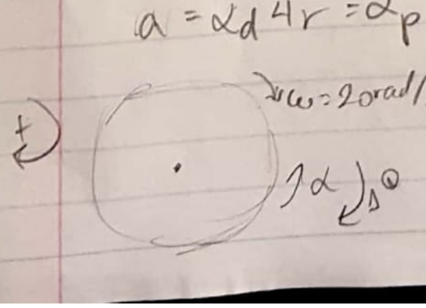
$-rT_1 + rT_2 = \frac{mr^2}{2}\alpha_p$

$a = \alpha_d 4r = \alpha_p r$

Disk: $\sum \vec{\tau_p} = \vec{I_p}\vec{\alpha_d}$

$\vec{\tau_{mg}} + \vec{\tau_{T_1}} = I_p \vec{\alpha_d}$

$(mg2R) - T_1(4R) = -I_p\alpha_d$



Drops by $\frac{1}{2}$ in 3s. Turns till stop?

$\vec{\omega_f} = \vec{\alpha}t + \vec{\omega_0}$

$\frac{1}{2}\vec{\omega_0} = \vec{\alpha}t + \vec{\omega_0}$

$\alpha = \frac{20}{6} = 3.33\,rad/s^2$

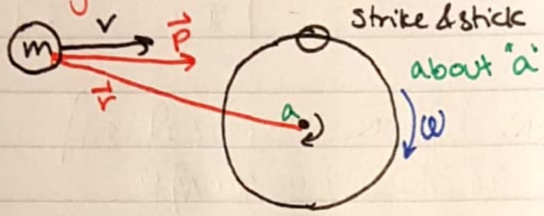
$\omega_f^2 - \omega_0^2 = 2\vec{\alpha}\cdot\Delta\theta$

$-20^2 = 2(3.33)\cdot\Delta\theta(-1)$

$\Delta\theta = \frac{-400}{6.66}$ Ans $= \frac{\Delta\theta}{2\pi}$

Angular Momentum:

Nov 3, 2017



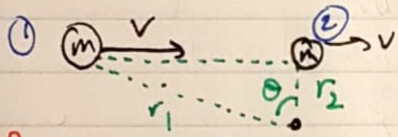
After collision has angular momentum

No external forces... so:

→ angular momentum must have existed before the collision as well "about a"

Since disk had no ang. momentum before, must have come from clay

$\vec{L}_a = \vec{r}_a \times \vec{p}$ "orbital"

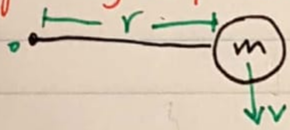


$\vec{L}_1 = \vec{r}_1 \times \vec{p} = r_1 p \sin\theta = r_2 p$

$\vec{L}_2 = \vec{r}_2 \times \vec{p} = r_2 \sin\theta p = r_2 p$

} no ext forces, so momentum is conserved.

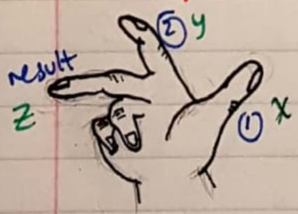
Spinning on fixed axis:

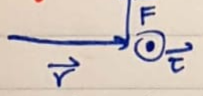


$$\begin{aligned} \vec{L}_o &= \vec{r} \times m\vec{v} \\ &= r m \omega r \\ &= m r^2 \omega \\ &\boxed{= I_o \omega} \end{aligned}$$

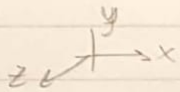
$\vec{L}_o = I_o \vec{\omega}$

Cross Products + Right hand Rule:





$\vec{\tau} = \vec{r} \times \vec{F}$



$\vec{r} = r\hat{i}$ $\vec{F} = F\hat{j}$

$\vec{\tau} = r\hat{i} \times F\hat{j} = rF(\hat{i} \times \hat{j}) = rF\hat{k}$

Conservation of Angular Momentum Nov 6, 2017.

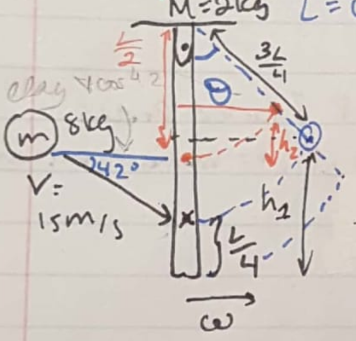
$$\frac{dL}{dt} = I\vec{\alpha} = \vec{\tau}_{net}$$

$\vec{\tau}_{net} = 0 \Rightarrow L$ is conserved.

$\vec{\tau}_{net}^{\,system} = 0 \Rightarrow L$ is constant.

Example:

1)



$\vec{L}_i = \vec{L}_f$

$\left(\frac{3L}{4}\right)(mv\sin 48°) = I_0\,\omega$

$= \left[\left(\frac{1}{3}ML^2\right) + \left(m\left(\frac{3}{4}\right)^2\right)\right]\omega$

$\omega =$ ____

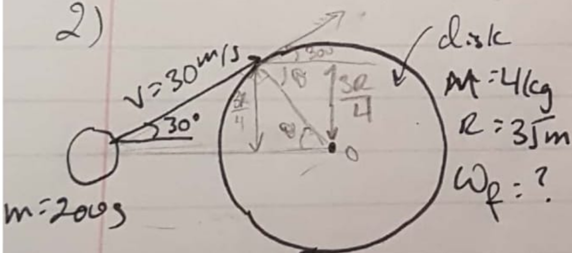
$\frac{1}{2}I_0\omega^2 = (m+M)gh_{cm}$

$= mgh_1 + Mgh_2$

$h_1 = \frac{3L}{4} - \frac{3L}{4}\cos\theta$

$h_2 = \frac{L}{2} - \frac{L}{2}\cos\theta$ c of m

2)



$\vec{L}_i = \vec{L}_f$

$R(mv\sin 78.6°) = I_0\omega_f$

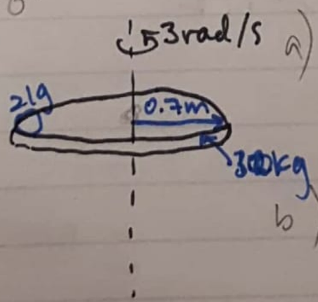
$Rmv\sin 78.6° = \left(\frac{MR^2}{2} + mr^2\right)\omega_f$

