

These numbers are the averages of three executions of each input file rounded to the nearest whole number, all raw data is below. All computation was done on the Linux machine.

	Unique Values	Size	stdSort	quickSelect1	quickSelect2	countingSort
test_input.txt	787	1K	39 $\mu$ s	17 $\mu$ s	22 $\mu$ s	118 $\mu$ s
test_input2.txt	3588	100K	4520 $\mu$ s	1471 $\mu$ s	1406 $\mu$ s	1479 $\mu$ s
test_input3.txt	5335	10M	341359 $\mu$ s	124933 $\mu$ s	125148 $\mu$ s	79782 $\mu$ s

#### Raw results

test_input.txt	stdSort	quickSelect1	quickSelect2	countingSort
Attempt one	40 $\mu$ s	18 $\mu$ s	24 $\mu$ s	129 $\mu$ s
Attempt two	43 $\mu$ s	17 $\mu$ s	21 $\mu$ s	116 $\mu$ s
Attempt three	34 $\mu$ s	15 $\mu$ s	20 $\mu$ s	109 $\mu$ s

test_input2.txt	stdSort	quickSelect1	quickSelect2	countingSort
Attempt one	4509 $\mu$ s	1450 $\mu$ s	1375 $\mu$ s	1442 $\mu$ s
Attempt two	4518 $\mu$ s	1485 $\mu$ s	1413 $\mu$ s	1500 $\mu$ s
Attempt three	4532 $\mu$ s	1478 $\mu$ s	1430 $\mu$ s	1496 $\mu$ s

test_input3.txt	stdSort	quickSelect1	quickSelect2	countingSort
Attempt one	341320 $\mu$ s	124981 $\mu$ s	125353 $\mu$ s	79867 $\mu$ s
Attempt two	341619 $\mu$ s	125322 $\mu$ s	125011 $\mu$ s	79691 $\mu$ s
Attempt three	341139 $\mu$ s	124496 $\mu$ s	125080 $\mu$ s	79787 $\mu$ s

The `stdSort` function's time complexity is dominated by `std::sort` which has an average and worst-case complexity of  $O(n \log n)$  for sorting  $n$  elements. While there are constant time ( $O(1)$ ) operations for calculations within `stdSort`, they are negligible compared to sorting. Overall, `stdSort` has a time complexity of  $O(n \log n)$ .

The `quickSelect1` function calculates the median (P50) using `quickSelectFunction`, which has an average-case time complexity of  $O(n)$ . However, it's crucial to note that while the average-case time complexity is linear, the worst-case time complexity can degrade to  $O(n^2)$ , especially if the pivot selection strategy consistently chooses poorly. Additionally, the utilization of insertion sort for small ranges adds efficiency to the algorithm for smaller datasets, further enhancing its practical performance. It also calculates P25 and P75, each requiring another call to `quickSelectFunction`. However, these calls operate on smaller partitions of the dataset, with sizes proportional to  $n/2$ . Therefore, each call has an average-case time complexity of  $O(n/2)$ . Finding the minimum and maximum values in the dataset involves iterating through a portion of the data, which contributes  $O(n)$  time complexity. Overall, the dominant time complexity comes from the calls to `quickSelectFunction`, making the average-case time complexity of `quickSelect1`  $O(n)$ .

The `quickSelect2` function efficiently determines the positions of minimum, P25, median (P50), P75, and maximum values for keys in constant time ( $O(1)$ ). These keys are then employed within the `quickSelectFunction` to retrieve the corresponding values after the array has undergone sorting. While the average-case time complexity of `quickSelectFunction` is linear at  $O(n)$ , it's important to acknowledge that the worst-case scenario can degrade to  $O(n^2)$ , particularly if the pivot selection strategy consistently performs poorly. Additionally, the integration of insertion sort for small ranges bolsters the algorithm's efficacy, particularly with smaller datasets. Despite the potential for quadratic time complexity in the worst-case scenario, the predominant time complexity of `quickSelect2` is driven by the call to `quickSelectFunction`, resulting in an average-case time complexity of  $O(n)$ .

The `countingSort` function operates in several distinct steps to calculate percentiles using counting sort. Initially, it counts the occurrences of each unique data value, utilizing an unordered map to store counts. This counting step runs in  $O(n)$  time, where  $n$  is the number of elements in the input data. Subsequently, it creates a vector of pairs from the hash map, which has a time complexity of  $O(u)$ , where  $u$  is the number of unique elements. Sorting this vector using `std::sort` takes  $O(u \log u)$  time. Following the sorting, the function calculates cumulative counts for quartiles and then determines quartile values based on these counts, each of which entails iterating through the sorted vector, resulting in  $O(u)$  time complexity for both steps. Therefore, the overall time complexity of the `countingSort` function is  $O(n + u \log u)$ , where  $n$  is the number of elements in the input data and  $u$  is the number of unique elements. Since  $u$  is bounded by  $n$ , this can be simplified to  $O(n \log n)$ .

Having fewer unique values and more copies of each value generally benefits sorting algorithms, as they can exploit the repetition to optimize their processes. Standard sorting algorithms like `stdSort` may improve due to reduced distinct comparisons and swaps needed. QuickSelect algorithms may exhibit improved performance due to reduced partitioning overhead in datasets with more duplicates. Counting sort remains efficient regardless of the number of duplicates, making it suitable for datasets with many copies of each value.

`stdSort` function:

Hypothesis: Standard sorting algorithms like `std::sort` typically benefit from having duplicates as they can exploit patterns in the data, resulting in more efficient sorting. With more copies of each value, there are fewer distinct comparisons and swaps needed, leading to faster sorting times.

`quickSelect1` and `quickSelect2` function:

Hypothesis: In datasets with more duplicates, the partitions created during the quickSelect algorithm tend to be smaller, leading to faster selection of the desired percentiles. Additionally, the use of insertion sort for small ranges further enhances efficiency for smaller datasets.

`countingSort` function:

Hypothesis: Counting sort's time complexity is unaffected by the number of duplicates, as it directly counts occurrences. Therefore, having more duplicates will not slow down the algorithm, and may even improve performance as it bypasses comparison-based operations.

Throughout the project, several notable observations emerged regarding the performance of different sorting and selection algorithms. Initially, there was an anticipation of significant runtime differences, particularly between theoretical expectations and practical results. However, as the input size increased, an unexpected trend surfaced: `countingSort`, which initially performed poorly, emerged as the most efficient, particularly benefiting from duplicate values in larger input sets.

Another intriguing observation was the divergence in execution times between `stdSort` and `countingSort`, despite both having  $O(n \log n)$  time complexity. While `stdSort` struggled with duplicates, `countingSort` excelled, demonstrating the importance of considering implementation details beyond theoretical complexities.

Furthermore, despite `quickSelect1` and `quickSelect2` sharing similar execution methods, their runtime remained consistently close across varying input sizes. This contrasted sharply with the considerable differences observed between `stdSort` and `countingSort`, underscoring the significance of implementation nuances.