Lab schedule
Learn

ECE105 - Introduction
Sep 8/2017

Practice:

$v_1 = 23\,m/s \quad \vec{a} = -9.8\,m/s^2 \quad v_2 = 0$

$d = v_2^2 - v_1^2 / 2a \quad = 27.0\,m$

$v_f^2 - v_0^2 = 2\vec{a}\Delta\vec{y}$

$0 - (23)^2 = 2|\vec{a}||\Delta\vec{y}|\cos 180 \quad -1$ this is where the - sign comes from (a is ~~not~~ -9.8)

$\Delta y = \dfrac{-23^2}{-2(9.8)}$

Definitions:

$\vec{v}_{avg} = \dfrac{\Delta\vec{x}}{\Delta t} \Big\} \lim_{\Delta t \to 0} \vec{v}_{avg} = \lim_{\Delta t \to 0} \dfrac{\Delta\vec{x}}{\Delta t} = \text{inst. } \vec{v} \Big\} \dfrac{d\vec{x}}{dt}$

$\vec{a}_{avg} = \dfrac{\Delta\vec{v}}{\Delta t} \Big\} \lim_{\Delta t \to 0} \vec{a}_{avg} = \lim_{\Delta t \to 0} \dfrac{\Delta\vec{v}}{\Delta t} = \text{inst. } \vec{a} \Big\} \dfrac{d\vec{v}}{dt}$

$\Delta\vec{x} = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t$

$\hookrightarrow \vec{x}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{x}_0$ (just made it Δd)

assuming constant $\vec{a}$

$x(t) = t^3 \quad X(t) = \int v\,dt + C$

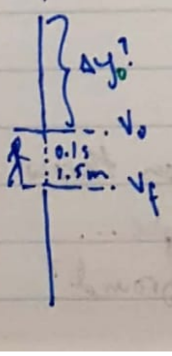
$v(t) = 3t^2 \quad = t^3 + C$ ← or x_0

$a = 6t \quad V(t) = \int a\,dt + v_0$ ← need const for initial values

Example:

You are sitting in your room. Stone dropped from top of building, you see it from window for 0.1s. From what height above ⊞ was dropped if ⊞ 1.5m

$v_1 = 0 \quad a = +9.8$



$\Delta\vec{y} = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t$

$1.5 = \frac{1}{2}(+9.8)(0.1)^2 + \vec{v}_0(0.1)$

$\vec{v}_0 = 14.51\,m/s$

$v_f^2 - v_0^2 = 2(9.8)\Delta y$

$14.51^2 - 0 = 19.6\Delta y$

$\Delta y =$

Scanned by CamScanner

Lecture 2 - Relative Motion

Sep 11/2017

Example:

1) You throw 2 balls simultaneously. one ↑ 30m/s one ↓ 15m/s. Distance between them 3 seconds later?

Ball 1 Δd: $\frac{1}{2}\vec{a}t^2 + v_0 t$
$= -\frac{1}{2}(9.8)(3)^2 + (30)(3)$
$= 45.9$ m

Ball 2 Δd: $\frac{1}{2}\vec{a}t^2 + v_0 t$
$= -\frac{1}{2}\vec{a}t^2 + v_0 t$
$= -\frac{1}{2}(9.8)(3)^2 + (-15)(3) = -89.1$

$|\Delta d_2 - \Delta d_1| = 135$ m

Best way ⇒ $\vec{v}_{12} = 45$ $\quad \vec{v}_{12} \times t = \Delta d = 135$

2) You let go of a ball and 0.5 seconds later you throw another straight down at 9.8m/s. $\vec{v}_{12}$ after 3 seconds?

ball 1 is going 4.9 m/s

4.9m/s, no matter what t is.

Relative Motion:

$\vec{v}_{12} = \vec{v}_1 - \vec{v}_2$

$\Delta\vec{x}_{12} = \Delta\vec{x}_1 - \Delta\vec{x}_2$

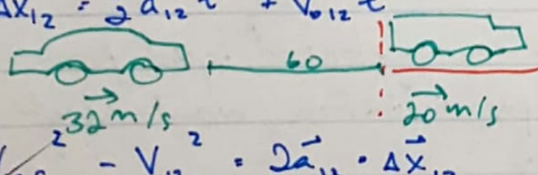
$\vec{a}_{12} = \vec{a}_1 - \vec{a}_2$

Example:

1) You are driving at 32 m/s, 60 m behind a truck moving at 20 m/s. Truck brakes at 4 m/s². What must be your $\vec{a}$ for no collision?

* Can't assume truck stops before you hit it.

$\Delta\vec{x}_{12} = \frac{1}{2}\vec{a}_{12}t^2 + v_{0_{12}}t$



need speeds to match at 60m.

60m

match $\cancel{v_{12f}^2} - v_{12_0}^2 = 2\vec{a}_{12}\cdot\Delta\vec{x}_{12}$

$-(v_{1_0} - v_{2_0})^2 = 2(\vec{a}_1 - \vec{a}_2)\cdot(\vec{x}_2 - \vec{x}_1)\cdot(-1)$

$-(12)^2 = -2(\vec{a}_1 - 4)(60)$

Find $\vec{a}_1$

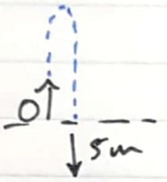
2) You are in an elevator moving up at 6 m/s (open roof), you throw an apple straight up at 25 m/s (rel. to you). Max height of apple...
a) rel to you? b) rel to original height above ground?

Projectile Motion:

Sep 13, 2017

Example:

1)



$v = 20\,m/s$ $v_f = ?$

$\Delta d_v = -5m$

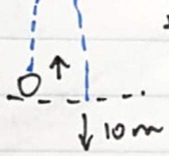
$\Delta d_v = \frac{v_f^2 - v_o^2}{2a}$

$-5 = \frac{v_f^2 - 20^2}{-2(9.8)}$

$v_f = 22.3\,m/s$ (down)

$v_f^2 - v_o^2 = 2|\vec{a}||\Delta d|(\cos\theta)$ — $\underbrace{\ }_{+ve}$ $\underbrace{\ }_{+1}$

2)



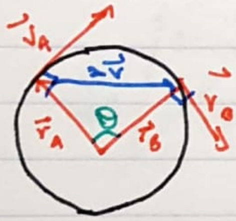
$v = 30\,m/s$ $t = ?$

$\Delta d_v = \frac{1}{2}\vec{a}t^2 + \vec{v}_o t$

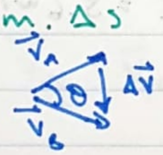
$-10 = \frac{1}{2}(-9.8)t^2 + (30)t$

$t =$

Circular Motion:



Sim. Δs



$\Delta\vec{v} \perp \Delta\vec{r} + // \Delta\vec{a}$

$a_r = \frac{v^2}{r}$

$s = \theta r$

$v = \omega r$

$a = \alpha r$

Example:

1) An object moves around a vertical circle starting at $\theta = 0$, $t = 0$, its speed is $10\,m/s$ 2 seconds later. What is the total $\vec{a}$ at $t = 2$?

Not constant $\vec{a}$. $\vec{a}_t$ $\vec{a}_r$ *Forgot to give $r = 7m$

Tangential $\vec{a}_t$: Use kin. equations to find $\vec{a}_t = 5\,m/s^2$

$\vec{a}_c + \vec{a}_t = \vec{a}$

↑ perpendicular.

$\vec{a}_c = \frac{v^2}{r} = \frac{100}{7}$

$|\vec{a}| = \sqrt{a_t^2 + a_c^2}$

$= \sqrt{25 + (\frac{100}{7})^2}$

Albert Wasef

ECE 105 Lab Introduction

Sep 14, 2017

Review guidlines on learn.

Read lab manual before lab

Submit on learn dropbox

↳ prelab1, data1, report1, lab1.

↳ Same file renamed.

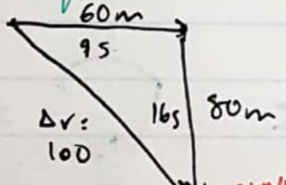
Lab 1 on 28th

Error Analysis - read on learn.

Example:

Sep 15, 2017

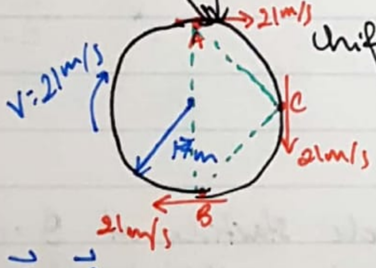
1)



$\vec{V}_{avg} = \frac{100}{25} = 4 m/s$

$V_{avg}\ (speed) = \frac{140}{25} = 5.6 m/s$

2)



Uniform.

Find: $\vec{V}_{avg_{AB}} = ?$ $\quad \vec{a}_{avg_{AB}} = ?$

$\vec{V}_{avg_{AC}} = ?$ $\quad \vec{a}_{avg_{AC}} = ?$

$V_{avg}\ (speed) = ?$

$\vec{A} \to \vec{B}$:

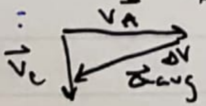
$\vec{V}_{avg} = \frac{2(17)}{(\pi r / 21)}$ ← Δd/Δt, so Δt = (Δd)($\vec{v}$) $= \frac{34}{(\pi r)(1/21)} (-\hat{j})$

$\vec{A} \to \vec{C}$:

$\vec{V}_{avg} = \left[\frac{(\sqrt{2})(17)}{(\pi r / (2)(21))}\right] \cos 45° \hat{i} + \left[\frac{(\sqrt{2})(17)}{(\pi r / (2)(21))}\right] \sin 45° \hat{j}$

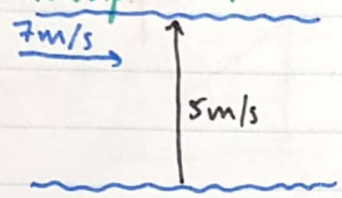
$\vec{a}_{avg}\ \vec{A} \to \vec{B} = \frac{42(-\hat{i})}{(\pi r / 21)}$ $\quad v_A \rightarrow \quad v_B \leftarrow \quad \therefore \vec{a} \leftarrow$

$\vec{a}_{avg}\ \vec{A} \to \vec{C}$:



$= \frac{\Delta \vec{V}}{\Delta t} = \frac{\vec{V}_c - \vec{V}_A}{\Delta t} = \frac{\quad}{(\pi r / (2)(21))}$

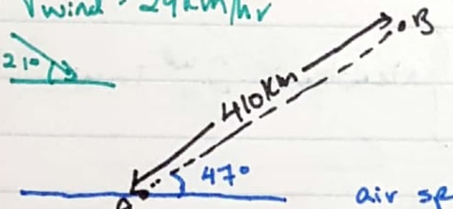
Example..?

1) 

Must aim straight across to cross in fastest time.

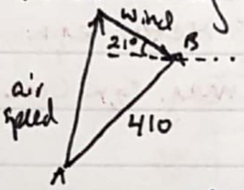
Bearing: where you aim
Heading: where you end up.

