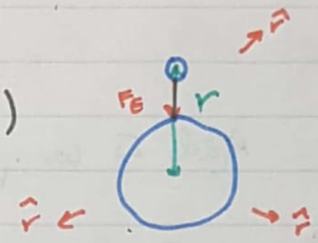
ECE106

Force and Field:

Jan 3, 2018

$$\vec{F}_E = \frac{G m_e m}{r^2}(-\hat{r})$$

$$= m\vec{g} \leftarrow \text{field}$$



$$\vec{g} = \frac{G m_E}{r^2}(-\hat{r})$$

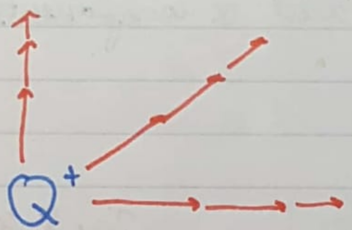
$$\vec{F}_{net} = m\vec{g}_{net}$$

Electrostatics:

$$\vec{F}_{Qq} = \frac{KQq}{r^2}(\hat{r})$$

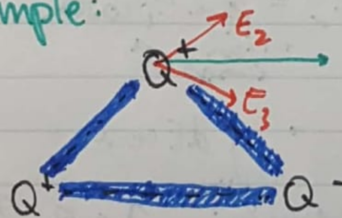
$= \vec{E}_Q q$ where $\vec{E}_Q = \frac{KQ}{r^2}\hat{r}$

$= q\vec{E}$ in general.



Example:

1) $\vec{E}$? F_{net}?

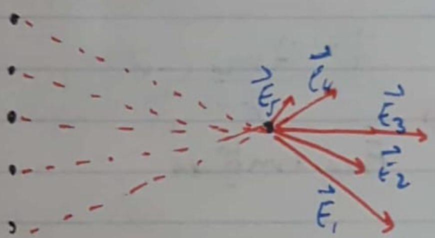


$$|\vec{E}_{net}| = (E_2 + E_3)\cos 60$$
$$= 2E\cos 60$$
$$= E$$

$E_{net}^2 = E_2^2 + E_3^2 + 2E_2E_3\cos\theta$

w/ θ is tail to tail.

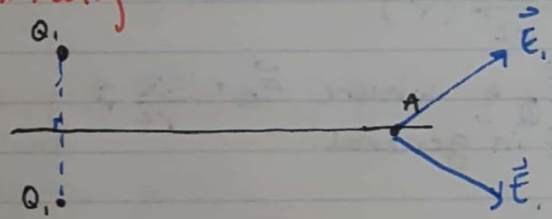
Point charge Model: Jan 5, 2018



Add $\bar{w}$ components

$$E = \frac{kQ}{r^2} = \frac{Q}{4\pi\varepsilon_0 r^2} \quad \left(k = \frac{1}{4\pi\varepsilon_0}\right)$$

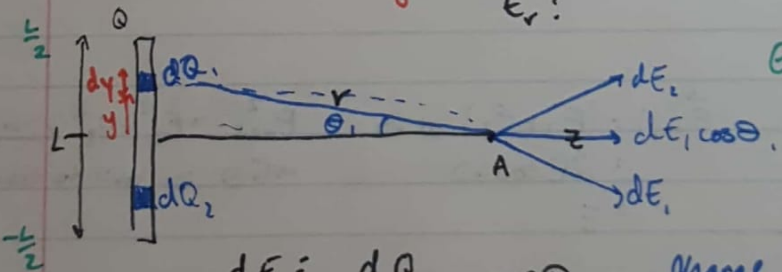
Symmetry:



just add x components

Continuous Charges: Jan 8, 2018

$\vec{E}_r$?



$$E_r \cong \sum dE_i \cos\theta_i = \lim_{i\to\infty} = \int_{line} dE\cos\theta$$

charge density: $\lambda = \frac{Q}{L} \Big\} dQ = \lambda dL$

$$dE_z = \frac{dQ}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{\lambda\, d_y \cos\theta}{4\pi\varepsilon_0 r^2}$$

$$\therefore E_z = \int_{line} dE_z = \int_{-L/2}^{L/2} \frac{\lambda\, d_y \cos\theta}{4\pi\varepsilon_0 r^2} = \int_{-L/2}^{L/2} \frac{x\, d_y}{4\pi\varepsilon_0 r^2}\left(\frac{z}{r}\right)$$

$$= \int_{-L/2}^{L/2} \frac{\lambda\, d_y\, z}{4\pi\varepsilon_0 (y^2+z^2)^{3/2}}$$

} Evaluate $\bar{w}$ tan sub.
$y = z\tan\alpha$

$y = z\tan\alpha$
$dy = z\sec^2\alpha\, d\alpha$

$(y^2+z^2)^{3/2} = (z^2(\tan^2\alpha+1))^{3/2}$
$= (z^2\sec^2\alpha)^{3/2}$
$= z^3\sec^3\alpha$

$$E_z = \int_{line} \frac{\lambda z^2 \sec^2\alpha\, d\alpha}{4\pi\varepsilon_0 z^3 \sec^3\alpha}$$

$\left(P_e = \lambda = \frac{Q}{L} = \text{Charge Density}\right)$

$$= \frac{\lambda}{4\pi\varepsilon_0 z}\int \frac{d\alpha}{\sec\alpha}$$
$$= \frac{\lambda}{4\pi\varepsilon_0 z} \sin\alpha \Big|_{line}$$
$$= \frac{\lambda}{4\pi\varepsilon_0 z}\left(\frac{y}{\sqrt{y^2+z^2}}\right)\Big|_{-L/2}^{L/2}$$
$$= \frac{\lambda L}{4\pi\varepsilon_0 z\sqrt{\frac{L^2}{4}+z^2}}$$

