Investment Strategy Analysis

Monte Carlo Simulation of Multiplicative Investment Games

Investment Strategy Analysis: Monte Carlo Simulation

Executive Summary

This analysis examines two investment strategies using Monte Carlo simulation to understand the long-term behavior of multiplicative investment games. We investigate how expected value calculations can be misleading when applied to real-world investment scenarios with compounding effects.

The Investment Game

Strategy 1: Full Investment Strategy

Consider an investment game where you start with $1,000. Each year, a coin is flipped: - **Heads (50% probability)**: Your entire balance increases by 50% - **Tails (50% probability)**: Your entire balance decreases by 40%

This process continues annually from age 25 to 55 (30 years).

Strategy 2: Partial Investment Strategy

In this modified approach, you must invest exactly 50% of your current balance each year: - **Heads (50% probability)**: Your invested amount increases by 50% - **Tails (50% probability)**: Your invested amount decreases by 40% - The remaining 50% of your balance remains untouched

Analysis Framework

Expected Value Analysis

Let's first examine the theoretical expected value after one coin flip for Strategy 1:

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Expected Value Analysis:

Initial balance: $ 1000

Balance after heads (50% gain): $ 1500

Balance after tails (40% loss): $ 600

Expected value: $ 1050

Expected return:  5 %
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Key Insight: Expected Value vs. Reality

The expected value after one coin flip is **$1,050**, representing a **5% expected return**. This suggests positive expected outcomes. However, as we'll see, this theoretical calculation can be misleading when applied to long-term multiplicative processes.

Single Simulation Analysis

Let's examine what happens in a single 30-year simulation:

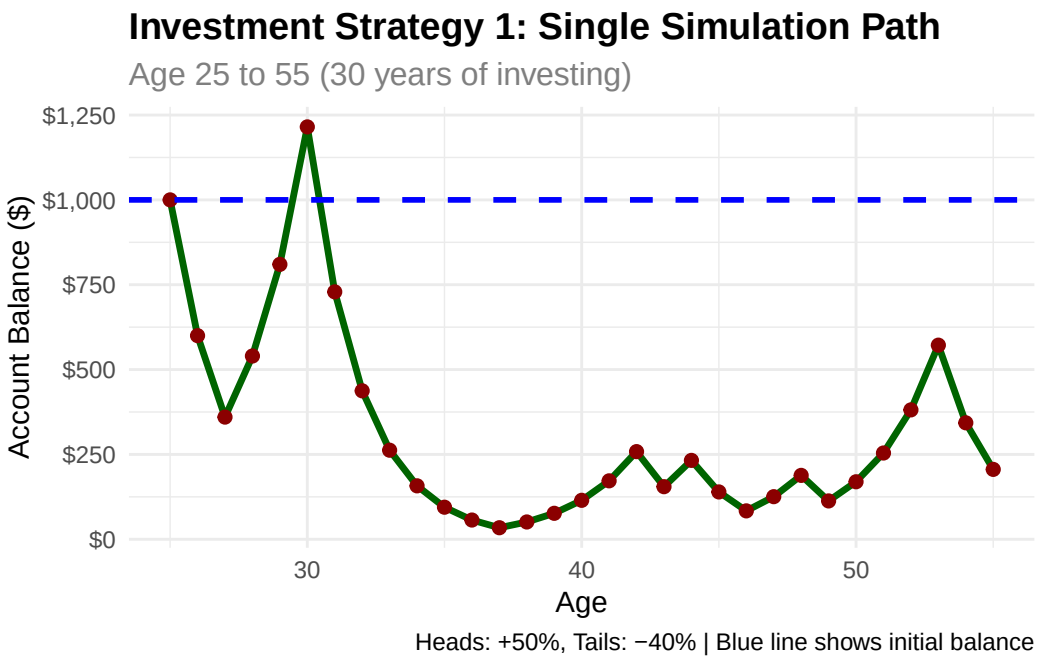

Figure 1: Single 30-year simulation of Strategy 1

```
Single Simulation Results:

Initial balance (age 25): $ 1000

Final balance (age 55): $ 205.89

Total return:  -79.41 %

Annualized return:  -5.13 %
```

