Project 3 – Dynamic Dependence with DCC-GARCH in Cryptocurrencies

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Contents

1. Objective

To model the dynamic and conditional dependence between Bitcoin and a set of altcoins using a DCC-GARCH framework, and to evaluate how conditional correlations evolve over time and respond to extreme market shocks.

2. Libraries & Setup

```
library(quantmod)
library(xts)
library(rugarch)
library(rmgarch)
library(tidyverse)
library(zoo)
library(ggplot2)
```

3. Data Download and Preparation

A multivariate dataset of daily cryptocurrency prices is constructed and aligned in time.

```
get_crypto_data <- function(symbol, start_date = "2025-01-01") {
tryCatch({
data <- getSymbols(
paste0(symbol, "-USD"),
src = "yahoo",
from = start_date,
auto.assign = FALSE
)
Cl(data)
}, error = function(e) {
message(paste("Downloading error:", symbol))
return(NULL)
})
}

btc  <- get_crypto_data("BTC")
eth  <- get_crypto_data("ETH")
link <- get_crypto_data("LINK")
zec  <- get_crypto_data("ZEC")

prices <- na.omit(merge(btc, eth, link, zec))
colnames(prices) <- c("BTC", "ETH", "LINK", "ZEC")

head(prices)
```

```
##                   BTC      ETH     LINK      ZEC
## 2025-01-01  94419.76 3353.504 21.67484 58.11317
## 2025-01-02  96886.88 3451.393 22.04044 59.39714
## 2025-01-03  98107.43 3605.010 23.39836 60.91785
```

```
## 2025-01-04  98236.23 3657.707 23.61336 59.18812
## 2025-01-05  98314.96 3634.104 23.63523 58.53623
## 2025-01-06 102078.09 3688.611 23.81981 60.16333
```

4. Log Returns

Daily log returns are computed and serve as the basis for dependence and volatility modeling.

```r
returns <- na.omit(diff(log(prices)))
summary(returns)
```

```
##      Index                 BTC                  ETH
##  Min.   :2025-01-02   Min.   :-0.0908227   Min.   :-0.1607306
##  1st Qu.:2025-03-31   1st Qu.:-0.0125000   1st Qu.:-0.0235762
##  Median :2025-06-27   Median :-0.0002815   Median : 0.0001428
##  Mean   :2025-06-27   Mean   :-0.0001982   Mean   :-0.0003402
##  3rd Qu.:2025-09-23   3rd Qu.: 0.0116691   3rd Qu.: 0.0195359
##  Max.   :2025-12-21   Max.   : 0.0912150   Max.   : 0.1972001
##       LINK                ZEC
##  Min.   :-0.237734   Min.   :-0.222889
##  1st Qu.:-0.028336   1st Qu.:-0.029082
##  Median :-0.001650   Median : 0.003598
##  Mean   :-0.001559   Mean   : 0.005770
##  3rd Qu.: 0.026852   3rd Qu.: 0.034111
##  Max.   : 0.163347   Max.   : 0.477598
```

5. Marginal Models: Univariate GARCH

Before modeling dynamic dependence, each asset's marginal volatility is modeled using a GARCH(1,1) specification.

```r
uspec <- ugarchspec(
variance.model = list(model = "sGARCH", garchOrder = c(1,1)),
mean.model = list(armaOrder = c(0,0), include.mean = TRUE),
distribution.model = "std"
)

mspec <- multispec(replicate(ncol(returns), uspec))
```

