Formal Methods for System Verification

Exercise 1

Consider the following two alternative PEPA specifications, S_0 and S_0', of a server:

$$\begin{aligned} S_0 &\stackrel{\text{def}}{=} (use, 2\rho).S_1 + (use, 3\rho).S_2 \\ S_1 &\stackrel{\text{def}}{=} (reset, \lambda).S_0 \\ S_2 &\stackrel{\text{def}}{=} (reset, \lambda).S_0 \end{aligned} \qquad \begin{aligned} S_0' &\stackrel{\text{def}}{=} (use, 5\rho).S_1' \\ S_1' &\stackrel{\text{def}}{=} (reset, \lambda).S_0' \end{aligned}$$

(a) Define the infinitesimal generator matrix $\mathbf{Q}$ of the Markov process underlying S_0.

(b) Define the infinitesimal generator matrix $\mathbf{Q}$ of the Markov process underlying S_0'.

(c) Determine whether S_0 and S_0' are strongly equivalent and in case of a positive answer exhibit a strong equivalence relation $\mathcal{R}$ such that $(S_0, S_0') \in \mathcal{R}$.

(d) For a given process $Client$, determine whether $S_0 \underset{\{use\}}{\bowtie} Client$ and $S_0' \underset{\{use\}}{\bowtie} Client$ are strongly equivalent. Justify the answer.

Exercise 2

Which are the main performance indices that you can compute given the steady state distribution of a PEPA process P? Give some examples.

Exercise 3

Consider the web service provided by YouTube which can serve the requests of different customers. Each customer may access to a locally available method, named *think*, (with probability p_1) or access to the remote web service (with probability $p_2 = 1 - p_1$). The local activities of the customers can be carried out independently of the web service. In contrast, the customer and the web service will cooperate when the customer requires a service offered by YouTube. Cooperation over given actions is reflected in the parallel composition by the cooperation set, $L = \{request, respond\}$.

$$\begin{aligned} YouTube &\stackrel{\text{def}}{=} (request, \top).(serve, \mu).(respond, \top).YouTube \\ Cust &\stackrel{\text{def}}{=} (think, p_1\lambda).(local, m).Cust + (think, p_2\lambda).(request, rq).(respond, rp).Cust \\ YouTube' &\stackrel{\text{def}}{=} YouTube/\{serve\} \end{aligned}$$

Consider the following systems:

$$\begin{aligned} Sys_0 &\stackrel{\text{def}}{=} (Cust \| Cust) \underset{L}{\bowtie} YouTube && \text{where } L = \{request, respond\} \\ Sys_1 &\stackrel{\text{def}}{=} (Cust \| Cust) \underset{L}{\bowtie} YouTube' && \text{where } L = \{request, respond\} \\ Sys_2 &\stackrel{\text{def}}{=} (Cust \| Cust)/\{serve\} \underset{L}{\bowtie} YouTube && \text{where } L = \{request, respond\} \\ Sys_3 &\stackrel{\text{def}}{=} ((Cust \| Cust) \underset{L}{\bowtie} YouTube)/\{serve\} && \text{where } L = \{request, respond\} \end{aligned}$$

(a) Determine whether Sys_0, Sys_2 are strongly equivalent. Justify the answer.

(b) Determine whether Sys_1, Sys_2 are strongly equivalent. Justify the answer.

(c) Determine whether Sys_1, Sys_3 are strongly equivalent. Justify the answer.

(d) Is it possibile to compute the steady state distribution of $(Cust \| YouTube \| Cust)$? Justify the answer.