2) $V_{wind} = 29 km/hr$

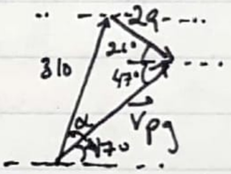


air speed = 310km/hr

a) How long does trip take?



$\vec{V}_{pg} = \vec{V}_{pa} + \vec{V}_{ag}$



$\frac{\sin 68}{310} = \frac{\sin \alpha}{29}$

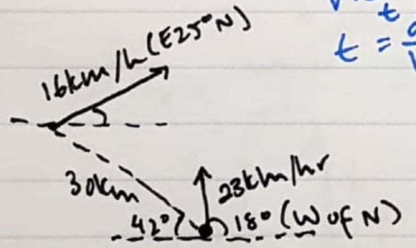
$\alpha = 4.98°$

$\frac{\sin 107°}{\vec{V}_{pg}} = \frac{\sin 68}{310}$

$V = \frac{d}{t}$

$t = \frac{d}{V} = \frac{410}{319.7} = 1.28 hr$ bearing: 51.98°

$\vec{V}_{pg} = 319.7 m/s$

3) 

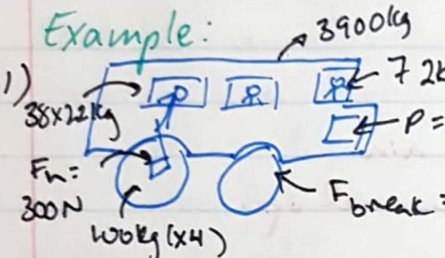
what is closest distance of approach?

Find relative V rel to time. Derive for rel. $\Delta \vec{d}$.

Forces: Sep 18, 2017

Example:

1)

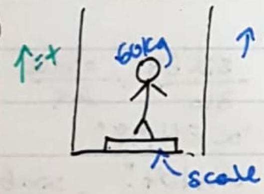


F_{net} = wheel?

$F_{net} = ma$
$= (100)(3.2)$
$= 320N$

} Newton's 2^{nd} law

2)



↑ $a = 7 m/s^2$

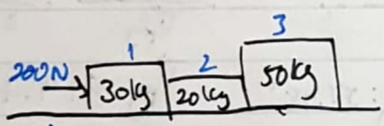
$\sum \vec{F}_y = m\vec{a}$

$\vec{N} + m\vec{g} = m\vec{a}$ ← vector eqn

$N - mg = ma$ ← mag. eqn (cos θ makes −)

$N = 1008N$

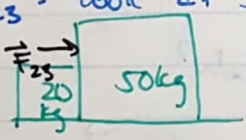
3)



$\sum F = ma = 200$

$a = 2 m/s^2$

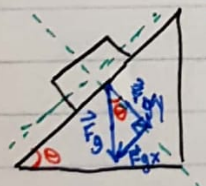
$\vec{F}_{23}$ = look at 3 in isolation



$\sum F = ma$
$= (2)(50)$
$= 100N$

Sep 20, 2017

*Grl2 physics? axis in direction of $\vec{a}$

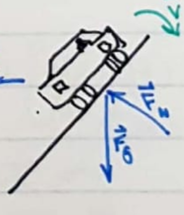


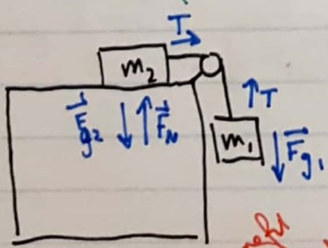
$\vec{F}_{gy} = -\vec{F}_N = \vec{F}_g \cos\theta$

Not a law!

$\vec{F}_{gx} = \vec{F}_g \sin\theta$

$\vec{F}_c$





*be careful w/ a direction

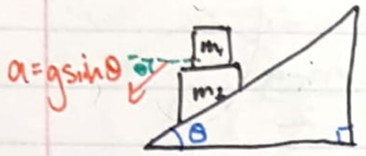
$\vec{F}_N = \vec{F}_{g2}$

$\sum \vec{F}_x = ma$

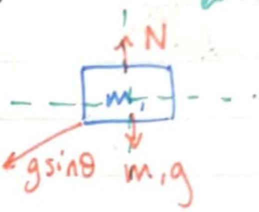
$T = m_2 a$ or $m_1 a = F_{g1} - T$ (depends on $\vec{a}$)

$a = \frac{m_1 g}{m_1 + m_2}$ $T =$

Find the $\vec{N}$ between m_1 and m_2.



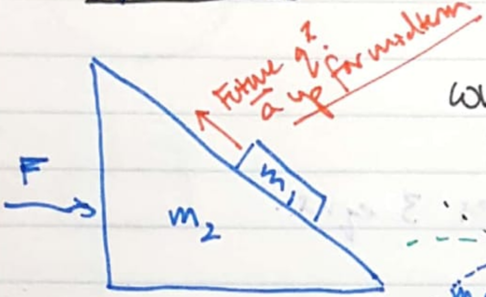
break up $\vec{a}$ into components



$\vec{N} + m_1\vec{g} = m\vec{a}_y$

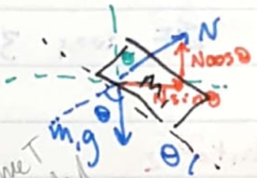
$N = m_1 g - m_1 g \sin^2\theta$

$= m_1 g \cos^2\theta$



What does F = ? so m_1 does not slide on m_2

$F = (m_1 + m_2)a$



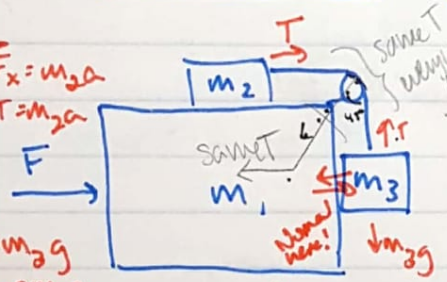
~~$N = m_1 g\cos\theta$~~ more $\vec{F}$ applied

$\Sigma F_y = m\vec{a}_y$ $\qquad$ $\Sigma F_x = m\vec{a}_x = 0$

$\boxed{N\sin\theta = m_1 a}$ $\quad$ $\boxed{N\cos\theta = m_1 g}$ $\quad$ $\boxed{a = g\tan\theta}$

$\Sigma \vec{F}_x = m_2 a$

$T = m_2 a$



1) What force do you have to push m_1 w/ so m_3 does not slide on m_1?

2) You let system go. Find $\vec{a}$ of m_1 ←.

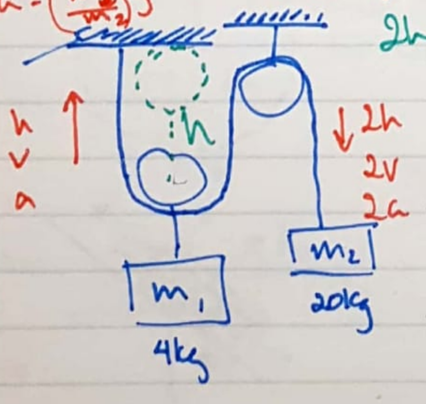
m_1 feels tension ← since pulley is part of mass.

$T = m_3 g$

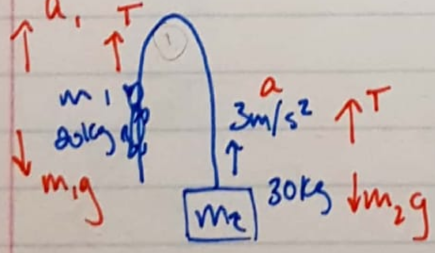
$m_3 g = m_2 a$

$a = \left(\frac{m_3}{m_2}\right)g$ $\qquad \therefore F = (m_1 + m_2 + m_3)\left(\frac{m_3}{m_2}\right)g$

2h slack avail, so $m_2 \downarrow 2h$.



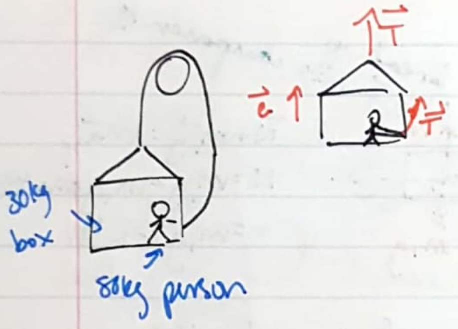
☆ Review grip notes ☆



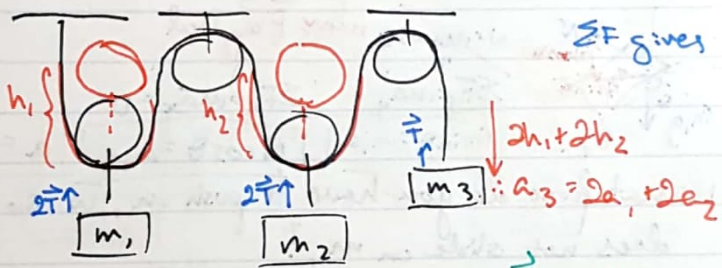
$\Sigma \vec{F}_{y\,m_2} = m_2\vec{a}_y = \vec{T} + m_2\vec{g} = m_2\vec{a}_2$

$\Sigma F_{y\,m_1} = m_1\vec{a}_y = \vec{T} + m_1\vec{g} = m_1 a_2$

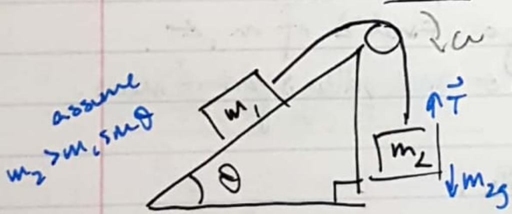
$a_1 = \dfrac{m_2 g - m_1 g + m_2 a}{m_1}$



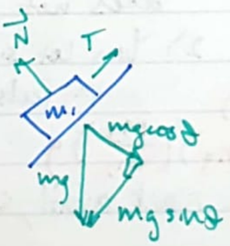
$$\sum \vec{F}_y = m\vec{a}_y$$
$$2\vec{T} + m_T\vec{g} = m_T\vec{a}$$
$$T = \frac{m_T g + m_T a}{2} = \frac{m_T g}{2}$$

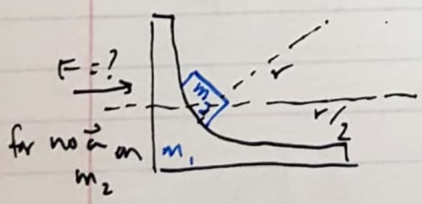


ΣF gives 3 eqns.