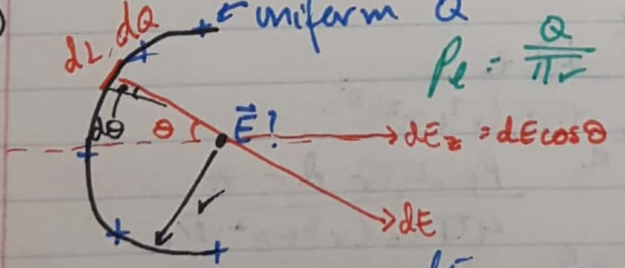
$\lim_{L \gg z} = 2$ for $\frac{L}{\sqrt{\frac{L^2}{4}+z^2}}$

$$\approx \frac{\lambda}{2\pi\varepsilon_0 z}$$

Example: Jan 10, 2018

1) uniform Q

$P_e = \frac{Q}{\pi r}$



$d\vec{E}_z = \frac{dQ}{4\pi\varepsilon_0 r^2}\cos\theta$

$dQ = P_e\, dL = P_e r\, d\theta$

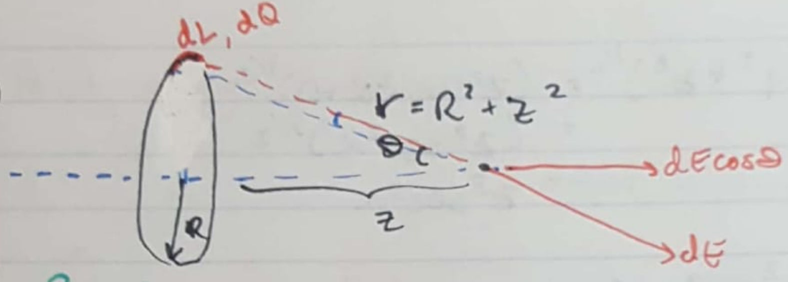
sub back in.

$$dE_z = \frac{P_e r}{4\pi r^2 \varepsilon_0}\cos\theta\, d\theta$$

$$E_z = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\lambda}{4\pi\varepsilon_0 r}\cos\theta\, d\theta$$

$$= \frac{\lambda}{2\pi\varepsilon_0 r}$$

2)



$\rho_\ell = \frac{Q}{2\pi R}$

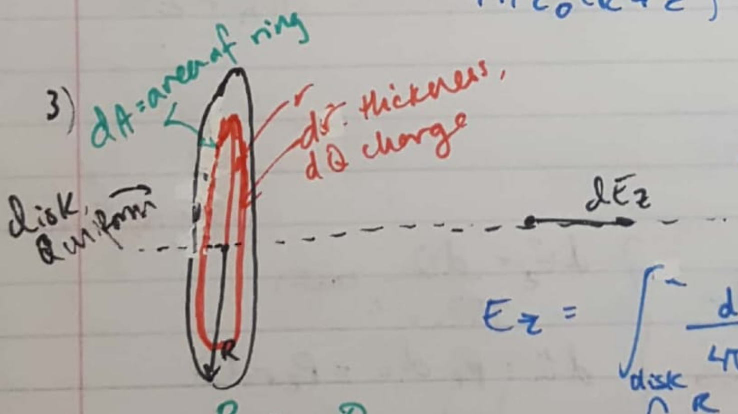
$$dE_z = \frac{dQ}{4\pi\varepsilon_0 r^2}\cos\theta$$

$$= \frac{dQ\ z}{4\pi\varepsilon_0 r^3} = \frac{dQ\ z}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}}$$

$$E_z = \int_{ring} \frac{z\,dQ}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}} = \left(\int_{ring} dQ\right)\frac{z}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}}$$

$$= \frac{Qz}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}}$$

3)



Jan 15, 2018

$$dE_z = \frac{dQ\,z}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}$$

$$E_z = \int_{disk} \frac{dQ\,z}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}$$

$\rho_s = \frac{Q}{\pi r^2}$

$$= \int_0^R \frac{\rho_s 2\pi r\,dr\,z}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}$$

$dQ = \rho_s(\text{Area})$
$= \rho_s(2\pi r\,dr)$

$$= \frac{\rho_s z}{2\varepsilon_0}\int_0^R \frac{r\,dr}{(r^2+z^2)^{3/2}}$$

u-sub $u = r^2+z^2$

$$\hookrightarrow = \frac{-2z\rho_s}{4\varepsilon_0}\left(\frac{1}{\sqrt{r^2+z^2}}\right)\Bigg|_0^R$$

$$= \frac{z\rho_s}{2\varepsilon_0}\left[\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}}\right]$$

$$\lim_{R\to\infty} E_z = \frac{\rho_s}{2\varepsilon_0}$$ } distance doesn't matter

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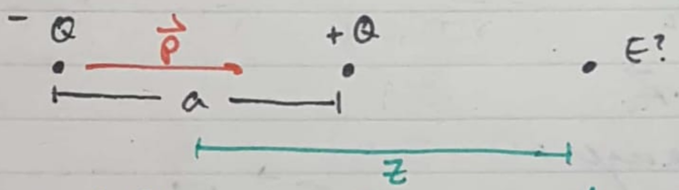
Summary of Field Calculations: Jan 17, 2018

Point charge: $\dfrac{Q}{4\pi\varepsilon_0 r^2}$

Infinite line: $\dfrac{\rho_l}{2\pi\varepsilon_0 r}$

Infinite sheet: $\dfrac{\rho_s}{2\varepsilon_0}$

Field due to Dipole:



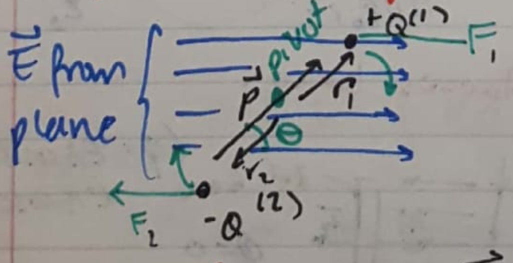
$$E = \frac{kQ}{(z-\frac{a}{2})^2} - \frac{kQ}{(z+\frac{a}{2})^2} \qquad \lim_{z \gg a} E_d = \frac{Qa}{2\pi\varepsilon_0 z^3}$$

$$= \frac{2azQ}{4\pi\varepsilon_0 [z^2 - \frac{a^2}{4}]^2}$$

$p = Qa$, so $\lim_{z \gg a} \vec{E}_d = \dfrac{\vec{p}}{2\pi\varepsilon_0 z^3}$ ← Electric Dipole moment

↳ points in $\vec{E}_d$ direction, ⟶ (+ charge closer, so)

Torque on a Dipole:



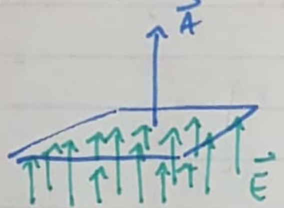
$\vec{p}$: - ⟶ +

$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2$$
$$= \tau_1 + \tau_2$$
$$= r_1 \sin\theta F + r_2 \sin\theta F \qquad r = \frac{a}{2}, \text{so...}$$
$$= aF\sin\theta$$
$$= pE\sin\theta$$

$$\therefore \vec{\tau} = \vec{p} \times \vec{E}$$

Gauss's Law:

