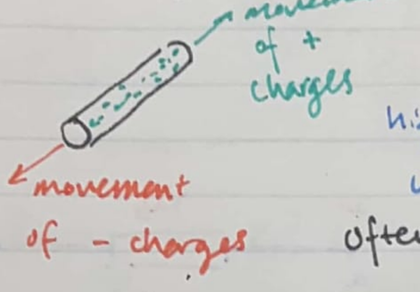
ECE 140

Jan 4, 2018

Current:

Moving charges.

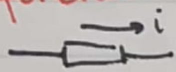
$$\boxed{i_{(t)} = \frac{dq_{(t)}}{dt}}$$ ← charge in coulomb (C)



high school way $\left\{ i = \frac{Q}{t} \right.$ in APs

often need to assume direction.

Reference Direction:



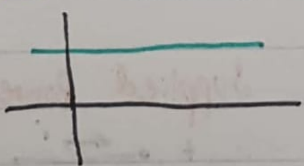
if i +, flows →
else, flows ←

Charge:

$$q_{(t)} = q(t_0) + \int_{t_0}^{t} i(\tau)\,d\tau$$

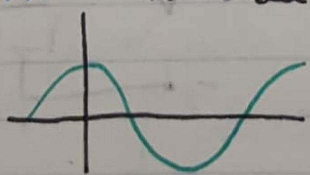
Types of Current:

1) DC - constant wrt time.



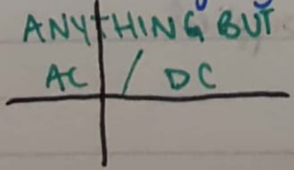
$i = 5A$

2) AC - sinusoidal



$i = 2\sin(5t)$

3) Time Varying:



$i = 5e^{-2t}$

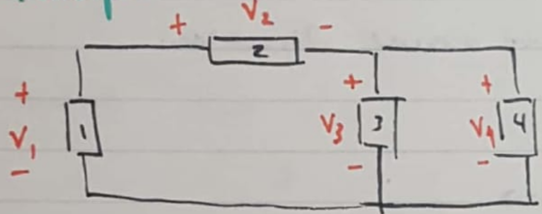
Voltage:

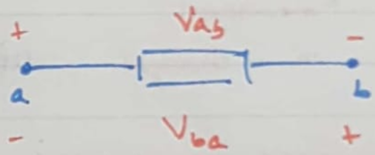
$V_{(t)} = \frac{dW_{(t)}}{dq}$ ← Energy (J), ← charge in Coulombs (C)

$\left(\frac{E}{Q} \text{ from high school}\right)$

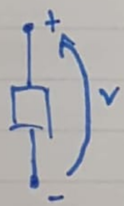
AKA emf, PD

Example:





$V_{ab} = -V_{ba}$



← Also sometimes used.

Power:

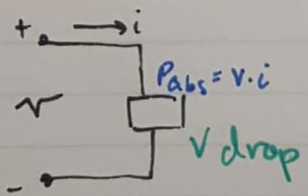
Jan 8, 2018

$p(t) = \frac{dw}{dt}$ ← J $= \frac{dw}{dq} \cdot \frac{dq}{dt} = v \cdot i$

↑ watts

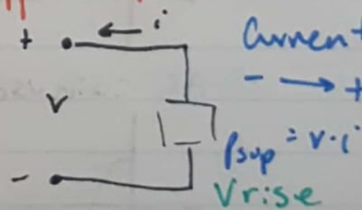
Absorbed Power:

Current + → - of V



$P_{abs} = -P_{sup}$

Supplied Power:



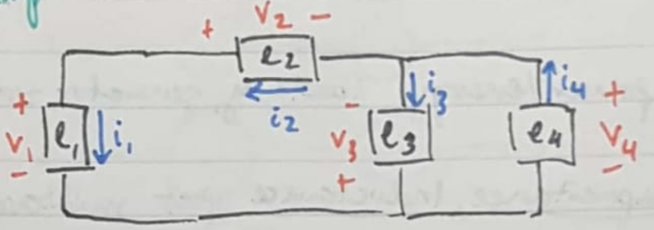
Current - → + of v

Passive sign convention ↑

+ → i → -

If $P_{abs} > 0$, actually absorbed. < 0 supplied.

Example:



$i_1 = i_2 = 5A$ $\quad V_1 = 8V$

$i_3 = -2A$ $\quad V_2 = -2V$ $\quad V_4 = 10V$

$i_4 = 3A$ $\quad V_3 = -10V$

a) Power for e_1? Not specified? Assume abs.

$P_{abs} = V_1 i_1 = 40W$ $\quad$ +40, so actually abs.

b) P_{abs} for e_2?

$P_{abs} = -V_2 i_2 = 10W$ $\quad$ +10, so actually abs.

c) P_{sup} for e_3?

$P_{sup} = V_3 i_3 = 20W$ $\quad$ +20, so actually sup.

4) P_{abs} for e_4?

$P_{abs} = -V_4 i_4 = -30W$ $\quad$ -30, so actually sup.

Conserved

Conservation of Power/Energy:

$$\Sigma P_{abs} = \Sigma P_{sup} \qquad \Sigma P = 0$$

Energy:

$$w = w(t_2) - w(t_1) = \int_{t_1}^{t_2} p(\tau)d\tau$$ number

Assume $t_1 = -\infty$, $w(t_1) = 0$.

$$w(t) = \int_{-\infty}^{t} p(\tau)d\tau$$ function of t.

Circuit Elements: Jan 9, 2018

1) Active:

↳ Generates power/energy (battery, generator, transistor)

2) Passive:

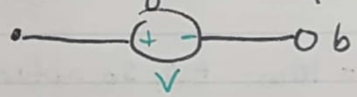
↳ Resistance, Capacitance, Inductance post midterm.

↳ Absorbs / stores energy.

I, V independent of each other

3) Independent Voltage Source:

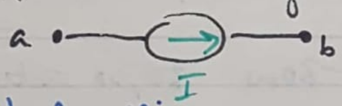
↳ Specific v. regardless of i. (Not constant, but independent)



$V_{ab} = V$, no matter i

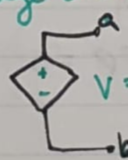
4) Independent Current Source:

↳ Specific current regardless of v. (" ")



5) Dependent Sources:

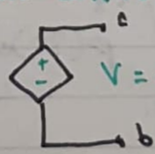
a) Voltage-controlled Voltage Source:



$V = \alpha V_x$ α = constant. V_x somewhere in rest of circuit.

control V.

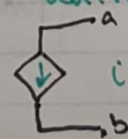
b) Current-controlled Voltage Source:



$V = r \cdot i_x$ Same deal

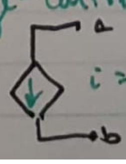
control current

c) Current-controlled Current Source:



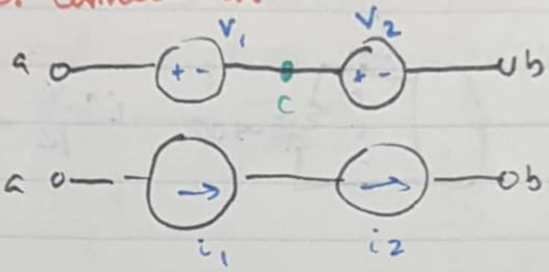
$i = \beta \cdot i_x$ Same

d) Voltage-controlled Current Source:



$i = g \cdot V_x$ Same

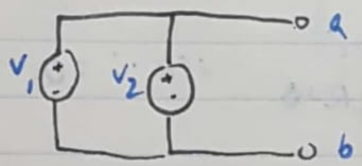
Series Connection:



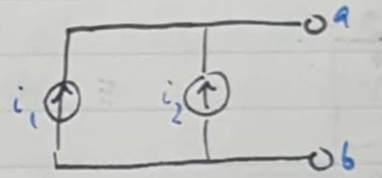
Only v_1, v_2 connected at c.

i_1 should $= i_2$ (else invalid)

Parallel connection:



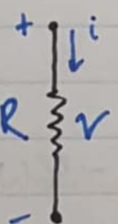
v_1 should $= v_2$
(else invalid)



valid

"Parallel" - connected btwn same two nodes (a, b).

Resistance and Ohm's Law:

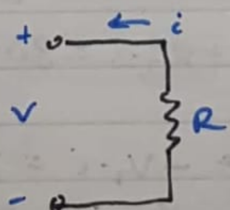


$\boxed{v = R \cdot i}$ $\quad R \geq 0$.

$\hookrightarrow \; i = \frac{1}{R} \cdot v$

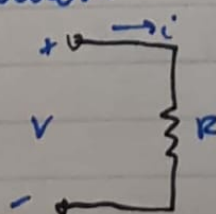
$= G \cdot v$ where G = conductance in Siemens (S or Ω^{-1})

Note:



$v = -R \cdot i$ for this
(watch for current direction)

Power:



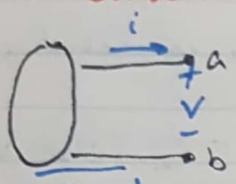
$P_{abs} = v \cdot i$

$= R \cdot i^2$

$= \frac{v^2}{R}$

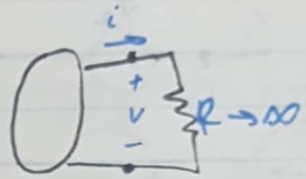
≥ 0 b/c of ^{2} } resistors always absorb power.

Open Circuit: Jan 10, 2018



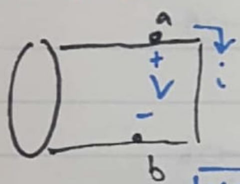
$\boxed{i = 0}$

$\equiv$



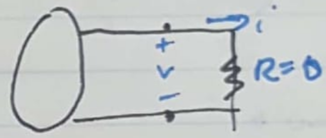
$i = \frac{V}{R} \to 0.$

Short circuit:



$\boxed{v = 0}$

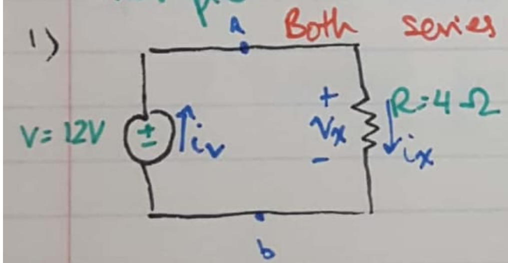
$\equiv$



$V = R \cdot i = 0.$

Example:

1) Both series + parallel



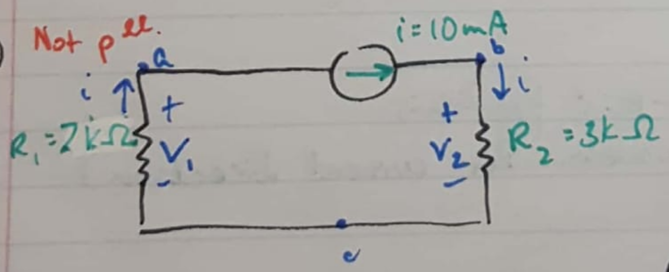
$V_x = 12V$ (−12V if $\overset{-}{}\,\overset{+}{}$)

$i_x = \frac{V_x}{R} = 3A$

$i_v = i_x = 3A.$

$P_{R,abs} = 12 \cdot 3 = 36W$ and $P_{v,abs} = -V \cdot i = -36W.$

2) Not p ll.



$V_1 = -R_1 \cdot i = -20V$

$V_2 = R_2 \cdot i = 30V$

$P_{R_1 abs} = -V_1 \cdot i = 200mW$

$P_{R_2 abs} = V_2 \cdot i = 300mW$

Kirchhoff's Current Law (KCL):

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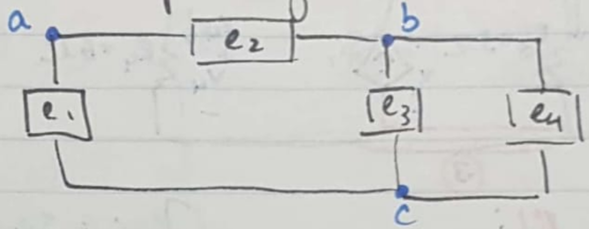
A node: pt or junction where 2+ elements connected.

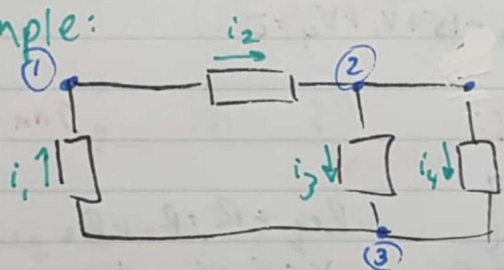


KCL: Sum of all currents at any node = 0.

$\boxed{\sum_n i_n = 0}$ $\quad$ i enter = i leaving node.

Example:

1)



1) $i_1 = i_2$
2) $i_2 = i_3 + i_4$
3) $i_3 + i_4 = i_1$

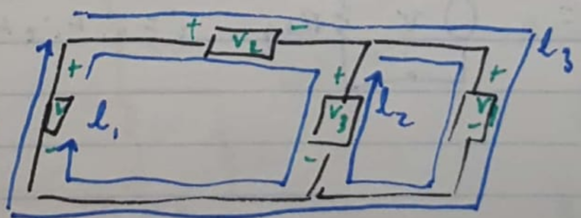
Note linear independence/dependence

N nodes give N-1 independent KCL eqn.

Kirchhoff's Voltage Law (KVL):

A loop: closed path where no element is encountered more than once.

mesh: loop w/o any loops inside it.



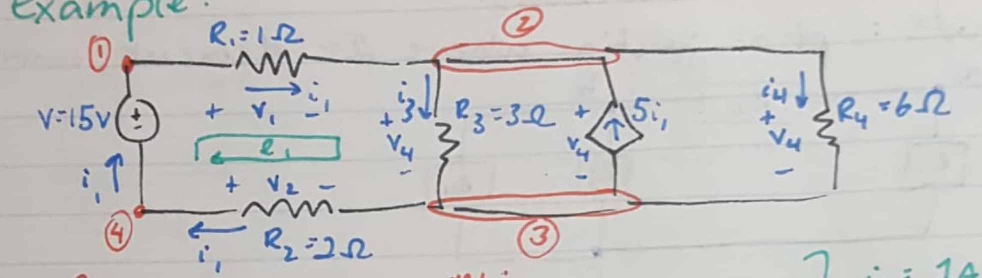
1) $V_1 = V_2 + V_3$
2) $V_3 = V_4$
3) $V_2 + V_4 = V_1$

KVL: Sum of all voltages around any loop is 0.

