

	Arrivo	Size	departe	departe
1)	0	20	20	32
2)	5	2	22	10.5
3)	6	10	32	27.5
			$\frac{20+17+26}{3}$	$\frac{32+5.5+21.5}{3}$
			$= 21.5$	$= 19.65$

Time	J1	J2	J3
0	20	/	/
5	15	2	/
6	14.5	1.5	10
10.5	13	Departure	8.5
27.5	4.5		Departure
32	Departure		

$\lambda = 0.25$
 $\mu = 0.2$

λ arrival rate

X is r.v. that models the size of the job

$$F(x) = P\{X \leq x\}$$

c.d.f. of the job size

$$f(x) = F'(x)$$

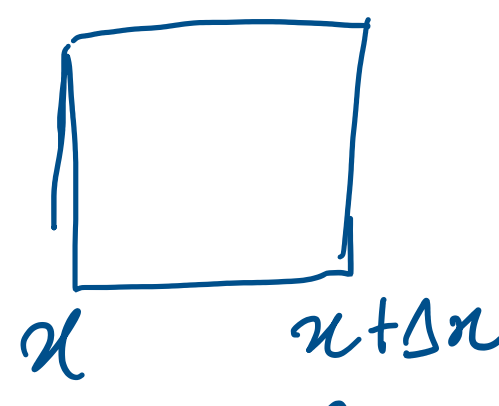
$$E[X] = \frac{1}{\mu}$$

Assumption $\lambda < \mu$ (stability)

$N(x)$: is the expected number of jobs in the system that have received at most x amount of work

$T(x)$: expected time spent in the system by a job of size x .

APPROACH 1



$$N = X R$$

$$\frac{N(x + \Delta x) - N(x)}{\Delta x} = \frac{[T(x + \Delta x) - T(x)] \cdot \lambda (1 - F(x))}{\Delta x}$$

$$\Delta x \rightarrow 0$$

$$N'(x) = T'(x) \lambda (1 - F(x))$$

$$T'(x) \lambda (1 - F(x)) = \lambda (1 - F(x))$$

$$T'(x) = \frac{\lambda}{\lambda}$$

APPROACH 2

$$N(x) = K \cdot \int_0^x P\{X > y\} dy$$

$$N'(x) = K P\{X > x\} = K (1 - F(x))$$

$$T'(x) = \frac{K}{\lambda}$$

$$\int T'(x) dx = \int \frac{K}{\lambda} dx$$

$$T(x) = \frac{Kx}{\lambda} + C$$

$$T(0) = 0 \Rightarrow C = 0$$

$$T(x) = \frac{Kx}{\lambda}$$

$$\lim_{x \rightarrow \infty} T(x) = \lim_{x \rightarrow \infty} \frac{x}{1-\rho}$$

$$\lim_{x \rightarrow \infty} \frac{Kx}{\lambda} = \lim_{x \rightarrow \infty} \frac{x}{1-\rho}$$

$$\lim_{x \rightarrow \infty} \frac{\frac{Kx}{\lambda}}{\frac{x}{1-\rho}} = 1$$

$$\lim_{x \rightarrow \infty} \frac{Kx}{\lambda} \cdot \frac{1-\rho}{x} = 1$$

$$\frac{K(1-\rho)}{\lambda} = 1 \Rightarrow K = \frac{\lambda}{1-\rho}$$

$$T(x) = \frac{\lambda}{1-\rho} \cdot \frac{x}{\lambda} = \frac{x}{1-\rho}$$

$$T(x) = \frac{x}{1-\rho}$$

Very important relation!!!