Solve ω_f

$\sin\theta = \frac{\frac{3}{4}R}{R} = \frac{3}{4}$

$\beta = \theta + 30°$

3)



a) $\vec{L}_i = \vec{L}_f$

$I_0\omega_1 = I_0\omega_f$

$\left(\frac{MR^2}{2} + mR^2\right)\omega_1 = \left(\frac{MR^2}{2}\right)\omega_f$

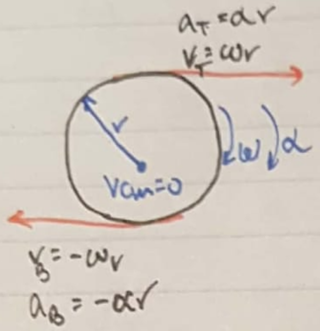
b) $W = \Delta K_E$

$= \frac{1}{2}I_f\omega_f^2 - \frac{1}{2}I_i\omega_i^2$

$= \frac{1}{2}\left(\frac{MR^2}{2}\right)\omega_f^2 - \frac{1}{2}\left(\frac{MR^2}{2} + mR^2\right)\omega_i^2$

Find ω when bug at center. Bug walks inwards. Work done by bug?

4)



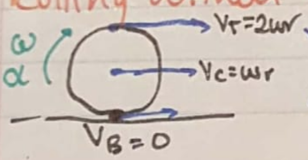
$v_T = v_{cm} + \omega r$ $\qquad$ $a_T = a_{cm} + \alpha r$

$v_B = v_{cm} - \omega r$ $\qquad$ $a_B = a_{cm} - \alpha r$

$\boxed{v_T = v_B + 2\omega r}$ $\qquad$ $\boxed{a_T = a_B + 2\alpha r}$

* fixed axis

Rolling Without Slipping: $\qquad$ Nov 10, 2017



$v_B = v_{cm} - \omega r = 0$

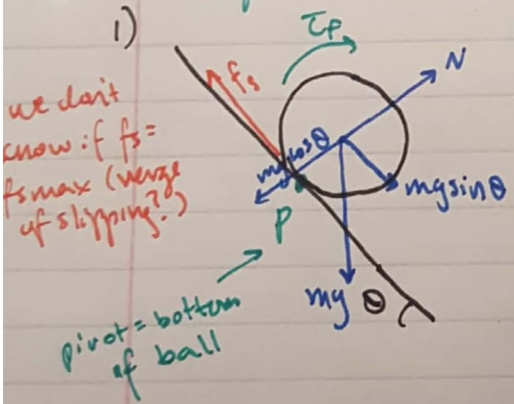
so $\boxed{v_{cm} = \omega r}$ $\rightarrow$ $v_T = \omega r + \omega r$

$= 2\omega r$ $\qquad$ * axis moving at ωr

$a_{cm} = \alpha r \quad a_B = 0 \quad a_T = 2\alpha r$

Example:

1)



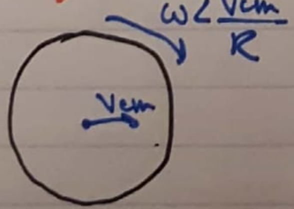
$mg\sin\theta$ no τ b/c center of mass.

$\downarrow$

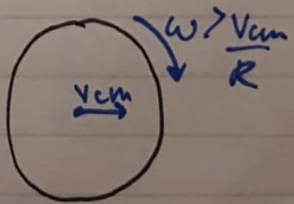
$\tau_{cm} = \tau_F$

$\tau_b = \tau_{mg\sin\theta}$

Rolling with Slipping:

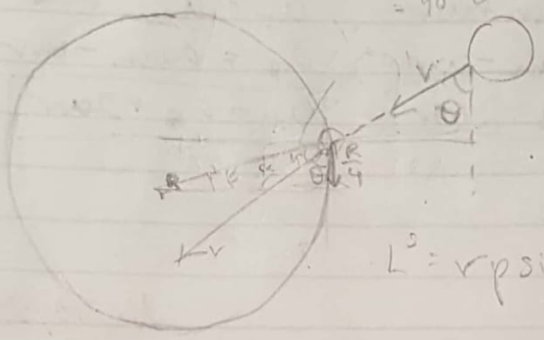


Skidding



Slipping

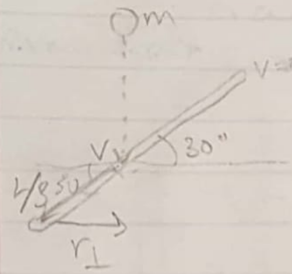
Review Session Nov 9, 2017



$\sin\beta = \frac{R/4}{R} = \frac{1}{4}$

$\varepsilon = 180 - \beta - 90 - \theta$
$= 90 - \beta - \theta$

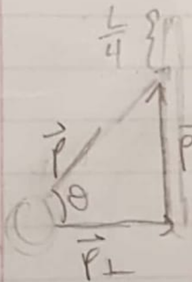
$L^{\circ} = rp\sin(90-\beta-\theta)$



$\vec{L}_i = \vec{L}_f$

$\frac{L}{3} mv\sin 120 = \left(\frac{1}{3}mL^2 + \frac{m\left(\frac{L}{3}\right)^2}{2}\right)\omega$

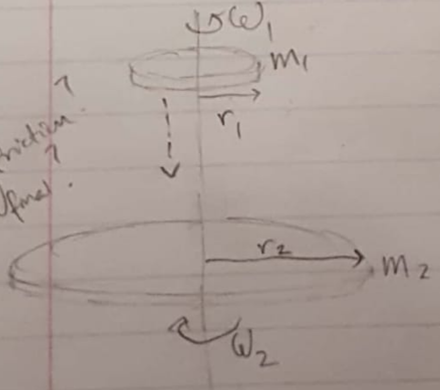
or. $\vec{L} = r_\perp p$
$= L/3 \cos 30\, mv$



$\vec{L} = \vec{r} \times \vec{p}$
$= \vec{r} \times \vec{p}_\perp$
$= rp\cos\theta$
$= \frac{3L}{4} mv\cos\theta$

+↑

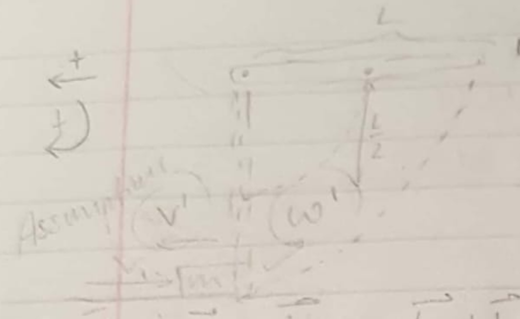
Wfriction ?
Wfinal ?