$a = ?$

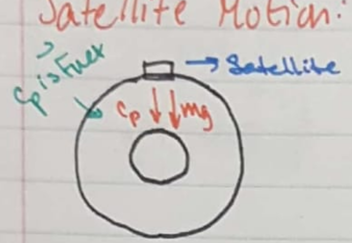


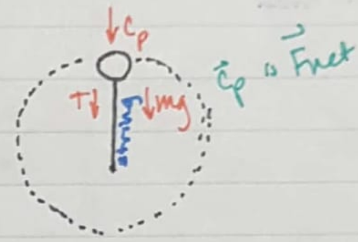


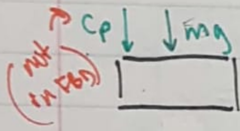
$(m_1+m_2)\,g\tan\theta$

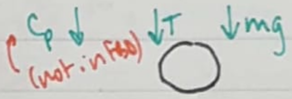
Satellite Motion:

Sep 25, 2017

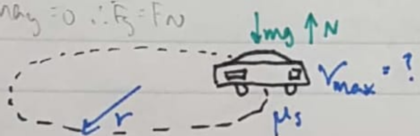








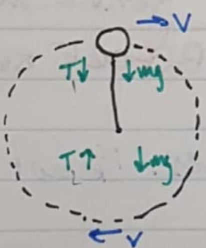
$\Sigma \vec{F}_y = m\vec{a}_y = 0 \therefore F_g = F_N$



$\Sigma \vec{F}_r = \frac{mv^2}{r} = \mu_s \vec{F}_N = \mu_s F_g = \mu_s m g$

$\frac{mv^2}{r} = \mu_s mg$

$v = \sqrt{\mu_s r g}$



vertical O. Find ΔT from top and bottom.

Top: $\Sigma \vec{F}_r = \frac{mv^2}{r} = T_1 + mg$ $\quad T_1 = \frac{mv^2}{r} - mg$

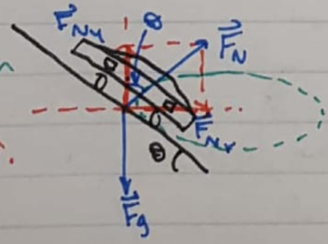
Bottom: $\Sigma \vec{F}_r = \frac{mv^2}{r} = T_2 - mg$ $\quad T_2 = \frac{mv^2}{r} + mg$

$\Delta T = T_1 - T_2$

$= \frac{mv^2}{r} - mg - \left(\frac{mv^2}{r} + mg\right)$

$= -2mg$

no friction
v = ?



$\vec{N} \neq mg\cos\theta$!

$\Sigma \vec{F}_y = \vec{F}_{N_y} + \vec{F}_g$

$\therefore \vec{F}_{N_y} = \vec{F}_g = mg$

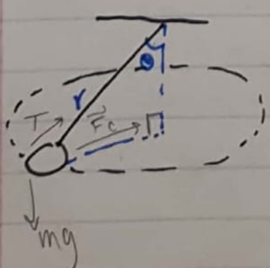
$F_N \cos\theta = mg$

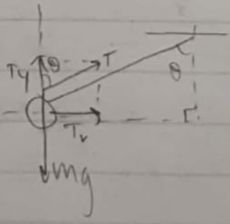
$\Sigma \vec{F}_r = m\vec{a}_r$

$\vec{F}_{N_r} = m\frac{v^2}{r}$

$F_N \sin\theta = \frac{mv^2}{r}$

$\tan\theta = \frac{v^2}{rg}$





$\Sigma \vec{F}_y = 0$

$\vec{T}_y + m\vec{g} = 0$, so $T\cos\theta = mg$ (2)

$\Sigma \vec{F}_r = m\vec{a}_r$

$\vec{T}_r = m\frac{v^2}{r}$

(1) $T\sin\theta = \frac{mv^2}{r}$

$\tan\theta = \frac{v^2}{rg}$

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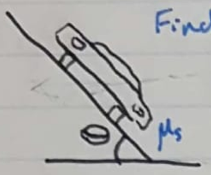
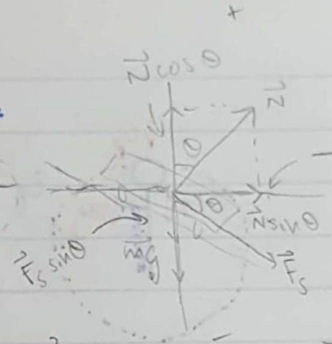


Find V_{max}



$\vec{F_s} = \vec{F}_{s\,max} = \mu_s F_N$

$\sum \vec{F_x} = m\vec{a_x} = m\frac{v^2}{r}$

$\vec{F_N}\sin\theta + \vec{F_s}\cos\theta = \frac{mv^2}{r}$

$\vec{F_N}\sin\theta + \mu_s\vec{F_N}\cos\theta = \frac{mv^2}{r}$

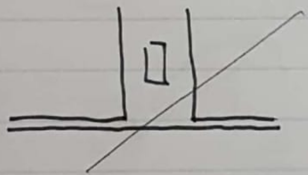
÷ $F_N\cos\theta - \mu_s F_N\sin\theta = mg$

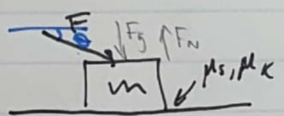
$\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} = \frac{v^2}{rg}$

$\sum \vec{F_y} = m\vec{a_y} = 0$

$\vec{F_N}\cos\theta + \vec{F_s}\sin\theta + \vec{F_g} = 0$

$F_N\cos\theta - \mu_s F_N\sin\theta = mg$





*Don't assume this object is gonna move! Need to establish this BEFORE solving.

$f_{s\,max} = F_N\mu_s = (mg + F\sin\theta)\mu_s$

$F\cos\theta \geq F_{s\,max}$ ∴ object moves. F_k not F_s

$\sum \vec{F_x} = m\vec{a_x} = \vec{F}\cos\theta + \vec{F_k}$

$= \vec{F}\cos\theta + \vec{F_N}\mu_k$

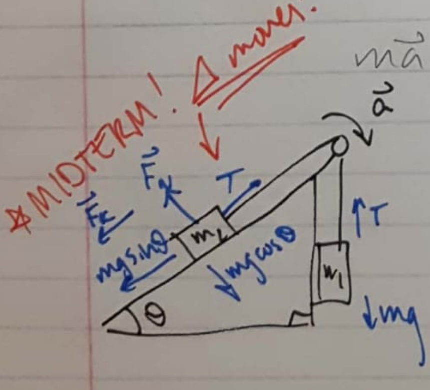
$m\vec{a_x} = F\cos\theta - F_N\mu_k$

$\sum \vec{F_y} = m\vec{a_y} = 0 = \vec{F}\sin\theta + \vec{F_N} + \vec{F_g}$

$\vec{F_N} = \vec{F}\sin\theta + mg$

$m\vec{a_x} = F\cos\theta - (\vec{F}\sin\theta + mg)\mu_k$

$= F\cos\theta - F\sin\theta\mu_k - mg\mu_k$

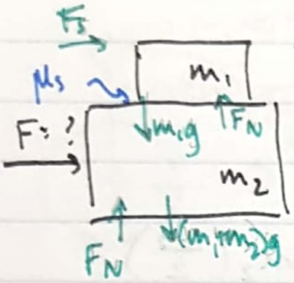


$T - m_2 g\sin\theta - f_k = m_2 a$

$f_k = F_N\mu_k = mg\cos\theta\,\mu_k$

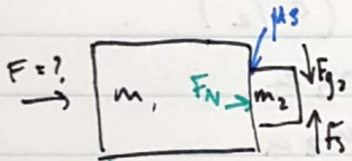
$T - m_1 g = -m_1 a$

Sep 29, 2017



$F_{max} = ?$ So m_1 doesn't slide on m_2

$$F_s = ma$$
$$F_N \mu_s = ma$$
$$mg\mu_s = ma$$
$$a = g\mu_s \quad \therefore \quad F = (m_1+m_2)g\mu_s$$

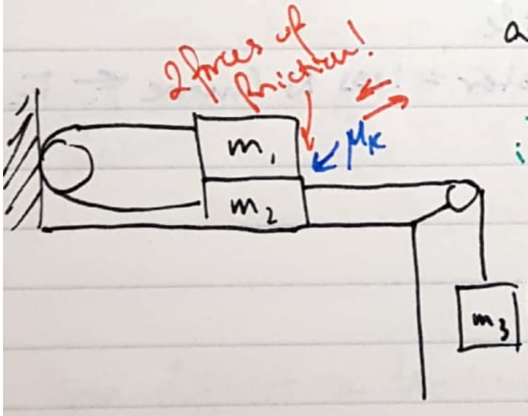


$F_{min} = ?$ so m_2 doesn't move ↑↓ ?

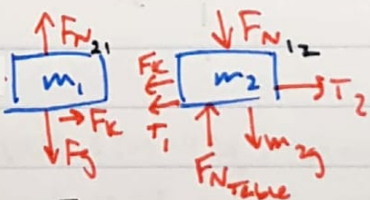
$$\Sigma \vec{F}_x = m_2 a$$
$$F_N = m_2 a$$

$$F_s = F_{g_2}$$
$$F_N \mu_s = F_{g_2} = m_2 g$$
$$m_2 a \mu_s = m_2 g$$
$$a = g/\mu_s$$

$$F = (m_1+m_2)a$$
$$= \frac{g(m_1+m_2)}{\mu_s}$$



$$i)\ m_3 g - 2m_1 g\mu_K = (m_1+m_2+m_3)a$$



$$F_{N_{12}} = F_{N_{21}} = m_1 g$$

$m_1)$ $\Sigma \vec{F}_y = 0 \Rightarrow \vec{F}_{N_{21}} = m_1 g$

$$\Sigma \vec{F}_x = m_1 \vec{a}_x$$
$$\vec{T}_1 + \vec{f}_K = m_1 \vec{a}$$
$$T - m_1 g\mu_K = m_1 a \quad (1)$$

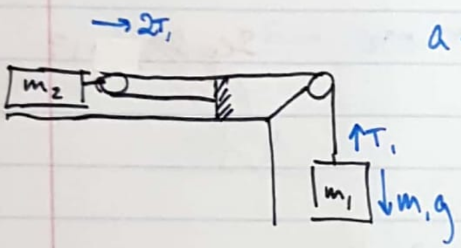
$m_2)$ $\Sigma \vec{F}_x = m_2 \vec{a}_x$

$$\vec{T}_2 + \vec{T}_1 + \vec{F}_K = m_2 \vec{a}$$
$$T_2 - T_1 - F_K = m_2 a$$
$$T_2 - T_1 - m_1 g\mu_K = m_2 a$$

$m_3)$ $\Sigma \vec{F}_y = m_3 \vec{a}_y$

$$\vec{T}_2 + m_3\vec{g} = m_3 \vec{a}_y$$
$$m_3 g - T_2 = m_3 a \quad (3)$$

Now, add all 3 eqns to get i).



