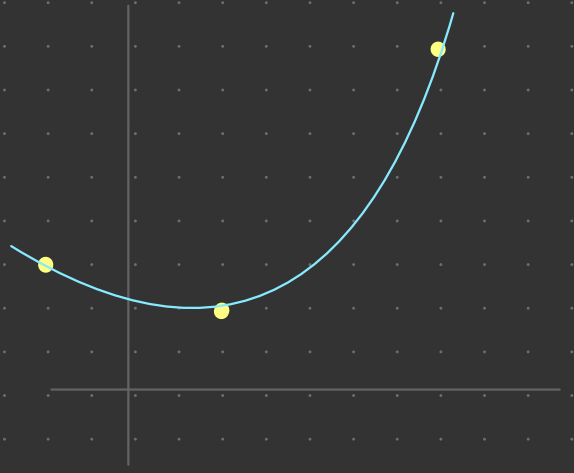


# Interpolation

Equally spaced

$x$	$y$
1	2.4
3	1.20
5	3.36
7	7.20



All methods use a similar approach.

$x$	$y$	$\Delta$	$\Delta^2$	$\Delta^3$	$\Delta^4$	$\Delta^5$
$x_0$ 15	$y_0$ 0.2588	$\Delta y_0$				
$x_1$ 20	$y_1$ 0.3420	$\Delta y_1$				
$x_2$ 25	$y_2$ 0.4226	$\Delta y_2$				
$x_3$ 30	$y_3$ 0.5	$\Delta y_3$				
$x_4$ 35	$y_4$ 0.5735	$\Delta y_4$				
$x_5$ 40	$y_5$ 0.6427					

$$\Delta y_0 = y_1 - y_0$$

$$\Delta y_1 = y_2 - y_1$$

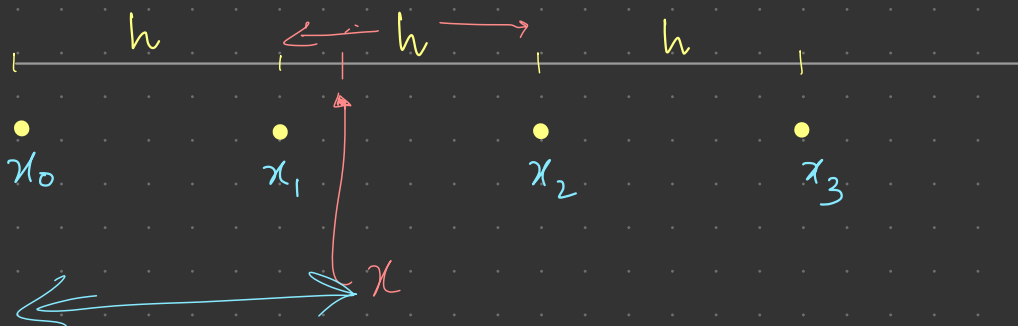
"first forward diff."

$$\Delta^2 y_0 = \Delta \Delta y_0 = \Delta (y_2 - y_1) = \Delta y_1 - \Delta y_0$$

"second forward diff."

$x$	$y$	$\Delta$	$\Delta^2$	$\Delta^3$	$\Delta^4$	$\Delta^5$
$x_0$ 15	$y_0$ 0.2588					
		$\Delta y_0$ 0.0832				
$x_1$ 20	$y_1$ 0.3420		$\Delta^2 y_0$ -0.002			
		$\Delta y_1$ 0.0805		$\Delta^3 y_0$ -0.0006		
$x_2$ 25	$y_2$ 0.4226		$\Delta^2 y_1$ -0.003		$\Delta^4 y_0$ 0.00002	
		$\Delta y_2$ 0.0773		$\Delta^3 y_1$ -0.0005		$\Delta^5 y_0$ 0.0000041
$x_3$ 30	$y_3$ 0.5		$\Delta^2 y_2$ -0.003		$\Delta^4 y_1$ 0.00002	
		$\Delta y_3$ 0.0735		$\Delta^3 y_2$ -0.0005		
$x_4$ 35	$y_4$ 0.5735		$\Delta^2 y_3$ -0.004			
		$\Delta y_4$ 0.0692				
$x_5$ 40	$y_5$ 0.6427					

What are we interested in? — "Interpolation".



"how far is  $x$  from closest (lower) point?"

$$p = \frac{x - x_0}{h} = \frac{22 - 15}{5} = 7/5$$

# Newton's forward Interpolation

$$\begin{aligned}
 y_n(x) = & y_0 \\
 & + p \Delta y_0 \\
 & + \frac{p(p-1)}{2!} \Delta^2 y_0 \\
 & + \frac{p(p-1)(p-2)}{3!} \Delta^3 y_0 \\
 & + \dots \\
 & + \frac{p(p-1)(p-2) \dots (p-(n-1))}{n!} \Delta^n y_0
 \end{aligned}$$

Example:

$$x = 3.5$$

$$y_0 = 3.818$$

$$x_0 = 2$$

$$h = 1$$

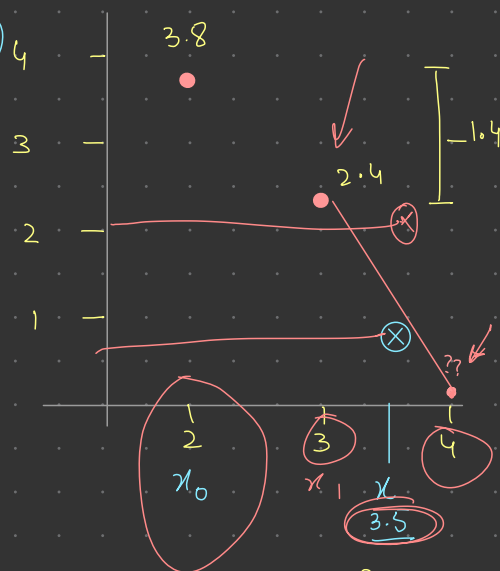
$$\Delta y_0 = -1.395$$

$$\Delta^2 y_0 = -2.0550$$

$$\Delta^3 y_0 = 3.738$$

$$y(x) = ?$$

$$p = \frac{3.5 - 2}{1} = 1.5$$



$$y(x) = y_0 + p \Delta y_0 + \frac{p(p-1)}{2!} \Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} \Delta^3 y_0$$

$$= 3.818 + (1.5)(-1.395) + \frac{(1.5)(0.5)(-2.0550)}{2!} + \frac{(1.5)(0.5)(-0.5)(3.7)}{3!}$$

$$= 3.818 - 2.0925 - 0.7706 - 0.2336$$

$$= 0.7213$$