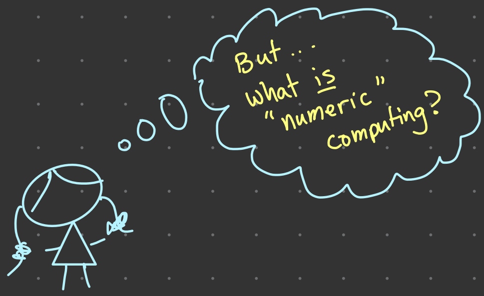


and why study it?

- Isn't all computing "numeric"?
  - not really ... (automata?)
- Numeric "methods"
  - as opposed to analytical methods
- Numeric computing simply borrowed the name

Study Plan

- Traditional "numeric methods" contents
    - Representation / Errors
    - Solving non-linear equations
    - Solving linear equations
    - Interpolation
    - Numeric differentiation / integration

- Python !
- Matrix and vector representation
- Machine learning "helpers"
- Libraries

"The purpose of computation is insight, not numbers"

– Richard Hamming

Let's begin by discussing the concept of numbers!

What types are there?

— Scalars

What is a scalar?

— "Has no direction" ...

- Real
  - transcendental
  - algebraic
    - rational
      - whole
      - fractions
    - irrational
- Imaginary

set of natural #s

$x = 2$ $\qquad x \in \mathbb{N}$ $\qquad \mathbb{N} = \{ \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \}$

$y = 1.92$ $\qquad y \in \mathbb{R}$

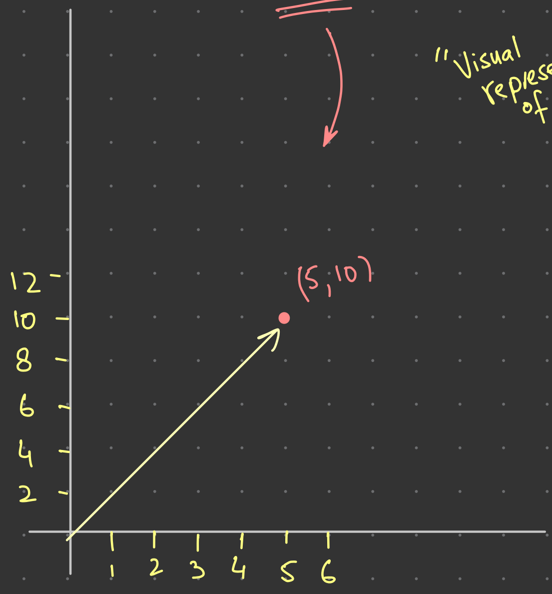
x

Representing vectors (in code)

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix} \leftarrow \begin{pmatrix} 2 \\ 14 \end{pmatrix}$$

<u>Not</u> a vector!!

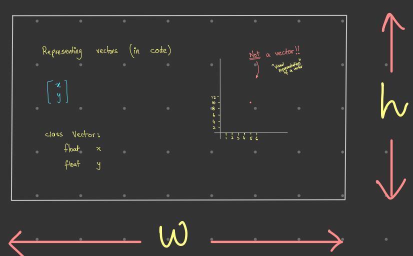
"Visual representation" of a vector




```
class Vector:
    float  x
    float  y

    add (vector b):
        c = Vector()
        c.x = self.x + b.x
        c.y = self.y + b.y
        return c

    # and so on ...
```



$v = \begin{bmatrix} h \\ w \end{bmatrix}$ ⇝ Does <u>not</u> have a direction!

$v \in \mathbb{R}^2$

$v \in \mathbb{R}$

$R^2 = R \times R$

$p \in R^3$

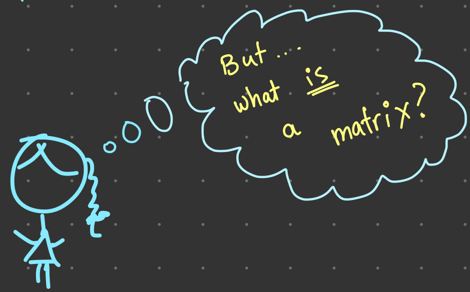
$(\quad , \quad)$

$(\quad , \quad)$

$(\quad , \quad)$

How about Matrices ?

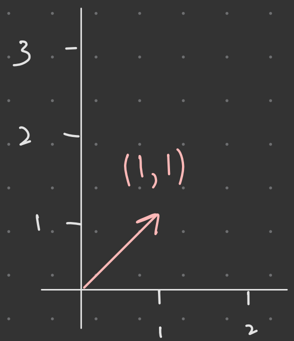
But ...