$\vec{L}_i = \vec{L}_f$

$\frac{m_1 r_1^2}{2}\omega_1 - \frac{m_2 r_2^2}{2}\omega_2 = \omega_{final}\left(\frac{m_1 r_1^2}{2} + \frac{m_2 r_2^2}{2}\right)$ assumption made.

$W_{friction} = \Delta ke$
$= k_f - k_i$
$= \frac{1}{2} I_f \omega_f^2 - \frac{1}{2}\left(I_{i_1}\omega_1^2 + I_{i_2}\omega_2^2\right)$



M $v_b = 0$

elastic (energy conserved)

mom: $\vec{L}_i = \vec{L}_f$ $\quad \vec{L}_1 + \vec{L}_2 = \vec{L}_1' + \vec{L}_2'$ $\quad -L_1 + L_2 = L_1' - L_2'$

① $-Lmv_1 + \frac{1}{3}ML^2\omega_i = Lmv_2 - \frac{1}{3}ML^2\omega'$

eng: ② $Mg\left(\frac{L}{2}\right) = \frac{1}{2}\left(\frac{1}{3}ML^2\right)(\omega)^2$ $\quad (mg\Delta h = \frac{1}{2} I_o\omega^2)$

eng 2: $K_i = K_f$ (during collision)

$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 = \frac{1}{2}mv'^2 + \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega'^2$

$\frac{1}{2}m(v^2 - v'^2) = \frac{1}{6}ML^2(\omega'^2 - \omega^2)$

③ $\frac{1}{2}m(v-v')(v+v') = \frac{1}{6}ML^2(\omega'-\omega)(\omega'+\omega)$

From ①: $mL(v+v') = \frac{1}{3}ML^2(\omega'+\omega)$

↳ Divide ③ by ①

* Retry ④ $(v_1 - v') = L(\omega' - \omega)$

From ①: $mL(v_1+v') = \frac{1}{3}ML^2(\omega'+\omega)$

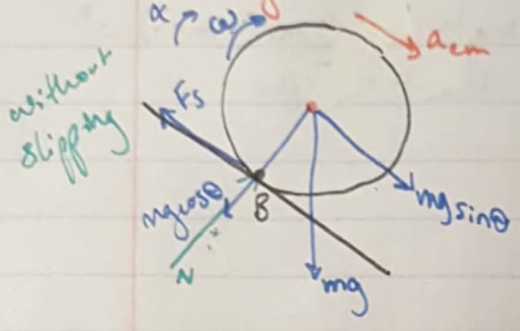
④: $(v_1 - v') = L(\omega' - \omega)$

$mL(v_1 - v') = mL^2(\omega' - \omega)$ add with 1

↳ $2mLv = \frac{1}{3}ML^2\omega' + \frac{1}{3}ML^2\omega + mL^2\omega' - mL^2\omega$

$$\omega' = \frac{2mLv + mL^2\omega - \frac{1}{3}ML^2\omega}{\frac{1}{3}ML^2 + mL^2}$$

Rolling Motion:



Approach A)

$\Sigma F_x = m\vec{a}_x$ $\qquad \Sigma\vec{\tau}_{cm} = I_{cm}\vec{\alpha}$ $\qquad$ ③ $a_{cm} = \alpha r$ By rolling w/o slip

① $mg\sin\theta - f_s = ma_{cm}$ $\qquad \tau_{f_s} = I_{cm}\alpha$

② $rf_s = I_{cm}\alpha$

$\cancel{r}(\cancel{m}g\sin\theta - \cancel{m}a) = \frac{2}{5}\cancel{m}r^{\cancel{2}}\left(\frac{a_{cm}}{\cancel{r}}\right)$

$5g\sin\theta - 5a_{cm} = 2a_{cm}$: $a_{cm} = \frac{5}{7}g\sin\theta$

$a_{cm} = \dfrac{mg\sin\theta}{\left(\frac{I_{cm}}{r^2} + m\right)}$

Approach B) (can't use when skidding)

$\Sigma\vec{\tau}_B = I_B\vec{\alpha}$

$\vec{\tau}_{mgx} = I_B\vec{\alpha}$

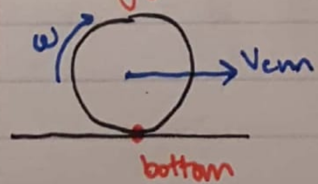
① $rmg\sin\theta = I_B\alpha$

② $a_{cm} = \alpha r = \dfrac{r^2 mg\sin\theta}{I_b}$ $\quad$ $\parallel$ axis theorem: $I_b = I_{cm} + mr^2$

$a_{cm} = \dfrac{mg\sin\theta}{\left(\frac{I_{cm}}{r^2} + m\right)}$

Energy in Rolling:

Nov 12, 2017



A) $\frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$ $\qquad$ B) $\frac{1}{2}I_b\omega^2$ (no trans at bottom)

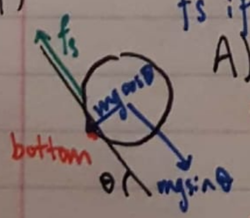
$= \frac{1}{2}(I_{cm} + mR^2)\omega^2$

Same thing $= \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$

$= \frac{7}{10}mv_{cm}^2$

Example:

1) f_s if object a) ball? b) cylinder? (Rolling no slipping)



A) $\Sigma\vec{F}_x = m\vec{a}_{cm}$

① $mg\sin\theta - f_s = ma_{cm}$

$\Sigma\vec{\tau}_{cm} = I_{cm}\vec{\alpha}$

② $rf_s = I_{cm}\alpha$

③ $a_{cm} = \alpha r$

B) $\Sigma\vec{\tau}_b = I_b\vec{\alpha}$

$rmg\sin\theta = (I_{cm} + mr^2)\alpha$

① $\alpha = \dfrac{mg\sin\theta}{I_{cm} + mr^2}$ $\quad$ ② $a_{cm} = \alpha r$ $\quad$ $a_{cm} = \dfrac{mg\sin\theta}{I_{cm}/r^2 + m}$

Ball: $a_{cm} = \frac{5}{7}g\sin\theta$, so $f_s = \frac{2}{7}g\sin\theta m$

Disk: $a_{cm} = \frac{2}{3}g\sin\theta$, so $f_s = \frac{1}{3}mg\sin\theta$

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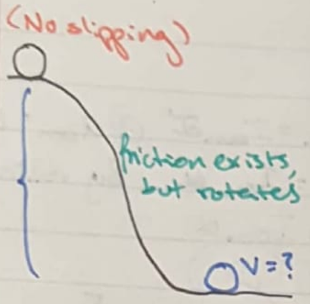
2) 

