

Q:1

$$f(n) = 3n^2 + 21n + 40$$

prove for $\Omega(n^3)$, $O(n^3)$, $\Theta(n^3)$

\Rightarrow for $O(n^3)$

$$f(n) \leq O(n^3)$$

\Rightarrow mathematically

yes, it is $O(n^3)$

because,

$$3n^2 + 21n + 40 \leq c \cdot n^3$$

$\therefore c = +ve$

\Rightarrow but according to Computer

it is wrong, because

in order to

compute property

it should

be $O(n^3)$

\Rightarrow for $\Omega(n^3)$

$$\text{as } \Omega(n^3) \leq f(n)$$

Ω mean it is tight lower bound
who's growth is linear in nature.

$$\Omega(n^3) \leq 3n^2 + 21n + 40$$

So it disatisfies the property
because

(left hand side is greater than
right hand side)
it totally disatisfied.

\Rightarrow for $O(n^3)$

where $f(n) = 3n^2 + 21n + 40$

$$c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$$

$$c_1 \cdot n^3 \leq 3n^2 + 21n + 40 \leq c_2 \cdot n^3$$

dividing n^2 both sides

$$c_1 \cdot n \leq 3n + 21 + \frac{40}{n} \leq c_2 \cdot n$$

So it satisfies the property because (\leq) $c_1(n)$ always less than or equal to $f(n)$ and for right hand side it is

also same that $c_2 \cdot n$ is greater than or equal to $f(n)$.

Q: 2

let $T()$ for $x=1$ to n do n
begin

let $L()$ $y=x$ while $y>1$ do
 $y = y/2$
end

first inner while loop:

$$L(y) = \cancel{(x+1)} \sum_{x=1}^y 1 = x+1$$

for outerloop

$$\text{for } T() = \sum_{i=1}^n T(x)$$

$$= \sum_{i=1}^n x + 1$$

$$= \sum_{i=1}^n x + \sum_{i=1}^n 1$$

$$= \frac{2i(2i+1)}{2} + 2i$$

$$= 2i^2 + 3i$$

\Rightarrow ignore lower order term

$$= 2i^2 \quad \text{let } i=n$$

$$= 2n^2$$

$$= O(n^2)$$

\therefore ignore constant

$$= O(n^2)$$