

## Ícaro Nunes - ICPC Library

Lembretes:

- Ler com cuidado
- Observar se arrays/segs/etc estão com tamanho de acordo com as constraints dadas e calculadas
- Checar se um grafo de entrada pode ter arestas repetidas
- Revisar assumptions feitas
- Observar casos base/edge

Formulas:

- Expected value of geometric distribution (expected number of failed Bernoulli trials before the first success, where  $p$  is the probability of a single success)  $= \frac{1-p}{p}$
- Sum of arithmetic sequence:
  - $n \times \frac{(a_1 + a_n)}{2}$
  - $\frac{n}{2} \times [2a_1 + (n-1) \times d]$
  - Where:  $a_1$  = first term,  $d$  = common difference,  $n$  = number of terms in the range
- Law of sines:
  - $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$
  - Where  $x$  = length of side opposed to angle  $X$
- Law of cosines:
  - $a^2 = b^2 + c^2 - 2bc \times \cos(A)$
  - Where  $a$  = length of side opposed to angle  $A$

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**1 Data Structures****1.1 Union-Find**

```
// union find code
int parent[1123456];
int ranks[1123456];

int disjoint = 0;

// the use of ct and newSet()
// is useful in problems
// where the initial number
// of disjoint sets is unknown
// such as in Hoshen-Kopelman
int ct = 0;
int newSet() {
    ct++;
    disjoint++;
    parent[ct] = ct;
    ranks[ct] = 1;

    return ct;
}

// for "regular" disjoint-sets problems
// one must use this function to
// initialize the union_find
inline void init_union_find(int sz) {
    disjoint = sz;

    for(int i=1; i<=sz; i++) {
        parent[i] = i;
        ranks[i] = 1;
    }
}

// find using compression
int find(const int& v) {
    int root = parent[v];

    while(root != parent[root]) {
        parent[root] = parent[parent[root]];
        root = parent[root];
    }

    parent[v] = root;

    return root;
}

// merge using ranks
void merge(const int& a, const int& b) {
    int root_a = find(a);
    int root_b = find(b);

    if(root_a == root_b)
        return;

    if(ranks[root_b] > ranks[root_a]) {
        parent[root_a] = root_b;
    }
}
```

```

    ranks[root_b] += ranks[root_a];
} else {
    parent[root_b] = root_a;
    ranks[root_a] += ranks[root_b];
}

disjoint--;
}
// end union find code

```

## 1.2 Iterative Segment Tree

```

#include <vector>
#include <optional>

// segment tree - 0 indexed !
// space: 4n
// time:
// - range query: O(logn)
// - lower_bound: O(logn)
// - update: O(logn)
template<typename T, class Op>
struct STree {
    int size;

    // treesize -> smallest power
    // of 2 that is >= size
    // (the actual space taken by
    // the tree is 2*treesize)
    int ts;

    // items -> pointer to the
    // start of the leaves of
    // the segtree (implemented
    // as a perfect binary tree)
    T* it;

    // nodes -> pointer to the
    // whole tree, with an extra
    // unused node (nd[0])
    T* nd;

    // operation struct, must be
    // implemented as:
    // struct Op {
    //     T neutral;
    //     T operator()(const T& a, const T& b) {}
    // }
    Op op;

    STree(const std::vector<T>& from) {
        ts=1;

        while(ts < from.size())
            ts <<=1;

        this->size = from.size();
        nd = new T[2*ts];
        it = nd+ts;

        for(int i=0; i<2*ts; i++)
            nd[i] = op.neutral;

        for(int i=0; i<size; i++)
            it[i] = from[i];

        for(int i=2*ts-1; i>1; i--)
            nd[i>1] = op(nd[i], nd[i>>1]);
    }

    void update(int idx, T val) {
        int i = ts+idx;

        nd[i] = val;
        i = i>>1;

        while(i>0) {
            nd[i] = op(nd[i>>1], nd[(i>>1)+1]);
            i>>=1;
        }
    }

    T query(int l, int r) {
        T resl = op.neutral;
        T resr = op.neutral;