$$f(x) = P'(x)$$

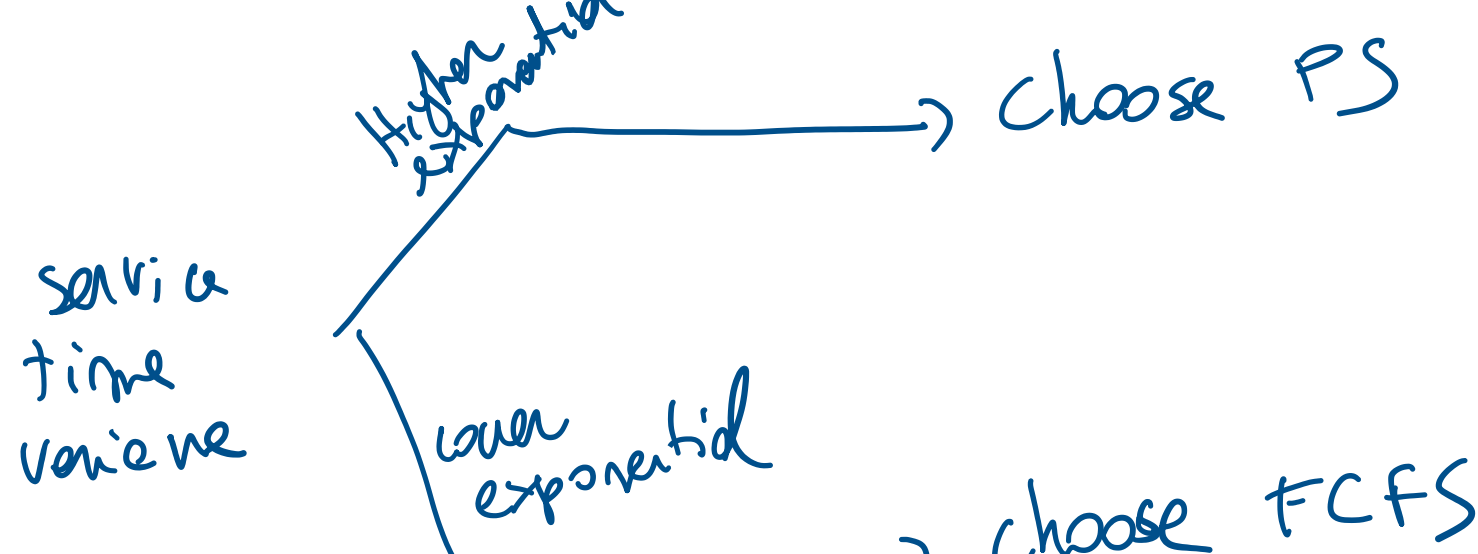
$$E[R] = \bar{R} = \int_0^{\infty} T(x) f(x) dx$$

$$= \int_0^{\infty} \frac{x}{1-\rho} f(x) dx = \frac{1}{1-\rho} \int_0^{\infty} x f(x) dx =$$

$$= \frac{E[X]}{1-\rho} = \frac{\frac{1}{\mu}}{1-\rho} = \frac{\frac{1}{\mu}}{1-\frac{\lambda}{\mu}} =$$

$$= \frac{1}{\mu} \cdot \frac{\mu}{\mu - \lambda} = \frac{1}{\mu - \lambda}$$

$$\bar{R} = \frac{1}{\mu - \lambda}$$



λ arrival rate

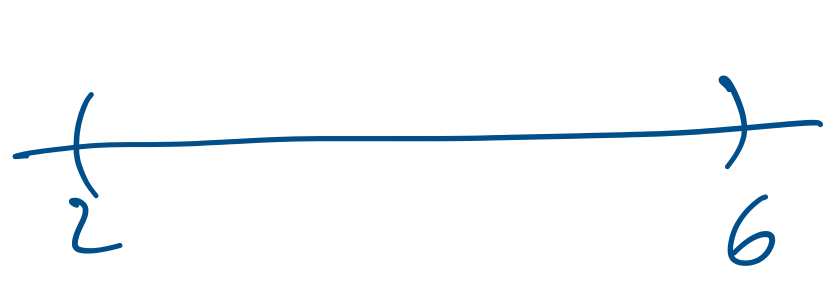
service time is unif in (2, 6) seconds

1) Find the stability condition

2) Decide if PS or FCFS is better

3) Compute the expected response time for the 2 disciplines at 80% of utilization

$$X \sim U(2, 6)$$



$$E[X] = \frac{2+6}{2} = 4s$$

$$\mu = \frac{1}{4} = 0.25 \text{ s}^{-1}$$

$$\lambda < \mu \Rightarrow \lambda < 0.25 \text{ s}^{-1}$$

$$\text{Var}[X] = \frac{(6-2)^2}{12} = \frac{16}{12} = \frac{4}{3} = 1.3 \text{ s}^2 \text{ unif}$$

find the variance of an exponential distribution whose mean is 4s $\Rightarrow \mu = 0.25$

$$Y \sim \exp(\mu) \text{ Var}[Y] = \frac{1}{0.25^2} = 16 \text{ s}^2$$

2) Choose FCFS because $1.3 < 16$

$$\bar{R}_{FCFS} = \frac{(\text{Var}[X] + \frac{1}{\mu^2}) \lambda}{2(1-\rho)} = \frac{(1.3 + 16) \cdot 0.2}{2(1-0.8)} = 12.5$$

$$\bar{R}_{PS} = \frac{1}{0.25 - 0.2} = \frac{1}{0.05} = 20s$$