6. DCC-GARCH Model

6.1 DCC Specification

```r
dcc_spec <- dccspec(
uspec = mspec,
dccOrder = c(1,1),
distribution = "mvnorm"
)
```

6.2 Model Estimation

```r
dcc_fit <- dccfit(
spec = dcc_spec,
data = returns
)

dcc_fit
```

```
##
## *---------------------------------*
## *          DCC GARCH Fit          *
## *---------------------------------*
##
## Distribution         :  mvnorm
## Model                :  DCC(1,1)
## No. Parameters       :  28
## [VAR GARCH DCC UncQ] : [0+20+2+6]
## No. Series           :  4
## No. Obs.             :  353
## Log-Likelihood       :  3064.257
## Av.Log-Likelihood    :  8.68
##
## Optimal Parameters
## -----------------------------------
##                 Estimate  Std. Error     t value Pr(>|t|)
## [BTC].mu        0.000274    0.000969    0.283038 0.777147
## [BTC].omega     0.000035    0.000057    0.627054 0.530624
## [BTC].alpha1    0.082557    0.067348    1.225824 0.220265
## [BTC].beta1     0.851905    0.162271    5.249889 0.000000
## [BTC].shape     4.540207    1.042060    4.356954 0.000013
## [ETH].mu       -0.000829    0.001742   -0.476079 0.634018
## [ETH].omega     0.000002    0.000000    3.420506 0.000625
## [ETH].alpha1    0.000000    0.000240    0.000015 0.999988
## [ETH].beta1     0.999000    0.000316 3160.105646 0.000000
## [ETH].shape     3.759318    0.401715    9.358174 0.000000
## [LINK].mu      -0.001175    0.002345   -0.501011 0.616363
## [LINK].omega    0.000002    0.000001    1.741935 0.081520
## [LINK].alpha1   0.000000    0.000158    0.000057 0.999954
## [LINK].beta1    0.999000    0.000206 4840.616121 0.000000
## [LINK].shape    5.329080    0.938487    5.678374 0.000000
## [ZEC].mu        0.000899    0.002567    0.350388 0.726047
## [ZEC].omega     0.000049    0.000043    1.138308 0.254992
## [ZEC].alpha1    0.057438    0.027007    2.126772 0.033439
## [ZEC].beta1     0.935569    0.028973   32.290839 0.000000
## [ZEC].shape     4.539328    1.229274    3.692690 0.000222
## [Joint]dcca1    0.013546    0.006169    2.195587 0.028122
## [Joint]dccb1    0.923676    0.030699   30.087686 0.000000
##
## Information Criteria
## ---------------------
##
## Akaike       -17.203
```

```
## Bayes        -16.896
## Shibata      -17.214
## Hannan-Quinn -17.081
##
##
## Elapsed time : 1.661286
```

7. Dynamic Conditional Correlations

Conditional correlations implied by the DCC-GARCH model are extracted and analyzed.