```

```

        r++;

        for(l += ts, r += ts; l<r; l>>=1, r>>=1) {
            if(l&1) resl = op(resl, nd[l++]);
            if(r&1) resr = op(nd[--r], resr);
        }

        return op(resl, resr);
    }

    ~STree() {
        delete nd;
    }

    // search for first occurrence to the right of strt
    // of a value greater than or equal to v
    int lower_boundr(int strt, T v) {
        int i=ts+strt;

        // accumulated value
        T acc=nd[i];

        while(i>1 and !(acc >= v)) {
            if(!(i&1) and op(acc, nd[i+1])>=v) break;
            if(!(i&1)) acc = op(acc, nd[i+1]);
            i>>=1;
        }

        if(i==1) return size;
        if(!(acc >= v)) i++;

        while(i<ts) {
            if(op(acc, nd[i<<1])>=v) i <=< 1;
            else acc = op(acc, nd[i<<1]), i = (i<<1) + 1;
        }

        return i-ts;
    }
};

```

## 1.3 SparseTable/RMQ

```

using namespace std;

template<typename T, class Op>
struct SparseTable {
    const static int LOG = 23;
    vector<T> st[LOG];
    Op op;

    SparseTable(const vector<T>& f) {
        const int& n = f.size();

        st[0] = f;

        int cursor = 1;
        for(int l=1; l<LOG; l++) {
            if((cursor << 1) > n) break;

            st[l].resize(n);

            for(int i=0; i<n; i++)
                st[l][i] = (i+cursor<n) ? op(st[l-1][i], st[l-1][i+cursor]) : st[l-1][i];

            cursor <<= 1;
        }
    }

    virtual T query(int l, int r) {
        T res = op.neutral;

        while(l<=r) {
            int len = r-l+1;
            int log = 32 - __builtin_clz(len) - 1;
            res = op(res, st[log][l]);
            l += (1 << log);
        }

        return res;
    }
};

```

```
// RMQ struct
// - range queries in O(1)
// - works for Operations that allow juxtaposition,
//   like min, max and gcd
template<typename T, class Op>
struct RMQ {
    T query(int l, int r) override {
        int len = r-l+1;
        int log = 32 - __builtin_clz(len) - 1;
        T res = this->st[log][l];
        res = this->op(res, st[log][r-(1 << log)+1]);
        return res;
    }
};

// example maximizer for numeric types
template<typename T>
struct Max {
    T neutral = 0;
    T operator()(const T& a, const T& b) {
        return max(a, b);
    }
};

// example minimizer for numeric types
template<typename T>
struct Min {
    T neutral = numeric_limits<T>::max();
    T operator()(const T& a, const T& b) {
        return min(a, b);
    }
};
```

## 1.4 Mo's Algorithm

```
#include <vector>
using namespace std;

const int BLKSIZE=1e2;

// OPTIONAL
inline int64_t hilbertOrder(int x, int y, int pow, int rotate) {
    if (pow == 0) {
        return 0;
    }
    int hpow = 1 << (pow-1);
    int seg = (x < hpow) ? (
        (y < hpow) ? 0 : 3
    ) : (
        (y < hpow) ? 1 : 2
    );
    seg = (seg + rotate) & 3;
    const int rotateDelta[4] = {3, 0, 0, 1};
    int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
    int nrot = (rotate + rotateDelta[seg]) & 3;
    int64_t subSquareSize = int64_t(1) << (2*pow - 2);
    int64_t ans = seg * subSquareSize;
    int64_t add = hilbertOrder(nx, ny, pow-1, nrot);
    ans += (seg == 1 || seg == 2) ? add : (subSquareSize - add - 1);
    return ans;
}
//-----

struct Query {
    int l, r, idx;
    int64_t ord;

    Query(int l, int r, int idx) : l(l), r(r), idx(idx) {
        ord = hilbertOrder(l, r, 21, 0);
    }

    bool operator < (const Query& other) const {
        // return ord < other.ord;
        int i1 = l/BLKSIZE, i2 = other.l/BLKSIZE;
        if(i1 == i2) return (i1%2 ? r > other.r : r < other.r);
        return i1 < i2;
    }
};

int add(int i) {
    // TODO
}
```

```
int remove(int i) {
    // TODO
}

void process(const vector<Query>& queries) {
    vector<int> res(queries.size());

    for(const auto& query: queries) {
        while(l > query.l)
            add(--l);

        while(r < query.r)
            add(++r);

        while(l < query.l)
            remove(l++);

        while(r > query.r)
            remove(r--);

        res[query.idx] = ans;
    }
}
```

