

These numbers are the averages of three executions of each input file rounded to the nearest whole number, all raw data is at the end of the report. All computation was done on the Linux machine.

	Vector implementation	Linked list implementation	Heap implementation	AVL tree implementation
input1.txt	532 μ s	9109 μ s	234 μ s	10030 μ s
Input2.txt	5661 μ s	217369 μ s	757 μ s	277495 μ s
Input3.txt	86334 μ s	7269102 μ s	3533 μ s	8984802 μ s

vectorMedian

Insertion into the sorted vector is executed using `std::upper_bound`, taking $O(\log n)$ time per insertion, where n represents the current size of the vector. With k insertions, the total insertion complexity becomes $O(k \log n)$. Subsequently, finding and removing medians from the sorted vector involves $O(n)$ time per operation in the worst case, with at most k removal operations, leading to a total complexity of $O(k O(n))$. Thus, the overall time complexity of the `vectorMedian` function is $O(k * n)$ in its simplest form, where k is the number of instructions provided.

listMedian

Inserting into the sorted list requires finding the correct position, typically taking $O(n)$ time where n is the current size of the list, though, on average, this can be considered $O(n/2)$. Similarly, finding and removing the median, which involves traversing half of the list elements, has a time complexity of $O(n)$. Consequently, for a sequence of k instructions, the total time complexity is $O(k * n)$, where k is the number of instructions and n is the size of the list at any given point. Therefore, the overall time complexity of the algorithm is $O(k * n)$.

heapMedian

It iterates through each instruction in the instructions vector, resulting in a time complexity of $O(k)$, where k is the number of instructions. Within each iteration, the function performs operations such as pushing, popping, and balancing the priority queues, each of which has a time complexity of $O(\log n)$, where n is the size of the priority queue. As a result, the overall time complexity of the algorithm is $O(k \log n)$.

treeMedian

Each insertion or removal operation in AVL trees takes $O(\log n)$ time, where n is the number of elements in the tree. The function iterates through each instruction in the input vector, leading to a time complexity of $O(k \log n)$, where k represents the number of instructions. Rebalancing AVL trees in the worst case also takes $O(\log n)$ time per operation. Additionally, calculating and storing medians has a time complexity of $O(k)$. Combining all these factors, the overall average-case time complexity of the function is $O(k \log n)$.

Analyzing the algorithms for finding the median `vectorMedian`, `listMedian`, `heapMedian`, and `treeMedian` reveals trade-offs between speed and memory usage. When dealing with large datasets, `vectorMedian` and `heapMedian` shine due to their impressive time to run. However, `heapMedian` takes the crown for its superior performance due to its more efficient runtime of $O(k \log n)$. This is because heap operations have lower inherent costs compared to manipulating vectors. On the other hand, `listMedian` and `treeMedian` come with a higher time to run with complexities of $O(k * n)$ and $O(k \log n)$ respectively. `ListMedian` suffers from inefficiencies due to linear search and deletion within a linked list. While `TreeMedian` offers efficiency gains over `listMedian`, maintaining balanced AVL trees introduces some overhead. Additionally, frequent calls to the `size` function within conditional statements might negate some of the AVL tree's performance benefits, as determining the size has a time complexity of $O(n)$.

While all four algorithms use additional space proportional to the input size ($O(n)$), runtime becomes increasingly important for larger datasets. This is especially true for algorithms with higher time complexities, where the slowdown becomes more significant as the data grows. Therefore, the best choice depends on a careful evaluation of not just time and space complexity, but also how runtime behaves as the input size increases. In ascending order of runtime for growing datasets, we have: `heapMedian`, `vectorMedian`, and `listMedian`, with `treeMedian` being the slowest of the four.

Initially, I expected significant differences between implementations, particularly when comparing `vectorMedian` and `listMedian`. However, their core structures were surprisingly similar. Likewise, `heapMedian` and `treeMedian` both utilize two underlying containers, exhibiting a similar design pattern. Furthermore, I was surprised by how few helper functions were necessary – only two for the AVL tree class. Exploring the STL offered valuable insights into the capabilities of base container classes like vectors, lists, and heaps. The most unexpected finding was the dramatic difference in runtime between implementations as input sizes increased. Despite all having similar time complexity with `vectorMedian` and `listMedian` having $O(k * n)$, and `heapMedian` and `treeMedian` having $O(k \log n)$ time complexity, their execution times diverged considerably with larger inputs. This finding highlights the importance of considering implementation details beyond just theoretical time complexity. Furthermore, execution times varied significantly across machines with different architectures.

Raw results

input1.txt	Vector implementation	Linked list implementation	Heap implementation	AVL tree implementation
Attempt one	555 μ s	9168 μ s	233 μ s	10123 μ s
Attempt two	496 μ s	8896 μ s	235 μ s	9807 μ s
Attempt three	544 μ s	9264 μ s	234 μ s	10160 μ s

Input2.txt	Vector implementation	Linked list implementation	Heap implementation	AVL tree implementation
Attempt one	5638 μ s	213804 μ s	757 μ s	280819 μ s
Attempt two	5680 μ s	205409 μ s	757 μ s	273593 μ s
Attempt three	5665 μ s	232893 μ s	758 μ s	278074 μ s

Input3.txt	Vector implementation	Linked list implementation	Heap implementation	AVL tree implementation
Attempt one	84001 μ s	7340897 μ s	3539 μ s	9035226 μ s
Attempt two	93656 μ s	7136742 μ s	3531 μ s	8894266 μ s
Attempt three	81346 μ s	7329666 μ s	3528 μ s	9024914 μ s