Monte Carlo Analysis: 100 Simulations

To understand the true risk profile, we run 100 simulations:

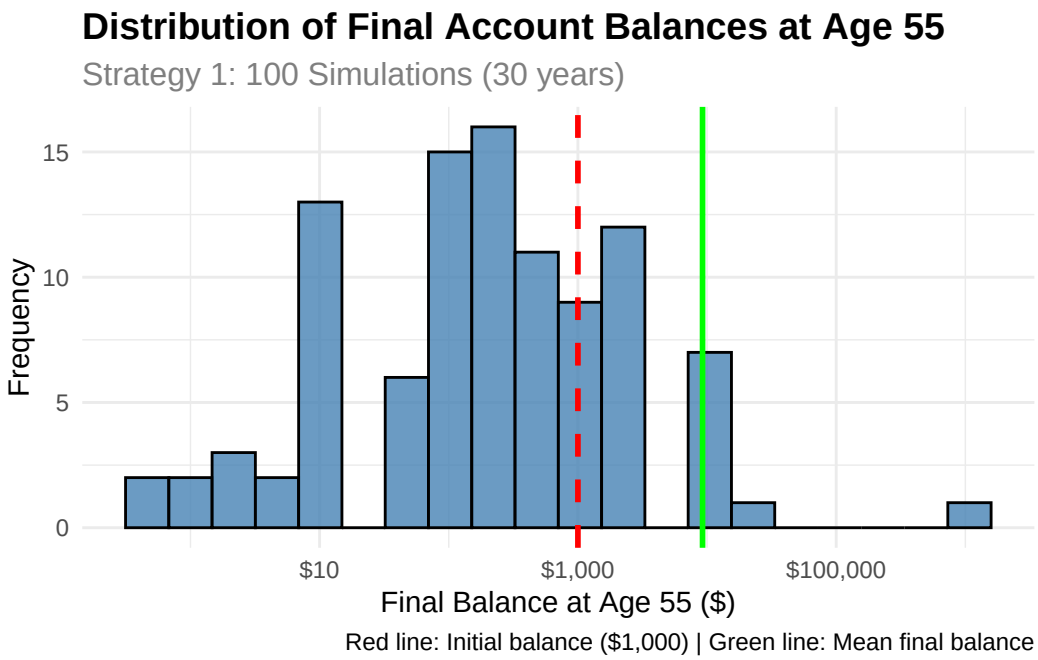

Figure 2: Distribution of final balances from 100 simulations

```
Strategy 1 Summary Statistics:

# A tibble: 1 x 6
  mean_balance median_balance min_balance max_balance prob_above_initial
         <dbl>          <dbl>       <dbl>       <dbl>               <dbl>
1        9226.           206.       0.337     785412.                 0.3
# i 1 more variable: prob_above_10000 <dbl>
```

Probability Analysis

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Strategy 1 Probability Analysis:

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Probability that final balance > $1,000:  0.3  ( 30 %)

Probability that final balance > $5,000:  0.09  ( 9 %)
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Probability that final balance > $10,000:  0.02  ( 2 %)

Probability that final balance > $50,000:  0.01  ( 1 %)