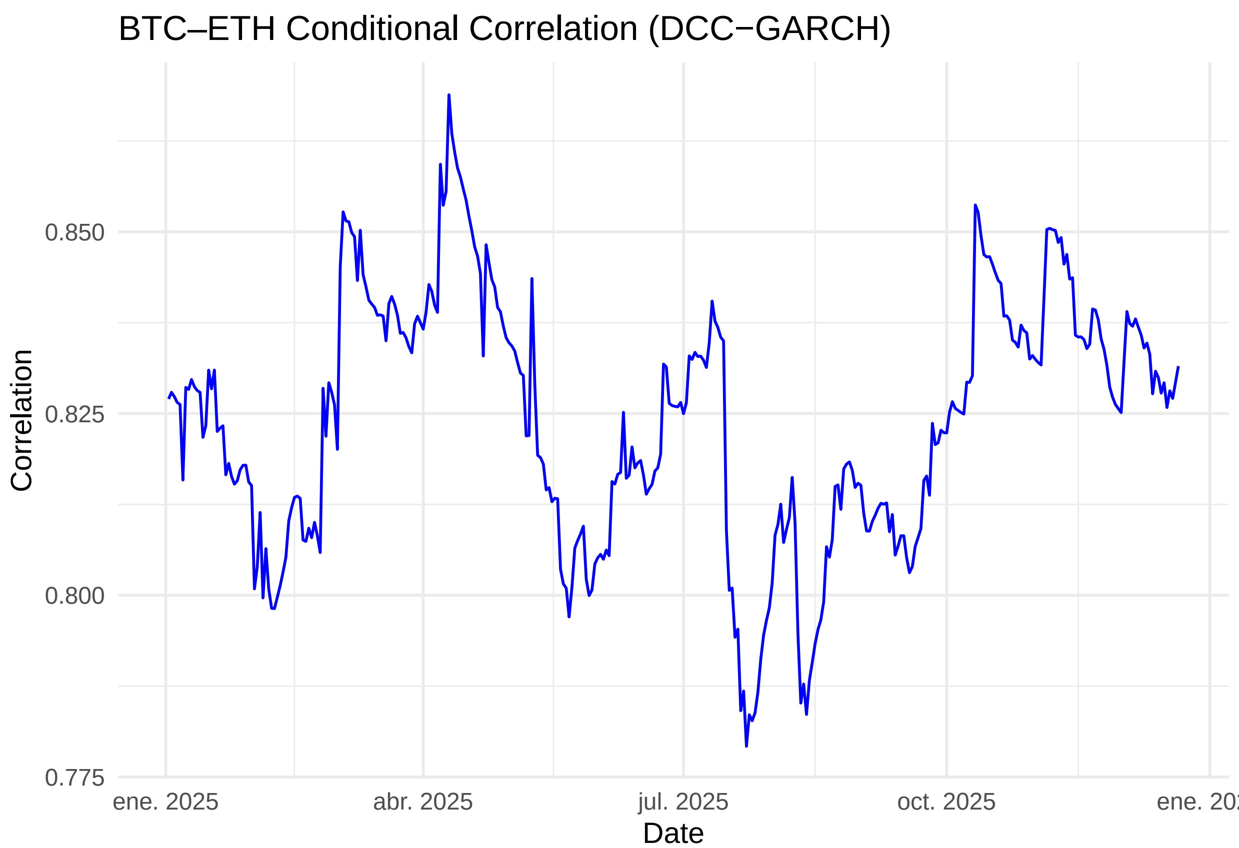
```
dcc_corr <- rcor(dcc_fit)

# BTC-ETH conditional correlation

btc_eth_corr <- dcc_corr[1, 2, ]

dcc_df <- data.frame(
Date = index(returns),
BTC_ETH = btc_eth_corr
)

ggplot(dcc_df, aes(x = Date, y = BTC_ETH)) +
geom_line(color = "blue") +
labs(
title = "BTC-ETH Conditional Correlation (DCC-GARCH)",
y = "Correlation",
x = "Date"
) +
theme_minimal()
```



8. Extension: Dynamic Dependence and BTC Shocks

This section evaluates whether conditional correlations increase during extreme BTC market movements, providing evidence of contagion.

8.1 Identification of BTC shocks

BTC shocks are defined as returns exceeding the 90th percentile of absolute standardized returns.

```
# Standardized BTC returns

btc_std <- scale(returns$BTC)

# Shock threshold (90th percentile)

shock_threshold <- quantile(abs(btc_std), 0.9)

btc_shock_indicator <- abs(btc_std) > shock_threshold
table(btc_shock_indicator)
```

```
## btc_shock_indicator
## FALSE  TRUE
##   317    36
```

8.2 Conditional Correlation During Shocks

```r
dcc_df$Shock <- btc_shock_indicator

# Average conditional correlations

mean_corr_total <- mean(dcc_df$BTC_ETH, na.rm = TRUE)
mean_corr_shock <- mean(dcc_df$BTC_ETH[dcc_df$Shock], na.rm = TRUE)


cat("Average conditional correlation (full sample):",
    round(mean_corr_total, 3), "\n")
```