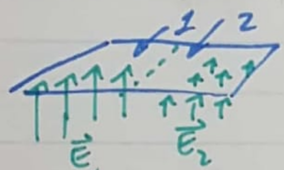
Flux:



$$\boxed{\varphi = \vec{E}\cdot\vec{A} = |\vec{E}||\vec{A}|\cos\theta}$$

If uneven E:

Add up ind. flux

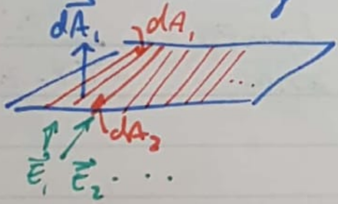


$\vec{A} = \vec{A}_1 + \vec{A}_2$

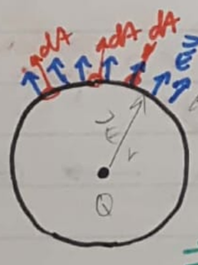
$\varphi = \vec{E}_1\cdot\vec{A}_1 + \vec{E}_2\cdot\vec{A}_2$

$$\varphi = \sum_i \vec{E}_i\cdot\vec{A}_i$$

Continuous changes:



$$\varphi \approx \sum_i \vec{E}_i\cdot d\vec{A}_i = \int_{\text{area}} \vec{E}\cdot d\vec{A}$$



$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}$

$|\vec{A}| = 4\pi r^2$

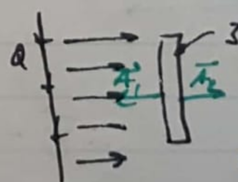
$\theta = 0$ and $\vec{E}$ same everywhere

$$\varphi = \oint \vec{E}\cdot d\vec{A}$$
$$= |\vec{E}_1||d\vec{A}_1| + |\vec{E}_2||d\vec{A}_2| + \dots$$
$$= |\vec{E}|(dA_1 + dA_2 + \dots)$$
$$= |\vec{E}|\cdot|\vec{A}|$$
$$= \frac{Q}{4\pi\varepsilon_0 r^2}\cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\boxed{\varphi_{\substack{\text{enclosed}\\\text{surface}\\(3D)}} = \frac{Q_{enc}}{\varepsilon_0}}$$ Gauss's Law.

$\varphi = \frac{5Q}{\varepsilon_0}$

Area vector always points away from encl.



$(\varphi = EA\cos 180 + EA\cos 0)$

$\varphi = 0$
(no enclosed charges)

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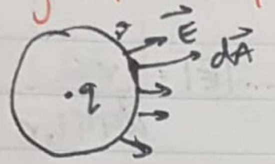
Using GL to find $\vec{E}$ from P.C: Jan 19, 2018



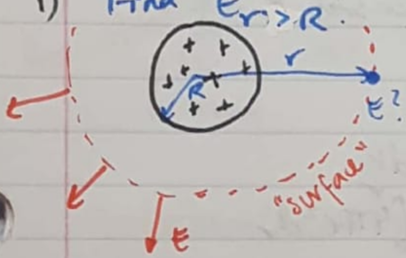
$\varphi_e = \frac{Q_{enc}}{\varepsilon_0} = \oint \vec{E}\cdot d\vec{A}$

$= \oint |\vec{E}||d\vec{A}|\cos\theta \leftarrow \theta = 0$ e'where

$= \oint |\vec{E}||d\vec{A}| = |\vec{E}|\oint |d\vec{A}| = |\vec{E}||\vec{A}|$ etc.

Example:

1) Find $E_{r>R}$.



$\varphi = \frac{Q_{enc}}{\varepsilon_0}$

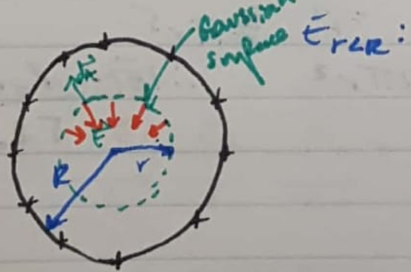
$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$ } Becomes same as pt. charge.

$E 4\pi r^2 = \frac{Q}{\varepsilon_0}$ ← Same as charged shell

Note: PC, sphere, shell, w/ uniform dist. behave same (r>R)

Field inside (r≤R):

Uniform shell:



$E_{r<R}$: $\varphi = \frac{Q_{enc}}{\varepsilon_0}$

$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} = 0 . = -|\vec{E}|\oint dA = 0$

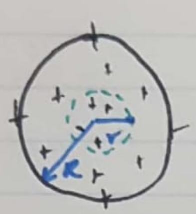
$\therefore \vec{E}_r = 0$ ← only if symmetry

Non-uniform shell: $\varphi = 0$, but $\vec{E} \neq 0$!!!

← $\vec{E}$ can't be 0, no symmetry.

↳ No sym? Can't param. $\vec{E}$, aka bring it out of integral.

uniform sphere:



$$\Phi_e = \frac{Q_{enc}}{\varepsilon_0} = \frac{r^3 Q}{R^3 \varepsilon_0}$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{r^3 Q}{R^3 \varepsilon_0} \quad \text{etc...} \quad |\vec{E}| = \frac{rQ}{R^3 4\pi\varepsilon_0}$$

$$Q_{enc} = \rho_V\left(\tfrac{4}{3}\pi r^3\right), \quad \rho_V = \frac{Q}{\frac{4}{3}\pi R^3} \quad \text{(sphere)}$$

(Gauss)

More Charge Density:

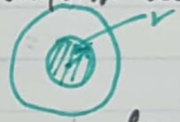
Jan 22, 2018

$$Q_{line} = \int_{line} dQ = \int_{line} \rho_L \, dL$$

$$Q_{plane} = \int_{plane} dQ = \int_{plane} \rho_S \, dS$$

$$Q_{sphere} = \int_{sphere} dQ = \int_{sphere} \rho_V \, dV$$

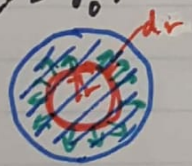
Uniform density ρ_V



$$Q_{(r)} = \int \rho_V \, dV = \rho_V \int_0^r 4\pi r^2 \, dr = \rho_V \frac{4\pi}{3} r^3$$

Example:

1) $\rho_V = \rho_0 r^2$ $\quad E_{r<R}$?