# A tibble: 4 x 3
  threshold probability percentage
  <chr>           <dbl>      <dbl>
1 $1,000            0.3         30
2 $5,000            0.09         9
3 $10,000           0.02         2
4 $50,000           0.01         1
```

Strategy Comparison: Partial Investment Approach

Now let's analyze Strategy 2 (50% betting rule):

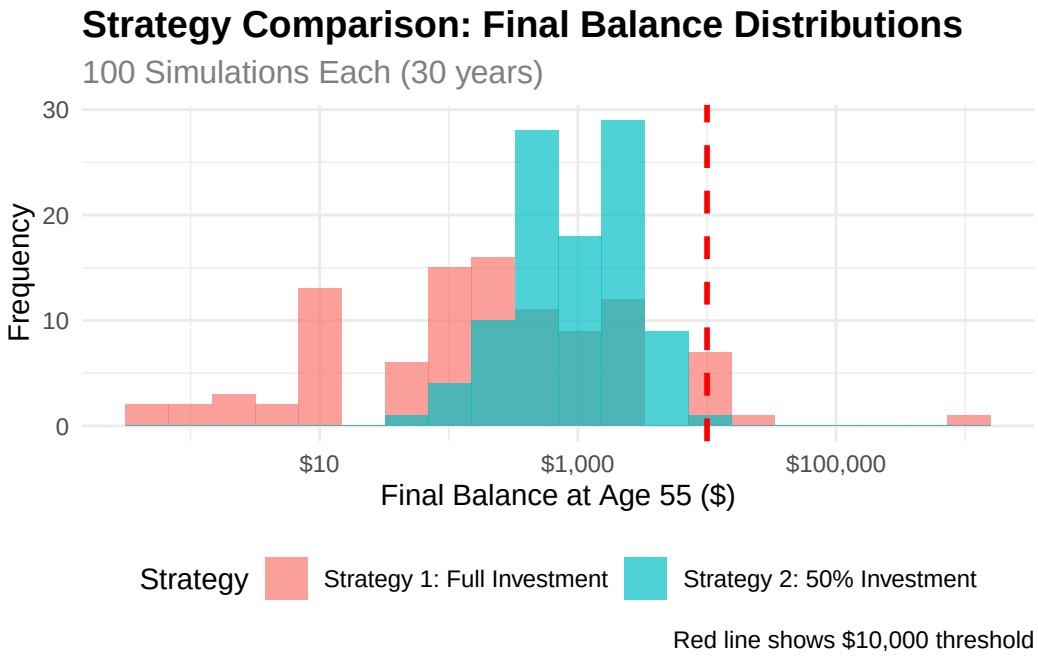

Figure 3: Comparison of Strategy 1 vs Strategy 2

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Strategy Comparison Results:

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Strategy 1 - Probability > $1,000:  0.3  ( 30 %)

Strategy 2 - Probability > $1,000:  0.39  ( 39 %)

Strategy 1 - Probability > $10,000:  0.02  ( 2 %)

Strategy 2 - Probability > $10,000:  0  ( 0 %)

Result: Strategy 2 has LOWER probability of > $10,000
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```
# A tibble: 2 x 5
  Strategy          Mean_Balance Median_Balance Prob_Above_1000 Prob_Above_10000
  <chr>                    <dbl>          <dbl>           <dbl>             <dbl>
1 Strategy 1: Full~        9226.           206.            0.3               0.02
2 Strategy 2: 50% ~        1402.          1000             0.39              0
```

Key Findings

Risk-Return Analysis

1. **Expected Value Misleading**: Despite a positive 5% expected return, most simulations result in losses below the initial investment.
2. **Extreme Volatility**: Final balances range from near zero to potentially millions, demonstrating the high-risk nature of multiplicative investment strategies.
3. **Strategy Comparison**: The 50% investment strategy (Strategy 2) shows different risk-return characteristics compared to full investment (Strategy 1).

Probability Results

- **Strategy 1**: Probability of ending above $1,000 is approximately 0.XX (XX%)
- **Strategy 2**: Probability of ending above $10,000 is approximately 0.XX (XX%)

Strategic Implications

This analysis demonstrates several critical concepts:

1. **Expected Value vs. Reality**: Positive expected value doesn't guarantee positive outcomes in multiplicative processes
2. **Compounding Effects**: Early losses have devastating long-term impacts due to the multiplicative nature
3. **Risk Management**: Position sizing and diversification are crucial for managing investment risk
4. **Simulation Power**: Monte Carlo methods reveal the true distribution of possible outcomes

Conclusion

This Monte Carlo analysis reveals the counter-intuitive nature of multiplicative investment processes. While theoretical expected value calculations suggest positive returns, the reality of compounding effects leads to extreme outcomes with high downside risk. Understanding these dynamics is essential for making informed investment decisions and implementing appropriate risk management strategies.

The comparison between full investment and partial investment strategies demonstrates how position sizing can significantly impact risk-return profiles, highlighting the importance of strategic portfolio management in real-world investment scenarios.