$\boxed{\sum_n V_n = 0}$ $\quad$ $V_{drops} = V_{rises}$

N meshes give N independent KVL eqn.

Example:



OL:

$V_1 = R_1 i_1$

$V_2 = -R_2 i_1$

$V_4 = R_3 i_3$

$V_4 = R_4 i_4$

KCL:

Only need node 2

$i_1 + 5i_1 = i_3 + i_4$

KVL: Only need l_1

$-V_2 - 15 + V_1 + V_4 = 0$

$i_1 = 1A \quad V_1 = 1V$

$i_4 = 2A \quad V_2 = -2V$

$i_3 = 4A \quad V_4 = 12V$

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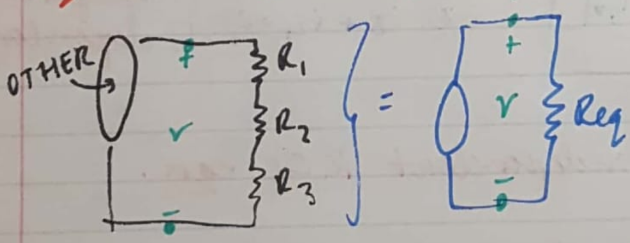
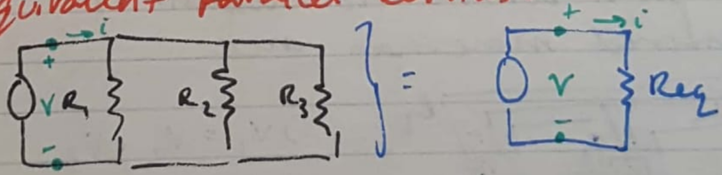
Equivalent Series Resistors:



$R_{eq} = R_1 + R_2 + R_3$

$v = v_1 + v_2 + v_3$

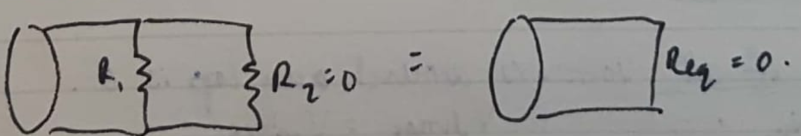
Equivalent Parallel Resistors:



$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

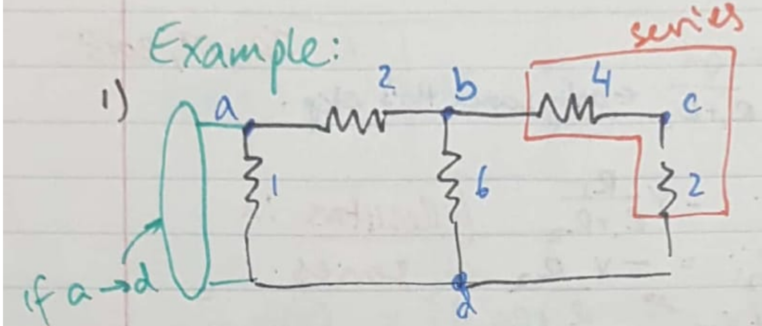
$i = i_1 + i_2 + i_3$

Note:



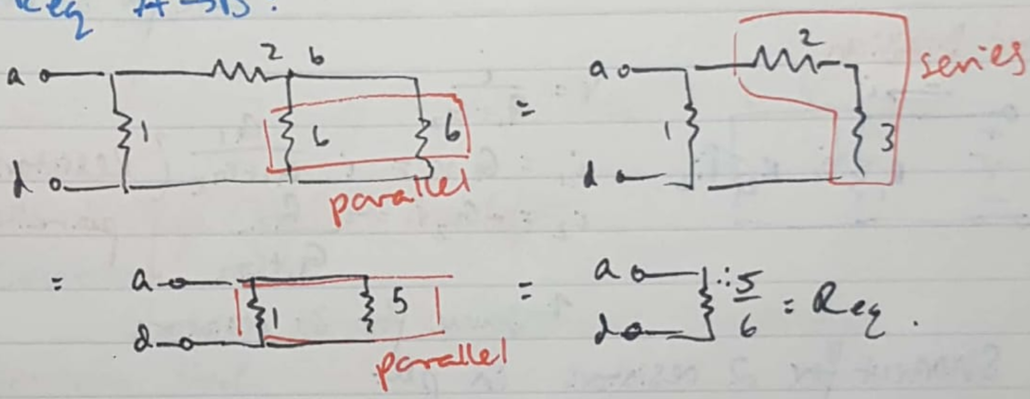
Example:

1)

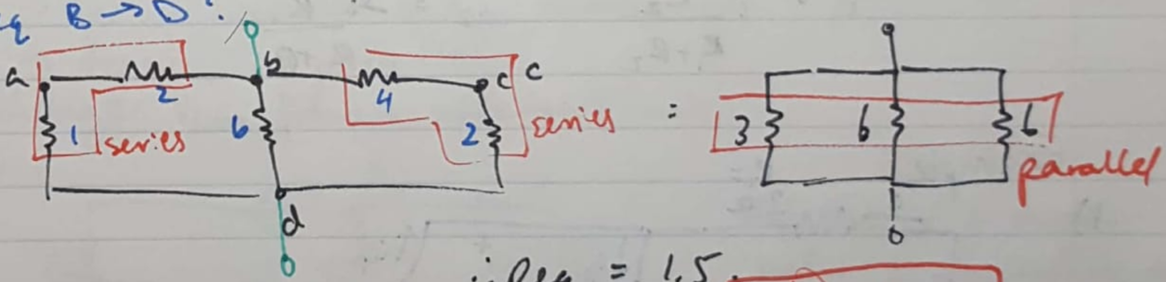


Should draw rest of circuit on left to show 1,2 not in series.
→ depends on nodes you want (a,d)

a) R_{eq} A→D?

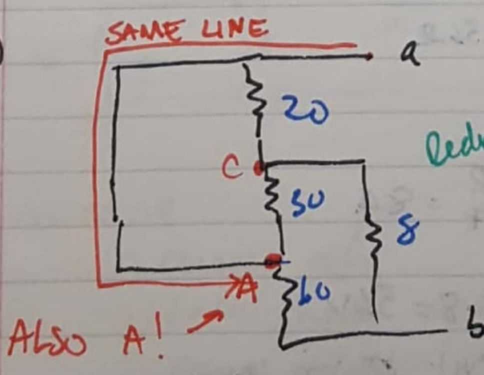


b) R_{eq} B→D?

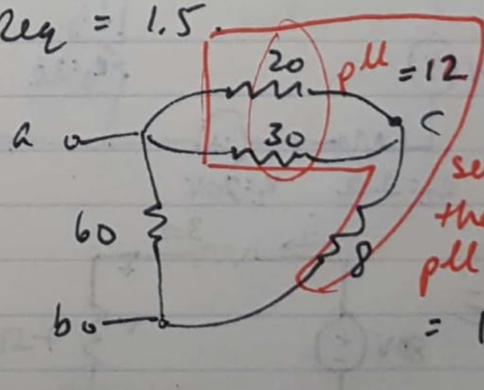


$\therefore R_{eq} = 1.5$.

2)



Redraw:



series, = 20
then
pll w/ 60
= 15 = R_{eq}.

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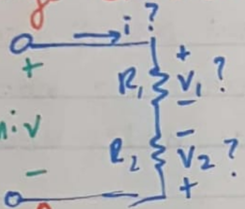
Voltage Division:

Given: v



$i = \frac{v}{R_1 + R_2}$ ← only saves this step.

$v_1 = R_1 i = v \frac{R_1}{R_1 + R_2}$

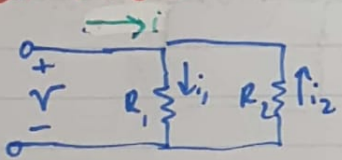
$v_2 = -R_2 i = -v \frac{R_2}{R_1 + R_2}$

based on +V−

Resistors in series.

↑ same for 3+ resistors

Current Division:



$v = \frac{i}{G_1 + G_2}$

$i_1 = G_1 v = i \frac{G_1}{G_1 + G_2}$

$i_2 = -G_2 v = -i \frac{G_2}{G_1 + G_2}$

Resistors in parallel

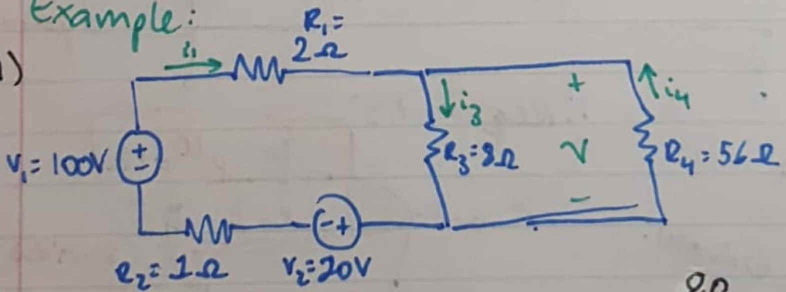
↑ same for 3+ resistors

Shortcut for 2 resistors in ∥:

$i_1 = i \frac{R_2}{R_1 + R_2}$ $\qquad i_2 = -i \frac{R_1}{R_1 + R_2}$

Example:

1)



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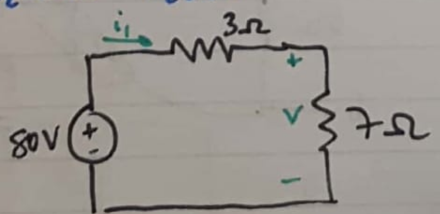


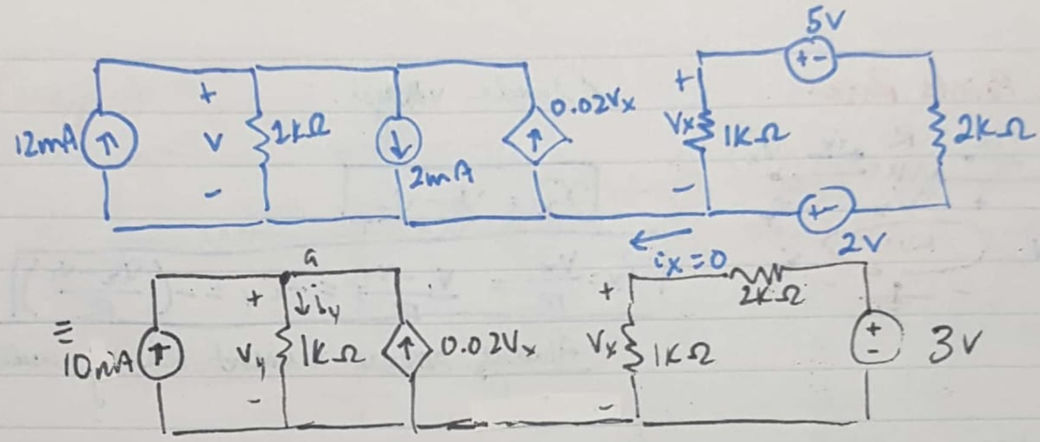
$i_1 = \frac{80}{3+7} = 8A$

$v = 7 \cdot 8 = 56V$

Now, just Ohm's law.

Or, CD: $i_3 = i_1 \cdot \frac{R_4}{R_3 + R_4}$ $\qquad i_4 = -i_1 \cdot \frac{R_3}{R_3 + R_4}$

2)



By VD: $V_x = 3 \cdot \frac{1}{1+2} = 1V$ Then KCL $V_y = 30V$

Node-Voltage (Nodal) Analysis:

Essential Node:

→ Node where 3+ elements connected.

Reference Node:

→ One of essential nodes chosen arb.

→ Assumed 0 potential.

Node Voltage: V_{drop} from a node → ref. node.

Example:

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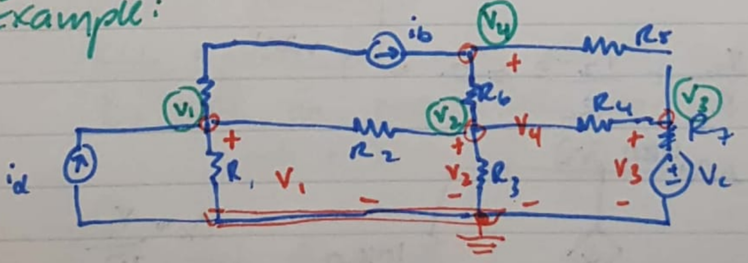
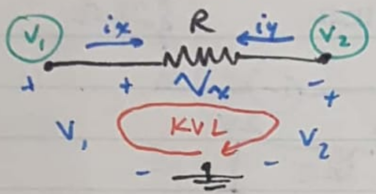


# of node V's @ essential nodes = # of ind. eqns. (4)

Procedure:

1) choose ref.
2) Label node voltages
3) Assign currents to branches (optional)
4) KCL at each non-ref essential node.
5) Ohm's law + KVL to get all I in terms of Voltages

Three Points Rule:



node voltages.