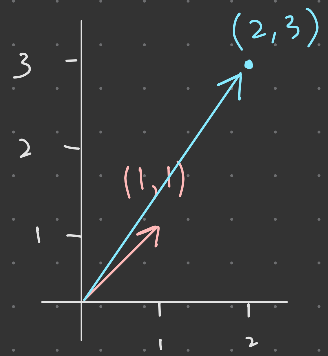
Specific case:

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix} = \begin{bmatrix}0\\ 1\end{bmatrix}$$

$$v = \begin{bmatrix}1\\ 1\end{bmatrix}$$



$$M = \begin{bmatrix}2 & 0\\ 0 & 3\end{bmatrix}$$

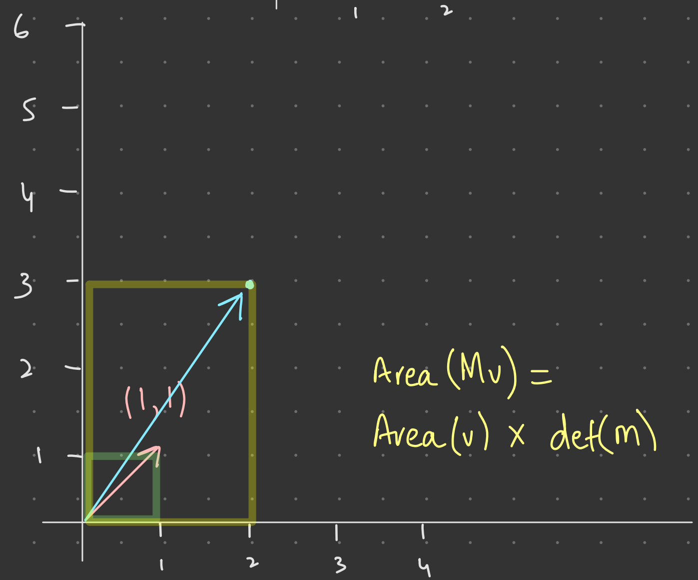


$$Mv = \begin{bmatrix}2 & 0\\ 0 & 3\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}$$

$$= \begin{bmatrix}2\\ 3\end{bmatrix}$$

"Transformation"

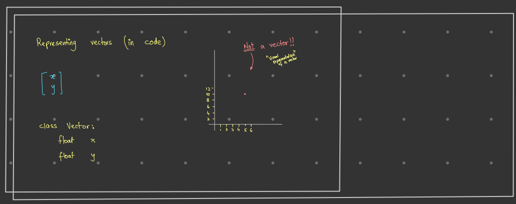
(determinant captures its essence)



Area(Mv) = Area(v) x det(m)

General View:

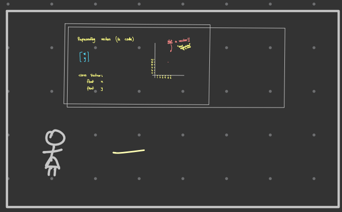
$\begin{bmatrix} h_1 \\ w_1 \end{bmatrix} \qquad \begin{bmatrix} h_2 \\ w_2 \end{bmatrix}$



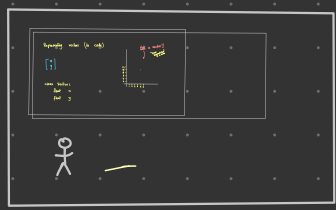
or $\quad m = \begin{bmatrix} h_1 & h_2 \\ w_1 & w_2 \end{bmatrix}$ → Matrix

$m \in \mathbb{R}^{2\times 2}$

$\begin{bmatrix} h_1 & h_2 \\ w_1 & w_2 \end{bmatrix}$



Room 01



Room 02

$t = \underbrace{\begin{bmatrix} h_1^{(1)} & h_2^{(1)} \\ w_1^{(1)} & w_2^{(1)} \end{bmatrix} \begin{bmatrix} h_1^{(2)} & h_2^{(2)} \\ w_1^{(2)} & w_2^{(2)} \end{bmatrix} \ldots}_{\text{This is called a "Tensor"}}$

$t \in \mathbb{R}^{2\times 2\times 4}$ → 4 Rooms

"3-Dimensional tensor"

info about one board; # of boards in a room; # Room; # blocks

$2 \times 2 \times 4 \times 2$

Shapes of vectors/matrices/tensors.

$$\begin{bmatrix}2\\5\\9\end{bmatrix} \rightsquigarrow$$

Vector of 3 elements

1D Tensor

1D Vector

"Vector"

$(3,) \neq (1,1)$

$$v = \begin{bmatrix}2 & 1\\3 & 7\\9 & 5\end{bmatrix} \rightsquigarrow$$

2D Tensor

"Matrix"

2D matrix

$(3,2)$

$v \in \mathbb{R}^{3\times 2}$

$2 \times 4 \times 3$

$$\begin{bmatrix}\begin{bmatrix}2 & 7 & 3 & 5\\5 & 9 & 2 & 8\end{bmatrix}\\\begin{bmatrix}1 & 2 & 5 & 3\\5 & 2 & 6 & 4\end{bmatrix}\\\begin{bmatrix}7 & 4 & 2 & 6\\9 & 3 & 1 & 8\end{bmatrix}\end{bmatrix}$$

3D Tensor

$(3,2,4)$

$\mathbb{R}^{3\times 2\times 4}$

Looks like this $\rightsquigarrow$

```
[[[2 7 3 5]
  [5 9 2 8]]
 [[1 2 5 3]
  [5 2 6 4]]
 [[7 4 2 6]
  [9 3 1 8]]]
```