## 2 Graph

### 2.1 Dijkstra

```
#include <algorithm>
#include <limits>
#include <queue>
#include <stack>
#include <vector>

template<typename T>
void dijkstra(const std::vector<std::pair<T, int>>& graph, bool* visited, T* dist, int* parent, int root
    , int n) {
    for(int i=1; i<=n; i++)
        dist[i] = std::numeric_limits<T>::max();

    std::priority_queue<std::pair<T, int>, std::vector<std::pair<T, int>>, std::greater<std::pair<T, int>
        >>> pq;

    pq.push({ 0, root });
    dist[root] = 0;
    parent[root] = root;

    while(!pq.empty()) {
        auto [dst, node] = pq.top();
        pq.pop();

        if(visited[node])
            continue;

        visited[node] = true;

        const std::vector<std::pair<T, int>>& adj = graph[node];

        for(const auto& [c_dst, child]: adj) {
            if(visited[child])
                continue;

            T newdst = dist[node] + c_dst;

            if(dist[child] > newdst) {
                dist[child] = newdst;
                pq.push({ newdst, child });
            }
        }
    }

    // retrieve_path SEEMS TO BE NOT WORKING
    //
    // std::vector<int> retrieve_path(int tgt, int* parent) {
    //     std::vector<int> ans;
    //     int curr = tgt;
    //     ans.push_back(tgt);
    //
    //     while(curr != parent[curr]) {
    //         ans.push_back(parent[curr]);
    //     }
```

```
// curr = parent[curr];
// }
//
// std::reverse(ans.begin(), ans.end());
//
// return ans;
// }
```

## 2.2 Toposort

```
#include <vector>
#include <stack>
#include <queue>

bool dfs(const std::vector<int>* graph, bool* visited, int* currtrav, std::stack<int>& ans, int node) {
    visited[node] = true;

    const std::vector<int>& adj = graph[node];

    bool poss = true;

    for(const int& child: adj) {
        if(currtrav[child] == currtrav[node]) {
            poss = false;
            break;
        }

        if(!visited[child]) {
            currtrav[child] = currtrav[node];

            if(!dfs(graph, visited, currtrav, ans, child)) {
                poss = false;
                break;
            }
        }
    }

    if(!poss)
        return false;

    currtrav[node] = -1;
    ans.push(node);
    return true;
}

// dfs-based toposort
// returns an empty vector in case it finds a cycle
// (which means there is no valid topological order
// for the graph)
//
// PS. the array currtrav represents the origin node
// for the current traversal, it will be useful for
// finding if there are cycles
//
// PS2. removed std::optional since it only compiles on
// C++17 above, and judges such as SPOJ only supports up
// to C++14
std::vector<int> tarjan(const std::vector<int>* graph, bool* visited, int* currtrav, int n) {
    for(int i=1; i<=n; i++)
        currtrav[i] = -1;

    bool poss = true;

    std::stack<int> ans;

    for(int i=1; i<=n; i++) {
        if(visited[i])
            continue;

        currtrav[i] = i;

        if(!dfs(graph, visited, currtrav, ans, i)) {
            poss = false;
            break;
        }
    }

    if(poss) {
        std::vector<int> v;

        while(!ans.empty()) {
            v.push_back(ans.top());
            ans.pop();
        }
    }
}
```

```
        return v;
    } else
        return std::vector<int>();
}

// "bfs-based" toposort
// returns an empty vector in case it finds a cycle
// (which means there is no valid topological order
// for the graph)
//
// PS. removed std::optional since it only compiles on
// C++17 above, and judges such as SPOJ only supports up
// to C++14
std::vector<int> kahn(const std::vector<int>* graph, bool* visited, int* indeg, int n) {
    std::queue<int> q;

    for(int i=1; i<=n; i++) {
        if(indeg[i] == 0)
            q.push(i);
    }

    std::vector<int> ans;

    while(!q.empty()) {
        int node = q.front();
        q.pop();

        if(visited[node])
            continue;

        visited[node] = true;
        ans.push_back(node);

        const std::vector<int>& adj = graph[node];

        for(const int& child: adj) {
            indeg[child]--;

            if(indeg[child] == 0)
                q.push(child);
        }
    }

    if(ans.size() == n)
        return ans;
    else
        return std::vector<int>();
}
```