$$\Phi_e = \frac{Q_{enc}}{\varepsilon_0}$$

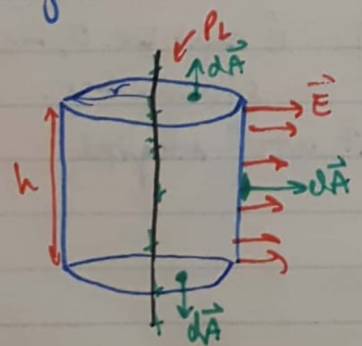
$$Q_{enc} = \int_0^r \rho_0 r^2 \cdot 4\pi r^2 \, dr = \rho_0 \int 4\pi r^4 \, dr = \rho_0 \frac{4}{5}\pi r^5$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{\rho_0 4\pi r^5}{5\varepsilon_0} = E\, 4\pi r^2 \qquad \therefore |\vec{E}| = \frac{\rho_0 r^3}{5\varepsilon_0}$$

Symmetries:

(Spherical for PC)

cylindrical:



$$\Phi_e = \cancel{\Phi_{top}} + \cancel{\Phi_{bottom}} + \Phi_{walls} \quad (\cos 90 = 0)$$

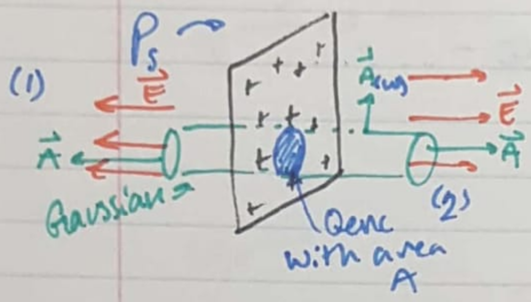
$$|\vec{E}| \int_{walls} |d\vec{A}| = \frac{Q_{enc}}{\varepsilon_0}$$

$$|\vec{E}|\, 2\pi r h = \frac{\rho_L \cdot h}{\varepsilon_0}$$

$$E = \frac{\rho_L}{2\pi\varepsilon_0 r}$$

Jan 24, 2018

(1)



$\varphi_1 + \varphi_2 + \varphi_w = \frac{Q_{enc}}{\varepsilon_0}$

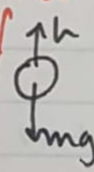
$\vec{E} \cdot \vec{A} + \vec{E} \cdot \vec{A} = Q_{enc}/\varepsilon_0$

$2EA = \frac{Q_{enc}}{\varepsilon_0}$

$E = \frac{\rho_s \cdot A}{2\varepsilon_0 \cdot A} = \frac{\rho_s}{2\varepsilon_0}$

Potential:

Gravitational

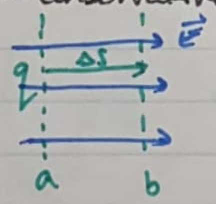


$\frac{\Delta U}{m} = gh = \Delta V_g$ (change in gravitational potential).

$*\Delta U_g = -W_g$, always. $W_{force} = \Delta U \Leftrightarrow \Delta K = 0$.

Generalization:

$\Delta U_{conservative} = -W_{conservative}$.



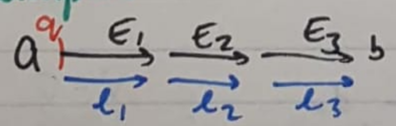
$\Delta U_e = -W_{F_e} = -\vec{F}_e \cdot \Delta\vec{s}$ *only if F_{const} (plane)

$= -\int_a^b q\vec{E} \cdot d\vec{\ell}$

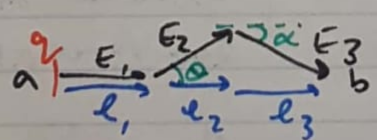
$\Delta V = -\int_a^b \vec{E} \cdot d\vec{\ell}$

Examples:

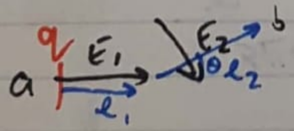
	W_F?	ΔU?	ΔV?

1) 

$q(E_1\ell_1 + E_2\ell_2 + E_3\ell_3)$ $\quad$ $-q(E_1\ell_1 + E_2\ell_2 + E_3\ell_3)$ $\quad$ $-E_1\ell_1, -E_2\ell_2 - E_3\ell_3$

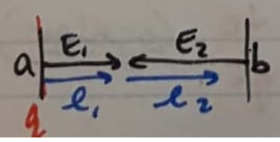
2) 

$\Delta V = -E_1\ell_1\cos\theta - E_2\ell_2\cos\theta - E_3\ell_3\cos\alpha$

3) 

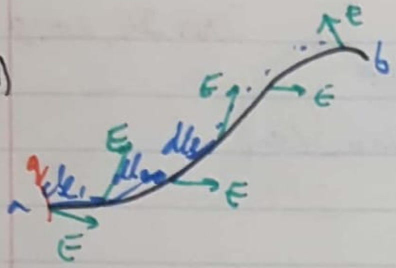
$\Delta U = -qE_1\ell_1 - qE_2\ell_2\cos\theta$ $\quad$ $\theta > 90$, $\cos\theta = -ve$

$\Delta V = -E_1\ell_1 - E_2\ell_2\cos\theta$

3.5) 

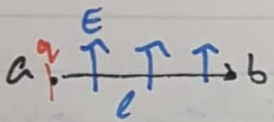
$\Delta V = -E_1\ell_1 + E_2\ell_2$ (gain potential

4)



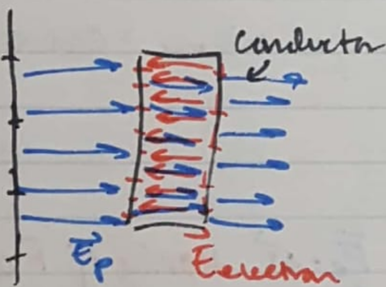
$$\Delta V = -E_1 \cdot dl_1 - E_2 \cdot dl_2 - \dots\dots$$
$$= -\int_a^b \vec{E} \cdot d\vec{l}$$

5)

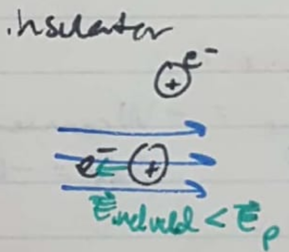


$\Delta V = \Delta U = 0.$ $(\cos 90)$

6)

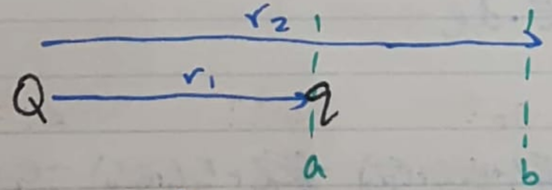


$\vec{E}_{inside} = 0$ (free electrons make field)



$E_{inside} = 0$, so $\Delta V = 0$ everywhere in a conductor.