$a = ?$

$$\sum \vec{F}_{m_1 y} = m_1 \vec{a}_y$$

$$m_1 g - T = m_1 \vec{a}_y \quad (1)$$

$$\sum \vec{F}_{m_2 x} = m_2 \vec{a}_x = m_2\left(\frac{\vec{a}_y}{2}\right)$$

$$2T = m_2\left(\frac{\vec{a}_y}{2}\right) \quad (2)$$

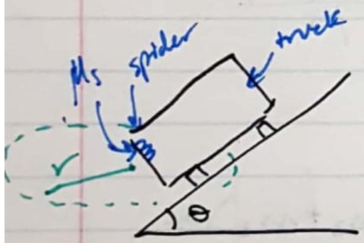
$$\frac{4m_1 g}{4m_1 + m_2} = a$$

$$2(m_1 g - m_1 \vec{a}) = m_2(\vec{a}/2)$$

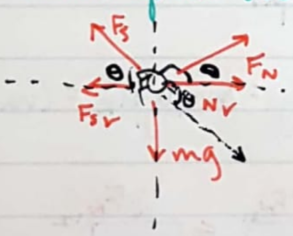
$$4(m_1)(g - a) = m_2 a$$

$$4m_1 g - 4m_1 a = m_2 a$$

$$4m_1 g = m_2 a + 4m_1 a$$

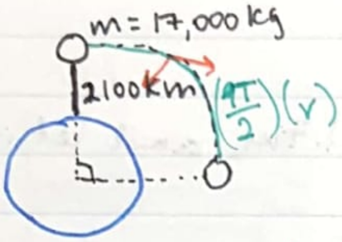


Vmax so spider doesn't slide



Faster = less N (more $\leftarrow F_c$)

Oct 2, 2017.



Find work done by gravity.

$W_T = \Delta K_e = 0$!

or... $W_T = \vec{F} \cdot \Delta\vec{d} = |\vec{F}||\vec{d}|\cos\theta \leftarrow \cos 90° = 0.$

These are $\perp$

$\boxed{\text{Total work}^{\text{on object}} = \text{change in}\ {}^{\text{object's}}\ \text{kinetic energy}}$

You hurl a cooper mini ($m = 690kg$) at 6m/s @ Mansour. He stops over a distance of 3m then hurls it back @ same $\vec{v}$. W?

$W_T = \Delta K_e = 0$, again.

F, d stay the same:

$W_{F_1} = \vec{F} \cdot \vec{d} = |\vec{F}||\vec{d}|\cos 180° \leftarrow -1$ when you stop

$W_{F_2} = \vec{F} \cdot \vec{d} = |\vec{F}||\vec{d}|\cos 0° \leftarrow +1$ when you release

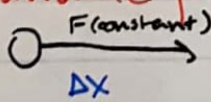
$\Delta W_T = W_{F_1} + W_{F_2}$ cancel out.

You deflect a hockey puck ($m = 42g$) moving at 32m/s by 30 degrees. You hit it at 20° to its original direction so the final speed is same as initial. W?

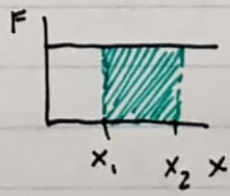
$\therefore$ No change in kinetic energy

$W_T = \Delta E_k = 0$, again.

Variable Forces in 1D:

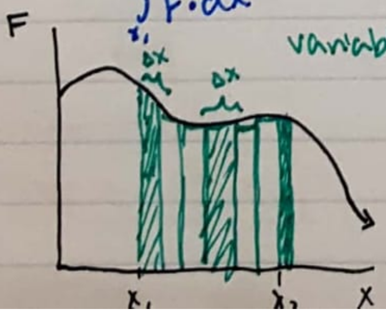


$w = |\vec{F}||\vec{d}|\cos 0 = |\vec{F}||\vec{d}|$

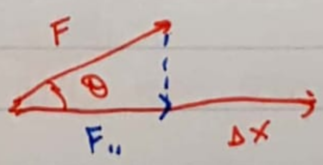


w = area under curve.

$\int_{x_1}^{x_2} \vec{F} \cdot d\vec{x}$



variable F



$W = |\vec{F}||\vec{\Delta x}|\cos\theta$

$= \vec{F}_{\parallel}\Delta x$

Example:

1) You lift a heavy ball from ground ↑ 1.7m and let it go at 7m/s.
12kg

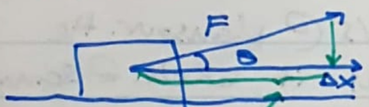
$W_T = \Delta K$ ← actually final - initial

cos θ changes sign

$W_{you} + W_g = \Delta K$

$W_{you} = \frac{1}{2}mv^2 + m\vec{g}\cdot\vec{h}$

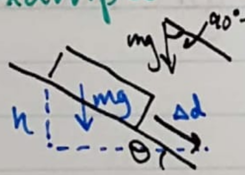
Work in 2D:



$F_{\parallel}$, or $F\cos\theta$.

Example:

1)



$W_g = ?$

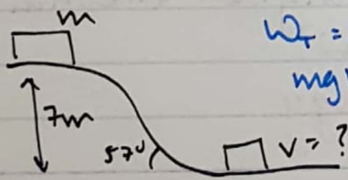
$W_g = m\vec{g}\cdot\Delta\vec{s}$

$= mg\Delta s(\sin\theta)$

$= mgh$

moral of the story: work done by gravity? all that matters is vertical displacement!

2)

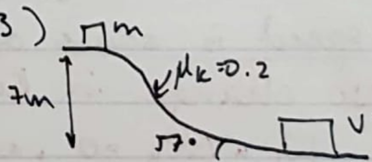


$W_T = \Delta K_e$

$mgh = \frac{1}{2}mv^2$

$v = \sqrt{2gh}$

3)



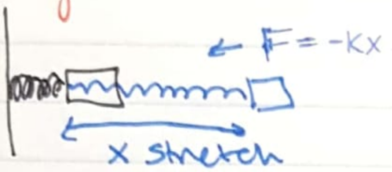
$W_T = \Delta K_e$

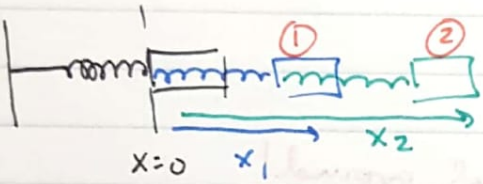
$W_g + W_f = \Delta K_e$

$mgh + (-mg\cos\theta\,\mu_k)\left(\frac{h}{\sin 57°}\right) = \frac{1}{2}mv^2$

Springs

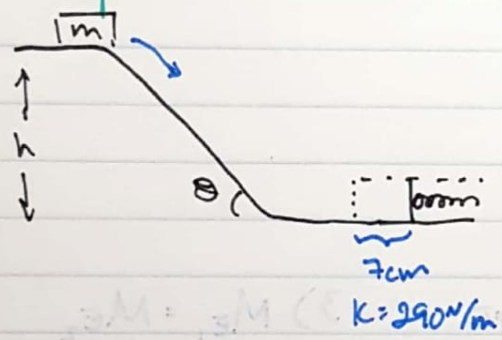
Oct 4, 2017





$$W_s = \int_{x_1}^{x_2} -kx\,dx \; \boxed{= \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2}$$

Example:

1) 

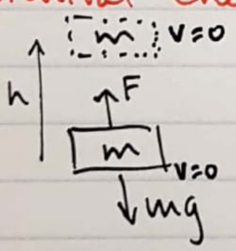
Block slides and hits already compressed spring. Max comp. of string?

$W_T = \cancel{\Delta K_e}^{\,0}$ $\quad W_T = W_g + W_s = 0$

$mgh + \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2 = 0$

$(12)(9.8)(0.9) = \frac{1}{2}(290)(0.07)^2 = \frac{1}{2}(290)x_2^2$

Potential Energy:

A) 

$W_F = \vec{F} \cdot \vec{h} = mgh$

$W_g = m\vec{g} \cdot \vec{h} = -mgh.$

$\Delta P_E = -W_g$. ← this applies for any cons. force.

↳ $= \Delta U_g$.

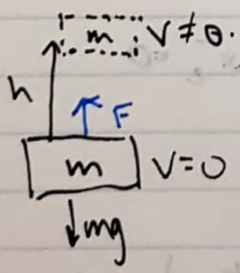
→ Conservative Forces:

↳ Can get ΔE back.

① ↳ $\boxed{F_F}$ is not conservative (can't get heat back).

② ↳ $\boxed{F_{app}}$ by humans

} ☆

B) 

$W_T = \Delta K_e$

$W_F + W_g = \Delta K_e \rightarrow W_{noncons.} + W_{cons.} = \Delta K_e$

$W_F - \Delta U_g = \Delta K_e \quad\quad W_{nc} - U_c = \Delta K_e$

$W_F = \underbrace{\Delta K_e + \Delta U_g}_{= \Delta M_E \text{ mech. energy}}$.

$$\boxed{W_{non.c} = \Delta M_E}$$

$$\boxed{\begin{aligned} W_T &= \Delta K \\ W_C &= -\Delta U_c \\ W_{NC} &= \Delta M_E \end{aligned}}$$

$W_{nc} = \Delta M_E$ VERY IMPORTANT

If $W_{nc} = 0$

$\Rightarrow \Delta M_E = 0$, so

1) $\Delta K + \Delta U = 0$

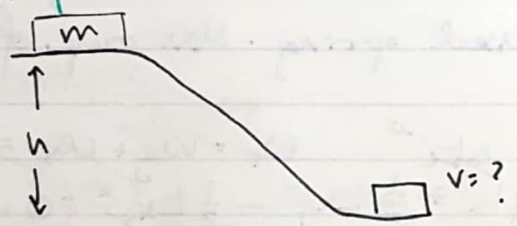
2) $M_{E_1} = M_{E_2} = M_{E_3} \ldots$

So...

$$\boxed{\begin{aligned} \Delta U_s &= -W_s \\ &= \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2 \end{aligned}}$$

switched around!

Example:

1) 

1) $W_T = \Delta K$

$W_g = \Delta K$

$mgh = \frac{1}{2}mv^2$

2) $W_{nc} = \Delta M_e = 0$

$\Delta M_E = 0$

$\Delta K + \Delta U = 0$

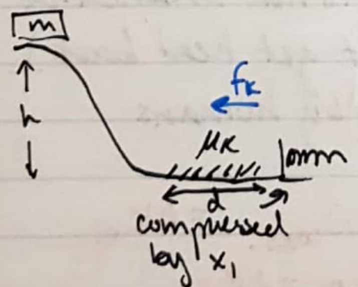
$\Delta K = -\Delta U$

$\Delta K = -(-mgh)$

$\frac{1}{2}mv^2 = mgh$

3) $M_{E_1} = M_{E_2}$

$mgh = \frac{1}{2}mv^2$

2) 

1) $W_T = \Delta K_e$

$W_g + W_F + W_s = \Delta K_e$

$mgh - mg\mu_K d + \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2 = 0$

2) $W_{nc} = \Delta M_e$

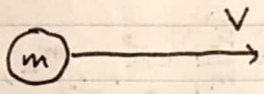
$-mg\mu_K d = \Delta K_e + \Delta U$

$-mg\mu_K d = -mgh + \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$

Momentum:

Oct 6, 2017.