A) $mgh = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}mv_{cm}^2$ ← friction did this

$mgh = \frac{7}{10}mv_{cm}^2$

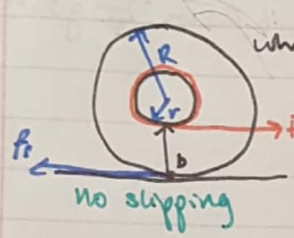
$v_{cm} = \sqrt{\frac{10}{7}gh}$

* less than $\sqrt{2gh}$

B) $mgh = \frac{1}{2}I_b\omega^2$

$mgh = \frac{1}{2}\left(\frac{2}{5}mr^2 + mr^2\right)\omega^2$

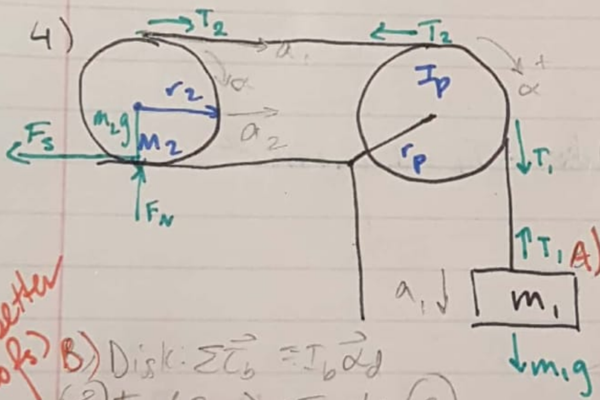
$v_{cm} = \sqrt{\frac{10}{7}gh}$

3) 

which way? ↔

$\sum\vec{\tau}_b = I_b\vec{\alpha}$

$\vec{\tau}_F = I_b\vec{\alpha}$

4) 

Nov 15, 2017

m_1: $m_1\vec{a}_1 = m_1g - T_1$ ①

Pulley: $\sum\vec{\tau}_o = I_p\vec{\alpha}_p$

$r_pT_1 - r_pT_2 = I_p\alpha_p$ ②

A) Disk: $\sum\tau_o = I_o\alpha_d$

$r_2T_2 + r_2F_f = \left(\frac{2}{5}M_2r_2^2\right)\alpha_d$ ③

$a_1 = \alpha_p r_p$ ④

$m_2\vec{a}_2 = T_2 - F_s$ ⑤

$a_1 = 2\alpha_d r_2$ ⑥ $a_2 = \alpha_d r$ ⑦

B) is better (no fs)

B) Disk: $\sum\vec{\tau}_b = I_b\vec{\alpha}_d$

③ $T_2(2r_2) = I_b\alpha_d$ ③

Review Session:

① $T_1 = m_1g - m_1a$ ③ $T_2 = \frac{I_b}{2r_2}\alpha_d$

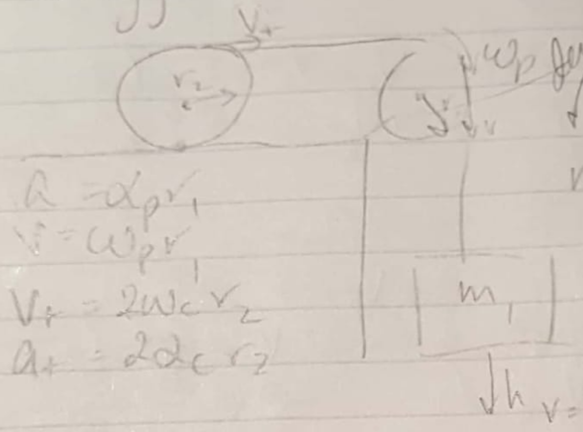
↳ ②: $r_p(m_1g - m_1a) - r_p\left(\frac{I_b}{2r_2}\right)\alpha_d = I_p\alpha_p$

$r_p(m_1g - m_1a) - r_p\left(\frac{I_b}{2r_2}\right)\left(\frac{a_1}{2r_2}\right) = I_p\left(\frac{a_1}{r_p}\right)$

$$a_1 = \frac{m_1g}{m_1 + \frac{I_p}{r_p^2} + \frac{I_b}{4r_2^2}} \quad \Big\} \quad I_b = \frac{M_2r_2^2}{2} + M_2r_2^2$$

Review Session
Energy Method

1)



derive:

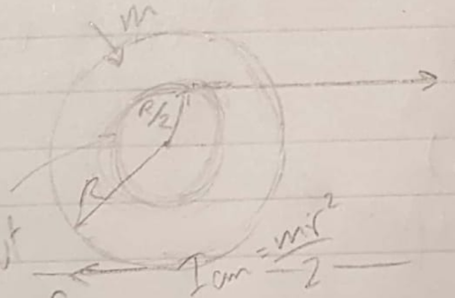
$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I_p\omega_p^2 + \frac{1}{2}I_b\omega_c^2$$

$$mgv = mva + I_p\omega_p\alpha_p + I_b\omega_c\alpha_c$$

$$mg\cancel{v} = m\cancel{v}a + I_p\left(\frac{\cancel{v}}{r_1}\right)\left(\frac{a}{r_1}\right) + I_b\left(\frac{\cancel{v}}{2r_2}\right)\left(\frac{a}{2r_2}\right)$$

$a = \alpha_p r_1$
$v = \omega_p r_1$
$V_t = 2\omega_c r_2$
$a_t = 2\alpha_c r_2$

2)



friction?

$$\Sigma \vec{\tau}_b = I_b \vec{\alpha}$$

$$\left(\frac{3R}{2}\right)F = \left(\frac{mR^2}{2} + mR^2\right)\alpha$$

$$\frac{3}{2}F = \frac{3}{2}mR^2\alpha$$

$$F = mR\alpha = Ma$$

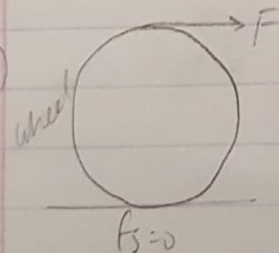
(assumption) $\Sigma \vec{F}_x = m\vec{a}_{x_{cm}}$

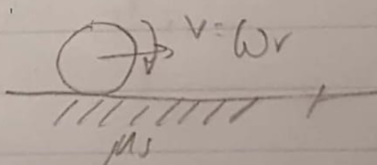
$F - f_s = m a_{cm}$

$\therefore f_s = 0$.