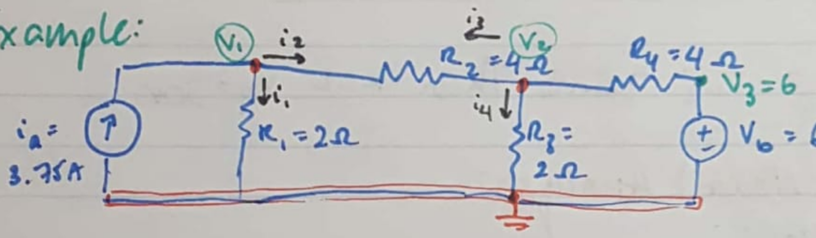
$$\boxed{V_x = V_1 - V_2}$$

$$\boxed{i_x = \frac{V_x}{R} = \frac{V_1 - V_2}{R} = -i_y = -\left(\frac{V_2 - V_1}{R}\right)}$$

↳ Always assume current leaving node

Example:

1)



KCL 1: $3.75 = i_1 + i_2$

$i_1 + i_2 = -3.75$ ← enter = -ve, leave = +ve

3pt rule $\frac{V_1}{2} + \frac{V_1 - V_2}{4} = 3.75$

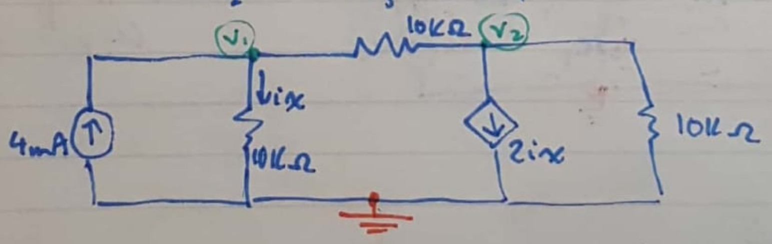
$0.75V_1 - 0.25V_2 = 3.75$

KCL2:

$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_2 - 6}{R_4} = 0.$

can solve now.

2)



$i_x = \frac{V_1}{10}$

KCL 1: $-4mA + \frac{V_1}{10k} + \frac{V_1 - V_2}{10k} = 0$

$\frac{2}{10}V_1 - \frac{1}{10}V_2 = 4$

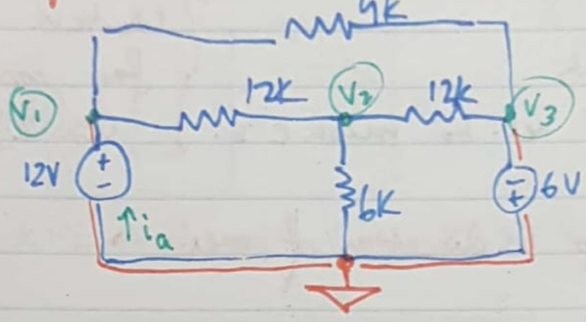
KCL2: $\frac{V_2 - V_1}{10} + 2i_x + \frac{V_2}{10} = 0$

Solve.
$V_1 = 16V$
$V_2 = -8V$

Exceptions:

Jan 22, 2018

1)



can't write current w/o resistance

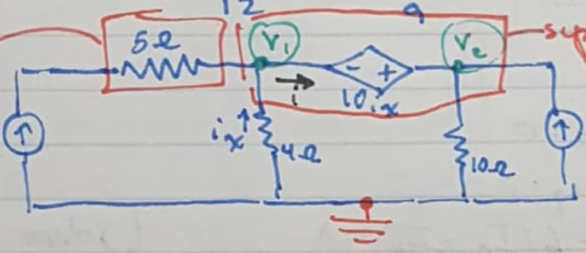
$V_1 = 12V$ $V_3 = -6V$ and KCL for 2

$V_2 = 1.6V$

To get i_a, KCL v_1:

$$i_a = \frac{V_1 - V_2}{12} + \frac{V_1 - V_3}{9} = 2.875 \text{ mA}$$

2)



$i_x = \frac{-V_1}{4}$

has no effect on V_1, V_2, etc...

KCL 1: $-4 + \frac{V_1}{4} + i = 0$

KCL 2: $-i + \frac{V_2}{10} - 2 = 0$

$V_2 - V_1 = 10\left(\frac{-V_1}{4}\right)$. Now solve.

Supernode Shortcut:

KCL applies for any closed surface.

Do KCL on supernode to avoid using i.

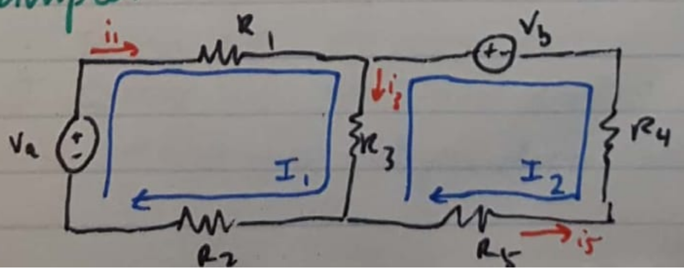
$$-4 + \frac{V_1}{4} + \frac{V_2}{10} - 2 = 0.$$

Mesh Current Analysis:

Jan 23, 2018

Mesh Current - Imaginary current, one direction + value.

Example:



$I_1 I_2$ mesh currents

branch currents

$i_1 = I_1$ $i_3 = I_1 - I_2$ $i_5 = -I_2$

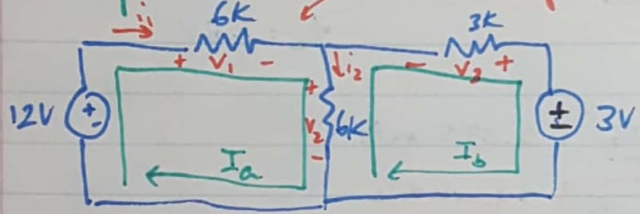
Procedure:

1) Draw + label mesh c's
2) KVL around each mesh
3) O'L and KCL to get V rel. to mesh c's.

} Watch for control (ex 2)

Example: Assume Vdrop in direction of mesh →.

1)



KVL I_a: $-12 + V_1 + V_2 = 0$

$-12 + 6i_1 + 6i_2 = 0$

$-12 + 6I_a + 6(I_a - I_b) = 0$

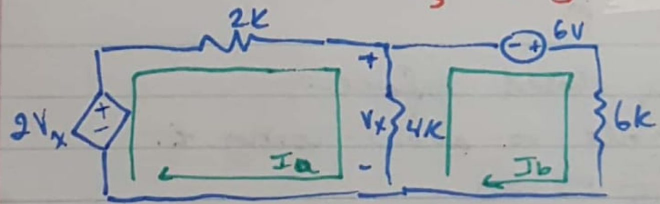
KVL I_b (short form): $6(I_b - I_a) + 3I_b = -3$

} solve for $I_a = \frac{5}{4}$ mA, $I_b = \frac{1}{2}$ mA

Can solve e'thang w/ I_b, I_a (ex: $i_2 = I_a - I_b$)

→ $V_3 = -3I_b$

2)



$V_x = 4(I_a - I_b)$

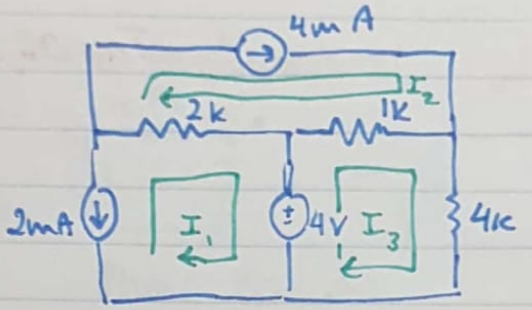
KVL I_a: $-2V_x + 2I_a + V_x = 0$

KVL I_b: $4(I_b - I_a) - 6 + 6I_b = 0$.

} solve for $I_a = 6$mA, $I_b = 3$mA

Jan 24, 2018

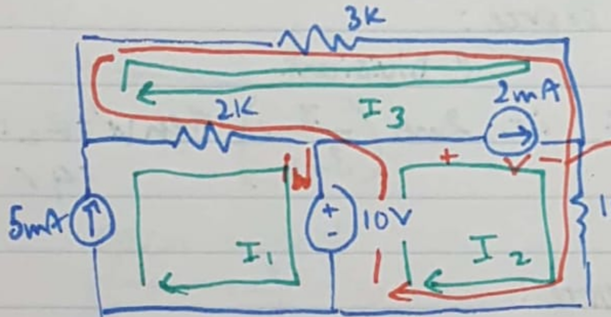
3)



★ $I_1 = -2mA$
$I_2 = 4mA$ } Don't need KVL for these.

KVL 3: $-4 + (I_3 - I_2) + 4I_3$

4)



$I_1 = 5mA$

KVL 2: $-10 + V + I_2 = 0$
KVL 3: $3I_3 - V + 2(I_3 - I_1) = 0$
$I_2 - I_3 = 2$

} $I_2 = 5mA$
$I_3 = 3mA$

Supermesh Shortcut:

KVL: $-10 + 2(I_3 - I_1) + 3I_3 + I_2 = 0$ (KVL 2 + KVL 3)

$i_b = I_1 - I_2 = 0 \Rightarrow$ removing this branch makes no dif.

Principle of Superposition:

Current/Voltage in a linear circuit can be found as the alg. sum of the indv. contributions of each indep. source acting alone.

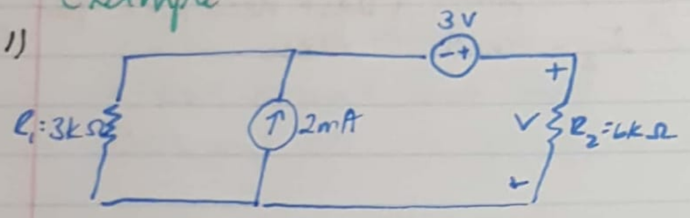
Deactivating Indep. Sources:

V source = 0. $\Rightarrow$ Short circuit.
I source = 0. $\Rightarrow$ Open circuit.

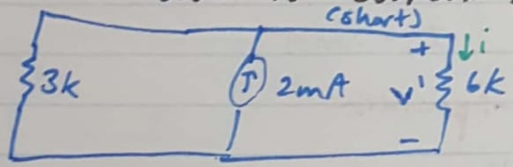
★ Not for dependent sources!.!!!!

Example: Jan 26, 2018

1)



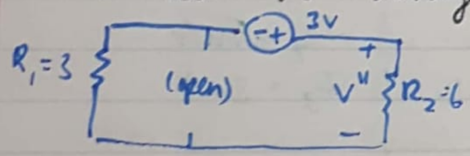
V' due to current source:



C Division:

$i = 2mA\left(\frac{3}{3+6}\right)$ then $V' = R_2 i$
$= 4V$

V'' due to voltage source:

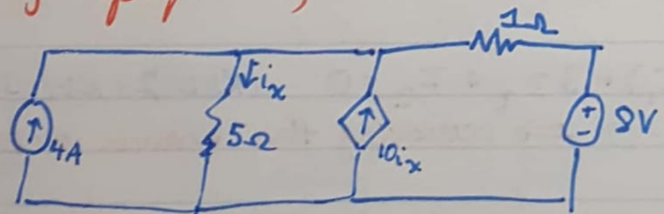


V division:

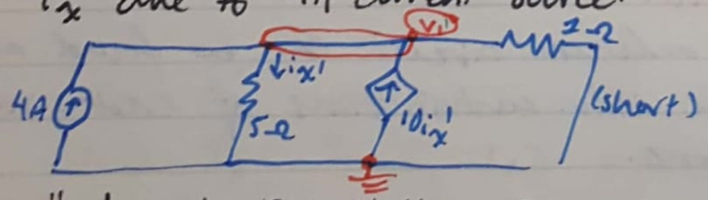
$V'' = \frac{(3)(R_2)}{(R_1+R_2)} = 2V$

By superposition, $V = V' + V'' = 6V$

2)



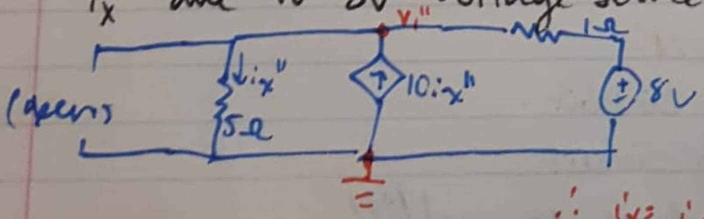
i_x' due to 4A current source:



KCL V_1': $-4 + \frac{V_1'}{5} - 10 \cdot \frac{V_1'}{5} + V_1' = 0$

$V_1' = -5V$, $i_x' = -1A$

i_x'' due to 8V voltage source:



KCL V_1'': $\frac{V_1''}{5} - 10 i_x'' + \frac{V_1'' - 8}{1} = 0$

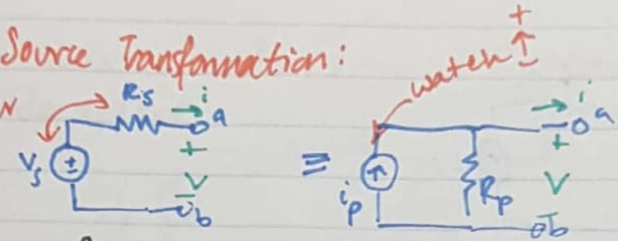
$V_1'' = -10V$, $i_x'' = -2A$

$\therefore i_x = i_x' + i_x'' = -3A$ by superposition.