$\vec{p} = m\vec{v}$



$$\frac{dp}{dt} = \frac{d(m\vec{v})}{dt} = \vec{v}\frac{dm}{dt} + m\frac{d\vec{v}}{dt} = \vec{F}_{net}$$

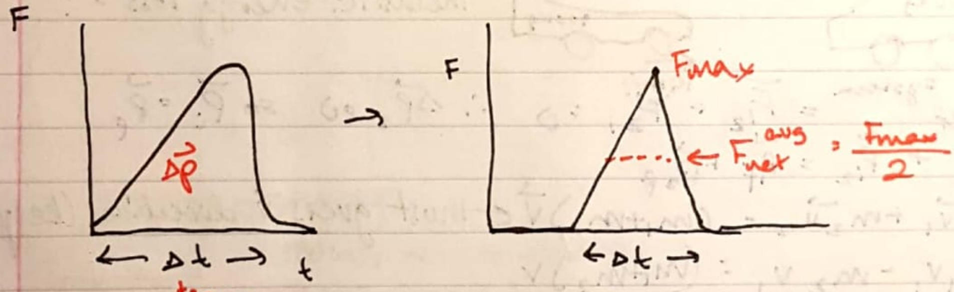
If m is constant, this = 0, so $F_{net} = ma$.

$dp = \vec{F}dt$

$\Delta\vec{p} = \int_{t_1}^{t_2} \vec{F}\,dt = \vec{F}_{net}^{\,avg}\,\Delta t.$

$\vec{F}_{net}^{\,avg} = \frac{\Delta\vec{p}}{\Delta t}$ $\Big\}$ $\Delta\vec{p}$:

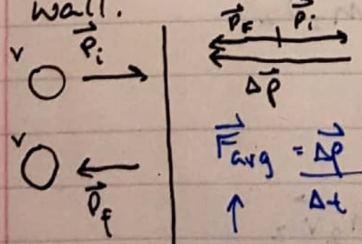
Impulse is Δmomentum



$$\Delta\vec{p} = \vec{I} = \int_{T_1}^{T_2} \vec{F}\cdot dt = \vec{F}_{avg}\,\Delta t$$

Example:

1) You throw a ball which strikes a wall horizontally. The impact lasts 70 ms, m = 33 g. V_{in} and V_{out} = 42 m/s. Find F_{avg} of ball on wall.



$\vec{F}_{avg} = \frac{\Delta\vec{p}}{\Delta t}$

↑

Wall→ball

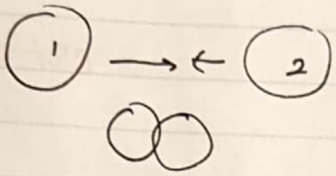
$$= \frac{\vec{P}_f - \vec{P}_i}{\Delta t}$$

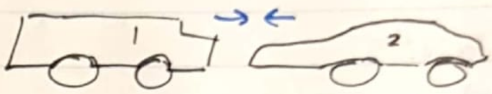
$$= \frac{mv - (-mv)}{\Delta t}$$

$$= \frac{2mv}{\Delta t}$$

$\vec{F}_{b\to w} = -\vec{F}_{w\to b}$

Scanned by CamScanner

2) 



on $m_1 \div \frac{\vec{F}_{21}}{m_1} = \vec{a}_1$, m_2, $\frac{\vec{F}_{12}}{m_2} = \vec{a}_2$

$F^{net}_{system} = \vec{F}_{12} + \vec{F}_{21} = 0.$

If $F^{net}_{system} = 0$, $\Delta\vec{P}_s = 0.$

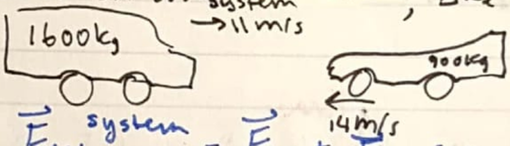
Collisions:

Inelastic collisions:

→ momentum$_{system}$ conserved, ΔK_e not.

entire system!

"complete inelastic"
→ cars become a clump

Ex: 

Inelastic Energy loss = ?

$\vec{F}^{system}_{net} = \vec{F}_{12} + \vec{F}_{21} = 0$ $\therefore \Delta\vec{P}_s = 0$ so $\vec{P}_i = \vec{P}_f$

$\vec{P}_{i_1} + \vec{P}_{i_2} = \vec{P}_{1f} + \vec{P}_{2f}$

$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1+m_2)\vec{v}$ ← must guess $\vec{v}$ direction (keep +)

$m_1v_1 - m_2v_1 = (m_1+m_2)v$

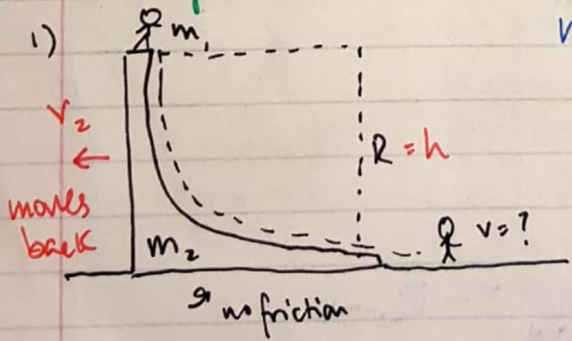
$v = 2m/s$

$E_{loss} = K_{ei} - K_{ef}$

$= \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v^2\right) - \left(\frac{1}{2}(m_1+m_2)v^2\right)$

$= 180{,}000J$

Example:

1) 

$mgh = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$m_1v_1 = m_2v_2$

If moves:

$\vec{P}_{Tx_1} = \vec{P}_{Tx_2}$

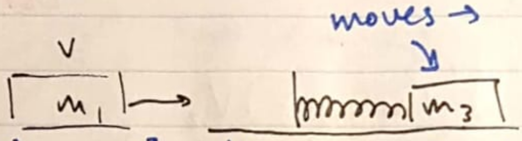
$0 = m_1\vec{v}_1 + m_2\vec{v}_2$

$m_1v_1 = m_2v_2$

$m_1gR = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$m_1gR = \frac{1}{2}m_1\left(\frac{m_2}{m_1}\right)^2 v_2^2 + \frac{1}{2}m_2v_2^2$

2) max comp?



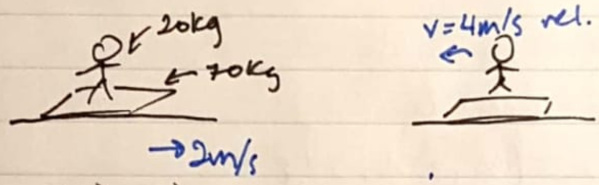
↳ when speeds are equal
↳ both move as 1 object

$\frac{1}{2}m_1v_1^2 = \frac{1}{2}(m_1+m_2)v^2 + \frac{1}{2}kx_{max}^2$

energy eqn.

$m_1v_1 = (m_1+m_2)v$ ← $\vec{P}_{T_1} = \vec{P}_{T_2}$

3)



Final sled speed?

$\vec{P}_{T_1} = \vec{P}_{T_2}$

$(90)(2) = (x+4)(20) + (70)(x)$

$x = 2.9\ m/s$

assume ←

OR ↗

$m_T\vec{v} = m_k\vec{v}_k + m_s\vec{v}_s$

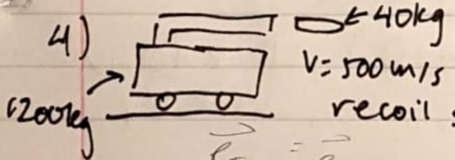
$m_T v = -m_k v_k + m_s v_s$

$\vec{v}_{ks} = \vec{v}_k - \vec{v}_s$ → +

$-4 = -v_k - x$ sub

$4 - x = v_k$

4)



V = 500 m/s rel. to cannon

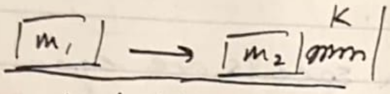
recoil speed?

$\vec{P}_{T_1} = \vec{P}_{T_2}$

$(1240)(0) = (1200)(x) + (40)(500)$

$x = -16.67\ m/s$

ECE Midterm Review Session Oct 13, 2017.



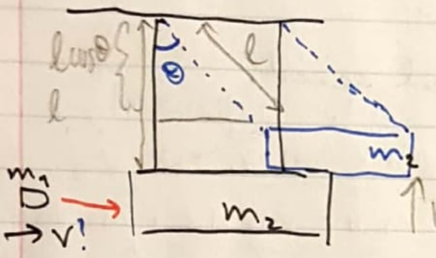
inelastic. x_{max}?

$\vec{P}_{T_1} = \vec{P}_{T_2}$

$m_1 v_1 = (m_1 + m_2) V_2$

$\frac{1}{2}(m_1+m_2)V_2^2 = \frac{1}{2} k x^2$

Energy not conserved during collisions (after, yes).



$h = l - l\cos\theta$

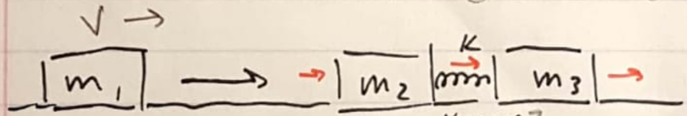
$m_1 v_1 = (m_1 + m_2) v_2$

$v_2 = \left(\frac{m_1 v_1}{m_1 + m_2}\right)$

$\frac{1}{2}(m_1+m_2) v_2^2 = (m_1+m_2) g \Delta h$

$v_2^2 = 2g(l - l\cos\theta)$

$\left(\frac{m_1 v_1}{m_1 + m_2}\right)^2 = 2g(l - l\cos\theta)$ All known but V_1



xmax?

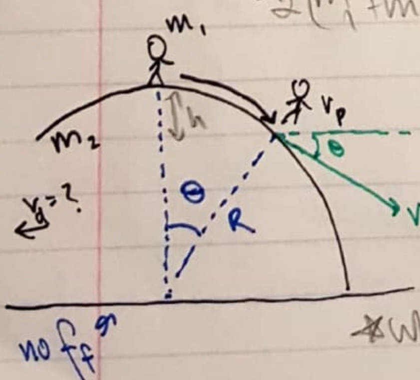
Inelastic

$\vec{P}_{T_1} = \vec{P}_{T_2}$

$m_1 v_1 = (m_1 + m_2) v_2$ and at max amp, same V, so $m_1 v_1 = (m_1 + m_2 + m_3) v_3$

Energy now:

$\frac{1}{2}(m_1 + m_2) v_2^2 = \frac{1}{2}(m_1 + m_2 + m_3) V_3^2 + \frac{1}{2} k x_{max}^2$

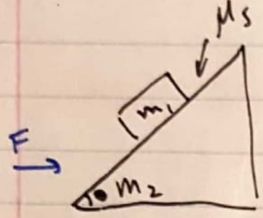


$h = R - R\cos\theta$ $\quad mg(R - R\cos\theta) = \frac{1}{2} m_1 v_p^2 + \frac{1}{2} m_d V_d^2$

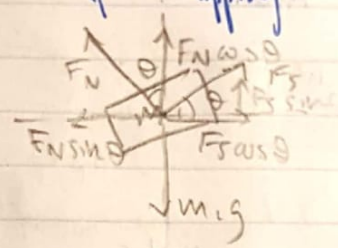
$\vec{P}_{x_i} = \vec{P}_{x_f}$

$0 = m_1 v_p \cos\theta - m_d v_d$ ← solve

*when does he fly off?



Fmax before slipping?