*No friction needed for rolling in this case

3)

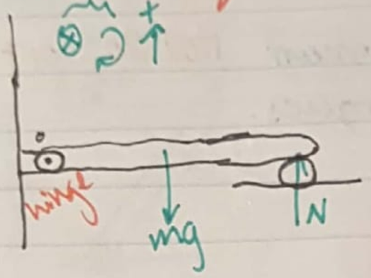


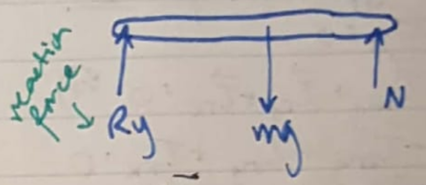


ice (no f)

Static Equillibrium:

1)





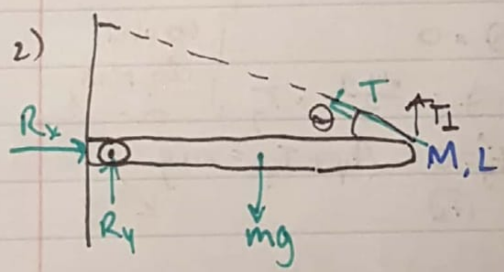
$\tau = rF\sin\theta$
$= rF_\perp$
$= r_\perp F$

$\sum \vec{\tau}_o = 0 \cdot \vec{\tau}_{mg} + \vec{\tau}_N$
$\tau_{mg} - \tau_N = 0$
$\frac{L}{2}mg - LN = 0$
$N = \frac{mg}{2}$

$\sum \vec{F}_y = 0$
$\vec{R}_y + m\vec{g} + \vec{N} = 0$
$R_y - mg + N = 0$
$R_y = mg - \frac{mg}{2} = \frac{mg}{2}$

Free hinge? Doesn't support torque at base.

2)

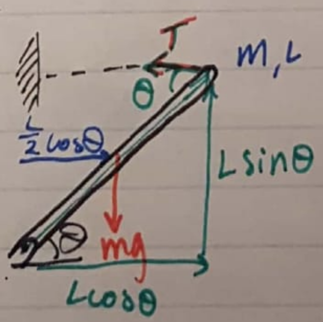


$\sum \tau_o = 0$
$\frac{L}{2}mg - LT\sin\theta = 0$
$mg = 2T\sin\theta$
$\sum \vec{F}_x = 0$
$R_x = T\cos\theta$

$\sum \vec{F}_y = 0$
$R_y + T\sin\theta = mg$

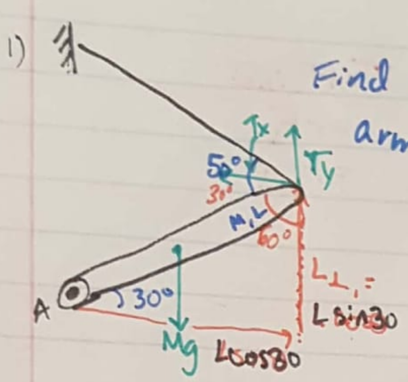
Since $T\sin\theta = \frac{mg}{2}$ and $T\cos\theta = R_x$,
$\tan\theta = \frac{mg}{2R_x}$
$R_x = \frac{mg}{2\tan\theta}$

3)



$\sum \vec{\tau}_o = 0$
$\tau_{mg} + \vec{\tau}_T = 0$
$\frac{L\cos\theta\, mg}{2} = TL\sin\theta$
$mg\cos\theta = 2T\sin\theta$

1)



Find unknown forces using moment arm approach for all torques. Nov 20, 2017

$\Sigma \tau_A = 0$

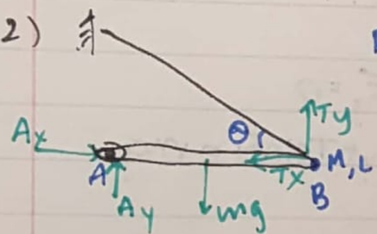
$\vec{\tau}_{mg} + \vec{\tau}_{Ty} + \vec{\tau}_{Tx} = 0$ (+)

$-\tau_{mg} + \tau_{Ty} + \tau_{Tx} = 0$

$-(\frac{L}{2}\cos 30) + (L\cos 30)T_y + (L\sin 30)T_x = 0$

$\cos 30 T_y + \sin 30 T_x = \frac{1}{2}\cos 30$

2)



Find A_y. $\Sigma \vec{\tau}_A \cdot I_A \vec{\alpha} = 0$

$\frac{L}{2} mg - LT\sin\theta = 0$

$T\sin\theta = \frac{mg}{2} = T_y$

could do this, or

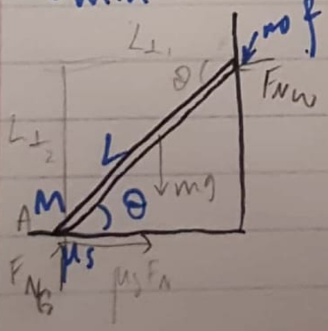
$\Sigma \vec{F}_x = 0 .\ A_x = T_x$ $\quad \Sigma \vec{F}_y = 0$ $\quad A_y + T_y = mg$

$\Sigma \vec{\tau}_B = 0$ $\quad \vec{\tau}_{Ay} + \vec{\tau}_{mg} = 0$ (+)

$-\tau_{mg} + \tau_{Ay} = -LA_y + \frac{L}{2}mg = 0$

$A_y = \frac{mg}{2}$

3) θ_{min}?



Nov 22, 2017

$\Sigma \vec{\tau}_A = 0$ (+)

$\vec{\tau}_{mg} + \vec{\tau}_{Nw} = 0$

$\tau_{mg} = \tau_{Nw}$

$mg(\frac{L}{2}\cos\theta) = N_w(L\sin\theta)$

$N_w = \frac{mg}{2\tan\theta}$ $\quad \mu_s mg = \frac{mg}{2\tan\theta}$ $\quad \tan\theta = \frac{1}{2\mu_s}$.