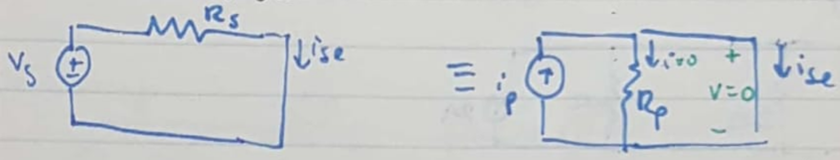
Scanned by CamScanner

Source Transformation:

order doesn't matter

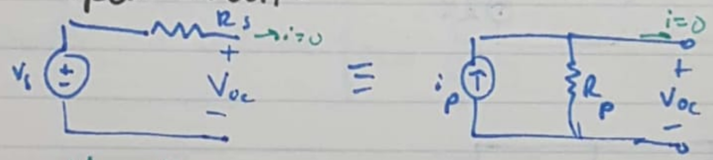


Short Circuit:



$i_{sc} = \frac{V_S}{R_S}$ $\qquad$ $i_{sc} = i_p$ $\quad$ $\therefore i_p = \frac{V_S}{R_S}$

Open circuit:

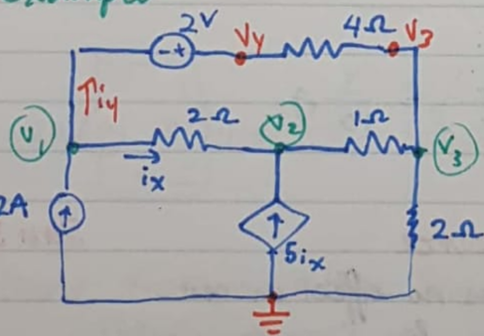


$V_{oc} = V_S$ $\qquad$ $V_{oc} = R_p \cdot i_p$ $\quad$ $\therefore R_p = R_S$

unrelated

just practice

Example:



$V_y = 2 + V_1$ $\qquad$ $i_x = \frac{V_1 - V_2}{2}$

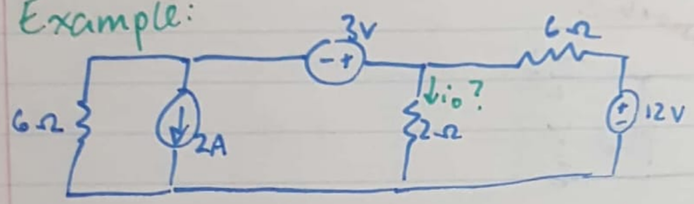
KCL1: $-2 + \frac{V_1 - V_2}{2} + \frac{V_1 + 2 - V_3}{4} = 0$

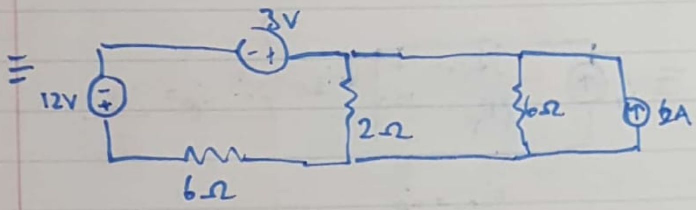
KCL2: $-5i_x + V_2 - V_3 - i_x = 0$

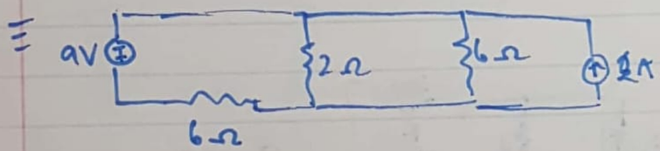
KCL3: $\frac{V_3}{2} + V_3 - V_2 + \frac{V_3 - V_1 - 2}{4} = 0$

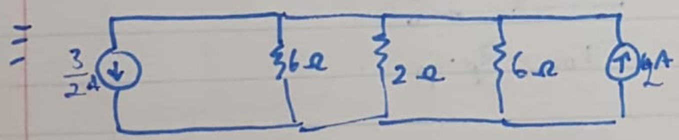
Scanned by CamScanner

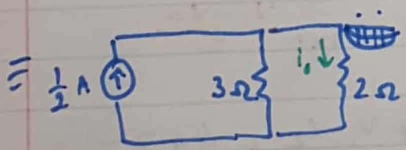
Example:











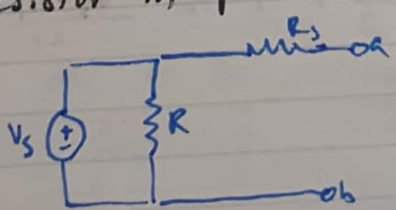
c' Division:

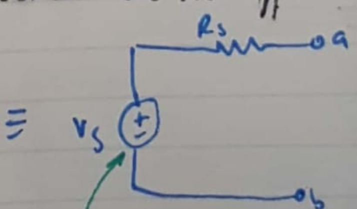
$$i_o = \frac{1}{2}\left(\frac{3}{5}\right) = \frac{3}{10} A$$

Resister in p^{ll} w̄ Voltage source

Jan 31, 2018

Resistor in p^{ll} w̄ V source has no effect on rest.

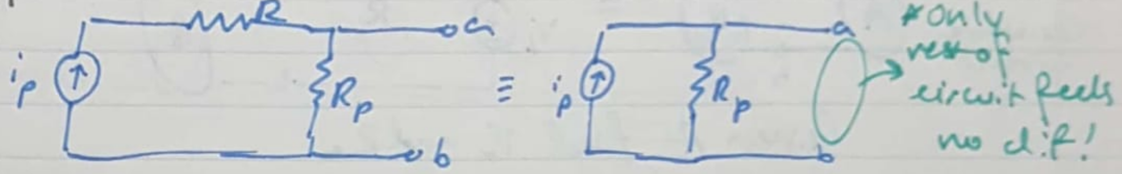




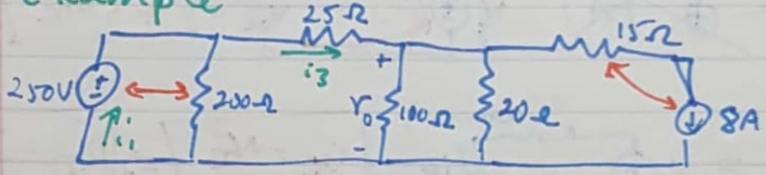
* has dif current, but rest of circut has no dif.

Resistor in Series with Current Source:

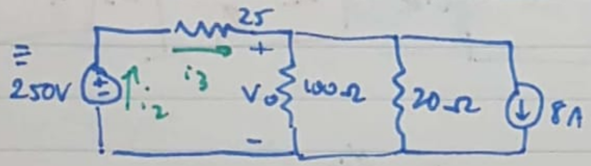
No effect



Example:

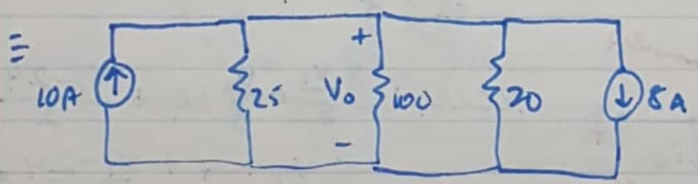


Find V_0 w̄ source transformation.



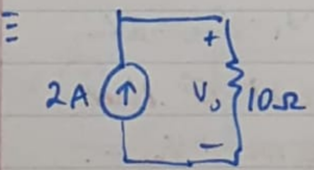
Now, source transformation.

$\boxed{i_1 \neq i_2}$!!!!



$i_3 = \frac{250-20}{25}$

$i_1 = i_3 + \frac{250}{200}$



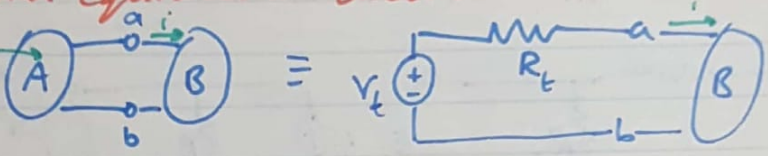
∴ $V_0 = 20V$

Source Transformation Can't always be Used:

→ When there's no V_{source} in series w̄ R or I_{source} in p^{ll} w̄ R.

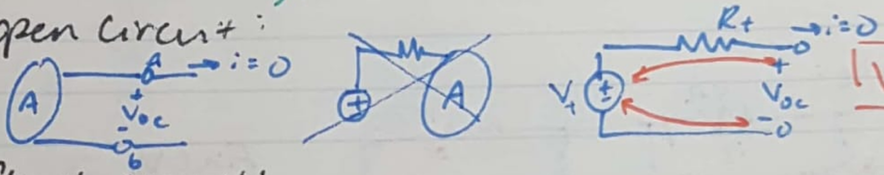
Thevenin Equivalent circuits:

Linear Resistive Circuit



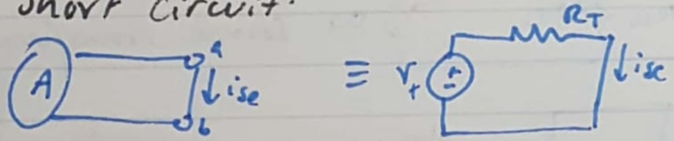
Given A, find V_t and R_t

open circuit:



$\boxed{V_t = V_{oc}}$

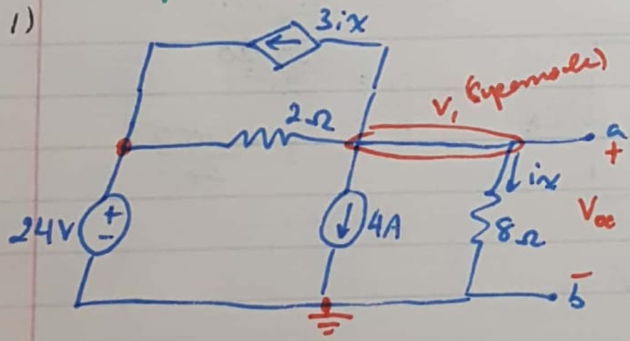
Short circuit:



$\boxed{i_{sc} = \frac{V_t}{R_t}} \Rightarrow \boxed{R_T = \frac{V_{oc}}{i_{sc}}}$

Example:

1)



$i_x = \frac{V_1}{8}$.

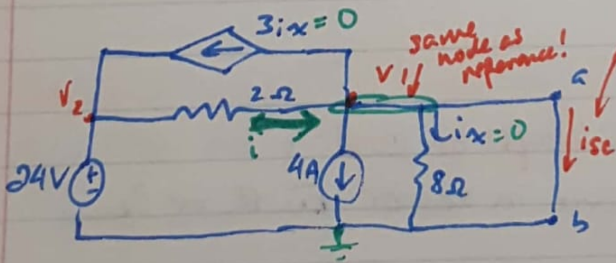
KCL V_1: $3\frac{V_1}{8} + \frac{V_1}{8} + 4 + \frac{V_1 - 24}{2} = 0$

$\frac{1}{2}V_1 + 4 + \frac{V_1 - 24}{2} = 0$

$V_1 = V_{oc} = 8V$.

Redraw circuit w/ short.

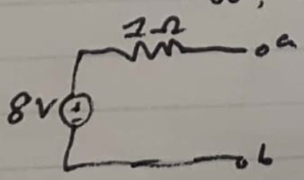
direction based on $+V_{oc}-$.



$i = \frac{V_2 - V_1}{2}$ But $V_1 = 0$ (short). $= 12A$

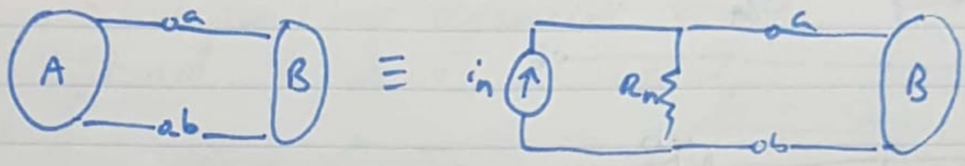
KCL ①: $i_{sc} = i - 4 - 0 - 0$. $i_{sc} = 8A$.

Now, $R_T = 1\Omega$.

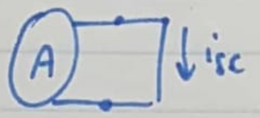


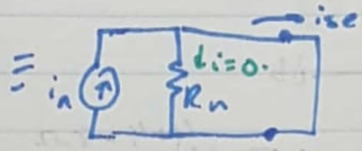
Norton Equivalent Circuit:

Feb 2, 2018.



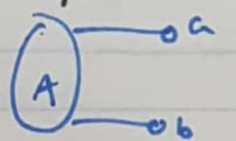
Short:

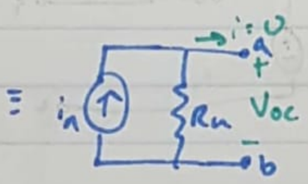




$\boxed{i_n = i_{sc}}$

Open:

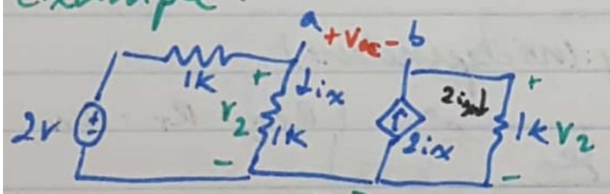




$V_{oc} = R_n \cdot i_{sc}$

$\boxed{R_n = \frac{V_{oc}}{i_{sc}}}$ Same as thev.

Example:

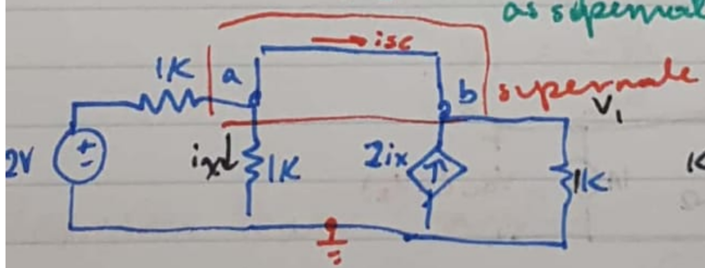


$\vec{i} = 0$ (take right half as supernode)

$V_1 = \frac{(2)(1)}{(2)} = 1V$ (V' division)

$i_x = \frac{1V}{1k} = 1mA$. $\therefore V_{oc} = V_1 - V_2$

$V_2 = (1k)(2i_x) = 2V$. $= -1V$



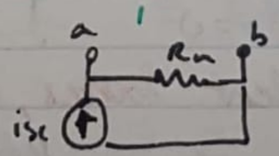
$i_x = \frac{V_1}{1} = V_1$

KCL V_1: $\frac{V_1 - 2}{1} + \frac{V_1}{1} - 2\frac{V_1}{1} + \frac{V_1}{1} = 0$

$V_1 = 2V$.

