1. The given set $A = \{0,1\}$ forms a group for only the boolean XOR operation. Operations AND and OR do not posess the identity element

2. The set W fails to satisfy the inverse element requirement for a group. There is no element "b" that can combine with any "a" to get the identity element.

3. Switching the operations would fail to meet the necessary criteria of a ring, which includes having an abelian group under one operation and a semigroup, along with the distributive property linking them.

4. Bezouts identity: $\gcd(a,n) = x \cdot a + y \cdot n$

One could use Bezouts identity to find the multiplicative inverse of an integer by assuming the above equation equals 1, and the entire equation is modulo n. Thus, $y \times n \bmod n$ equals 0 and x modulo n equals a^{-1}.

$\gcd(47, 97) = 1 = x \cdot 47 + y \cdot 97$

$97 = 2 \cdot 47 + 3$

$47 = 15 \cdot 3 + 2$

$3 = 1 \cdot 2 + 1$

$2 = 1 \cdot 2 + 0$

$1 = 3 - 1 \cdot 2$

$1 = 3 - 1(47 - 15 \cdot 3)$

$1 = 3 \cdot 16 - 47 \cdot 1$

$1 = (97 - 47 \cdot 2) \cdot 16 - 47$

$1 = 97 \cdot 16 - 47 \cdot 33$

Inverse $= -33$

Remap: $97 - 33 = \boxed{64}$

5. (a) $28x = 34 \pmod{37}$

Modular Inverse using Euclid:

$1 = 28x + 37y$

$37 = 1 \cdot 28 + 9$

$28 = 3 \cdot 9 + 1$

$9 = 9 \cdot 1 + 0$

$\gcd = 9$

$1 = 28 - 3 \cdot 9$

$1 = 28 - 3 \cdot (37 - 1 \cdot 28)$

$1 = 28 - 3 \cdot 37 + 3 \cdot 28$

$1 = \underline{4} \cdot 28 - 3 \cdot 37$

Inverse $= \boxed{4}$

$x = (34 \cdot 4) \bmod 37 = \boxed{25}$

(b) $19x = 42 \pmod{43}$

$1 = 19x + 43y$

$43 = 19 \cdot 2 + 5$

$19 = 3 \cdot 5 + 4$

$5 = 1\cdot 4 + 1 \rightarrow 1 = 5 - (1\cdot 4)$

$4 = 1\cdot 4 + 0$

$1 = 5 - 1(19 - 3\cdot 5) = 5 - 1\cdot 19 + 3\cdot 5 = 4\cdot 5 - 1\cdot 19$

$1 = 4\cdot(43 - 19\cdot 2) - 1\cdot 19 = 4\cdot 43 - 8\cdot 19 - 1\cdot 19$

$1 = 4\cdot 43 - 9\cdot 19$

Inverse $= -9 \rightarrow$ Remap: $43 - 9 = 34$

$x = (42\cdot 34) \bmod 43 = \boxed{9}$

(c) $54x \equiv 69 \pmod{79}$

$1 = 54x + 79y$

$79 = 54\cdot 1 + 25$

$54 = 25\cdot 2 + 4$

$25 = 4\cdot 6 + 1 \rightarrow 1 = 25 - 4\cdot 6$

$4 = 1\cdot 4 + 0$

$1 = 25 - (54 - 25\cdot 2)6 = 25 - 6\cdot 54 + 12\cdot 25 = 13\cdot 25 - 6\cdot 54$