## 2.3 Prim/Kruskal

```
#include <queue>
#include <vector>
#include "../data-structure/union_find/union_find.h"

template<typename T>
T prim(const std::vector<std::pair<T, int>>* graph, bool* visited, int root) {
    using pTi = std::pair<T, int>;
    std::priority_queue<pTi, std::vector<pTi>, std::greater<pTi>> pq;

    T total = 0;

    pq.push({0, root});

    while(!pq.empty()) {
        auto [cost, node] = pq.top();
        pq.pop();

        if(visited[node])
            continue;

        visited[node] = true;
        total += cost;

        const std::vector<pTi>& adj = graph[node];

        for(const auto& [c_dist, child]: adj) {
            if(visited[child])
                continue;

            pq.push({c_dist, child});
        }
    }
}
```

```

    return total;
}

// for kruskal
template<typename T>
struct edge {
    int a, b;
    T c;

    bool operator<(const edge<T>& other) const {
        return c < other.c;
    }
};

template<typename T>
T kruskal(std::vector<edge<T>>& edges, int n) {

    init_union_find(n);

    sort(edges.begin(), edges.end());

    T total = 0;

    for(const edge<T>& e: edges) {
        int root_a = find(e.a);
        int root_b = find(e.b);

        if(root_a != root_b) {
            total += e.c;
            merge(root_a, root_b);
        }
    }

    return total;
}

```

## 2.4 Floyd-Warshall

```

#include <cstdint>
#include <limits>

template<typename T, std::size_t N, std::size_t M>
using Matrix = T[N][M];

// the adjacency matrix passed to floyd-warshall must contain
// weight of INF (numeric_limits<T>::max()) on pairs of nodes
// that are not adjacent, and 0 on the main diagonal
template<typename T, std::size_t N, std::size_t M>
void floydWarshall(const Matrix<T, N, M>& graph, Matrix<T, N, M>& dist, int n) {
    const T INF = std::numeric_limits<T>::max();

    for(int i=1; i<=n; i++)
        for(int j=1; j<=n; j++)
            dist[i][j] = graph[i][j];

    // k -> intermediate node
    for(int k=1; k<=n; k++) {
        // i -> source
        for(int i=1; i<=n; i++) {
            // j -> target
            for(int j=1; j<=n; j++) {
                T newdist;

                if(dist[i][k] == INF || dist[k][j] == INF)
                    newdist = INF;
                else
                    newdist = dist[i][k] + dist[k][j];

                dist[i][j] = min(dist[i][j], newdist);
            }
        }
    }
}

```

## 3 Math

### 3.1 FFT (recursive)

```

#include <iostream>
#include <math>

```

```

const long double PI = std::arccos(-1);

template<typename T>
struct Complex {
    T a, b;

    Complex(T a, T b) {
        this->a = a;
        this->b = b;
    }

    Complex(T a) {
        this->a = a;
        this->b = 0;
    }

    Complex operator+(const Complex& other) const {
        return Complex(a+other.a, b+other.b);
    }

    Complex operator-(const Complex& other) const {
        return Complex(a-other.a, b-other.b);
    }

    Complex operator*(const T& real) const {
        return Complex(a*real, b*real);
    }

    Complex operator*(const Complex& other) const {
        return Complex(a*other.a - b*other.b, a*other.b + b*other.a);
    }
};