KCL: $i_{sc} = \frac{2 - V_1}{1} - \frac{V_1}{1} = -2mA$. $\Rightarrow R_n = \frac{-1}{-2mA} = 0.5k\Omega$

$\equiv$

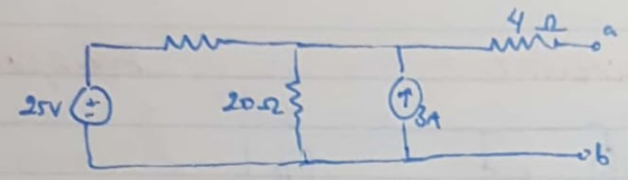


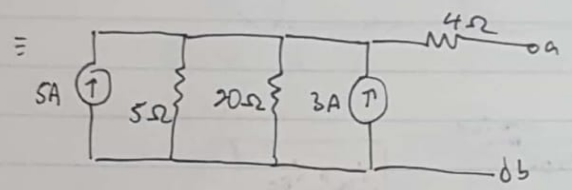
Midterm:

→ up to source transformation

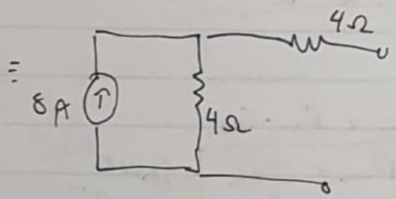
2)

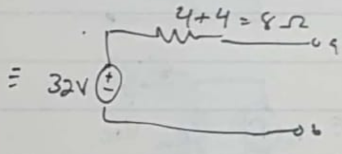
Feb 4, 2018





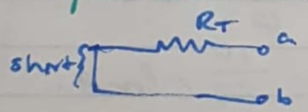
Source Transformation

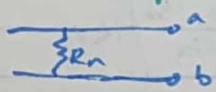




Finding R_T or R_N with R_{eq}:

→ Independent sources only. (no dependent).

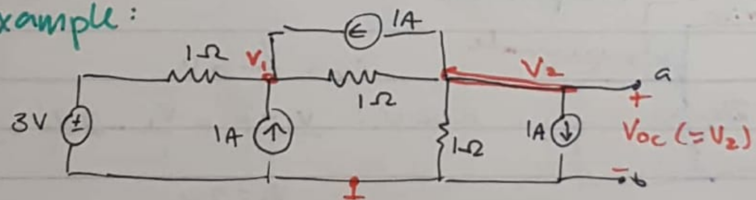




$\Big\}\ R_{eq} = R_T = R_N$

Example:

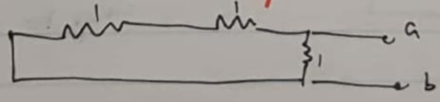
1)



$$KCL\ 1: \ -1-1+\frac{V_1-3}{1}+\frac{V_1-V_2}{1}=0$$
$$KCL\ 2: \ 1+1+\frac{V_2}{1}+\frac{V_2-V_1}{1}=0$$

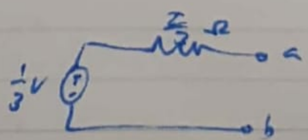
$\Big\}\ V_{oc} = V_2 = \frac{1}{3}V.$

Since only independent sources, $R_T = R_{eq}$. Deactivate sources.



$R_{eq} = \frac{2}{3}\ \Omega$

∴ T. equiv:



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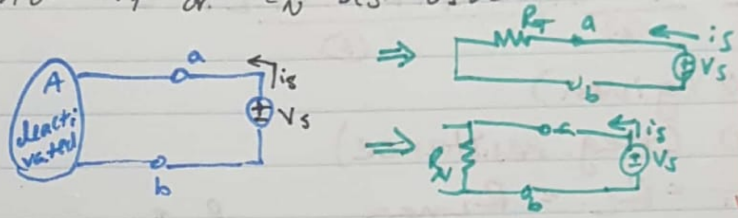
Finding R_T or R_N with External Sources:

↳ Independent/dependent sources.

↳ Find V_T or i_N as usual.

Can also do ⊕i_s, then find V_s

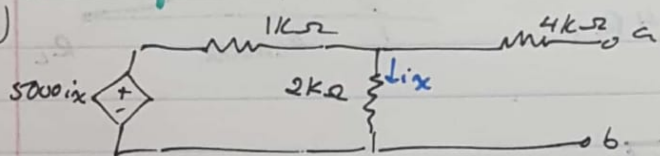


$$\boxed{R_T = \frac{V_s}{i_s}}$$

$$\boxed{R_N = \frac{V_s}{i_s}}$$

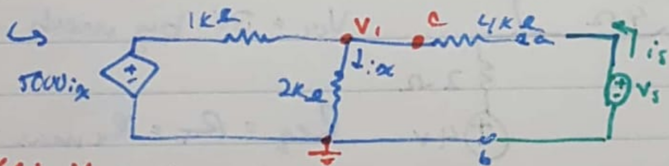
Example:

1)



$V_{oc}, i_{sc} = 0$ (no ind. sources)

↳



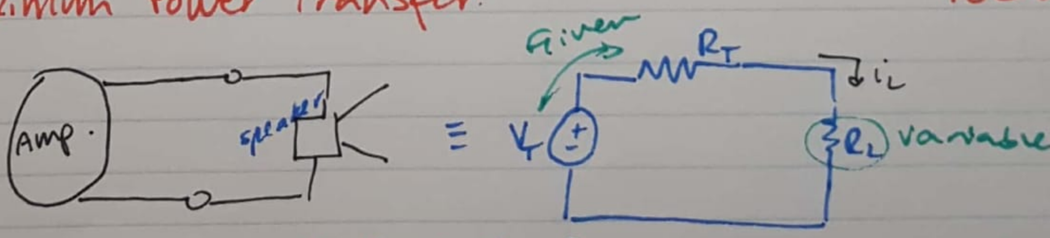
R_T' c→B = ~~1k Ω~~ :ö

KCL V_1: $\frac{V_1 - 5000 i_x}{1k} + \frac{V_1}{2k} + \frac{V_1 - V_s}{4} = 0$ { Sub $i_x = \frac{V_1}{2k}$

↳ $V_1 = -\frac{V_s}{3}$, $i_s = \frac{V_s - V_1}{4}$ ⇒ $\frac{V_s}{i_s} = 3k\Omega = R_T$

Maximum Power Transfer:

Feb 6, 2018



Given A, find R_L for max power

$$\boxed{i_L = \frac{V_T}{R_T + R_L}} \qquad \boxed{P_L = R_L \left(\frac{V_T}{R_T + R_L}\right)^2}$$ ← R_L variable, derive wrt R_L.

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$$\frac{dP_L}{dR_L} = \frac{V_T^2 (R_T+R_L)^2 - R_L V_T^2\, 2(R_T+R_L)}{(R_T+R_L)^4} = 0$$

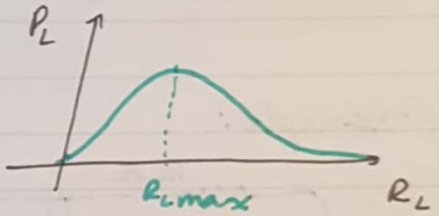
$$\Rightarrow \underbrace{V_T^2}_{①} \underbrace{(R_T+R_L)}_{②} \underbrace{[(R_T+R_L) - 2R_L]}_{③} = 0$$

① $\neq 0$ (given)

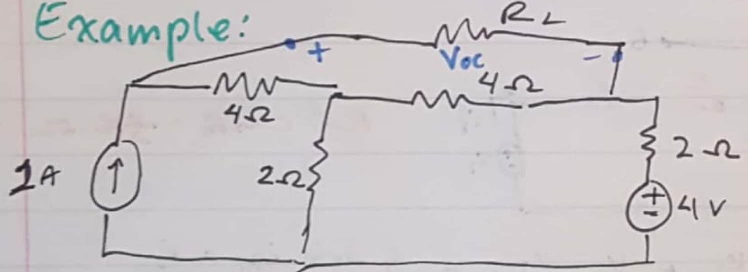
② $\neq 0$ (neg. resistance)

③ $R_T = R_L = R_{L\,max}$

So, $P_{L\,max} = \frac{R_T V_T^2}{(2R_T)^2} = \frac{V_T^2}{4R_T}$



Example:



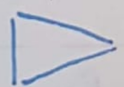
$V_{oc} = 3V$ by mesh analysis

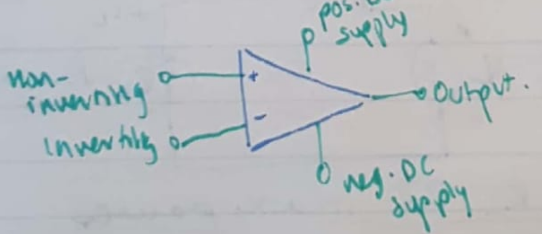
$R_{eq} = R_T = R_{L\,max} = 6\Omega$.

No dependent sources, so deactivate to find R_{eq}.

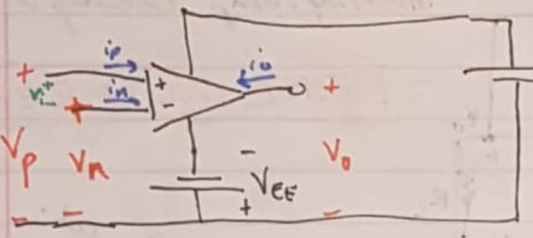
Operational Amplifier:

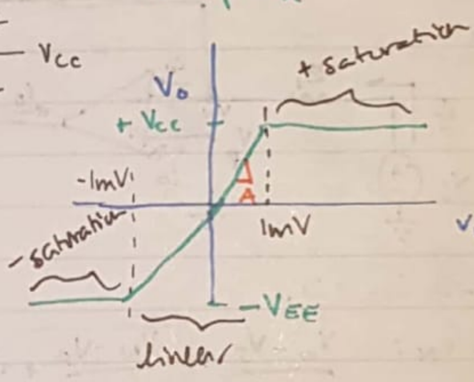
- Amplification, Integration, Addition/Subtraction, Differentiation, non-linear.

 Symbol.



Example:





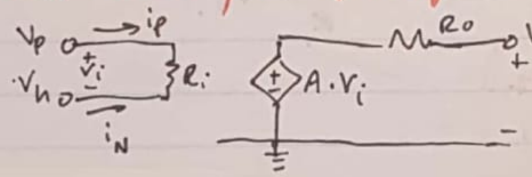
(Assume $V_{cc} = V_{ee} = 10V$)

A = slope (ampf).
→ Open-circuit voltage gain.

If $A = 10^4$

$V_i \approx 0$. $V_p \approx V_n$.

Linear Model for Op-Amps:



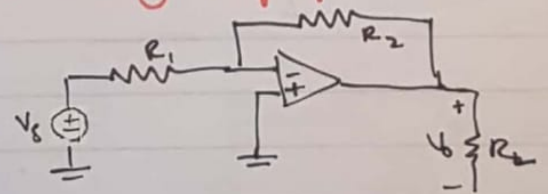
R_i = inp resistance
$= 10^5 - 10^{12}\ \Omega$

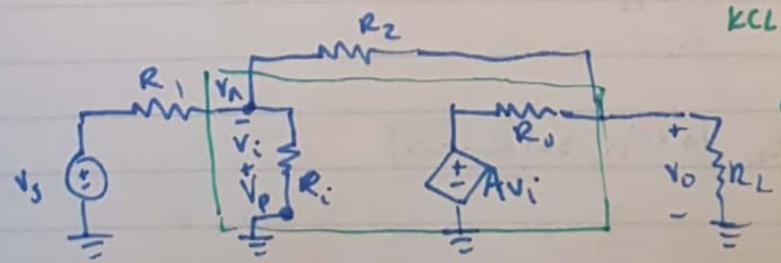
R_o = Out resistance
$= 1 - 150\ \Omega$

$A = 10^4 - 10^8$

$R_i, A \gg R_o$

Inverting Amplifier:





KCL V_n: $\frac{V_n - V_s}{R_1} + \frac{V_n}{R_i} + \frac{V_n - V_o}{R_2} = 0$

KCL V_o: $\frac{V_o - AV_i}{R_o} + \frac{V_o - V_N}{R_2} + \frac{V_o}{R_L} = 0$

where $V_i = V_p - V_N = -V_N$

If $R_1 = 1K\Omega$, $R_2 = 5$, $R_L = 1$, $R_i = 10^8\Omega$, $R_o = 50\Omega$, $A = 10^5$, then:

Complicated to solve: $V_o \simeq -5V_s$

Voltage Gain $\boxed{A_v \approx -5. = \frac{V_o}{V_s}}$

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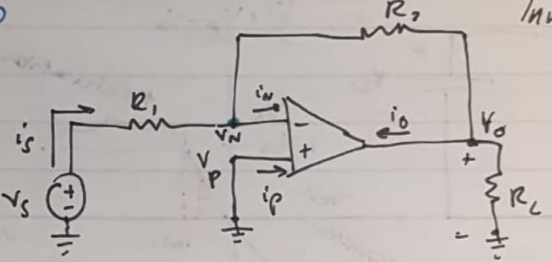
Ideal Model: Feb 28, 2018

$i_p = i_N = 0$
$V_p = V_N$

Inverting Amp example.

NEVER KCL @ REF

Start KCL w/ inputs.



$V_p = 0 \Rightarrow V_N = 0$.

KCL V_N: $\frac{V_N - V_S}{R_1} + \frac{V_N - V_o}{R_2} + i_N = 0$

$\frac{-V_S}{R_1} + \frac{-V_o}{R_2} = 0 \Rightarrow \boxed{V_o = \frac{-R_2}{R_1} V_S}$

same answer as linear model.

independent of R_L

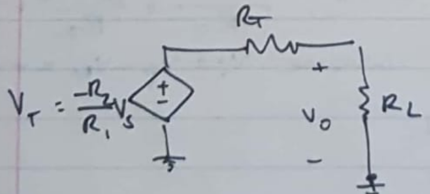
$\left(V_{gain} = \frac{V_o}{V_s} = \frac{-R_2}{R_1}\right)$

Input Resistance:

$R_{in} = \frac{V_S}{i_S}$, $i_S = \frac{V_S - \cancel{V_N}^{\,0}}{R_1} = \frac{V_S}{R_1}$. $\boxed{\therefore R_{in} = R_1}$

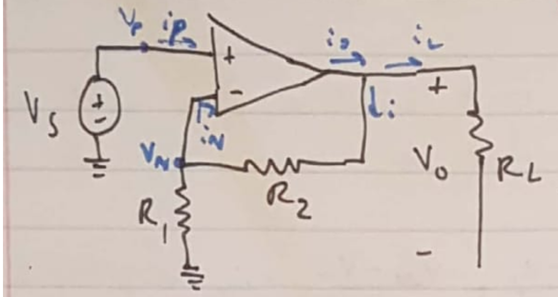
Output Resistance:

$R_{out} = R_T$ $V_{oc} = -\frac{R_2}{R_1} V_S$



$V_o = V_T$, so $\boxed{R_T = 0}$

Non-Inverting Amplifier:



$V_p = V_s = V_N$

KCL V_N: $\frac{V_N}{R_1} + \frac{V_N - V_o}{R_2} + i_N = 0$

$\frac{V_s}{R_1} + \frac{V_s - V_o}{R_2} = 0$

$R_2 V_s = R_1 V_o - R_1 V_s$

$V_o = \left(\frac{R_2 + R_1}{R_1}\right) V_s$.

