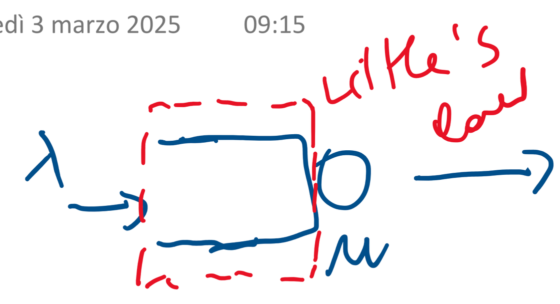
M/G/1

lunedì 3 marzo 2025 09:15



X is r.v. modelling the service time

$$E[X] = \frac{1}{\mu}$$

$\boxed{\lambda < \mu}$ stability condition

$$\overline{N_W} = \overline{W}\cdot\lambda$$

exp. waiting time

exp. # of jobs in the buffer

$$E[N_W] = E[W]\cdot\lambda$$

Let's use Pasta and derive the expected waiting time $\overline{W}$

We condition on the system state at arrival: empty / non-empty
$1-\rho$ / ρ

$$\rho \triangleq \frac{\lambda}{\mu}$$

- case empty

$$E[W \mid \text{system empty}] = 0$$

- case non empty

residual life of the job in service

$$E[W \mid \text{system non-empty}] = E[\xi] + E[N_W \mid \text{system non empty}]\cdot\frac{1}{\mu} \qquad \text{(PASTA)}$$

$$(*) \quad E[W] = 0\cdot(1-\rho) + \left(E[\xi] + E[N_W \mid \text{system non empty}]\frac{1}{\mu}\right)\cdot\rho$$

law of total probability

$$E[N_W] = \cancel{E[N_W \mid \text{system empty}](1-\rho)} + E[N_W \mid \text{system non empty}]\,\rho$$

$$E[N_W \mid \text{system non empty}] = \frac{E[N_W]}{\rho}$$

$$E[N_W] = E[W]\cdot\lambda$$

Little

$$* \quad E[W] = E[\xi]\cdot\rho + \frac{E[N_W]}{\cancel{\rho}}\cdot\cancel{\rho}\cdot\frac{1}{\mu}$$

$$E[W] = E[\xi]\cdot\rho + E[W]\cdot\frac{\lambda}{\mu}$$

$\frac{\lambda}{\mu} \to \rho$

$$E[W](1-\rho) = E[\xi]\cdot\rho$$

$$E[W] = \frac{E[X^2]}{2E[X]}\cdot\frac{\rho}{1-\rho} = \frac{E[X^2]}{2\frac{1}{\mu}}\cdot\frac{\rho}{1-\rho} = \frac{E[X^2]\cdot\cancel{\mu}\cdot\frac{\lambda}{\cancel{\mu}}}{2(1-\rho)}$$

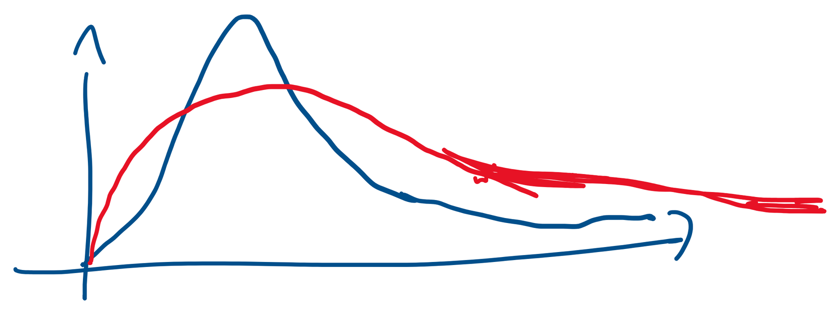
$$\boxed{\overline{W} = \frac{E[X^2]\,\lambda}{2(1-\rho)}}$$

$$\overline{R} = \overline{W} + \frac{1}{\mu}$$

$$\overline{N} = \lambda\cdot\overline{R}$$

$$\overline{W} = \frac{\left(Var[X] + \frac{1}{\mu^2}\right)\lambda}{2(1-\rho)}$$

?



Example deterministic service time $\Rightarrow Var[X] = 0$

$$\overline{R} = \frac{\frac{1}{\mu^2}\cdot\lambda}{2(1-\rho)} + \frac{1}{\mu} = \frac{\rho}{2\mu(1-\rho)} + \frac{1}{\mu} = \frac{\rho + 2(1-\rho)}{2\mu(1-\rho)}$$

$$= \frac{\rho + 2 - 2\rho}{2\mu(1-\rho)} = \frac{2-\rho}{2\mu(1-\rho)} = \frac{2-\frac{\lambda}{\mu}}{2\mu\left(1-\frac{\lambda}{\mu}\right)}$$

$$= \frac{\frac{2\mu-\lambda}{\cancel{\mu}}}{\frac{2\mu(\mu-\lambda)}{\cancel{\mu}}} = \frac{2\mu-\lambda}{2\mu(\mu-\lambda)}$$

$$\overline{R}_D = \frac{2-\rho}{2(\mu-\lambda)} \qquad \overline{R}_M = \frac{1}{\mu-\lambda}$$

$$= (2-\rho)\cdot\frac{1}{2}\,\overline{R}_M$$