$\Sigma F_y = 0 \quad N_g = mg$

$\Sigma F_x = 0 \quad N_W = F_{f\,max}$

$N_w = \mu_s N = \mu_s mg$

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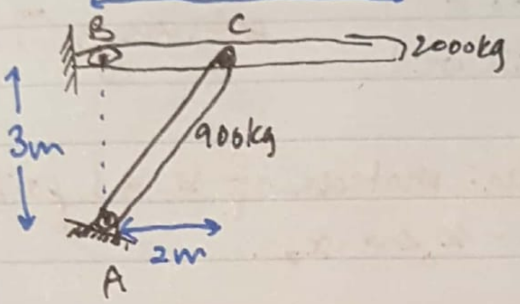
$. LN(\sin\theta\cos(\ldots) - \sin(\ldots)\cos\theta)$

$LN(\sin(20-(70)))$

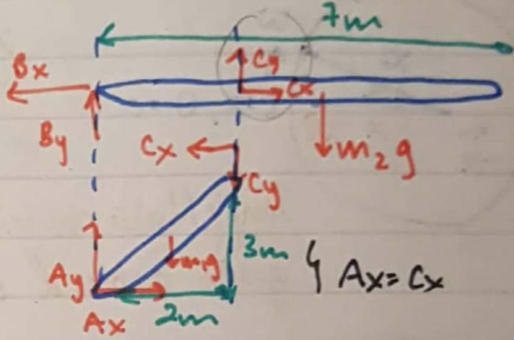
must be opposite

Support forces A, B, C

4)

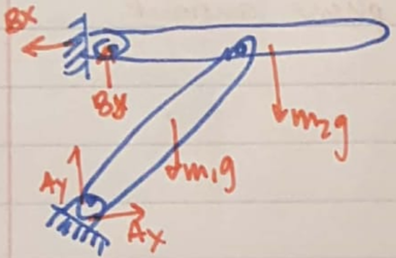


FBD:



{ Ax = Cx

Whole Body: (C is internal force)



$\Sigma \vec{\tau}_A = 0$

$\tau_{m_1g} + \tau_{m_2g} - \tau_{B_x} = 0$

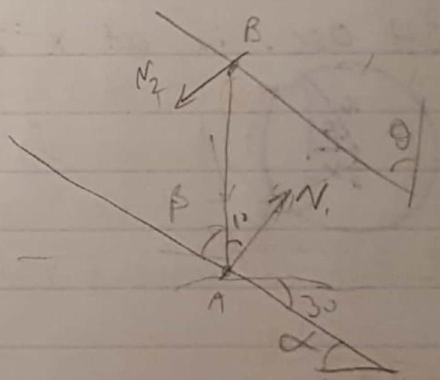
$m_1g(1) + m_2g(3.5) - B_x(3) = 0$

$B_x = 28093\,N$

$\Sigma \vec{F}_x = 0$

$B_x = A_x = C_x$.

3.



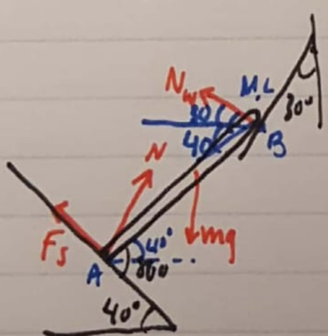
Individual Beam:

$\Sigma \vec{\tau}_A = 0$

$m_1g(1) - C_x(3) + C_y(2) = 0$

C_y: ___ . Done.

5)



$\Sigma \vec{\tau}_A = 0$

$\tau_{mg} = \tau_{N_{w(x)}}$

$mg\left(\frac{L}{2}\cos 40°\right) = N_{w(x)} L \sin 70$

N_w ✓.

$\Sigma \vec{F}_x = 0$

$-f_{sx} + N_x - N_{wx} = 0$

$-f_s\cos 40 + N\sin 40 - N_w\cos 30 = 0$

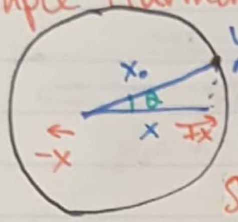
$\Sigma \vec{F}_y = 0$

$f_{sy} + N_y - mg + N_{wy} = 0$

$f_s\sin 40 + N\cos 40 - mg + N_w\sin 30 = 0$

2eqn 2unknowns

Simple Harmonic Motion:

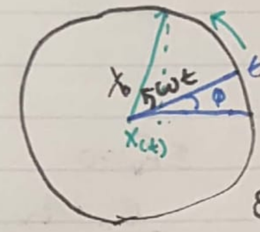


$t=0,\ \theta=0.$

$x = x_0 \cos\theta$
$= x_0 \cos(\omega t)$

*Always ↺ ω

Simple Harmonic Motion: motion of x as point goes around circle. $-x_0 \leftrightarrow x_0$.



$t=0$
$\theta=0.$

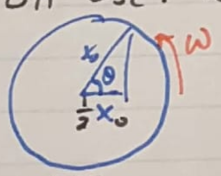
$x = x_0 \cos(\underbrace{\omega t + \varphi}_{\text{phase angle}})$ → phase angle.

φ → phase constant

$\theta_{(t)} = \omega t + \varphi$

Example:

1) A SH osc. is at $x = \frac{1}{2}x_0$ at $t=0$. $\varphi = ?$



moving away from eq ↺ (center)

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

$\omega(0) + \varphi = \frac{\pi}{3}, \frac{5\pi}{3}$

$\varphi = \frac{\pi}{3}, \frac{5\pi}{3}$

$\frac{1}{2}x_0 = x_0 \cos(\omega(0) + \varphi)$

$\varphi = \cos^{-1}(\frac{1}{2})$

$\varphi = \frac{\pi}{3}, \frac{5\pi}{3}$

* $\frac{5\pi}{3}$ is correct (Moving away from center)

Formula:

$x = x_0 (\cos(\omega t + \varphi))$

$v_{(t)} = \frac{d x_{(t)}}{dt} = \underbrace{-\omega x_0}_{v=\omega r} \sin(\omega t + \varphi)$

$+V_x = -\omega x_0 \sin(\omega t + \varphi)$

↳ At $t=0$, $\sin(\varphi) = -ve$.

$a_{(t)} = \frac{d^2(x_{(t)})}{dt^2} = \underbrace{-\omega^2 x_0}_{v^2/r = a} \cos(\omega t + \varphi)$

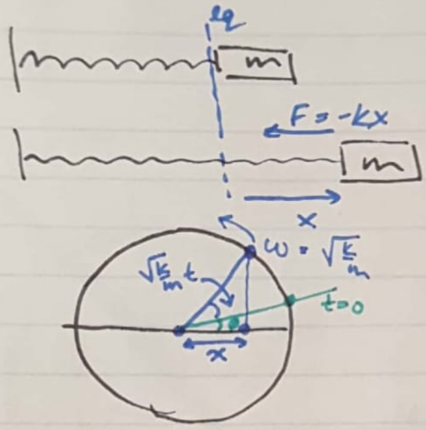
$$\underbrace{\frac{d^2 x_{(t)}}{dt^2} = -\omega^2 x_{(t)}}$$ } a always points to eq.

