M/G/1/PS

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	Arrival	Size	FIFO departure	PS departure
1)	0	20	20	$\boxed{32}$
2)	5	2	22	10.5
3)	6	10	$\boxed{32}$	27.5

FIFO: $\frac{20+17+26}{3} = 21\,s$

PS: $\frac{32+5.5+21.5}{3} = 19.6\,s$

$\boxed{PS}$ residual life

time	J1	J2	J3
0	20	/	/
5	15	2	/
6	14.5	1.5	10
10.5	13	Departure	8.5
27.5	4.5		Departure
32	Departure		

$x = 5$

$x = 30$

λ arrival rate

X is r.v. that models the size of the job

$F(x) = P_r\{X \le x\}$ c.d.f. of the job size

$f(x) = F'(x)\,dx$

$E[X] = \frac{1}{\mu}$

Assumption $\lambda < \mu$ (stability)

$N(x)$: is the expected number of jobs in the system that have received at most x amount of work

$T(x)$: expected time spent in the system by a job of size x.

APPROACH 1

$N = X R$

x $\quad x+\Delta x$

$$\frac{N(x+\Delta x) - N(x)}{\Delta x} = \frac{[T(x+\Delta x) - T(x)]\cdot \lambda(1-F(x))}{\Delta x}$$

$\Delta x \to 0$

$$\boxed{N'(x) = T'(x)\,\lambda\,(1-F(x))}$$

$T'(x)\,\lambda\,\cancel{(1-F(x))} = K\,\cancel{(1-F(x))}$

$T'(x) = \frac{K}{\lambda}$

APPROACH 2

$$N(x) = K\cdot\int_0^x P_r\{X > y\}\,dy$$

$$\boxed{N'(x) = K\,P_r\{X > x\} = K(1-F(x))}$$

$$T'(x) = \frac{K}{\lambda}$$

$$\int T'(x)\,dx = \int \frac{K}{\lambda}\,dx$$

$$T(x) = \frac{Kx}{\lambda} + C$$

$$T(0) = 0 \Rightarrow C = 0$$

$$T(x) = \frac{Kx}{\lambda}$$

$$\lim_{x\to\infty} T(x) = \lim_{x\to\infty}\frac{x}{1-\rho}$$

$$\lim_{x\to\infty}\frac{Kx}{\lambda} = \lim_{x\to\infty}\frac{x}{1-\rho}$$

$$\lim_{x\to\infty}\frac{\frac{Kx}{\lambda}}{\frac{x}{1-\rho}} = 1 \qquad \lim_{x\to\infty}\frac{K\cancel{x}}{\lambda}\cdot\frac{1-\rho}{\cancel{x}} = 1$$

$$\frac{K(1-\rho)}{\lambda} = 1 \Rightarrow K = \frac{\lambda}{1-\rho}$$

$$T(x) = \frac{\cancel{\lambda}}{1-\rho}\cdot\frac{x}{\cancel{\lambda}} = \frac{x}{1-\rho}$$

$\boxed{T(x) = \frac{x}{1-\rho}}$ very important relation!!!

$f(x) = F'(x)$

$$E[R] = \bar{R} = \int_0^\infty T(x)\,f(x)\,dx$$

$$\overset{!}{=} \int_0^\infty \frac{x}{1-\rho}\,f(x)\,dx = \frac{1}{1-\rho}\int_0^\infty x\,f(x)\,dx =$$

$$= \frac{E[X]}{1-\rho} = \frac{\frac{1}{\mu}}{1-\rho} = \frac{\frac{1}{\mu}}{1-\frac{\lambda}{\mu}} =$$

$$= \frac{1}{\cancel{\mu}}\cdot\frac{\cancel{\mu}}{\mu-\lambda} = \frac{1}{\mu-\lambda}$$

$$\boxed{\bar{R} = \frac{1}{\mu-\lambda}}$$

service time variance
- higher exponential → choose PS
- lower exponential → choose FCFS

λ arrival rate

service time is uniform in (2, 6) seconds

1) Find the stability condition
2) Decide if PS or FCFS is better
3) Compute the expected response time for the 2 disciplines at 80% of utilization

$X \sim U(2,6)$

2 ——— 6

$E[X] = \frac{6+2}{2} = 4\,s$

$\mu = \frac{1}{4} = 0.25$ j/s

1) $\lambda < \mu \Rightarrow \lambda < 0.25$ j/s

$Var[X] = \frac{(6-2)^2}{12} = \frac{16}{12} = \frac{4}{3} = 1.3\ s^2$ uniform

find the variance of an exponential distribution whose mean is 4 s $\Rightarrow \mu = 0.25$

$Y \sim exp(\mu)$ $\quad Var[Y] = \frac{1}{0.25^2} = 16\ s^2$

2) Choose FCFS because $1.3 < 16$

$$\bar{R}_{FCFS} = \frac{\left(Var[X] + \frac{1}{\mu^2}\right)\lambda}{2(1-\rho)} + 4 = \frac{(1.3+16)\cdot 0.2}{2(1-0.8)} + 4 = 12\ldots s$$

PR

$$\bar{R}_{PS} = \frac{1}{0.25-0.2} = \frac{1}{0.05} = 20\,s$$