$R_{in} = \frac{V_s}{i_s} = \frac{V_s}{i_p} \boxed{= \infty.}$ $\boxed{R_{out} = 0}$

→ $P_{sup} = 0$, so $P_L = \frac{V_o^2}{R_L}$

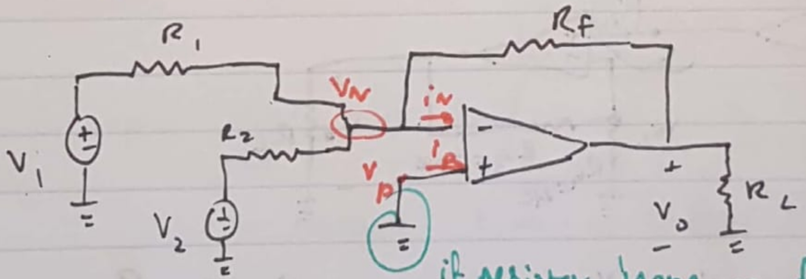
Gained power: $\frac{P_L}{P_s} = \infty$

→ $i_o = i + i_L = \frac{V_o - V_s}{R_2} + \frac{V_o}{R_L}$

Summing Amp:

March 2, 2018

$V_p = V_N = 0$ (ideal).

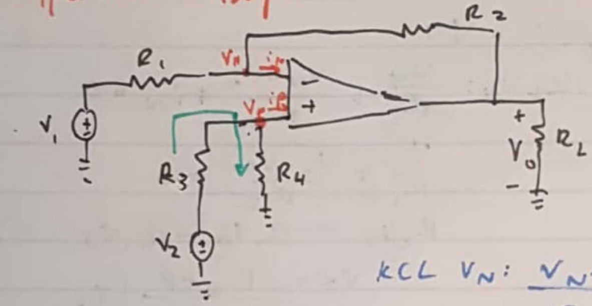


if resistor here, no dif: $i_p = 0$.

KCL V_N: $\frac{\cancel{V_N} - V_1}{R_1} + \frac{\cancel{V_N} - V_2}{R_2} + \frac{\cancel{V_N} - V_o}{R_F} + \cancel{i_N} = 0$

$V_o = -\left(\frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2\right)$ } hence the name.

Difference Amp:



ideal as well ($i_p = i_N = 0$, $V_p = V_N$)

KCL V_N: $\frac{V_N - V_1}{R_1} + \frac{V_N - V_0}{R_2} + i_N = 0$

$$V_0 = \left(\frac{R_2}{R_1} + 1\right) V_N - \frac{R_2}{R_1} V_1$$

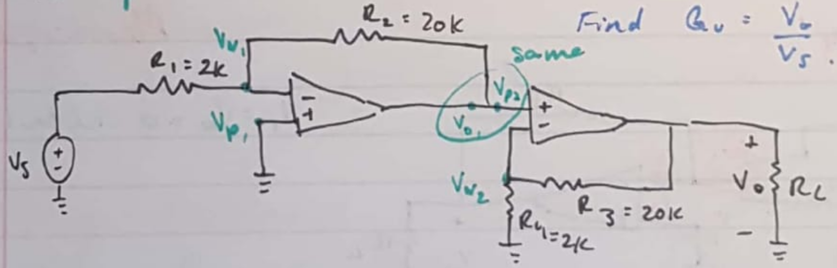
Since $i_p = 0$, R_3, R_4 in "series"

$V_p = \frac{R_4}{R_3 + R_4} \cdot V_2 = V_N$

$$= \frac{R_2}{R_1}\left[\frac{\left(1 + \frac{R_1}{R_2}\right) V_2}{\left(\frac{R_3}{R_4} + 1\right)} - V_1\right]$$

If $\frac{R_1}{R_2} = \frac{R_3}{R_4}$: $\boxed{V_0 = \frac{R_2}{R_1}(V_2 - V_1)}$

Example:



Find $G_v = \frac{V_0}{V_S}$.

KCL V_{N_1}: $\frac{\cancel{V_{N_1}} - V_S}{R_1} + \frac{\cancel{V_{N_1}} - V_{O_1}}{R_2} + \cancel{i_{N_1}} = 0 \Rightarrow V_{O_1} = -\frac{R_2}{R_1} V_S = V_{P_2} = V_{N_2}$

KCL V_{N_2}: $\frac{V_{N_2}}{R_4} + \frac{V_{N_2} - V_0}{R_3} + \cancel{i_{N_2}} = 0 \Rightarrow V_0 = \left(\frac{R_3}{R_4} + 1\right) V_{N_2}$

$$G_v = \frac{\text{that} \rightarrow}{V_S} = \left(\frac{R_3}{R_4} + 1\right)\left(-\frac{R_2}{R_1} V_S\right)$$

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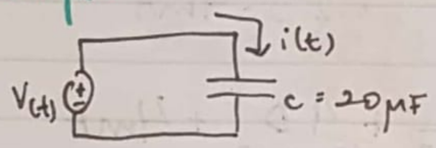
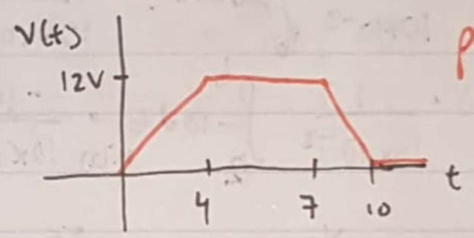
Capacitors: Mar 5, 2018

$q(t) = C\,V(t) \Rightarrow i(t) = C\frac{dV(t)}{dt} \Rightarrow V(t) = \int_{t_0}^{t} \frac{i(t)\,dt}{C} + V(t_0)$

1) If $V(t)$ constant, $i = 0$. (open circuit)
2) $\frac{dV(t)}{dt}$ must be defined (continuous V fcn)

Example:





piecewise cont. OK.

Find $i(t)$

$0 \le t \le 4$: $i(t) = 20 \cdot 3 = 60\,\mu A$

$4 < t \le 7$: $i(t) = 0$

$10 \ge t > 7$ $i(t) = -80\,\mu A$

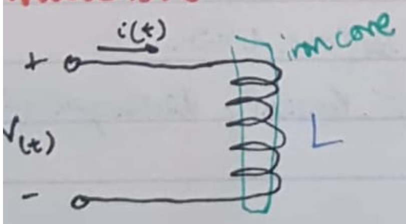
$t > 10$: $i(t) = 0$.

Power: $v(t) \cdot i(t) = \boxed{C \cdot V(t)\frac{dV(t)}{dt}}$.

Energy: $\frac{dw}{dt} = p(t) \Rightarrow \boxed{w = \frac{C}{2}\left[V^2(t_2) - V^2(t_1)\right]} = \frac{1}{2}C \cdot V^2(t)$

↳ Assume $t_1 \to -\infty \Rightarrow V(t_1) = 0 \Rightarrow t_2 = t$

Inductors: Mar 6, 2018



Ideal Model:

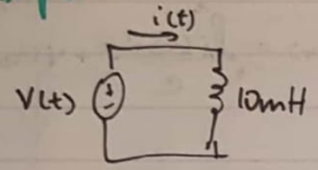
$V(t) = L\frac{di(t)}{dt}$ L in $\frac{V \cdot s}{A}$ (Henry's)

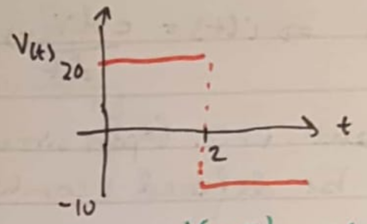
$i(t) = \frac{1}{L}\int_{t_0}^{t} V(t)\,dt + i(t_0)$

1) If $i(t)$ constant, $V = 0$ (short circuit)
2) $\frac{di(t)}{dt}$ must be defined (continuous i fcn)

$W(t) = \frac{1}{2}L\,i(t)^2$ $P(t) = L \cdot i(t)\frac{di(t)}{dt}$

Example:

1) 

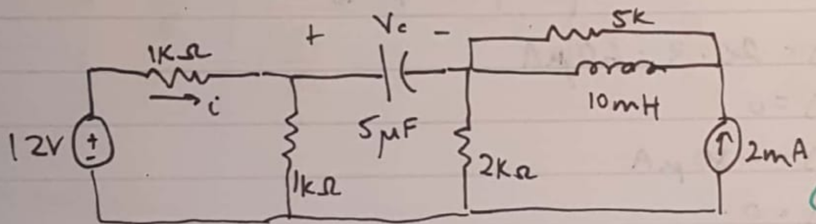


Find $i(t)$, $w(t)$ where $i(0^-) = 0$

$i(t)$ must be continuous, so $i(x^-) = i(x^+) = i(x)$

$0 \le t < 2$: $i(t) = \frac{1}{10\times10^{-3}} \int_0^t 20\,dt + 0 = \frac{20t}{10\times10^{-3}}$ $\quad i(2^-) = 4mA$

$2 \le t < \infty$: $i(t) = \frac{1}{10\times10^{-3}} \int_2^t -10\,dt + i(2) = \frac{-10t}{10\times10^{-3}} + \frac{10(2)}{10\times10^{-3}} + 4mA$

2) 

Find i, $V_c(t)$, $W_c(t)$, $W_L(t)$

Left for a long time.

Cap ⇒ open. Ind ⇒ short.

Inductor Relations:

Mar 7, 2018

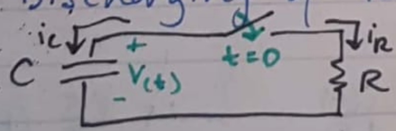
$L_{series} = L_1 + L_2 + L_3$

$L_{parallel} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$

Conductors switched.

Transient Analysis:

Discharging of RC circuit:



C charged to V_i before $t=0$.

RC = time const = τ. Basically discharged after 5τ

→ $R_{thevenin}$

$V(0^-) = V_i = V(0)$

For $t \ge 0$:

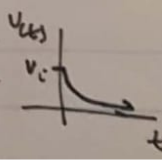
$i_c + i_R = 0$

$C\frac{dV(t)}{dt} + \frac{V(t)}{R} = 0$ Dif. eqn.

$\rightarrow RC\frac{dv}{dt} + V(t) = 0 \Rightarrow V(t) = Ke^{st}$

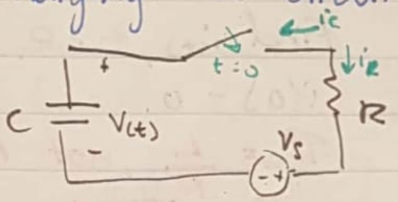
$RCKse^{st} + Ke^{st} = 0$

$s = \frac{-1}{RC}$, $K = V_i$. $\therefore V(t) = V_i e^{\frac{-1}{RC}t}$



Charging RC Circuit:

Mar 9, 2018



$V(0) = 0$.

$i_c + i_R = 0$

$$C\frac{dV}{dt} + \frac{V(t) - V_s}{R} = 0$$

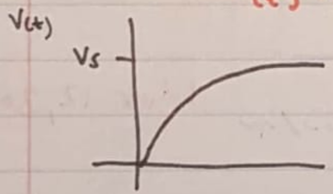
$$RC\frac{dV}{dt} + V(t) = V_s \Rightarrow \text{Assume } V(t) = K_1 e^{st} + K_2$$

Then $RC\left[K_1 s e^{st}\right] + K_1 e^{st} + K_2 = V_s$.

$$\underline{(RCs+1)}(K_1 e^{st}) + \underline{K_2} = \underline{0} + \underline{V_s}$$

$$s = \frac{-1}{RC} \quad \text{and} \quad K_2 = V_s$$

$$V(t) = K_1 e^{-t/RC} + V_s \Rightarrow = \underbrace{-V_s e^{-t/RC}}_{\text{transient}} + \underbrace{V_s}_{\text{steady-state}}$$



$V(5\tau) \approx V_s$

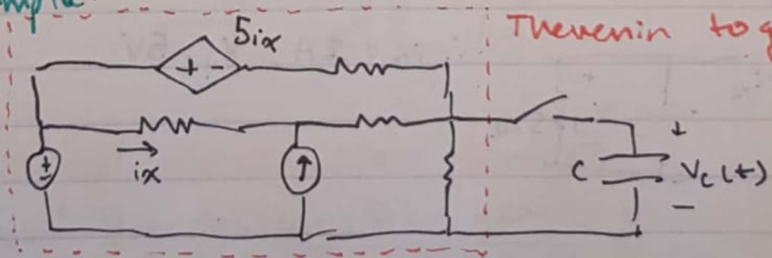
Shortcut: C behaves as O.C, so $i_R = 0 \Rightarrow V_R = 0 \Rightarrow V(t) = V_s$ steady state

General Formula:

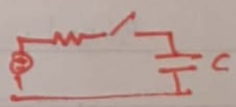
$$V_c(t) = \underbrace{V_c(\infty)}_{\text{Steady state}} + \left[\underbrace{V_c(t_0)}_{\text{Given/find.}} - V_c(\infty)\right] e^{-(t-t_0)/\tau}$$

Example:

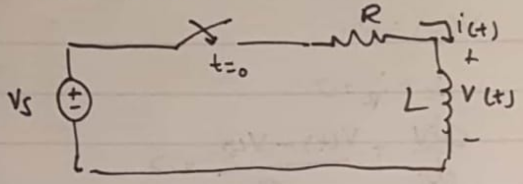
1)



Thevenin to get



RL circuit:



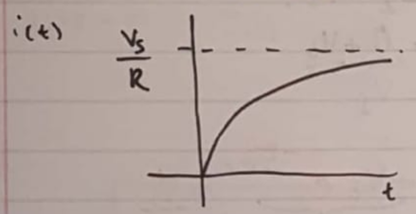
Find $i_{(t)}$, $t > 0$.