$F = (m_1 + m_2)a$

$\sum \vec{F}_x = m_1 a$

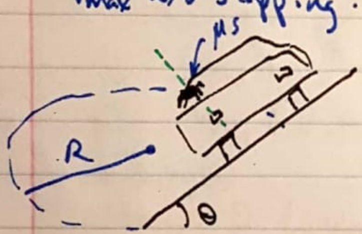
① $\mu_s F_N \cos\theta - F_N \sin\theta = m_1 a$

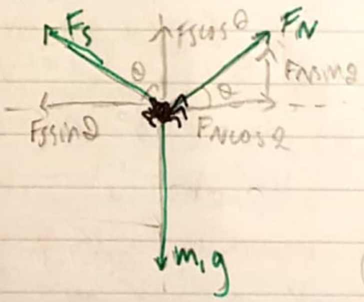
$\sum F_y = 0$

② $F_N \cos\theta + \mu_s F_N \sin\theta = m_1 g$

①/② $= \dfrac{\mu_s \cos\theta - \sin\theta}{\cos\theta + \mu_s \sin\theta} = \dfrac{a}{g}$

Vmax w/o slipping.





$\sum \vec{F}_x = \dfrac{mv^2}{r}$

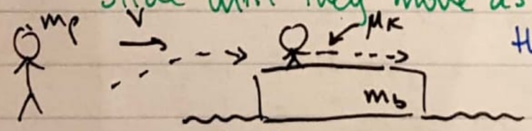
① $\mu_s F_N \sin\theta - F_N \cos\theta = \dfrac{mv^2}{r}$

$\sum \vec{F}_y = 0 \quad \mu_s F_N \cos\theta + F_N \sin\theta = m_1 g$

② $\mu_s F_N \cos\theta + F_N \sin\theta = m_1 g$

①/② $\dfrac{\mu_s \sin\theta - \cos\theta}{\mu_s \cos\theta + \sin\theta} = \dfrac{v^2}{rg}$

slide until they move as one



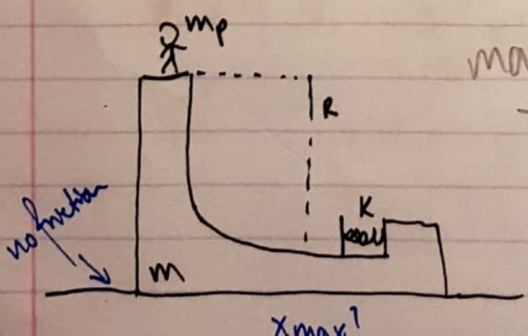
How far slide on block? rel to block

$\vec{p}_{T_1} = \vec{p}_{T_2}$

$m_p v_1 = (m_p + m_b) v_2$ move as one

no f

$\frac{1}{2} m_p v_1^2 = \frac{1}{2}(m_p + m_b) v_2^2 + m_p g \mu_K \cdot d$



Xmax?

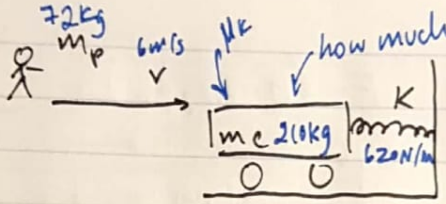
max comp when speeds are equal!

$\vec{p}_{T_1} = 0 = \vec{p}_{T_2}$

$0 = (m_1 + m_2) v_2$ So $v_2 = 0$.

$\therefore mgR = \frac{1}{2} k x^2$ since objects stop moving.

Scanned by CamScanner



how much does person slide on cart?

$x_{max} = ?$

$\vec{p}_{T_1} = \vec{p}_{T_2}$

$m_p v_p = (m_p + m_c) v_c$

After collision

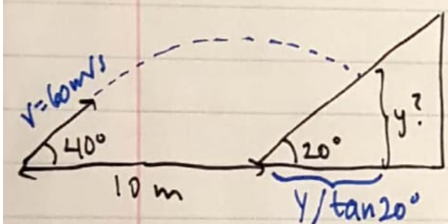
$\hookrightarrow \frac{1}{2}(m_c + m_p) v_c^2 = \frac{1}{2} k x_{max}^2$

Solve for x_{max}

$\frac{1}{2} m_p v_p^2 = mg\mu_k d + \frac{1}{2} k x_{max}^2$

Solve for d.

$\frac{1}{2}(m_c + m_p) v_c^2 + m_p g \mu_k d = \frac{1}{2} m_p v_p^2$



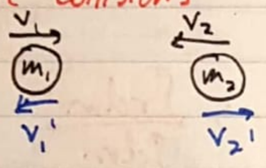
$d_x = v\cos\theta t$

$10 + \frac{y}{\tan 20°} = v\cos\theta t$

$d_y = v\sin\theta t - \frac{1}{2} a t^2$

$y = v\sin\theta t - \frac{1}{2} g t^2$

Elastic Collisions Oct 25, 2017.



Energy and momentum conserved.

$\xrightarrow{\text{momentum}} +$

1) Conserve momentum:

① $m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$ *even if 0, don't cancel yet.

2) Conserve energy:

② $\frac{1}{2}m_1\vec{v}_1^{\,2} + \frac{1}{2}m_2\vec{v}_2^{\,2} = \frac{1}{2}m_1\vec{v}_1'^{\,2} + \frac{1}{2}m_2\vec{v}_2'^{\,2}$ now they bounce off

→ Group by mass, choose + momentum, and assume unknown direction

① $m_1(v_1 + v_1') = m_2(v_2 + v_2')$

② $\frac{1}{2}(m_1)(v_1^2 - v_1'^2) = \frac{1}{2}m_2(v_2^2 - v_2'^2)$ } Group by mass

sub → $m_1(v_1 - v_1')(v_1 + v_1') = m_2(v_2 - v_2')(v_2 + v_2')$

divide by ① → $v_1 - v_1' = v_2' - v_2$

Now: 2 linear eqns, 2 unknowns:

① $m_1 v_1 + m_1 v_1' = m_2 v_2' + m_2 v_2$

② $v_1 - v_1' = v_2' - v_2$

Want: v_2'.

② $m_1 v_1 - m_1 v_1' = m_1 v_2' - m_1 v_2$ $(\times m_1)$

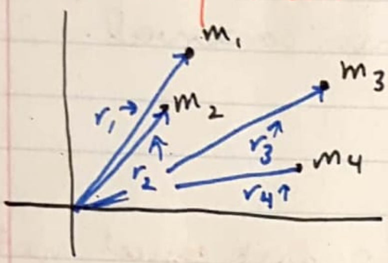
$-$② + ① $2m_1 v_1 = m_2 v_2' + m_2 v_2 + m_1 v_2' - m_1 v_2$

$v_2' = \dfrac{2m_1 v_1 - m_2 v_2 + m_1 v_2}{m_1 + m_2}$ Now find v_1' NOT by subbing in v_2'. Do same as above but $\times m_2$.

−ve answer? opposite assumption.

★ → If $v_1' = -5$? WRONG ASSUMPTION!

Centre of Mass:

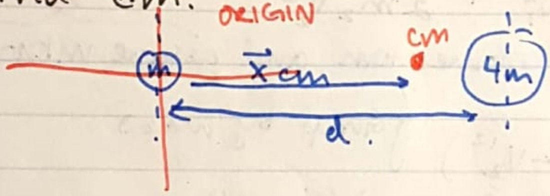


$$\vec{r}_{cm} = \frac{\sum_i m_i \vec{r_i}}{\sum_i m_i} \rightarrow \frac{\int r\,dm}{\int dm}$$

$$x: \vec{x}_{cm} = \frac{m_1\vec{x}_1 + m_2\vec{x}_2 \ldots}{m_1 + m_2 \ldots} \quad y, z \text{ etc.}$$

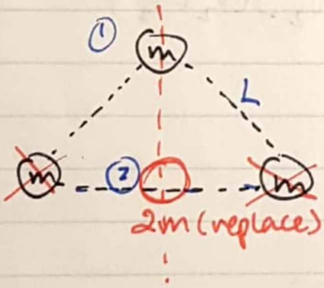
Example:

1) Find c.m.



$$\vec{x}_{cm} = \frac{m(0) + 4m(d)}{5m} = \frac{4}{5}d.$$

2) eq. Δ. 30°



$$\vec{y}_{cm} = \frac{m_1\vec{y}_1 + m_2\vec{y}_1}{m_1 + m_2} = \frac{m\left(\frac{\sqrt{3}}{2}\right)l + 2m(0)}{3m} = \frac{\sqrt{3}}{6}l.$$

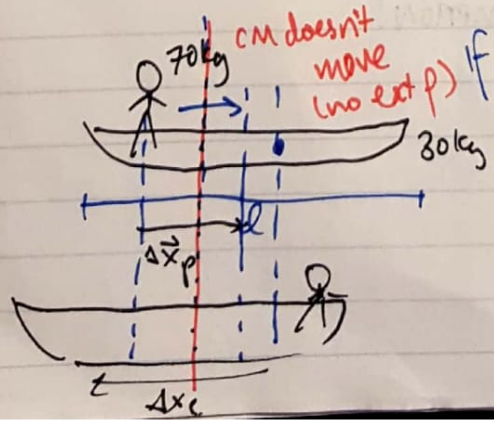
$$m_T \vec{r}_{cm} = m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots$$

$$m_T \vec{V}_{cm} = m_1\vec{v}_1 + m_1\vec{v}_2 + \ldots \quad \text{CM momentum}$$

$$m_T \vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots \quad \text{CM acceleration}$$

$$= \vec{F}_{net_1} + \vec{F}_{net_2} + \ldots \quad \text{EXTERNAL forces only.}$$

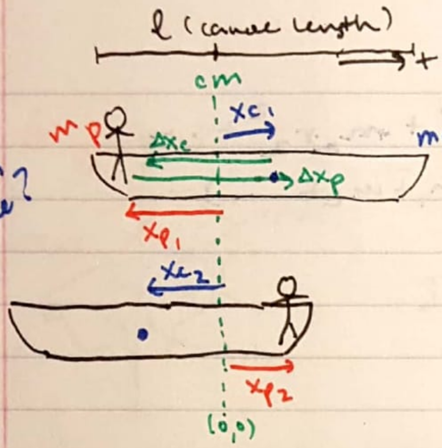
3)



If no ext forces, CM doesn't move!

$$m_p \Delta x_p = m_c \Delta x_c.$$

1) How far does canoe move?



both 0.

$m_T \vec{x}_{cm_1} = m_p \vec{x}_{p_1} + m_c \vec{x}_{c_1}$ Oct 25, 2017

$m_T \vec{x}_{cm_2} = m_p \vec{x}_{p_2} + m_c \vec{x}_{c_2}$

$0 = m_p \Delta \vec{x}_p + m_c \Delta \vec{x}_c$

can get from momentum b/c mom. conserved.
↳ Internal forces only.

$m_1 \vec{v}_1 + m_2 \vec{v}_2 = 0$ $\vec{p}_i = \vec{p}_f = 0$.

