Root Finding

Intermediate Value Theorem

Suppose f is continuous
on the closed interval $[a, b]$

and let N be any number
between $f(a)$ and $f(b)$
where $f(a) \neq f(b)$.

There exists a number c in (a, b)
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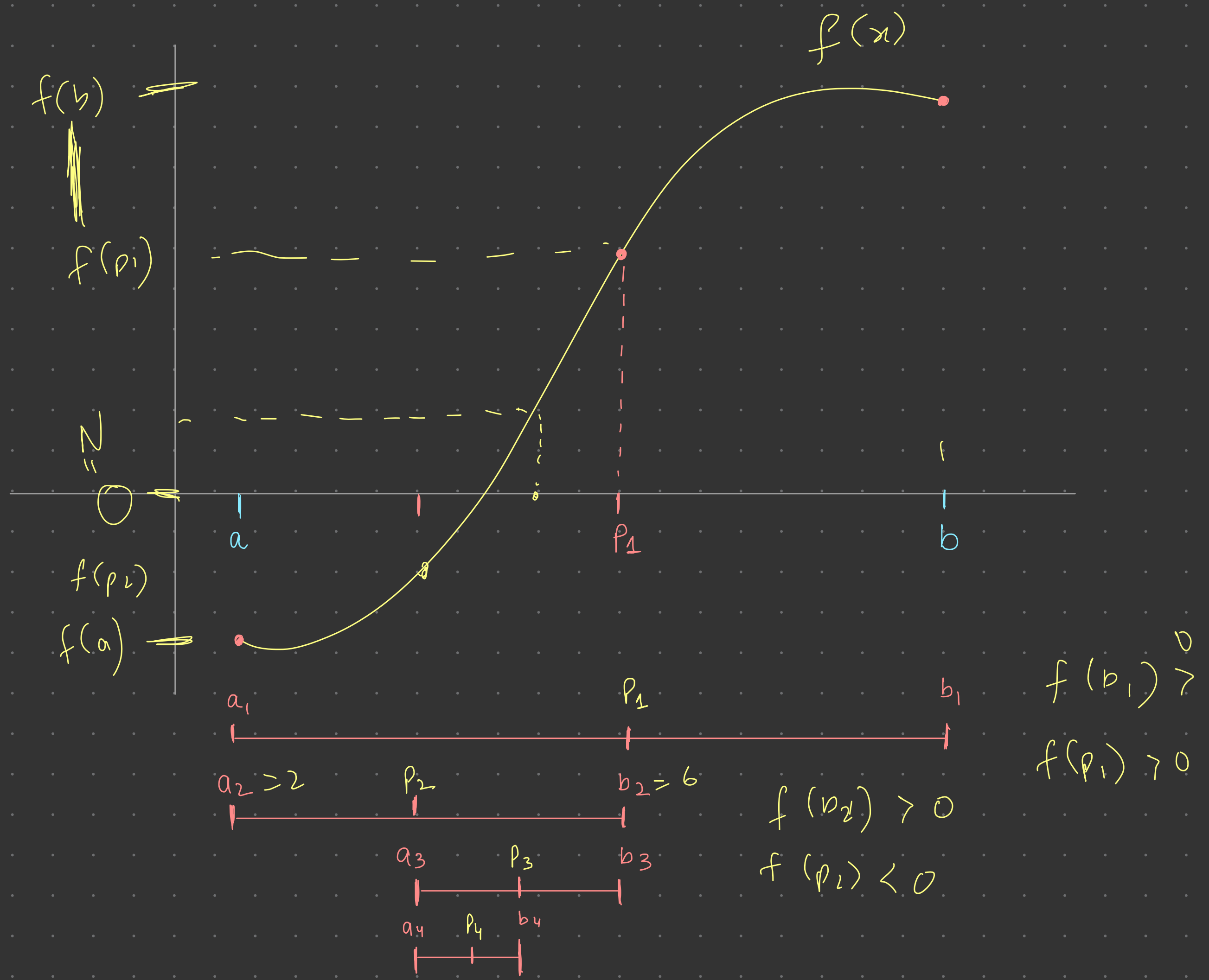
The Intermediate Value Theorem Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$ where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.

Finding Roots

Secant method



Algo:

1. Set $a_1 = a$, $b_1 = b$ (f(a) & f(b) must have different signs)

2. Set $p_1 = \dfrac{a_1 + b_1}{2}$

3. a) If $f(p_1) = 0$, <u>done</u>.

   b) If $f(p_1)$ has same sign as $f(a_1)$
   set $a_2 = p_1$ ($f(a_2)$ & $f(b_2)$ must end up with different signs)

   c) If $f(p_1)$ has same sign as $f(b_1)$
   set $b_2 = p_1$

4. Go to <u>2</u>.

Newton's Method

1. Start with a guess P_1 ($f'(P_1)$ must not be 0)

2. Calculate $P_n = P_{n-1} - \dfrac{f(P_{n-1})}{f'(P_{n-1})}$

3. Repeat untill "good enough"

$$f(x) = x^2 - 36$$
$$f'(x) = 2x$$

$$P_n = P_{n-1} - \left[\frac{(P_{n-1})^2 - 36}{2(P_{n-1})}\right]$$

$$P_1 = 2$$

$$P_2 = 2 - \left[\frac{2^2 - 36}{2(2)}\right] = 2 - \left[\frac{4 - 36}{4}\right]$$
$$= 2 - \left[\frac{-32}{4}\right]$$
$$= 10$$