7)



$$W_r = \int q \vec{E}_a \cdot \vec{dr} = q \int_{r_1}^{r_2} \frac{Q}{4\pi\varepsilon_0 r^2} dr$$

$$\Delta U = -q \int_{r_1}^{r_2} \frac{Q}{4\pi\varepsilon_0 r^2} dr$$

$$\Delta V = -\int_{r_1}^{r_2} \frac{Q}{4\pi\varepsilon_0 r^2} dr = \frac{Q}{4\pi\varepsilon_0 r_2} - \frac{Q}{4\pi\varepsilon_0 r_1}$$

$V = 0$ at $r = \infty$
$Q > 0$: $V = \infty$ at $r \to 0$
$Q < 0$: $V = -\infty$ at $r = 0$

$$V_Q(r) = \frac{Q}{4\pi\varepsilon_0 r}$$ } potential due to point charge. (scalar)

$$U_Q(r) = \frac{Qq}{4\pi\varepsilon_0 r}$$

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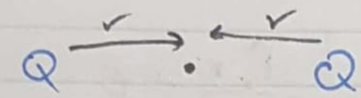
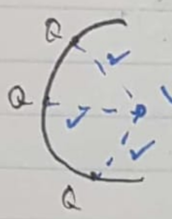
Jan 29, 2018

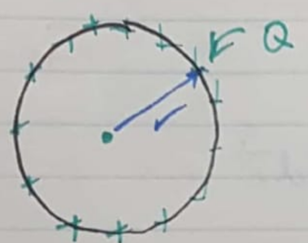
1)


$V = V_1 + V_2$

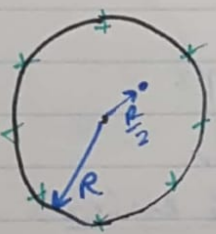
$V = \frac{2Q}{4\pi\varepsilon_0 r}$ (but $|\vec{E}| = 0$)

2)


$V = \frac{3Q}{4\pi\varepsilon_0 r}$ Total charge

3)


$V_{(0)} = \frac{Q}{4\pi\varepsilon_0 r}$ Total charge.

4)


Shell $r < R$

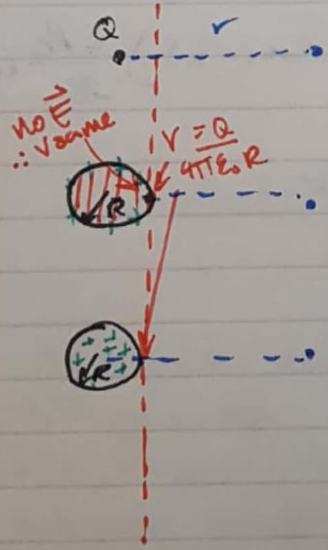
$V_{\frac{R}{2}} = ?$

$\Delta V = -\int_a^b \vec{E} \cdot d\vec{l}$

$\vec{E}_{inside} = 0$ (symm + Gauss)

$\therefore \Delta V = 0$, so

V same as center $\left(\frac{Q}{4\pi\varepsilon_0 r}\right)$

5)


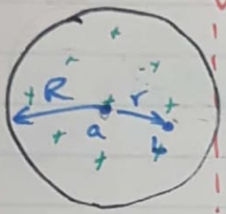
$E = \frac{Q}{4\pi\varepsilon_0 r^2}$ $V = \frac{Q}{4\pi\varepsilon_0 r}$

$r > R$

$E = \frac{Q}{4\pi\varepsilon_0 r^2}$ $V = \frac{Q}{4\pi\varepsilon_0 r}$

$E = \frac{Q}{4\pi\varepsilon_0 r^2}$ $V = \frac{Q}{4\pi\varepsilon_0 r}$

6)



$V = \frac{Q}{4\pi\varepsilon_0 R}$ ← use this as comp. at surface.

$$\Delta V = -\int_0^r \vec{E}\cdot d\vec{r} \qquad \frac{\rho_v r}{3\varepsilon_0} = E$$

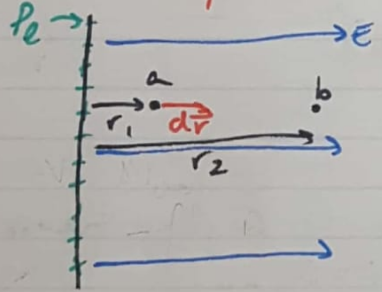
$$= -\int_0^r \frac{rQ}{R^3 4\pi\varepsilon_0}\,dr \qquad = -\int \frac{\rho_v r\cdot\hat{r}}{3\varepsilon_0}\cdot d\vec{r}$$

$$= -\left[\frac{r^2 Q}{R^3 8\pi\varepsilon_0}\right]_0^r \qquad = -\frac{\rho_v r^2}{6\varepsilon_0}\Big/_0^r$$

$$= -\frac{r^2 Q}{R^3 8\pi\varepsilon_0} \qquad = \frac{-\rho_v r^2}{6\varepsilon_0}$$

Potential from a Line:

Jan 31, 2018



$$\Delta V = -\int_{r_1}^{r_2} \frac{\rho_\ell \hat{r}}{2\pi\varepsilon_0 r}\cdot d\vec{r}$$

$$= -\int_{r_1}^{r_2} \frac{\rho_\ell\, dr}{2\pi\varepsilon_0 r}$$

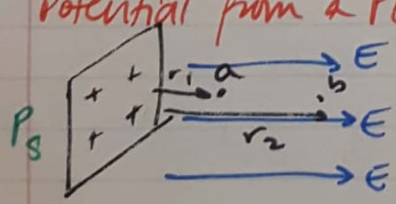
$$= -\left[\frac{\rho_\ell}{2\pi\varepsilon_0}\ln(r)\right]_{r_1}^{r_2}$$

$$= \frac{\rho_\ell \ln(r_1)}{2\pi\varepsilon_0} - \frac{\rho_\ell \ln(r_2)}{2\pi\varepsilon_0}$$

$$= \frac{\rho_\ell}{2\pi\varepsilon_0}\left(\ln(r_1) - \ln(r_2)\right) = \frac{\rho_\ell}{2\pi\varepsilon_0}\ln\left(\frac{r_1}{r_2}\right)$$

$$\Delta V = \frac{\rho_\ell}{2\pi\varepsilon_0}\ln\left(\frac{r_1}{r_2}\right)$$