$\Delta \vec{x}_{pc} = \Delta \vec{x}_p - \Delta \vec{x}_c$ $m_p \Delta x_p - m_c \Delta x_c = 0$ (→+) ①

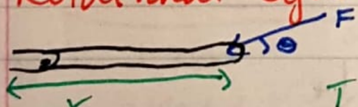
$l = \Delta x_p - (-\Delta x_c)$

② $= \Delta x_p + \Delta x_c$

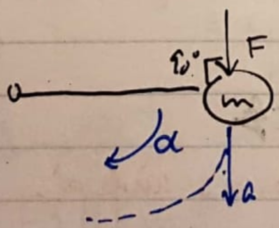
$\Delta x_p = \frac{m_c \Delta x_c}{m_p}$ → sub ② $l = \frac{m_c \Delta x_c}{m_p} + \Delta x_c$

$\Delta x_c = \frac{l}{1 + \frac{m_c}{m_p}}$

Rotational Dynamics: Torque



$\tau = rF\sin\theta$



Always specify pivot as subscript of τ

$F = ma$

$\tau = |r||F| = rma$

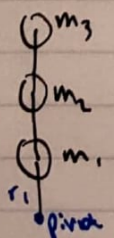
$\tau = rm\alpha r$

$\tau = mr^2\alpha$

$r \times F$ but $\sin 90 = 1$

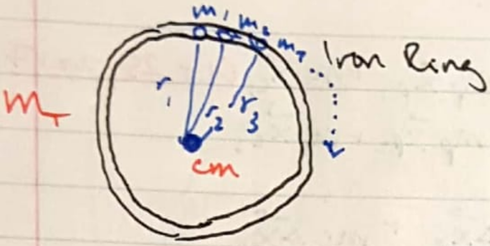
inertia: ability to resist angular acc. linear? m.

relationship btwn F and a. τ is a F.

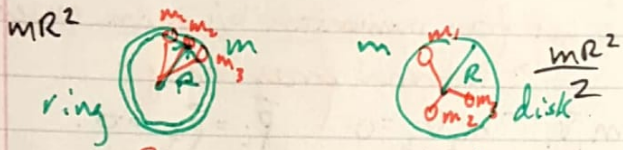


$\tau_{net} = \tau_1 + \tau_2 + \tau_3$
$= (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2)\alpha$
$= I_{total}\,\alpha$

Total Inertia $= m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$



$$I_{cm} = m_1 r_1^2 + m_2 r_2^2 + \ldots$$
$$= r^2 (m_1 + m_2 + \ldots)$$
$$= m_T r^2$$

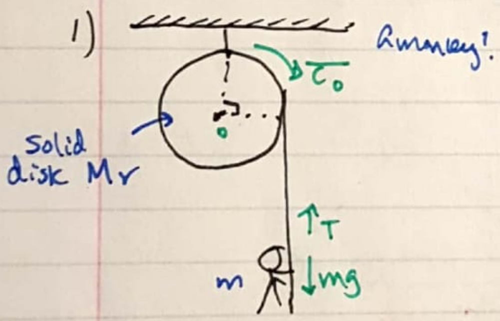


Same mass, but disk has much less inertia, since mass is spread so some has smaller r.

Disk has smaller r's. Ring has max r for every m_i.

Oct 30, 2017

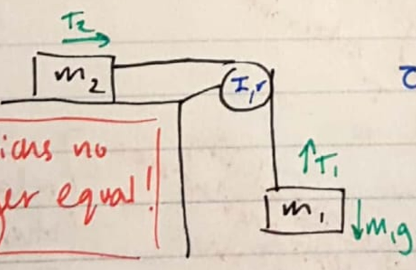
1)



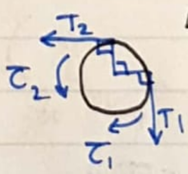
$$\Sigma \vec{\tau}_0 = \vec{I}_0 \alpha$$
$$rT = \frac{Mr^2}{2} \alpha$$
$$\frac{2T}{M} = \frac{a}{r}$$

$mg - T = ma$, so
$$a = \frac{mg}{\left(m + \frac{M}{2}\right)}$$

2)



Tensions no longer equal!

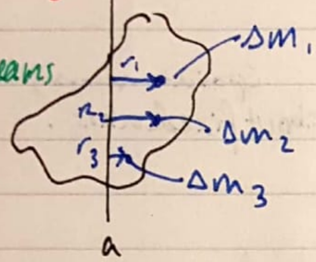


Let $+\circlearrowright$ $\rightarrow +$ $\downarrow +$
$$\Sigma \vec{\tau}_0 = \vec{I}_0 \alpha$$
$$\tau_1 - \tau_2 = I\alpha$$
$$r\tau_1 - r\tau_2 = I\frac{a}{r}$$

Sub T's rel. to m's and a's.

Nov 1, 2017

In general:



★ Δm means small value.

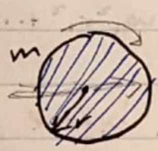
$I_a = ?$

$I_0 \cong \sum_i \Delta m_i r_i^2$ ← the same → $\int_{V_{object}} r^2 \, dm$

↳ lim becomes integral.

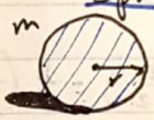
Can use integration to show:

disk



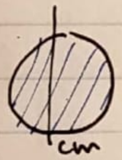
$\boxed{I_{cm} = \frac{mr^2}{2}}$

sphere



$\boxed{I_{cm} = \frac{2}{5} mr^2}$

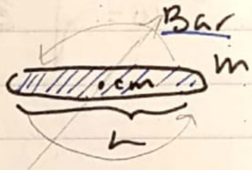
spherical shell



$\boxed{I_{cm} = \frac{2}{3} mr^2}$

(hollow sphere)

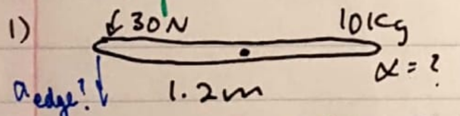
Bar



$\boxed{I_{cm} = \frac{1}{12} mL^2}$

Example:

1)



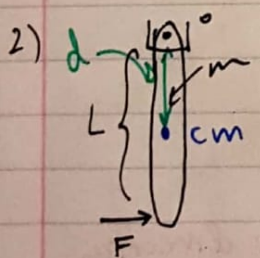
$\sum \vec{\tau}_0 = \vec{I}_0 \alpha$

$(r)(F) = (\frac{1}{12})(mL^2)\alpha \qquad \alpha = \frac{6F}{mL}$

$a_{edge} = \alpha r$

$= \frac{6F(\frac{L}{2})}{mL} = \frac{3F}{m}$ } makes sense, since bar ≠ point mass

2)



$\tau_0 = I_0 \alpha$

$LF = (I_{cm} + md^2)\alpha$

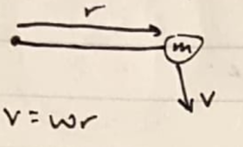
$LF = \left(\frac{mL^2}{3}\right)\alpha$

etc.

Parralel Axis Theorem: $\boxed{I_a = I_{cm} + md^2}$

↳ Two axis must be ∥

Rotational Energy:



$v = \omega r$

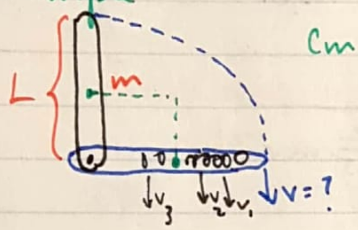
$K = \frac{1}{2} m v^2$

$= \frac{1}{2} m \omega^2 r^2$

$\boxed{= \frac{1}{2} I \omega^2}$ Rotational KE about fixed axis. (point mass)

Example:

1)



Cm fell by $\frac{L}{2}$. $mg\left(\frac{L}{2}\right) = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \ldots$

$= \frac{1}{2} m \left(\omega_1^2 r_1^2 + \omega_1^2 r_1^2 + \ldots\right)$

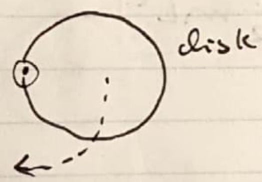
$= \frac{1}{2} I \omega^2$

$mg\left(\frac{L}{2}\right) = \frac{1}{2}\left(\frac{1}{3} m L^2\right)\omega^2$ from before

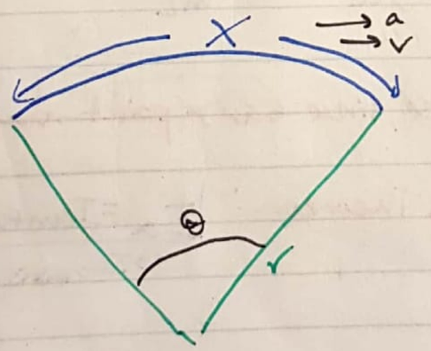
$\omega = \sqrt{\frac{3g}{L}}$ $\quad V = \omega L$

$V = \sqrt{3gL}$

2)



3)



$\Delta x = \vec{v}_1 t + \frac{1}{2} \vec{a} t^2 / r$

$\Delta \theta = \omega t + \frac{1}{2} \alpha t^2$

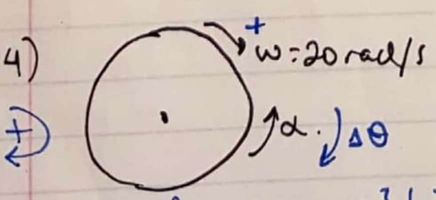
$\vec{V}_2 = \vec{V}_1 + \vec{a} t / r$

$\omega_2 = \omega_1 + \alpha t$

$V_f^2 - V_i^2 = 2\vec{a} \cdot \Delta \vec{d} / r^2$

$\omega_f^2 - \omega_i^2 = 2 \vec{\alpha} \cdot \Delta \theta$

4)



Stops after 2 rev. α? t?