$1 = 13(79 - 54\cdot 1) - 6\cdot 54$

$= 13\cdot 79 - 13\cdot 54 - 6\cdot 54 = 13\cdot 79 - 19\cdot 54$

Inverse $= -19 \rightarrow$ Remap: $-19 + 79\cdot 2 = 139$

$139 \bmod 79 = 60$

$x = 69\cdot 60 \bmod 79 = \boxed{32}$

(d) $153x \equiv 182 \pmod{271}$

$1 = 153x + 271y$

$271 = 153\cdot 1 + 118$

$153 = 118\cdot 1 + 35$

$118 = 35\cdot 3 + 13$

$35 = 13\cdot 2 + 9$

$13 = 9\cdot 1 + 4$

$9 = 4\cdot 2 + 1 \rightarrow 1 = 9 - 4\cdot 2$

$4 = 1\cdot 4 + 0$

$1 = 9 - (13 - 9\cdot 1)\cdot 2 = 9 - 13\cdot 2 + 9\cdot 2 = 3\cdot 9 - 13\cdot 2$

$1 = 3\cdot(35 - 13\cdot2) - 13\cdot2 = 3\cdot35 - 13\cdot6 - 13\cdot2 = 3\cdot35 - 13\cdot8$

$1 = 3\cdot35 - (118 - 35\cdot3)\cdot8 = 3\cdot35 - 8\cdot118 + 24\cdot35 = 27\cdot35 - 8\cdot118$

$1 = 27\cdot(153 - 118\cdot1) - 8\cdot118 = 27\cdot153 - 27\cdot118 - 8\cdot118 = 27\cdot153 - 35\cdot118$

$1 = 27\cdot153 - 35(271 - 153\cdot1) = 27\cdot153 - 35\cdot271 + 35\cdot153 = 62\cdot153 - 35\cdot271$

Inverse $= 62$

$X = (182\cdot62) \bmod 271 = \boxed{173}$

(c) $672x = 836 \pmod{997}$

$1 = 672x + 997y$

$997 = 672\cdot1 + 325$

$672 = 325\cdot2 + 22$

$325 = 22\cdot14 + 13$

$22 = 13\cdot1 + 9$

$13 = 9\cdot1 + 4$

$9 = 4\cdot2 + 1 \longrightarrow$ M.I. $= 408$

$4 = 1\cdot4$

$X = (836\cdot408) \bmod 997$

$= \boxed{114}$

6. GF(89) of $(54x^{10} - 62x^9 - 84x^8 + 70x^7 - 75x^6 + x^5 - 50x^3 + 84x^2 + 65x + 78) + (-67x^9 + 44x^8 - 26x^7 - 37x^6 + 61x^5 + 68x^4 + 22x^3 + 74x^2 + 87x + 38)$

$\Rightarrow \boxed{(54x^{10} + 49x^9 + 49x^8 + 44x^7 + 66x^6 + 62x^5 + 68x^4 + 61x^3 + 69x^2 + 63x + 27)}$

7. GF(11) of $(8x^3 + 6x^2 + 8x + 1) \times (3x^3 + 9x^2 + 7x + 5)$

$\Rightarrow 24x^6 + 72x^5 + 56x^4 + 40x^3 + 18x^5 + 54x^4 + 42x^3 + 30x^2 + 24x^3 + 72x^3 + 56x^2 + 40x + 3x^3$

$+9x^2+7x+5$

$= 24x^6 + 90x^5 + 134x^4 + 157x^3 + 95x^2 + 47x + 5 \rightarrow \bmod 2 =$

$\boxed{2x^6 + 2x^5 + 2x^4 + 3x^3 + 7x^2 + 3x + 5}$

8. (a) $(x^2+x+1) \times (x^2+x) = x^4 + x^3 + x^2 + x^3 + x^2 + x \rightarrow \bmod 2 = x^4 + x$

in $GF(2^3) \rightarrow x^3 = x + 1$

Therefore, $x^4 = x(x+1)$ sub into our expression $x^2 + x + x = \boxed{x^2}$

(b) $x^2 - (x^2+x+1) = -x - 1 \rightarrow \bmod 2 = \boxed{x+1}$

(c) $x^2+1 \,\overline{)\, x^2+x+1}$ quotient 1; $-(x^2+1)$; $x - 1 + 1$ $= 1 \ R \ \boxed{x}$