Potential from a Plane:



$$\Delta V = -\int_{r_1}^{r_2} \frac{\rho_s}{2\pi\varepsilon_0}\,dr$$

$$= \frac{-\rho_s}{2\pi\varepsilon_0}\Delta r$$

$$\Delta V = \frac{-\rho_s}{2\pi\varepsilon_0}\Delta r$$

Conductors in Electrostatic Equilibrium:

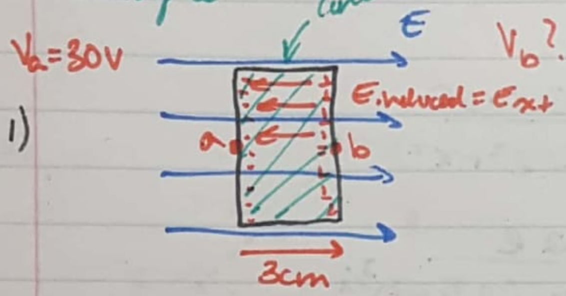
Conductors - free electrons.

→ Some are not as "bound" to nucleus.

↳ Will move in presence of field (current) ← external field.

↳ To be in eq, field inside conductor must = 0. always eq!

Example:



1) $\vec{E}_{inside} = 0$, so $\Delta V = 0$

$\therefore V_b = V_a = 30V$

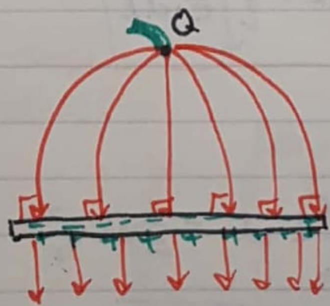
2) 

charge Q in conducting sphere.

$V_{\frac{R}{2}}$? E_{inside} must $= 0$.

↳ By Gauss's Law, charge must be at edge for $E=0$. ($Q_{enc}=0$)

→ $V_{r=0} = \frac{Q}{4\pi\epsilon_0 R}$. $\Delta V = 0 = \vec{E}$ so $V_{\frac{R}{2}} = \frac{Q}{4\pi\epsilon_0 R}$

3) 

Must intersect at 90°

so $E_{inside} = 0$ by shielding.

1) $\vec{E} = 0$ inside conductor
2) charge on surface
3) E just outside $= \sigma_{local}/\epsilon_0$
4) $\vec{E}$ normal to surface
5) Same V everywhere
6) Pointy areas have less charge, more density, more field. O-o

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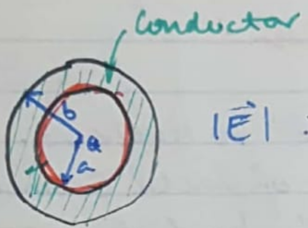
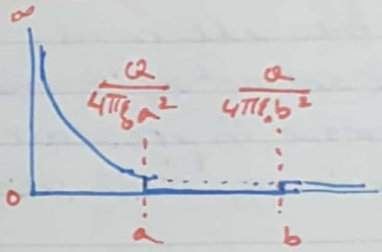
4)



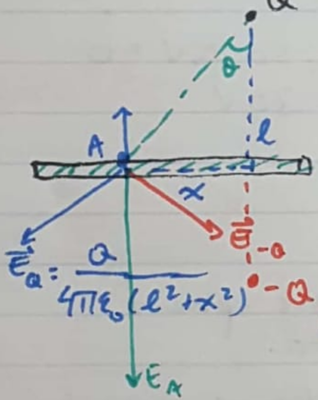
○ electrons w/ $-Q$ charge

$|\vec{E}|$:



Same as P.C but O gap.

5) E_A?



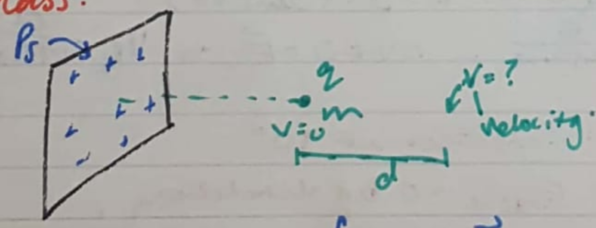
$-Q$ image created by conductor

$$E_A = \frac{2Q\cos\theta}{4\pi\varepsilon_0(l^2+x^2)}$$

$$= \frac{2Ql}{4\pi\varepsilon_0(l^2+x^2)^{3/2}}$$

Mass:

Feb 2, 2018.



$\Delta K = -\Delta U$

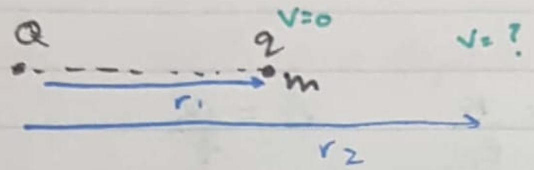
$$\boxed{\Delta K = q\int_a^b \vec{E}\cdot d\vec{l}}$$

$$\Delta U = -q\int \vec{E}_{plane}\, d\vec{l}$$

$$\Delta K = q\int \frac{P_s}{2\varepsilon_0}\,dl = \frac{q P_s}{2\varepsilon_0} d = \frac{1}{2} m v_2^2$$

$$v = \sqrt{\frac{q P_s}{m\varepsilon_0}}$$

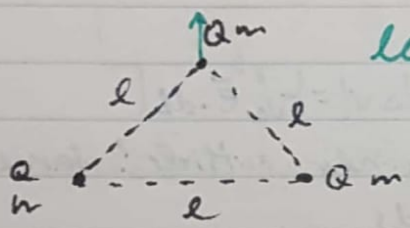
1)



$$\Delta K = -\Delta U = q\int_{r_1}^{r_2} \frac{Q}{4\pi\varepsilon_0 r^2}\,dr$$

$$= \frac{Qq}{4\pi\varepsilon_0 r_1} - \frac{Qq}{4\pi\varepsilon_0 r_2} = \frac{1}{2}mv_2^2 - 0.$$

2)

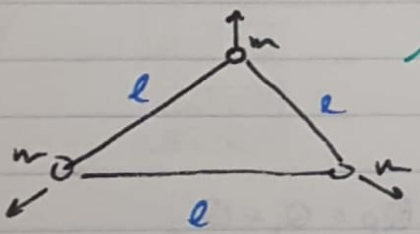


let one charge go. Speed @ ∞?

$$\Delta K = -\Delta U = -(U_f - U_i)$$
$$\frac{1}{2}mv^2 = -\left(\frac{Qq}{4\pi\varepsilon_0 l} - \frac{3Qq}{4\pi\varepsilon_0 l}\right)$$

whole system.