$i(0) = 0$.

$\tau = \frac{L}{R_{thevenin}}$, not RC here.

$R \cdot i_{(t)} + L \frac{di}{dt} = V_s$

$\frac{L}{R} \frac{di}{dt} + i_{(t)} = \frac{V_s}{R}$ } $i_{(t)} = -\frac{V_s}{R} e^{-t/\tau} + \frac{V_s}{R}$.



$V_{(t)} = L \frac{di_{(t)}}{dt}$

$= L\left[0 - \frac{V_s}{R}\left(-\frac{1}{\tau}\right)e^{-t/\tau}\right]$

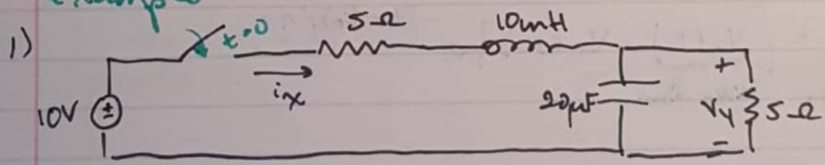
$= V_s e^{-t/\tau}$

Generalization: Mar 12, 2018

$i_L(t) = i_L(\infty) + [i_L(t_0) - i_L(\infty)]e^{-(t-t_0)/\tau}$

Example:

1)

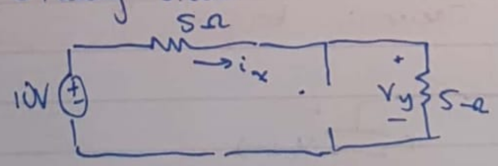


$t >> 0$, find i_x, V_y.

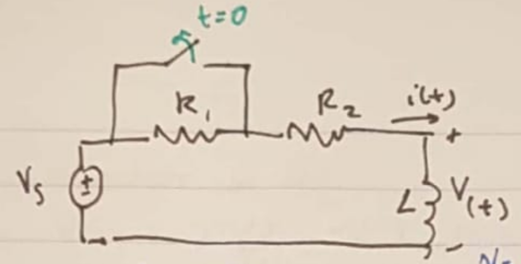
↳ $>> 0 \Rightarrow$ steady state

$> 0 \Rightarrow$ transient

Steady state:

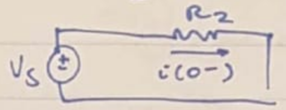


$\therefore i_x = 1A$, $V_y = 5V$

2) 

Switch closed for long time. Opened at $t=0$. Find $i_{(t)}$, $V_{(t)}$ for $t \geq 0$.

$t_0 = 0$. $i_L(0) = \frac{V_s}{R_2}$, since S.S. at $t=0^-$.



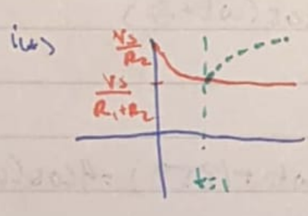
$\tau = \frac{L}{R_T} = \frac{L}{R_1+R_2}$

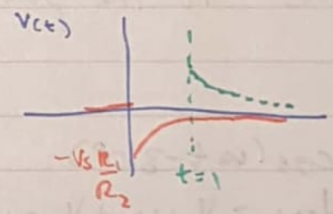
$i_L(\infty) = \frac{V_s}{R_1+R_2}$ (S.S. with switch open).

$\therefore i_L(t) = \frac{V_s}{R_1+R_2} + \left[\frac{V_s}{R_2} - \frac{V_s}{R_1+R_2}\right] e^{-(t-0)/\tau}$ ①

$V_{(t)} = L\frac{di_{(t)}}{dt} = L\left[\frac{-1}{\tau}\left(\frac{V_s}{R_2} - \frac{V_s}{R_1+R_2}\right)e^{-t/\tau}\right]$

$= -V_s \frac{R_1}{R_2} e^{-t/\tau}$





If switch closed at $t=1$, graph is same until $t=1$.
→ Then $t_0 = 1$, and use eqn. find $i(1^-)$ with ①.
↳ τ becomes $\frac{L}{R_2}$

Complex Numbers: Mar 14, 2018

$j = \sqrt{-1}$ $\quad z = x + jy = re^{j\theta} = r\angle\theta = r(\cos\theta + j\sin\theta)$

$r = \sqrt{x^2+y^2}$ $\quad \theta = \tan^{-1}\left(\frac{y}{x}\right)$

Sinusoidal Steady-state Analysis:

$V_{(t)} = \underline{V_m}\cos(\underline{\omega}t + \theta)$ $\quad -\infty < t < \infty$

↳ max value/magnitude. ↳ Ang freq. rad/s.

Period $T = \frac{2\pi}{\omega_{(rad)}}$

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Some identities:

$\sin\theta = \cos(\theta - \frac{\pi}{2})$

$-\sin\theta = \cos(\theta + \frac{\pi}{2})$

$-\cos\theta = \cos(\theta \pm \frac{\pi}{2})$

Example:

1) $V_{(t)} = 3\sin(20t+30°)$ $\quad 3 = V_m,\ \omega = 20,\ \theta = -60°.$

$= 3\cos(20t-60°)$

Phasors:

$\tilde{V} = V_m e^{j\theta}$ is the phasor for $V_{(t)} = V_m\cos(\omega t+\theta)$

Example:

1) $V_1(t) = 3\cos(\omega t - 30°)$ $\quad V_2(t) = 4\sin(\omega t + 135°) = 4\cos(\omega t + 45°)$

$V_{(t)} = V_1(t) + V_2(t)$

Get in C.N. rectangular form.

$\tilde{V}_1 = 3e^{-j30}$ $\quad \tilde{V}_2 = 4e^{j45}$

$= 3(\cos(-30) + j\sin(-30))$ $\quad = 4(\cos 45 + j\sin 45)$

$= 2.598 - 1.5j$ $\quad = 2.828 + 2.828j.$

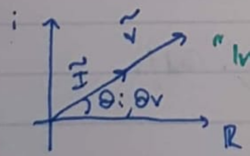
$\tilde{V} = \tilde{V}_1 + \tilde{V}_2 =$ ____ . $\tan^{-1}(\frac{y}{x})$ to get θ. $\sqrt{x^2+y^2} = r$.

$V_{(t)} = 5.586\cos(\omega t + 13.75°)$

Current AC: Mar 19, 2018

$i(t) = I_m\cos(\omega t + \theta_i)$ $\quad \tilde{I} = I_m e^{j\theta_i}$ $\quad V_m = R I_m$

$V(t) = R\,i(t)$ $\quad \tilde{V} = V_m e^{j\theta_v} = R\tilde{I}$ $\quad \theta_v = \theta_i$



"In phase"
$\theta_i = \theta_v$

Inductors in AC:

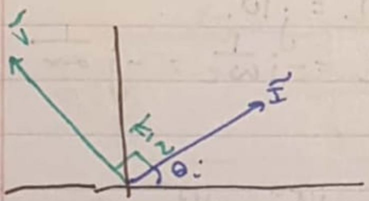
$$V(t) = L\frac{di}{dt} = -L I_m \omega \sin(\omega t + \theta_i)$$

$$= \underbrace{\omega L I_m}_{V_m} \cos(\omega t + \underbrace{\theta_i + \tfrac{\pi}{2}}_{\theta_v})$$

$$\tilde{V} = \omega L I_m e^{j(\theta_i + \frac{\pi}{2})} = j\omega L \tilde{I} \qquad (e^{j\frac{\pi}{2}} = j)$$

Inductor Impedence $Z_L = j\omega L$

$$\tilde{V} = Z_L \tilde{I}$$



$\tilde{V}$ "leads" by $\frac{\pi}{2}$.

$\tilde{I}$ "lags" by $\frac{\pi}{2}$

Capacitors in AC:

$$i(t) = \frac{dv}{dt} \qquad \tilde{I} = j\omega C \tilde{V} \Rightarrow \tilde{V} = \underbrace{\frac{1}{j\omega C}}_{} \tilde{I} = -j\frac{1}{\omega C}$$

capacitor impedance $= -j\frac{1}{\omega C} = Z_c$

$$V_m = \frac{1}{\omega C} I_m$$

$$\theta_v = \theta_i - \frac{\pi}{2}$$

General Impedance:

$Z = \underset{\text{resistance}}{R} + j\underline{X} \leftarrow$ reactance $\qquad Y = \frac{1}{Z} = \underset{\text{conductance}}{\underline{G}} + j\underline{B} \leftarrow$ susceptance

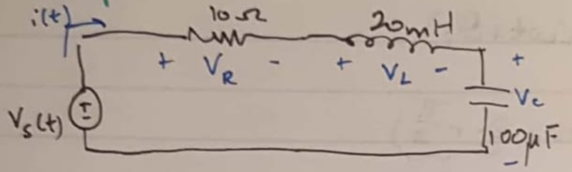
→ KCL: $\sum \tilde{I}_n = 0$. KVL: $\sum \tilde{V}_n = 0$

→ Series: $Z_1 + Z_2 = Z_{eq}$

→ Parallel: $Z_1 \parallel Z_2 = Z_{eq} = \dfrac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}}$

Example: Mar 20, 2018

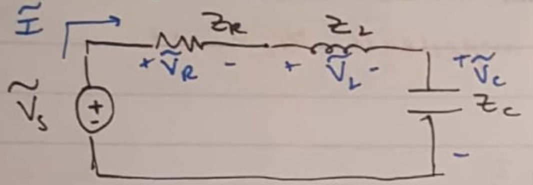
1)



$V_s(t) = 20\cos(500t + 15°)$.

Find $i(t)$, $V_R(t)$, $V_L(t)$, $V_c(t)$

Phasor Domain:



$\tilde{V}_s = 20e^{j15°}$

$Z_R = R = 10\,\Omega$

$Z_L = j\omega L = j10.$

$Z_c = \frac{1}{j\omega c} = -j\frac{1}{\omega c} = -j20$

KVL: $\tilde{V}_R + \tilde{V}_L + \tilde{V}_c = \tilde{V}_s$

$\tilde{I}(Z_R + Z_L + Z_c) = \tilde{V}_s$

$$\tilde{I} = \frac{20\angle 15°}{10 + j10 - j20} = \frac{20e^{j15°}}{\sqrt{10^2+10^2}\,\angle \tan^{-1}\left(\frac{-10}{10}\right)} = \frac{20e^{j15°}}{10\sqrt{2}\,e^{j-45°}} = \sqrt{2}\,e^{j60°}$$

$\therefore I(t) = \sqrt{2}\cos(500t + 60°)$

$\therefore V_R(t) = Ri(t) = 10\sqrt{2}\cos(500t + 60°)$

$\tilde{V}_L = (j\omega L)\tilde{I} = (10j)(\sqrt{2}e^{j60°})$

$= (10e^{j90°})(\sqrt{2}e^{j60°}) = 10\sqrt{2}e^{j150°}$

$\therefore V_L(t) = 10\sqrt{2}\cos(500t + 150°)$

$\tilde{V}_c = Z_c\tilde{I} = (20e^{j-90°})(\sqrt{2}e^{j60°})$

$V_c(t) = 20\sqrt{2}\cos(500t - 30°)$

If $c = 200\mu F$:

$Z_c = -j10$, then $Z_{eq} = 10$

$\rightarrow \tilde{I} = 2\angle 15°$ and $\tilde{V}_L + \tilde{V}_R = 0$

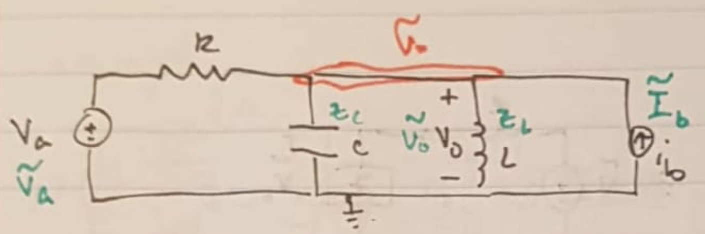
when is $Z_L + Z_c = 0$?

$Z_c = -Z_L$

$\omega^2 = \frac{1}{LC}$

$\omega = \frac{1}{\sqrt{LC}}$

2)

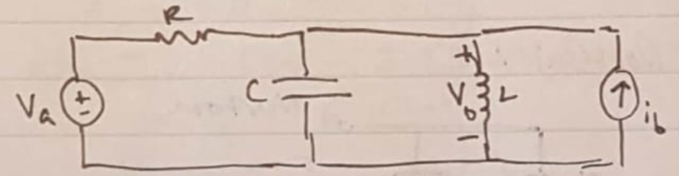


$V_a = 2\cos(1000t + 30°)$ Find V_o.
$i_b = \cos(1000t - 45°)$
$R = 4\Omega$ $C = 450\mu F$ $L = 2mH$

$\tilde V_a = 2e^{j30}$ $\tilde I_b = e^{-45j}$ $z_c = -j\frac{1}{\omega C} = -j4$ $z_L = j\omega L = j2$.

KVL: $\frac{\tilde V_o - \tilde V_a}{R} + \frac{\tilde V_o}{j4} + \frac{\tilde V_o}{j2} - \tilde I_b = 0$ Solve for $V_o = 3.5\cos(1000t + 83°)$

3)

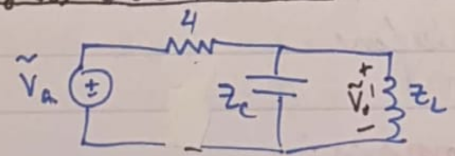


Mar 21, 2018

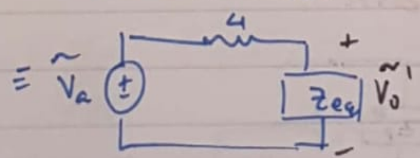
Same example, but $V_a = 2\cos(\underline{1000}t + 30°)$, $i_b = \cos(\underline{2000}t - 45°)$

Different freq. Use superposition.