$\omega_f^2 - \omega_i^2 = 2 \vec{\alpha} \cdot \Delta \theta$ opp direction

$-400 = 2(\alpha)(4\pi)(-1)$

$\alpha =$

$t = \frac{\vec{\omega}_2 - \vec{\omega}_1}{\vec{\alpha}}$ need sign convention

$= \frac{\omega_1}{\alpha}$

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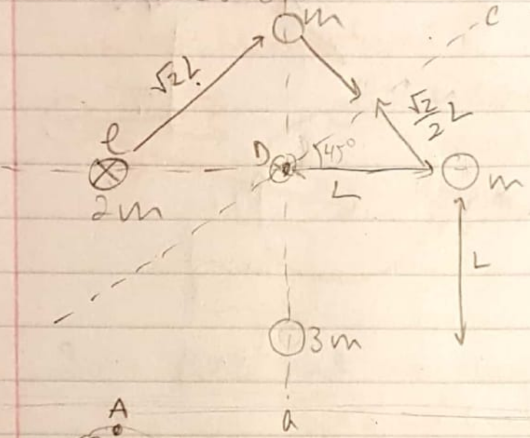
Review session

Nov 1, 2017



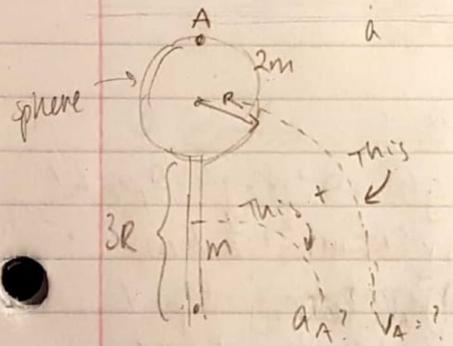
$I_a: mL^2 + 2mL^2 = 3mL^2$

$I_c: \frac{1}{2}mL^2 + \frac{1}{2}mL^2 + \frac{1}{2}3mL^2 = \frac{1}{2}L^2 3m$

$* \left(\frac{\sqrt{2}}{2}L\right)^2 = \left(\frac{1}{2}\right)L^2$

$I_d: mL^2 + mL^2 + 2mL^2 + 3mL^2$

$I_e: m(\sqrt{2}L)^2 + m(2L)^2 + 3m(\sqrt{2}L)^2 + 0$



$\Delta K = -(\Delta U)$ Can also do c of mass approach

$2mg(4r) + mg(1.5R) = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$

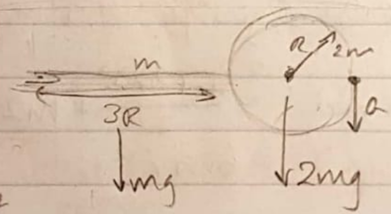
$9.5mgr = \frac{1}{2}\left(\frac{1}{3}m(3r)^2\right)\omega^2 + \frac{1}{2}\left(\frac{2}{5}mr^2 + 32mr^2\right)\omega^2$

$* \hookrightarrow I_{sphere} = I_{cm} + md^2 = \frac{2}{5}mr^2 + 2m(4r)^2$

$9.5gr = \frac{3}{2}r^2\omega^2 + \frac{82}{5}r^2\omega^2 = \frac{179}{10}r^2\omega^2$

$\omega^2 = \frac{95}{179}\frac{g}{R}$ $V_A = \omega 5r$

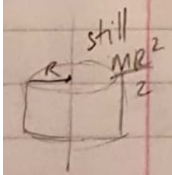
$I_o = \left\{\frac{64}{5}mr^2 + 3mr^2\right\}$
$I_1 + I_2$

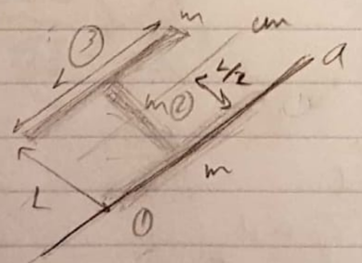


$a = \alpha 5r$ $\sum \tau_o = I_o\alpha$

$\tau_1 + \tau_2 = I_o\alpha$

$1.5Rmg + 4R(2mg) = I_o mR^2\alpha$





Thin bars

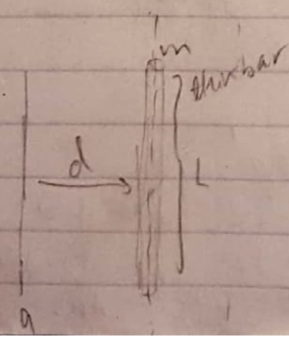
$I_a = I_1 + I_2 + I_3$

$= md_1^2 + md_2^2 + md^2$

$= \frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2 + md^2$

$= \frac{1}{12}mL^2 + \frac{1}{4}mL^2 + mL^2 = \frac{4}{3}mL^2$

I_a^{bar}?



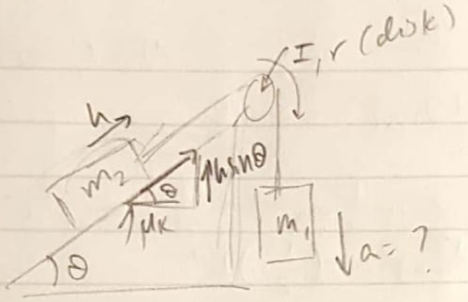
$I_a = I_{cm} + md^2$

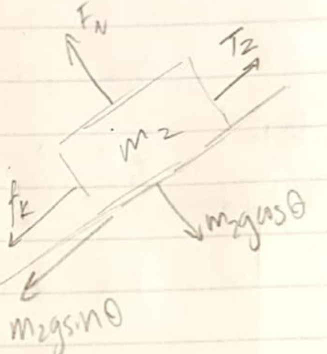
$= \frac{mr^2}{2} + md^2$

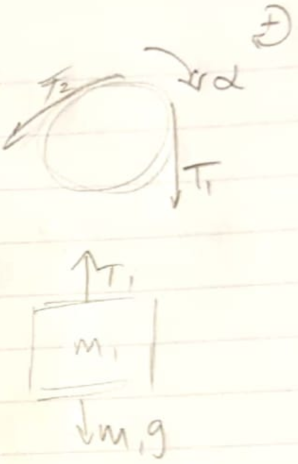
$= md^2$

or... $I_T = m_1d^2 + m_2d^2 \ldots$

$= md^2$







$m_1: \Sigma \vec{F_y} = m_1 \vec{a}$

① $m_1 g - T_1 = m_1 a$

$m_2: \Sigma \vec{F_x} = m_2 \vec{a_x}$

② $T_2 - m_2 g \sin\theta - m_2 g \cos\theta \mu_K = m_2 a$

Pulley: $\Sigma \vec{\tau_0} = \vec{I_0} \alpha$

$rT_1 - rT_2 = \frac{mr^2}{2}\left(\frac{a}{r}\right)$

③ $T_1 - T_2 = (ma)/2$

$m_2: \Sigma F_y = 0.$

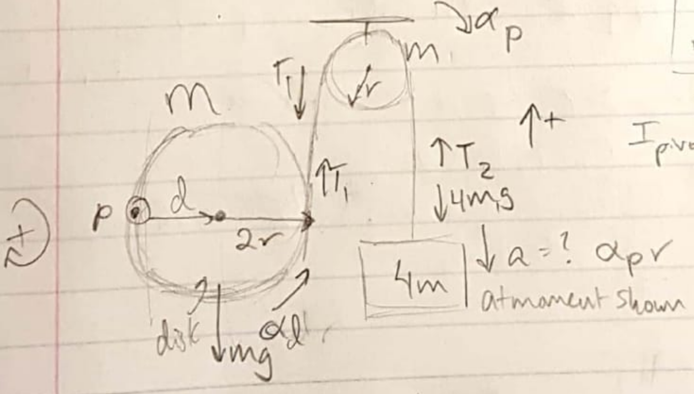
$F_N = m_2 g \cos\theta$

*One line solution:

$m_1 gh = m_2 gh \sin\theta + m_2 g \cos\theta \mu_K h + \frac{1}{2}m_1 v^2 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$\frac{d}{dt}: m_1 g v = m_2 g v \sin\theta + m_2 g \cos\theta \mu_K v + m_1 v a + m_2 v a + I\omega\alpha$

$m_1 g = m_2 g \sin\theta + m_2 g \cos\theta \mu_K + m_1 a + m_2 a + \frac{I}{r^2}$



$I_{pivot} = I_{cm} + md^2 = \frac{m(2r)^2}{2} + m(2r)^2 = 6mr^2$

$4m: 4mg - T_2 = 4ma$

Pulley: $\Sigma \vec{\tau_0} = \vec{I_0} \alpha_p$

$\vec{\tau_{T_1}} + \vec{\tau_{T_2}} = I_0 \alpha_p$

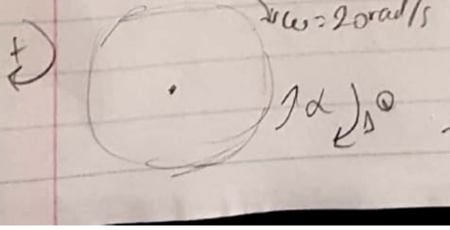
$-rT_1 + rT_2 = \frac{mr^2}{2}\alpha_p$

$a = \alpha_d 4r = \alpha_p r$

Disk: $\Sigma \vec{\tau_p} = \vec{I_p} \vec{\alpha_d}$

$\vec{\tau_{mg}} + \vec{\tau_{T_1}} = I_p \vec{\alpha_d}$

$(mg 2R) - T_1(4R) = -I_p \alpha_d$



Drops by $\frac{1}{2}$ in 3s. Turns till stop?

$\vec{\omega_f} = \vec{\alpha} t + \vec{\omega_0}$

$\frac{1}{2}\vec{\omega_0} = \vec{\alpha} t + \vec{\omega_0}$

$\alpha = \frac{20}{6} = 3.33 \text{ rad/s}^2$

$\omega_f^2 - \omega_0^2 = 2\vec{\alpha}\cdot\Delta\theta$

$-20^2 = 2(3.33)\cdot\Delta\theta(-1)$

$\Delta\theta = \frac{-400}{6.66}$

$\text{Ans} = \frac{\Delta\theta}{2\pi}$

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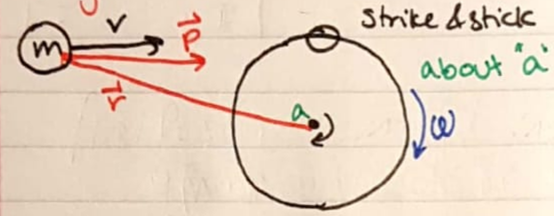
Angular Momentum:

Nov 3, 2017



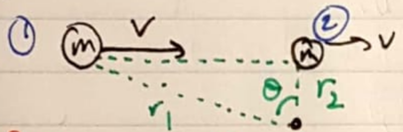
After collision has angular momentum about "a"

No external forces... so:

→ angular momentum must have existed before the collision as well "about a"

Since disk had no ang. momentum before, must have come from clay

$\vec{L}_a = \vec{r}_a \times \vec{p}$ "orbital"

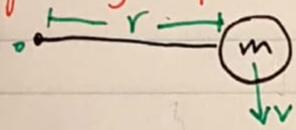


$\vec{L}_1 = \vec{r}_1 \times \vec{p} = r_1 p \sin\theta = r_2 p$

$\vec{L}_2 = \vec{r}_2 \times \vec{p} = r_2 \sin\theta\, p = r_2 p$

no ext forces, so momentum is conserved.

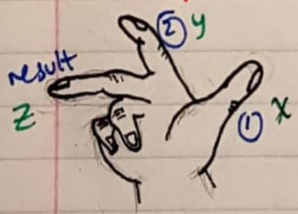
Spinning on fixed axis:

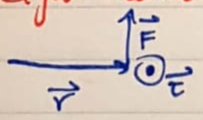


$$\vec{L}_0 = \vec{r} \times m\vec{v} = r m \omega r = m r^2 \omega = \boxed{I_0 \omega}$$

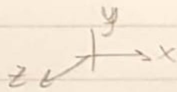
$$\vec{L}_0 = I_0 \vec{\omega}$$

Cross Products + Right hand Rule:





$\vec{\tau} = \vec{r} \times \vec{F}$



$\vec{r} = r\hat{i}$ $\quad \vec{F} = F\hat{j}$

$\vec{\tau} = r\hat{i} \times F\hat{j} = rF(\hat{i} \times \hat{j}) = rF\hat{k}$