3)



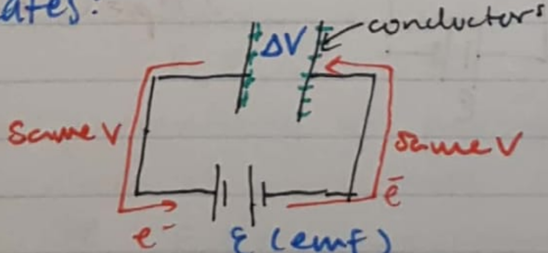
let all three charges go. Speed at ∞?

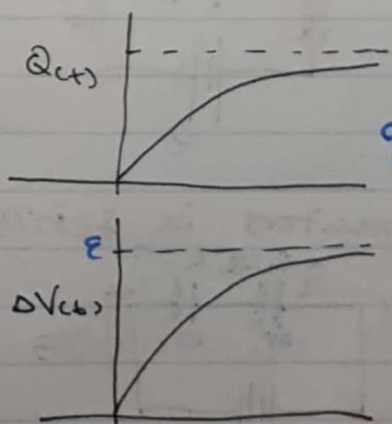
$$\Delta K = -\Delta U_s = -\left(0 - \frac{3Qq}{4\pi\varepsilon_0 l}\right) = 3\left(\frac{1}{2}mv^2\right)$$

Capacitance:

Feb 5, 2018

pll plates:



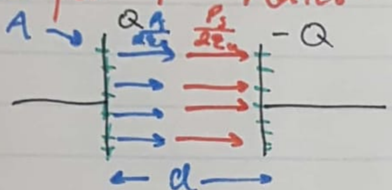


$$q_{(t)} = C\Delta V_{(t)}$$
$$= C\varepsilon$$

Battery will pump e^- from plate to plate until equillibrium.

All conductors, so ΔV across wire $= 0$.

C for P^{ll} Plates:

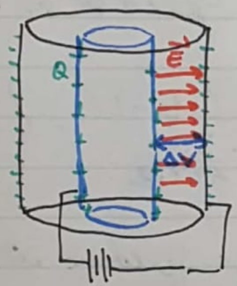


$$\vec{E} = \frac{\rho_s}{\varepsilon_0} \qquad \rho_s = \frac{Q}{A}$$

$$|\Delta V| = \left|-\int_a^b \vec{E}\cdot d\vec{l}\right| = \left|\frac{-\rho_s}{\varepsilon_0} d\right| = Ed.$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{\frac{\rho_s}{\varepsilon_0} d} = \frac{Q\varepsilon_0}{\rho_s d} = \frac{A^2\varepsilon_0}{Ad} = \boxed{\frac{A\varepsilon_0}{d}}$$

Cylindrical:



$Q = C\Delta V$

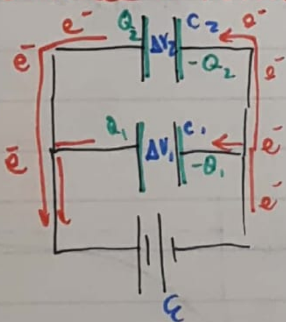
$Q = \int \rho_s\, ds \qquad |\Delta V| = \left|-\int_a^b \vec{E}\cdot d\vec{l}\right|$

Field from cylinder outside? Same as line.

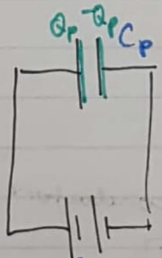
$$C = \frac{\int \rho_s\, ds}{\left|-\int(\vec{E}\cdot d\vec{l})\right|}$$

Capacitance in Parallel:

$\Delta V_1 = \Delta V_2 = \varepsilon$



≡



$Q_p = Q_1 + Q_2$

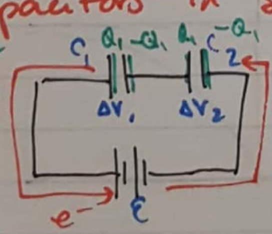
$\Delta V_p = \varepsilon = \Delta V_1 = \Delta V_2$

So, $C_p \Delta V_p = C_1\Delta V_1 + C_2 \Delta V_2$

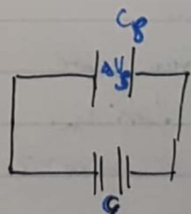
$C_p = C_1 + C_2$

Capacitors in series:

$-Q_1$ because G. shielding



≡



$\Delta V_s = \Delta V_1 + \Delta V_2 = \varepsilon$

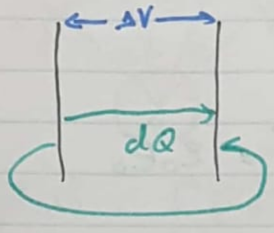
$\Delta V_s = \frac{q_1}{C_1} + \frac{q_2}{C_2} = \frac{Q_s}{C_s}$

But $Q_s = Q_2 = Q_1$

$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$

Energy Stored in a Capacitor:

Feb 7, 2018



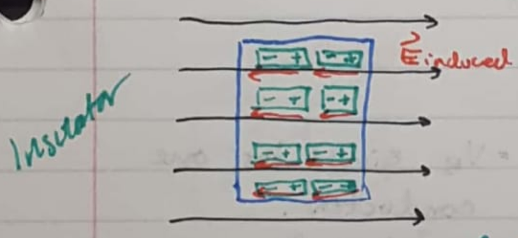
$dU = \Delta V dQ$

energy needed to bring dq across ΔV.

$$U \cong \sum_i du = \sum_i \Delta V dQ = \int_0^Q \Delta V dQ = \int_0^Q \frac{q}{C} dQ = \frac{Q^2}{2C}$$

$$\boxed{U = \frac{Q^2}{2C} = \frac{1}{2} C \Delta V^2 = \frac{1}{2} Q \Delta V}$$

Dielectrics:



$\vec{E} = \vec{E}_0 + \vec{E}_{ind}$

$= E_0 - E_{ind}$

$= \frac{E_0}{\varepsilon_r} = \frac{E_0}{K}$ ← rel. permativity / dielectric constant

Applies when $\vec{E}$ is perpendicular to boundary.