<u>$V_o'(t)$ due to V_a:</u>



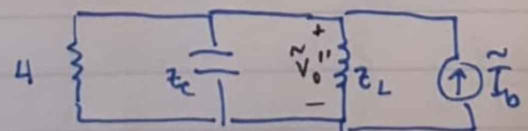
$\tilde V_a = 2\angle 30°$
$z_c = -j\frac{1}{\omega C} = -j4$
$z_L = j\omega L = j2$



$z_{eq} = \frac{z_c z_L}{z_c + z_L} = j4$

V division: $\tilde V_o' = \tilde V_a \left(\frac{z_{eq}}{z_{eq} + 4}\right) = \sqrt{2}\cos(1000t + 75°)$

<u>$V_o''(t)$ due to i_b:</u>



$\tilde I_b = 1\angle -45°$
$z_c = -j2$
$z_L = j4$ changed.

$z_{eq} = 4 \| z_c \| z_L$
$\tilde V_o(t) = z_{eq} \cdot \tilde I_b$
$\therefore$ $v_o''(t) = z_{eq} \tilde I_b =$

$\therefore$ $V_o(t) = V_o'(t) + V_o''(t) = \sqrt{2}\cos(1000t + 75°) + 2\sqrt{2}\cos(2000t - 90°)$
(can't combine).

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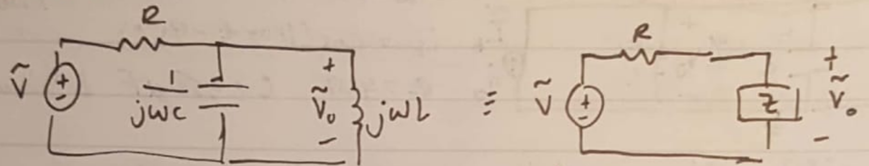
Remark:



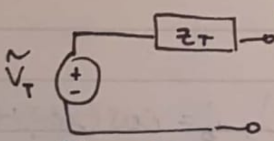
$$Z = \frac{1}{j\omega C} \parallel j\omega L = \frac{L/C}{j\left(\omega L - \frac{1}{\omega C}\right)}$$

If $\omega L = \frac{1}{\omega C} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$, $Z = \infty$ (open circuit) $\tilde{V}_o = \tilde{V}$

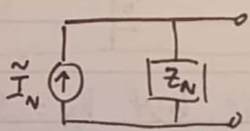
(parallel reasonances).

Thevenin + Norton Phasor Domain:

Thevenin



Norton

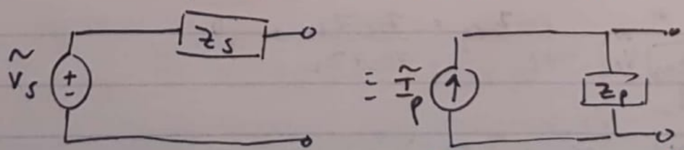


$\tilde{V}_{oc} = \tilde{V}_T$

$\tilde{I}_N = \tilde{I}_{sc}$

$Z_T = Z_N = \frac{\tilde{V}_{oc}}{\tilde{I}_{sc}}$ } same relations as before.

Source Transformation Phasor Domain:

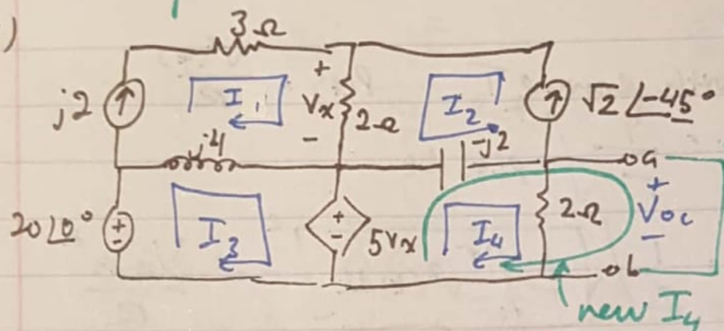


$Z_s = Z_p$ $\qquad \frac{\tilde{V}_s}{Z_s} = \tilde{I}_p$ $\qquad \tilde{V}_s = Z_p \cdot \tilde{I}_p$

Mar 23, 2018

Example:

1) Find Thevenin a-b.



$\downarrow I_{sc} = I_4$ (new I_4)

$I_1 = j2 = 2\angle 90°$

$I_2 = \sqrt{2}\angle -45° = 1-j$

$V_x = 2\cdot(I_1+I_2) = 2\cdot(1+j) = 2\sqrt{2}\angle 45°$

KVL I_4: $-5V_x - j2\cdot(I_2+I_4) + 2I_4 = 0$

$\rightarrow I_4 = 6\angle 90°$

$\therefore V_{oc} = 2\cdot I_4 = 12\angle 90°$

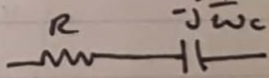
To find Isc, get short + adjust I_4.

I_1, I_2, V_x unchanged.

KVL I_4: $-5V_x - j2(I_2+I_4) = 0$

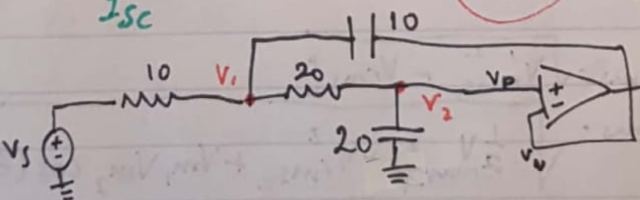
$\rightarrow I_4 = I_{sc} = 12/\sqrt{2}\ \angle 135°$

$Z_{TH} = \dfrac{V_{oc}}{I_{sc}} = \sqrt{2}\angle -45° = 1-j$ capacitive, since $-j$.



$Z_{TH} = R - j\frac{1}{\omega C}$

2)



Vfollower: $V_o = V_2$

Find V_o given $V_s = 12\cos(5000t)$

Convert to time domain.

$C_1 = 10nF = -j20$

$C_1 = 2nF = -j10$

$$\frac{V_1-V_s}{10} + \frac{V_1-V_2}{20} + \frac{V_1-V_o}{-j20} = 0 \quad (1)$$

$$\frac{V_2-V_1}{20} + \frac{V_2}{-j10} = 0 \quad (2)$$

$V_o = 4\angle -90°$

AC Power:

Root mean square Value:

Assume $V(t)$ periodic with period T. $\quad P(t) = \frac{V^2(t)}{R}$

"Avg Power" $P_{av} = \frac{1}{T}\int_0^T p(t)\,dt$

$$= \frac{1}{R}\Big[\underbrace{\sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}}_{\text{RMS VALUE}}\Big]^2 = \frac{V_{RMS}^2}{R}$$

$$\boxed{\therefore V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}}$$

RMS Value for Sinusoid:

Mar 26, 2018

$T = \frac{2\pi}{\omega} \qquad V(t) = V_m\cos(\omega t + \theta)$

$$V_{rms} = \sqrt{\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} V_m^2\cos^2(\omega t+\theta)\,dt}$$

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}} \qquad \boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$$

If $V(t) = V_1(t) + V_2(t)$:

1) If $\omega_1 \neq \omega_2$: $V_{rms} = \sqrt{V_{rms_1}^2 + V_{rms_2}^2}$

2) If $\omega_1 = \omega_2$: $V_{rms} = \sqrt{V_{rms_1}^2 + V_{rms_2}^2 + V_{m_1}V_{m_2}\cos(\theta_1 - \theta_2)}$

↳ Don't use 2. Convert to one sinusoid $\bar{w}$ phasors.

If $V(t)$ = constant?

$V_{rms} = V_{dc}$ (same as function)

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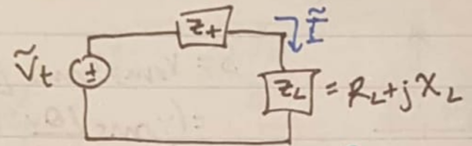
AC Max Power:



$P_{avg} = I_{rms}^2 \cdot |Z_L|$

$$P_{max} = \frac{|V_t|^2}{8R_T}$$

where $\boxed{Z_L = \overline{Z_T}}$

$R_L = R_T$

$X_L = -X_T$

If $Z_L = R_L$, then...

$Z_L = |Z_t| = \sqrt{R_t^2 + X_t^2}$

for max power.

AC power:

Let $V = V_m\cos(\omega t + \theta_v - \theta_i)$, $I = I_m\cos(\omega t)$.

Instantaneous power:

$p(t) = V(t) \cdot I(t)$

$= P - P\cos(2\omega t) + Q\sin(2\omega t)$

Active (Real) Power:

$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$= V_{rms} I_{rms} \cos(\theta_v - \theta_i)$

Avg Power:

$P_{av} = P$

Reactive Power:

$Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i)$

Power Factor:

Mar 28, 2016

$P_f = \cos(\theta_v - \theta_i)$

$P = V_{rms} I_{rms} P_f$

For resistors, $pf = \cos(0) = 1$

$P = V_{rms} \cdot I_{rms}$, $Q = 0$.

Inductors:

→ $\cos(\theta_v - \theta_i) = \cos 90^\circ = 0$

→ $\sin(\theta_v - \theta_i) = \sin 90 = 1$

$P = 0$

$Q = +V_{rms} \cdot I_{rms} = \frac{V_{rms}^2}{\omega L} = I_{rms}^2 \omega L$

Capacitors:

→ $\cos(\theta_v - \theta_i) = \cos -90 = 0$

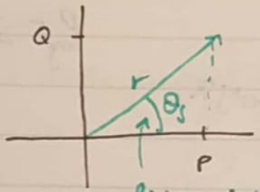
→ $\sin(\theta_v - \theta_i) = \sin -90 = -1$

$P = 0$

$Q = -V_{rms} \cdot I_{rms} = -\frac{I_{rms}^2}{\omega C} = -\omega C V_{rms}^2$

Complex Power:

$$S = P + jQ$$



Power triangle

$$S = \sqrt{P^2+Q^2}\ \angle \tan^{-1}\left(\frac{Q}{P}\right)$$
$$= \sqrt{P^2+Q^2}\ \angle(\theta_v - \theta_i)$$
$$= V_{rms} I_{rms} \angle(\theta_v - \theta_i)$$

$V_{rms} \cdot I_{rms} = |S| = \sqrt{P^2+Q^2}$ = Apparent Power.

$$S = V_{rms} I_{rms} \angle \theta_v - \theta_i$$
$$= \underbrace{(V_{rms} \angle \theta_v)}\ \underbrace{(I_{rms} \angle -\theta_i)}$$

$\frac{\tilde{V}}{\sqrt{2}} = \tilde{V}_{rms}$ $\quad \bar{\tilde{I}}_{rms} = \frac{\bar{\tilde{I}}}{\sqrt{2}}$

$$\boxed{\therefore S = \tilde{V}_{rms} \cdot \bar{\tilde{I}}_{rms}}$$
$$= \frac{1}{2} \tilde{V} \bar{\tilde{I}}$$

Units of Power:

- Active: P — Watt (W)
- Reactive: Q — Volt·Ampere Reactive (VAR)
- Complex: S — Volt·Ampere (VA)
- Apparent: $|S|$ — Volt·Ampere (VA)

Power for Impedance:

Apr 2, 2018

$$S = Z \cdot \tilde{I}_{rms}\ \bar{\tilde{I}}_{rms} = \underbrace{R I_{rms}^2}_{P} + j \underbrace{X I_{rms}^2}_{Q}$$

Power Triangle for Impedance:

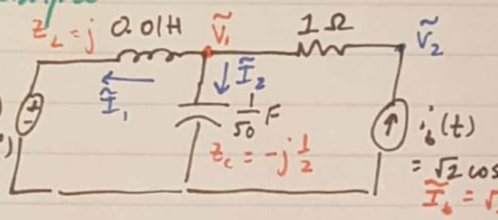
- $Q > 0 \Rightarrow$ Inductor
- $Q < 0 \Rightarrow$ Capacitor
- $Q = 0 \Rightarrow$ Resistor

Power Factor:

1) pf lagging –
   → $\tilde{I}$ lags $\tilde{V}$ (inductor) and $Q > 0$
2) pf leading –
   → $\tilde{I}$ leads $\tilde{V}$ (capacitor) and $Q < 0$

Example:

1)



Find P, Q for every element.

$= \cos(100t - 90°)$

$\tilde{V}_a = 1\angle -90° = -j$

$= \sqrt{2}\cos(100t + 45°)$

$\tilde{I}_b = \sqrt{2}\angle 45° = 1 + j$

$$\frac{V_1 + j}{j} + \frac{V_1}{-j\frac{1}{2}} = \sqrt{2}\angle 45 = 1 + j \Rightarrow \boxed{V_1 = 1\angle 0°}$$

$\tilde{V}_2 = 2 + j$ and $\tilde{I}_1 = \sqrt{2}\angle -45 = 1 + j$ and $\tilde{I}_2 = 2j$

Power for $\tilde{V}_a$:

$$P = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right)\cos(-90° + 45°) = \frac{1}{2}\ W$$

$$Q = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right)\sin(-90° + 45°) = -\frac{1}{2}\ VAR$$

Power for $\tilde{I}_b$:

(need $-\tilde{I}_b$) $= -1 - j$ $= \sqrt{2}\angle 45 - 180$

$$P = \left(\frac{\sqrt{5}}{\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right)\cos(26.57 - (-135)) = -\frac{3}{2}\ W$$

$$Q = \left(\frac{\sqrt{5}}{\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right)\sin(26.57 - (-135)) \quad \frac{1}{2}\ VAR$$

Power for L:

$P = 0W$

$V_L = \sqrt{2}\angle 45$

$$Q = \left(\frac{\sqrt{2}}{\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right)\sin(45 - (-45)) = 1\ VAR = \omega L I_{rms}^2$$

Power for C:

$P = 0W$

$$Q = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{2}{\sqrt{2}}\right)\sin(0 - 90) = -1\ VAR = \frac{-1}{\omega C} I_{rms}^2$$

Power for R:

$P = R I_{rms}^2 = 2W$

$Q = 0$

$\sum P = \sum Q = 0$