```

```

// works for a power of 2 sized vector
// T must be a floating point type because
// of the trigonometric operations
template<typename T>
vector<Complex<T>> fft(const vector<T>& p) {
    int n = p.size();

    vector<Complex<T>> ans(n);

    if(n == 1) {
        ans[0] = Complex<T>(p[0]);
        return ans;
    }

    T angle = (2*PI)/n;
    Complex<T> w = Complex<T>(std::cos(angle), std::sin(angle));

    vector<T> pe(n/2), po(n/2);
    for(int i=0; i<=n-2; i+=2)
        pe[i/2] = p[i], po[i/2] = p[i+1];

    vector<Complex<T>> ye = fft(pe), yo = fft(po);

    Complex<T> w_i = Complex<T>(T(1));

    for(int i=0; i<n/2; i++) {
        ans[i] = ye[i] + w_i*yo[i];
        ans[i+n/2] = ye[i] - w_i*yo[i];
        w_i = w_i*w;
    }

    return ans;
}

```

```

// works for a power of 2 sized vector
// T must be a floating point type because
// of the trigonometric operations
template<typename T>
vector<Complex<T>> ifft(const vector<Complex<T>>& p) {
    int n = p.size();

    vector<Complex<T>> ans(n);

    if(n == 1) {
        ans[0] = Complex<T>(p[0]);
        return ans;
    }

    T angle = (2*PI)/n;

```

```

Complex<T> w = (1/double(n)) * Complex<T>(std::cos(-angle), std::sin(-angle));
vector<T> pe(n/2), po(n/2);
for(int i=0; i<n-2; i+=2)
    pe[i/2] = p[i], po[i/2] = p[i+1];

vector<Complex<T>> ye = ifft(pe), yo = ifft(po);

Complex<T> w_i = Complex<T>(T(1));

for(int i=0; i<n/2; i++) {
    ans[i] = ye[i] + w_i*yo[i];
    ans[i+n/2] = ye[i] - w_i*yo[i];
    w_i = w_i*w;
}

return ans;
}

std::ostream& operator<<(std::ostream& os, const Complex<T>& c) {
    os << "Complex(" << c.a << ", " << c.b << ")";
    return os;
}

```

### 3.2 FFT (in-place)

```

using ld = long double;
using cd = complex<ld>;
const ld PI = acos(-1);

int reverse(int num, int lg_n) {
    int res = 0;
    for (int i = 0; i < lg_n; i++) {
        if (num & (1 << i))
            res |= 1 << (lg_n - 1 - i);
    }
    return res;
}

void fft(vector<cd> &a, bool invert) {
    int n = a.size();
    int lg_n = 0;
    while ((1 << lg_n) < n)
        lg_n++;

    for (int i = 0; i < n; i++) {
        if (i < reverse(i, lg_n))
            swap(a[i], a[reverse(i, lg_n)]);
    }

    for (int len = 2; len <= n; len <<= 1) {
        ld ang = 2 * PI / len * (invert ? -1 : 1);
        cd wlen(cos(ang), sin(ang));
        for (int i = 0; i < n; i += len) {
            cd w(1);
            for (int j = 0; j < len / 2; j++) {
                cd u = a[i+j], v = a[i+j+len/2] * w;
                a[i+j] = u + v;
                a[i+j+len/2] = u - v;
                w *= wlen;
            }
        }
    }

    if (invert) {
        for (cd &x : a)
            x /= n;
    }
}

```

### 3.3 Matrix Multiplication

```

#include <cstdlib>

template<typename T, std::size_t N>
using Matrix = T[N][N];

template<typename T, std::size_t N>
void cp(const Matrix<T, N>& src, Matrix<T, N>& tgt) {
    for(int i=0; i<N; i++)

