Question 1: (4 + 4 + 4 + 3 marks)

Find the roots of the following equations:

1) Solve the equation:

$$3x^4 = 4x^2(2x - 1)$$

Step-by-step solution: Start by distributing on the right-hand side:

$$3x^4 = 8x^3 - 4x^2$$

Move all terms to one side of the equation:

$$3x^4 - 8x^3 + 4x^2 = 0$$

Factor out the common factor:

$$x^2(3x^2 - 8x + 4) = 0$$

Now solve each part separately:

- From $x^2 = 0$, we get:

$$x = 0 \quad \text{(a repeated root)}$$

- Solve the quadratic $3x^2 - 8x + 4 = 0$ using the quadratic formula:

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4 \cdot 3 \cdot 4}}{2 \cdot 3} = \frac{8 \pm \sqrt{64 - 48}}{6} = \frac{8 \pm 4}{6}$$

  So the two additional solutions are:

$$x = \frac{12}{6} = 2 \quad \text{and} \quad x = \frac{4}{6} = \frac{2}{3}$$

Final Answer:

$$x = 0 \quad \text{(double root)}, \quad x = \frac{2}{3}, \quad x = 2$$

2) Solve using the square root method:

$$4x^2 + 24x - 8 = 0$$

Step-by-step solution: Divide the entire equation by 4 to simplify:

$$x^2 + 6x - 2 = 0$$

Move the constant term to the other side:

$$x^2 + 6x = 2$$

Now complete the square:

$$x^2 + 6x + 9 = 2 + 9 \Rightarrow (x+3)^2 = 11$$

Take the square root of both sides:

$$x + 3 = \pm\sqrt{11}$$

Finally, isolate x:

$$x = -3 \pm \sqrt{11}$$

Final Answer:

$$x = -3 + \sqrt{11}, \quad x = -3 - \sqrt{11}$$

Question 2:

Solve the following inequalities and express your answers using interval notation.

(i)

$$\left|\frac{7-3x}{2}\right| \leq 1$$

Solution:
Step 1: To remove the absolute value, recall the rule:

$$|A| \leq B \Rightarrow -B \leq A \leq B$$

Apply this rule to the expression:

$$-1 \leq \frac{7-3x}{2} \leq 1$$

Step 2: Multiply all parts of the inequality by 2 to eliminate the denominator:

$$-2 \leq 7 - 3x \leq 2$$

Step 3: Subtract 7 from all sides:

$$-9 \leq -3x \leq -5$$

Step 4: Now divide through by -3 (remember: flip the inequality signs):

$$3 \geq x \geq \frac{5}{3} \quad \Rightarrow \quad \frac{5}{3} \leq x \leq 3$$

Final Answer in Interval Notation:

$$\left[\frac{5}{3}, 3\right]$$

Check:

- For $x = 2$:

$$\left|\frac{7-6}{2}\right| = \left|\frac{1}{2}\right| = 0.5 \leq 1$$

- For $x = 3$:

$$\left|\frac{7-9}{2}\right| = \left|\frac{-2}{2}\right| = 1 \leq 1$$

(ii)

$$0 < 4x - 1 \leq 2$$

Solution:
Step 1: Interpret the compound inequality as two separate parts:

$$0 < 4x - 1 \quad \text{and} \quad 4x - 1 \leq 2$$

Step 2: Solve each one:

- From the first inequality:

$$0 < 4x - 1 \Rightarrow 1 < 4x \Rightarrow x > \frac{1}{4}$$

- From the second inequality:

$$4x - 1 \leq 2 \Rightarrow 4x \leq 3 \Rightarrow x \leq \frac{3}{4}$$

Step 3: The solution is the intersection:

$$\frac{1}{4} < x \leq \frac{3}{4}$$

Final Answer in Interval Notation:

$$\left(\frac{1}{4}, \frac{3}{4}\right]$$

Check:

- $x = \frac{1}{2}$:

$$4 \cdot \frac{1}{2} - 1 = 2 - 1 = 1 \in (0, 2] \Rightarrow \text{satisfies}$$

- $x = \frac{1}{4}$:

$$4 \cdot \frac{1}{4} - 1 = 1 - 1 = 0 \not> 0 \Rightarrow \text{does not satisfy}$$

- $x = \frac{3}{4}$:

$$4 \cdot \frac{3}{4} - 1 = 3 - 1 = 2 \leq 2 \Rightarrow \text{satisfies}$$

Question:

(1) Write the equation of the line that passes through point $A(0, -6)$ and is parallel to the y-axis.

Solution:

Step 1: Understand the Geometry A line parallel to the **y-axis** is a **vertical line**. Vertical lines have the general form:

$$x = k$$

Where k is a constant — the **x-coordinate** of any point the line passes through.

Step 2: Apply the Given Point Since the line passes through point $(0, -6)$, the x-coordinate is:

$$x = 0$$

Final Answer:

$$x = 0$$

(2) Write the equation of a circle with center at $(-2, 3)$ and passing through the point $(4, 5)$.

Solution:

Step 1: Standard Form of a Circle The equation of a circle with center (h, k) and radius r is:

$$(x - h)^2 + (y - k)^2 = r^2$$

Step 2: Plug in the Center Given the center is $(-2, 3)$, substitute:

$$(x + 2)^2 + (y - 3)^2 = r^2$$

Step 3: Use the Point to Find the Radius Substitute the point $(4, 5)$ into the equation:

$$(4 + 2)^2 + (5 - 3)^2 = r^2 \Rightarrow 6^2 + 2^2 = r^2 \Rightarrow 36 + 4 = r^2 \Rightarrow r^2 = 40$$

Final Equation of the Circle:

$$(x + 2)^2 + (y - 3)^2 = 40$$

Question 4: (6 + 4 + 5 marks)

a) Solve the logarithmic equation:

$$\log_5 x = \log_5 24 + \log_5 4 - \log_5 12$$

Step 1: Combine the logarithmic expressions

$$\log_5 24 + \log_5 4 = \log_5(24 \times 4) = \log_5 96$$

$$\log_5 96 - \log_5 12 = \log_5 \left(\frac{96}{12}\right) = \log_5 8$$

So, the equation becomes:

$$\log_5 x = \log_5 8$$

Step 2: Eliminate the logarithms Since the logarithmic expressions are equal:

$$x = 8$$

Final Answer:

$$\boxed{8}$$

b) Solve the exponential equation:

$$e^{5x-2} = 5$$

Step 1: Apply the natural logarithm

$$\ln(e^{5x-2}) = \ln(5)$$

Step 2: Use logarithmic identities

$$5x - 2 = \ln(5)$$

Step 3: Solve for x

$$5x = \ln(5) + 2 \Rightarrow x = \frac{\ln(5) + 2}{5}$$

Final Answer:

$$x = \frac{\ln(5) + 2}{5}$$

c) Find the vertex and axis of symmetry of the function:

$$f(x) = x^2 + 10x + 16$$

Step 1: Complete the square

$$f(x) = (x^2 + 10x) + 16 = (x^2 + 10x + 25 - 25) + 16 = (x + 5)^2 - 9$$

Step 2: Identify the vertex

$$\text{Vertex} = (-5, -9)$$

Step 3: Find the axis of symmetry

$$\text{Axis of symmetry} = x = -5$$

Final Answer:

- Vertex: $(-5, -9)$
- Axis of symmetry: $x = -5$