"Solution:" $x_{(t)} = x_0 \cos(\omega t + \varphi)$

$\left\{ \begin{array}{l} \frac{d^2 B}{dt^2} = -AB \\ B = B_0 \cos(\sqrt{A}\, t + \varphi) \end{array} \right.$

Harmonics, cont...:

Nov 27, 2017



$$-kx = ma$$

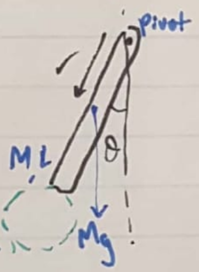
$$a = \frac{d^2x}{dt^2} = \frac{-k}{m}x$$

$$x = x_0 \cos\left(\sqrt{\frac{k}{m}}\,t + \phi\right)$$

Need to get this form:

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

Pendulums (Gravity only):



Review COM torques

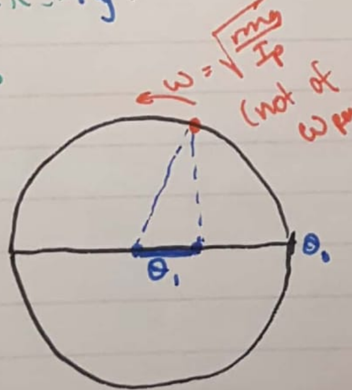
$$\Sigma \vec{\tau}_p = I_p \vec{\alpha} \quad \oplus$$

$$Mg\frac{L}{2}\sin\theta = I_p \alpha = I_p \frac{d^2\theta}{dt^2} = -rmg\sin\theta$$

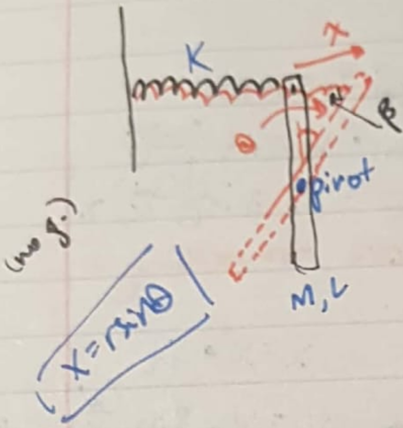
$$\frac{d^2\theta}{dt^2} = \frac{-rmg}{I_p}\sin\theta$$ For small θ, $\sin\theta \approx \theta$

$$\frac{d^2\theta}{dt^2} = \frac{-rmg}{I_p}\theta$$

$$\theta = \theta_0 \cos\left(\sqrt{\frac{rmg}{I_p}}\,t + \phi\right)$$



$$T = \frac{2\pi}{\omega}$$ since $$\omega = \frac{\Delta\theta}{\Delta t}$$



$$\sum \vec{\tau}_{0,s} = I_0 \vec{\alpha}$$

$$\vec{r} \times \vec{F}_s = I_0 \vec{\alpha}$$

$$r F_s \sin\phi = I_0 \alpha$$

$$r(-kx) \sin\phi = I_0 \alpha$$

$$\frac{-k r^2 \sin\theta \sin\phi}{I} = \frac{d^2\theta}{dt^2}$$

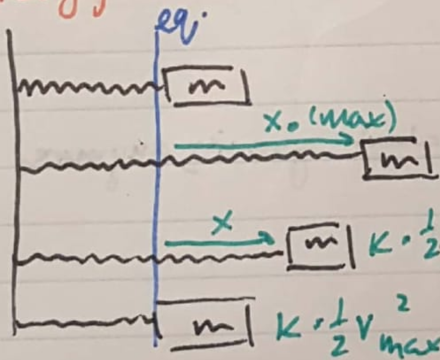
Small $\theta \Rightarrow \sin\phi = 1$.

$$\frac{d^2\theta}{dt^2} = \frac{-kr^2}{I}\theta$$

$$\theta = \theta_0 \cos\left(\sqrt{\frac{kr^2}{I}}\, t + \varphi\right)$$

Then $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{\sqrt{\frac{kr^2}{I}}}$

Energy: Nov 29, 2017



$ME_1 = ME_2 = ME_3$

$v = 0$, $K = 0$, $U = \frac{1}{2} k x_0^2$

$K = \frac{1}{2} m v^2$, $U = \frac{1}{2} k x^2$

$K = \frac{1}{2} v_{max}^2$, $U = 0$.

Example:

1) A spring-mass system does SHM where, at a point x, the energy is $\frac{1}{2}$ kinetic. x? "Half of the energy is kinetic"

$K = \frac{1}{2} ME$

$\frac{1}{2} m v^2$: Doesn't help.

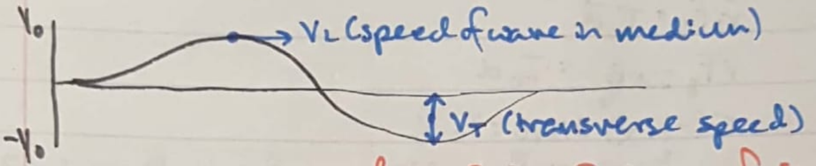
$U = \frac{1}{2} ME$ (the other half)

$\frac{1}{2} k x^2 = \frac{1}{2}\left(\frac{1}{2} k x_0^2\right)$

$x = \frac{\sqrt{2}}{2} x_0$

Scanned by CamScanner

Waves:



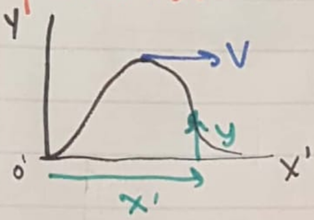
$y_{(t)} = y_0 \sin \omega t$

$y_{(x)} = y_0 \sin\left(\frac{\omega x}{v}\right)$

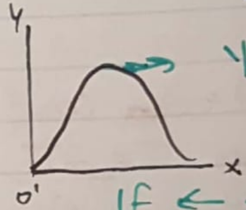
$= y_0 \sin\left[\left(\frac{\omega}{v}\right)x\right]$

from Point of view of someone travelling @ same speed as wave.