```

```

        for(int j=0; j<N; j++)
            tgt[i][j] = src[i][j];
    }

template<typename T, std::size_t N>
void matmul(const Matrix<T, N>& a, const Matrix<T, N>& b, Matrix<T, N>& dst, const int MOD) {
    Matrix<T, N> tmp;

    for(int i=0; i<N; i++) {
        for(int j=0; j<N; j++) {
            T sum = 0;

            for(int l=0; l<N; l++) {
                sum += (a[i][l]*b[l][j])%MOD;
                sum %= MOD;
            }

            tmp[i][j] = sum;
        }
    }

    cp(tmp, dst);
}

```

## 4 Geometry

### 4.1 Point2D

```

#include <bits/stdc++.h>
using namespace std;
using ll=long long;

template<typename T>
struct Point {
    T x,y;

    Point() = default;
    Point(T x, T y) : x(x), y(y) {}

    bool operator<(const Point& o) const {
        if(x == o.x) return y<o.y;
        return x<o.x;
    }

    // Point operator
    Point operator+(const Point& o) const {
        return Point(x+o.x, y+o.y);
    }

    Point operator-(const Point& o) const {
        return Point(x-o.x, y-o.y);
    }
};

template<typename T>
T dot(const Point<T>& a, const Point<T>& b) {
    return a.x*b.x + a.y*b.y;
}

template<typename T>
T cross(const Point<T>& a, const Point<T>& b) {
    return a.x*b.y - a.y*b.x;
}

using point = Point<ll>;

```

### 4.2 Convex Hull

```

#include <bits/stdc++.h>
#include <point.h>
using namespace std;

// Convex Hull, runs in O(n) for sorted list
// pts must be vector of at least 3 points
// SORTED with pair sort
// Returns: points of convex hull in ccw order
vector<point> convex_hull(const vector<point>& pts) {
    const int& n = pts.size();

    vector<point> hull = { pts[0], pts[1] };

    for(int i=2; i<n; i++) {
        while(hull.size() > 1 and cross(pts[i] - hull[hull.size()-2], hull.back() - hull[hull.size()-2]) < 0)

```

```

        hull.pop_back();
        hull.push_back(pts[i]);
    }

    int lhsz = hull.size();
    hull.push_back(pts[n-2]);

    for(int i=n-3; i>=0; i--) {
        while(hull.size() > 1 and cross(pts[i] - hull[hull.size()-2], hull.back() - hull[hull.size()-2]) < 0)
            hull.pop_back();
        hull.push_back(pts[i]);
    }
    hull.pop_back();

    return hull;
}

```

## 5 Number Theory

### 5.1 Divisors

```

#include <bits/stdc++.h>

// regular implementation
// - time complexity: O(sqrt(n))
// - space complexity: O(sqrt(n))
template<typename Integer>
std::vector<Integer> find_divisors(Integer val) {
    std::vector<Integer> v;
    std::stack<Integer> finals;

    Integer root = sqrt(val);

    // this less than or equal to is important
    // to avoid edge case like in find_divisors(20),
    // where sqrt(20) == 4, and therefore, in a
    // mistake I previously made, my algorithm was
    // checking "root" separately, only to check
    // if the square root was exact. Even though
    // 4 is not the exact square root of 20, it is
    // still a divisor of 20, and that version of
    // the algorithm would output { 1, 2, 10, 20 }
    // instead of { 1, 2, 4, 5, 10, 20 }. Another
    // fix was to keep the separate check, but
    // using ceil-rounded (instead of floor-rounded)
    // square root.
    for(Integer i=1; i<=root; i++) {
        if(val%i == 0) {
            v.push_back(i);

            Integer div = val/i;

            if(div != i)
                finals.push(div);
        }
    }

    while(!finals.empty()) {
        v.push_back(finals.top());
        finals.pop();
    }

    return v;
}