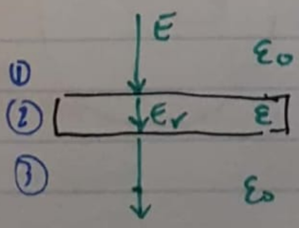
Gauss's Law for Dielectrics:

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{enc}}{\varepsilon_0 \varepsilon_r} = \frac{Q_{enc}}{\varepsilon}$$

ε_0: permativity of free space

ε_r: relative permativity (depends on material)

$\varepsilon = \varepsilon_0 \varepsilon_r$: permativity



$E_r(\text{inside dielectric}) = \frac{E}{\varepsilon_r}$ ← >1

E is not continuous across boundary.

$\vec{D} = \epsilon \vec{E}$ is continuous

1) $E = E_0$ $\quad D = \varepsilon_0 E_0$

2) $E = E_r = \frac{E_0}{\varepsilon_r}$ $\quad D = (\varepsilon_r \varepsilon_0)\frac{E_0}{\varepsilon_r} = \varepsilon_0 E_0$

3) $E = E_0$ $\quad D = \varepsilon_0 E_0$

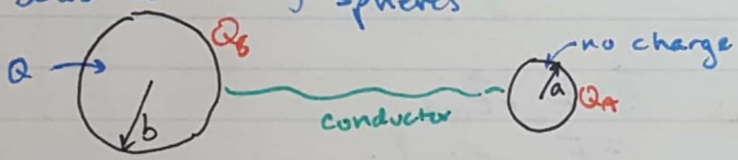
So, Gauss' Law for Dielectrics:

$$\oint \vec{D} \cdot d\vec{s} = Q_{enc}.$$

Midterm Review:

1) Solid conducting spheres

$V_{a/2}$ on small sphere

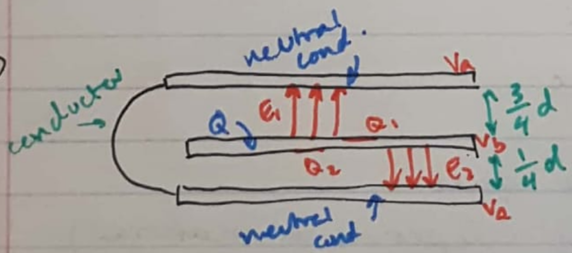


$V_B = \frac{Q_B}{4\pi\varepsilon_0 b}$ $\quad V_A = \frac{Q_a}{4\pi\varepsilon_0 a}$ $\Big\}$ At eq, $V_A = V_B$ since its one overall conductor.

$\hookrightarrow \frac{Q_B}{b} = \frac{Q_a}{a}$ $\quad Q_a + Q_B = Q$ $\Big\{ Q_A = \frac{Q}{\left(1+\frac{b}{a}\right)}$ $\Rightarrow V_A =$ ___

All over A.

2)



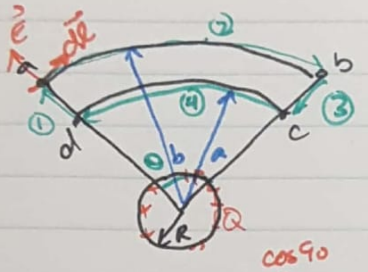
$Q_1 + Q_2 = Q$ $\quad E_1 = \frac{\rho_{s1}}{\varepsilon_0}$ $\quad E_2 = \frac{\rho_{s2}}{\varepsilon_0}$

$\Delta V_1 = \Delta V_2$

$$\left(\frac{\rho_{s1}}{\varepsilon_0}\right)\left(\frac{3d}{4}\right) = \frac{\rho_{s2}}{\varepsilon_0}\left(\frac{d}{4}\right) \Rightarrow \rho_{s1} = \frac{\rho_{s2}}{3}$$

$Q_1 = \frac{Q_2}{3} \Rightarrow Q_2 = \frac{3Q}{4}$

3)



$\oint \vec{E} d\vec{l} = 0.$

2, 4 are equipotential, no $|\vec{E}|$ change ($\cos 90$)
1, 3 cancel.

$$\oint \vec{E}\cdot d\vec{l} = \cancel{\int_a^b \vec{E}\cdot d\vec{l}}^{\cos 90} + \int_b^c \vec{E}\cdot d\vec{l} + \cancel{\int_c^d \vec{E}\cdot d\vec{l}}^{\cos 90} + \int_d^a \vec{E}\cdot d\vec{l}$$

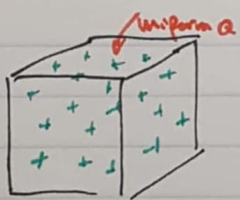
$$= 0 \text{ since } \int_d^a \vec{E}\cdot d\vec{l} = \frac{Q}{4\pi\varepsilon_0 d} - \frac{Q}{4\pi\varepsilon_0 a} = -\int_b^c \vec{E}\cdot d\vec{l}$$

4)

Φ_e through one face?

$\Phi_e = \dfrac{\underline{Q_{enc}}}{6\varepsilon_0}$ (6 faces)

6)

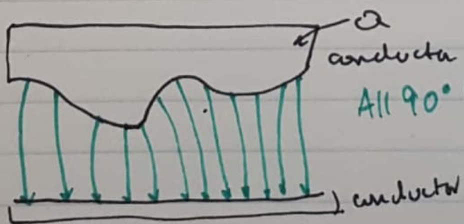


• E?

Gauss's Law doesn't work (E not $\perp$)

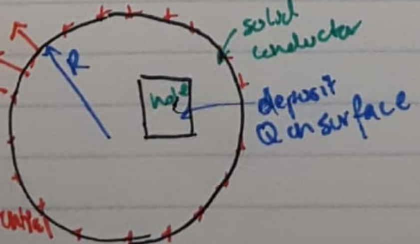
(would triple integral)

7)



All 90° for eq potential surface

8)



charge moves to be uniform b/c $E_{inside} = 0$, $E_{outside}$ = equipotential

a) $E_{in\ cube}$? b) $V_{center\ sphere}$? c) V_{cube}?

Field 0 inside, so a) = 0

b) = c) = $\dfrac{Q}{4\pi\varepsilon_0 R}$

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