9.1

Differentiating Parametric Equations

1) $x(t) = a\cos(t) + b\sin(t)$
$y(t) = 2a\sin(t) + t^2$

$$\frac{dx}{dt} = -a\sin(t) + b\cos(t)$$

$$\frac{dy}{dt} = 2a\cos(t) + 2t$$

$$\boxed{\frac{dy}{dt} \div \frac{dx}{dt} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{dy}{dx}}$$

$$\frac{dy}{dx} = \frac{2a\cos(t) + 2t}{-a\sin(t) + b\cos(t)}$$

2) $x(t) = ct$
$y(t) = c/t$

$$\frac{dx}{dt} = c, \quad \frac{dy}{dt} = \frac{-c}{t^2}$$

$$\frac{dy}{dx} = \frac{-c}{t^2} \div c = \frac{-c}{t^2} \times \frac{1}{c} = \frac{-1}{t^2}$$

3) $x(\theta) = a\cos^3\theta$, $y(\theta) = b\sin^3\theta$

$$\frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta, \quad \frac{dy}{d\theta} = 3b\sin^2\theta\cos\theta$$

$$\frac{dy}{dx} = \frac{3b\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta} = -\frac{b}{a}\tan\theta$$

* Chain ~~Implicit~~ differentiation:
Let $\cos\theta = z$, $f'(z^3) = 3z^2 = 3\cos^2\theta$
$f'(\cos\theta) = \sin\theta$

4) $x(t) = a\cos\left(\frac{t}{2}\right)$

$y(t) = b\sin\left(\frac{t}{2}\right)$

$\frac{dx}{dt} = -\frac{a}{2}\sin\left(\frac{t}{2}\right)$, $\frac{dy}{dx} = \frac{b}{2}\cos\left(\frac{t}{2}\right)$

$\frac{dy}{dx} = \frac{-b\cos(t/2)}{a\sin(t/2)} = -\frac{b}{a}\cot\left(\frac{t}{2}\right)$

5) $x = a(\cot(t) + t\sin(t))$

$y = a(\sin(t) - t\cos(t))$

* Product rule

$t\sin(t) = t \cdot \frac{d}{dt}(\sin(t)) + \sin(t) \cdot \frac{dt}{dt}$

$= t\cos(t) + \sin(t)$

(a) $\cancel{\frac{dx}{dt} =}$ $\left(\frac{1}{a}\right)\frac{dx}{dt} = -\csc^2(t) + t\cos(t) + \sin(t)$

$\frac{dx}{dt} = a(-\csc^2(t) + t\cos(t) + \sin(t))$

$\frac{1}{a}\frac{dy}{dt} = \cos(t) + t\sin(t) - \cos(t)$

$\frac{dy}{dt} = a(t\sin(t))$

$\frac{dy}{dx} = \frac{a(t\sin(t))}{a(-\csc^2(t) + t\cos(t) + \sin(t))} = \frac{t\sin(t)}{-\csc^2(t) + t\cos(t) + \sin(t)}$

6) $3x = t^3$, $2y = t^2$

$x = \frac{t^3}{3}$, $y = \frac{t^2}{2}$

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$\frac{dx}{dt} = t^2$, $\frac{dy}{dt} = \frac{2t}{2} = t$

$\frac{dy}{dx} = \frac{t}{t^2} = \frac{1}{t}$

7) $x = \sin(t)$ $y = \sin(2t)$

$\frac{dx}{dt} = \cos(t)$

$\frac{dy}{dt} = 2\cos(2t)$

$\frac{dy}{dx} = \frac{2\cos(2t)}{\cos(t)}$

8) $x = 2\sin(1+3t)$ $y = 2t^3$

$\frac{dx}{dt} = 3 \cdot 2\cos(1+3t) = 6\cos(1+3t)$

$\frac{dy}{dt} = 6t^2$

$\frac{dy}{dx} = \frac{t^2}{\cos(1+3t)}$

$\frac{dy}{dx}$ when $t = -\frac{1}{3}$ is $\boxed{1/9}$

$\frac{1}{9} * \frac{1}{\cos(0)} = \frac{1}{9} * 1 = \frac{1}{9}$

9) $x = 6\tan(t)$, $y = 2t+1$

$\frac{dx}{dt} = 6\sec^2(t)$, $\frac{dy}{dt} = 2 \Rightarrow \frac{dy}{dx} = \frac{1}{3\sec^2(t)} = \frac{\cos^2(t)}{3}$

10) $x = t^3 - t$, $y = \sqrt{3t+1}$

$\frac{dx}{dt} = 3t^2 - 1$

$\frac{dy}{dt} = \frac{3}{2\sqrt{3t+1}}$

$\frac{dy}{dx} = \frac{3}{(2)\sqrt{3t+1}\,(3t^2-1)}$

$\frac{dy}{dx}$ at $t=1$ is $\boxed{\frac{3}{8}}$

$\frac{3}{2 \cdot 2 \cdot 2} = \frac{3}{8}$

~~11) $x = 2\sin(1+3t)$~~

~~$y = 2t^3$~~

~~$\frac{dx}{dt} =$~~

11) $x = \ln(4t-3)$ $y = \frac{2}{t}$

$\frac{dx}{dt} = \frac{4}{4t-3}$, $\frac{dy}{dt} = \frac{-2}{t^2}$

$\frac{dy}{dx} = \frac{-2}{t^2} * \frac{4t-3}{4} = \frac{-4t+3}{2t^2}$

$\frac{dy}{dx}$ when $t=3$ is $\frac{-9}{18} = \boxed{-\frac{1}{2}}$