If still POV:



$y = y_0 \sin\left[\left(\frac{\omega}{v}\right)x'\right]$



$x = x' + vt$

$y = y_0 \sin\left(\frac{\omega}{v}x - \omega t\right)$

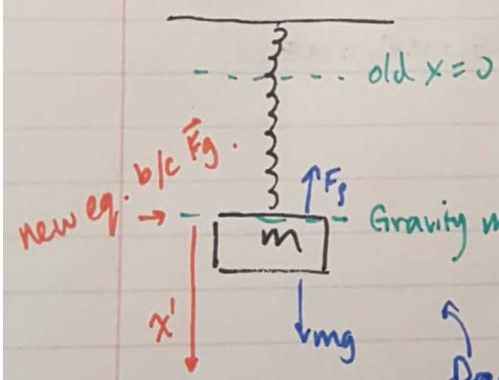
Let $k = \frac{\omega}{v}$

$y = y_0 \sin(kx - \omega t)$

If ← then $y = y_0 \sin(kx + \omega t)$

The Restoring force:

Nov 30, 2017



Gravity moved $x=0$ to $x = \frac{mg}{k}$.

(Don't worry about gravity anymore)

Derivation:

new $x=0$!

Just changed POV to new eq. point

$mg - kx = ma$

$a = g - \frac{k}{m}x$ Let $x = x' + \frac{mg}{k}$

$ma = -kx'$ $a = a'$, so (by $\frac{d^2x}{dt^2}$)

$ma = -kx$

Review Session

Nov 29, 2017

A particle (SHM) @ at time t it is at $x = \frac{1}{2}x_0$ moving away from eq. 0.4s later it has $\frac{1}{2}V_{max}$ moving towards eq. for first time. Find T.

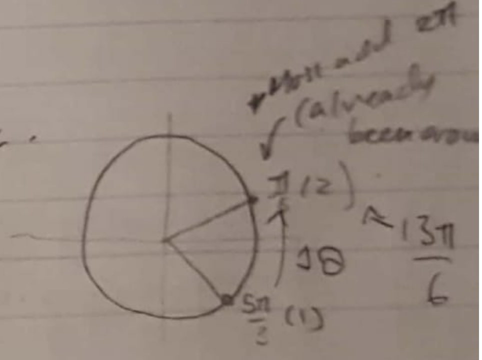
t_1: $x = x_0 \cos(\omega t_1 + \varphi)$

$\frac{1}{2} = \cos(\omega t_1 + \varphi)$ $\therefore \omega t_1 + \varphi = \cancel{\frac{\pi}{3}}$ $\frac{5\pi}{3}$ (away from eq).

$V_{(t_2)} = -\omega x_0 \sin(\omega t_2 + \varphi)$

$-\frac{1}{2}\omega x_0 = -\omega x_0 \sin(\omega t_2 + \varphi)$

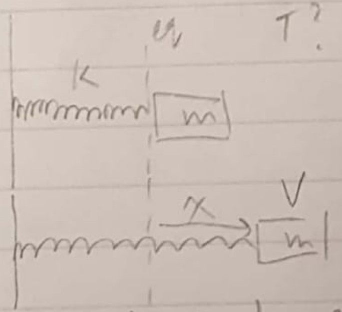
$\omega t_2 + \varphi = \frac{\pi}{6}$, $\cancel{\frac{5\pi}{6}}$ towards eq.



$\therefore$ $\omega t_2 + \varphi - \omega t_1 + \varphi = \frac{13\pi}{6} - \frac{5\pi}{3}$

$\omega(0.4) = \frac{\pi}{6}$

Shortcut: $\Delta\theta = \frac{\pi}{3} + \frac{\pi}{6}$ $\omega = \frac{\Delta\theta}{\Delta t} = \frac{\frac{\pi}{3} + \frac{\pi}{6}}{0.4}$ $T = \frac{2\pi}{\omega}$



$F = -kx = ma$

$a = \frac{-k}{m}x$

$\frac{d^2x}{dt^2} = -\frac{k}{m}x$

} Force approach

$ME = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$

$\frac{dME}{dt} = 0$ (conserved)

$kxv + mva = 0$

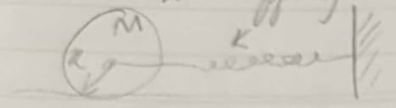
$kx = ma$ etc...

} energy approach

2100 - 169 / 640

1931 / 80 $\times 10^{-3}$

3) no slipping.



$ME = \frac{1}{2}kx^2 + \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}mv_{cm}^2$ or $\frac{1}{2}kx^2 + \frac{1}{2}I_b\omega^2$

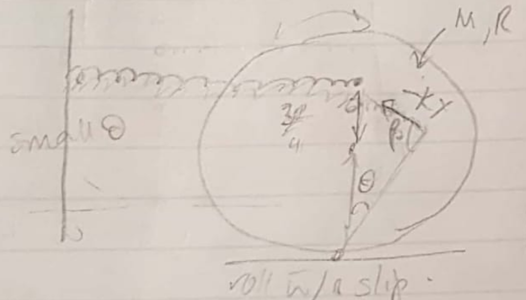
$\frac{d}{dt}ME = 0 \Rightarrow kxv_{cm} + I_{cm}\omega\alpha + mv_{cm}a_{cm} = 0$

$kx + I_{cm}\left(\frac{1}{R}\right)\left(\frac{a_{cm}}{R}\right) + ma_{cm}$

$kx + \frac{I_{cm}a_{cm}}{R^2} + ma_{cm} = 0$

$a_{cm}\left(\frac{I_{cm}}{R^2} + m\right) = -kx$

4)



$\Sigma\vec{\tau}_b = I_b\vec{\alpha}$

$\frac{7R}{4}(-kx)\sin\beta = \frac{3}{2}mR^2\alpha$

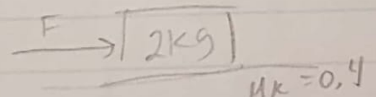
$x = \frac{7R}{4}\sin\theta$

small θ

$\left(\frac{7R}{4}\right)(-k)\left(\frac{7R}{4}\right)\sin\theta\cdot\sin\beta = \frac{3}{2}mR^2\frac{d^2\theta}{dt^2}$

$-\left(\frac{7R}{4}\right)^2 k\theta = \frac{3}{2}mR^2\frac{d^2\theta}{dt^2}$

5)



F = 70N Δt = 10ms. v?

Impulse = $\frac{\Delta \vec{p}}{\Delta t}$

$mv_f - mv_i = (F - f_k)\Delta t$

$2v_f = (70 - 2\cdot 9.8\cdot 0.4)(10^{-3})$