```

### 5.2 Fexp/Modular Inverse

```

template<typename T>
T fexp(T x, T exp, const T MOD) {
    T cursor = 1;
    T ans = 1;
    T currpow = x;

    while(cursor <= exp) {
        if(cursor & exp)
            ans = (ans%MOD) * (currpow%MOD) %MOD;

        cursor <=< 1;
        currpow = currpow%MOD;
        currpow = (currpow+currpow)%MOD;
    }
}

```

```

    }

    return ans;
}

template<typename T>
T inv(T x, const T MOD) {
    return fexp(x, MOD-2, MOD);
}

```

### 5.3 Gcd/Lcm

```

template<typename T>
T gcd(T a, T b) {
    if(a == b)
        return a;

    if(b > a)
        return gcd(b, a);

    if(!b) return a;
    return gcd(b, a%b);
}

template<typename T>
T lcm(T a, T b) {
    return a*b/gcd(a, b);
}

```

### 5.4 Sieve/Factors

```

#include <cmath>
#include <vector>

void sieve(bool* prime, std::vector<int>& primes, const int MAX_V) {
    for(int i=2; i<MAX_V; i++) {
        prime[i] = true;
    }

    int root = std::sqrt(MAX_V);

    for(int i=2; i<=root; i++) {
        if(!prime[i])
            continue;

        primes.push_back(i);

        for(int j=i; j <= MAX_V; j+=i) {
            prime[j] = false;
        }
    }

    for(int i=root; i<MAX_V; i++) {
        if(prime[i])
            primes.push_back(i);
    }
}

std::vector<std::pair<int, int>> factors(const int& x, const std::vector<int>& primes) {
    std::vector<std::pair<int, int>> res;

    int rem = x;

    for(const int& p: primes) {
        if(p > rem)
            break;

        if(rem % p != 0)
            continue;

        int amt = 0;

        while(rem % p == 0) {
            rem /= p;
            amt++;
        }

        res.push_back(std::make_pair(p, amt));
    }

    return res;
}

```

## 6 Others

### 6.1 Kadane

```
template<typename T>
T kadane(T* arr, int n) {
    T curr = arr[0];
    T best = curr;

    for(int i=1; i<n; i++) {
        curr = max(arr[i], curr + arr[i]);
        best = max(best, curr);
    }

    return best;
}
```

### 6.2 Z Function

```
#include <vector>
#include <string>

std::vector<int> zstring(const std::string& s) {
    std::vector<int> z(s.size(), 0);

    int left = 0, right = 0;

    for(int k=1; k < s.size(); k++) {
        if(k > right) {
            left = right = k;

            while(right < s.size() && s[right] == s[right - left])
                right++;

            z[k] = right - left;

            right--;
        } else /* k <= right */ {
            int k1 = k - left;

            if(z[k1] < right - k) {
                z[k] = z[k1];
            } else {
                left = k;

                while(right < s.size() && s[right] == s[right - left])
                    right++;

                z[k] = right - left;
                right--;
            }
        }
    }
}
```

```
        return z;
    }

    std::vector<int> match(const std::string& s, const std::string& pattern) {
        std::string pt = pattern + "$" + s;
        std::vector<int> z = zstring(pt);
        std::vector<int> res;

        for(int i=pattern.size() + 1; i < pt.size(); i++) {
            if(z[i] == pattern.size())
                res.push_back(i - pattern.size() - 1);
        }

        return res;
    }
}
```

## 7 Utils

### 7.1 Tester

```
import random
import subprocess

MAX_N = 100

def gen_case() -> str:
    return f"1\n"

random.seed((1 << 9) | 31)

for i in range(100):
    print()
    print()
    case = gen_case()
    print(f"Test #{i+1}: ")
    print(case)
    # test bruteforce
    bf = subprocess.run(['out/b'], input=case, encoding='ascii', capture_output=True)
    # test solution
    sol = subprocess.run(['out/m'], input=case, encoding='ascii', capture_output=True)

    bf_res = bf.stdout
    sol_res = sol.stdout

    print(f"bruteforce {bf_res}, solution {sol_res}")

    if bf_res == sol_res:
        print("accepted")
    else:
        print("WA")
        break
```