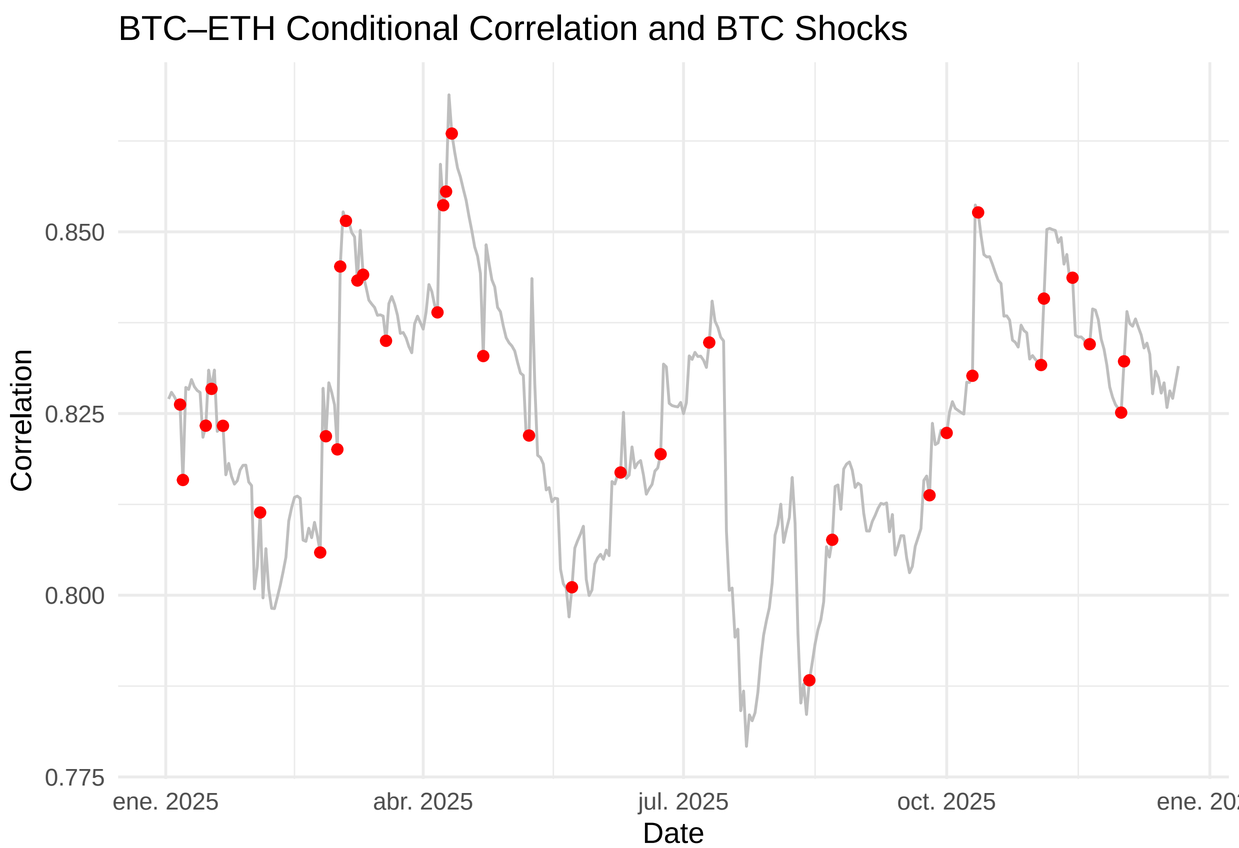
```
## Average conditional correlation (full sample): 0.824
```

```r
cat("Average conditional correlation during BTC shocks:",
    round(mean_corr_shock, 3), "\n")
```

```
## Average conditional correlation during BTC shocks: 0.829
```

8.3 Visualization: Shocks vs Conditional Correlation

```r
ggplot(dcc_df, aes(x = Date, y = BTC_ETH)) +
geom_line(color = "gray") +
geom_point(
data = subset(dcc_df, Shock == TRUE),
aes(y = BTC_ETH),
color = "red",
size = 1.5
) +
labs(
title = "BTC-ETH Conditional Correlation and BTC Shocks",
y = "Correlation",
x = "Date"
) +
theme_minimal()
```



9. Economic Interpretation

BTC–ETH correlations are time-varying and non-constant.

Conditional correlations increase during extreme BTC shocks, indicating contagion.

DCC-GARCH captures dependence dynamics that are not observable using simple rolling correlations.

Results are consistent with, but extend beyond, the unconditional dependence analysis in Project 2.

10. Conclusion

This project demonstrates the ability to:

Model multivariate systems with conditional volatility.

Separate marginal dynamics (GARCH) from dependence dynamics (DCC).

Analyze financial contagion in cryptocurrency markets.

Apply standard tools from modern financial econometrics (Engle, 2002).

Together with Projects 1 and 2, this work forms a coherent analytical sequence: univariate volatility → rolling dependence → conditional multivariate dependence.