Analysis and Approaches

Criterion A – Presentation

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	The exploration has some coherence or some organization.
2	The exploration has some coherence and shows some organization.
3	The exploration is coherent and well organized.
4	The exploration is coherent, well organized, and concise.

The "presentation" criterion assesses the organization and coherence of the exploration.

A **coherent** exploration is logically developed, easy to follow and meets its aim. This refers to the overall structure or framework, including introduction, body, conclusion and how well the different parts link to each other.

A **well-organized** exploration includes an introduction, describes the aim of the exploration and has a conclusion. Relevant graphs, tables and diagrams should accompany the work in the appropriate place and not be attached as appendices to the document. Appendices should be used to include information on large data sets, additional graphs, diagrams and tables.

A **concise** exploration does not show irrelevant or unnecessary repetitive calculations, graphs or descriptions.

The use of technology is not required but encouraged where appropriate. However, the use of analytic approaches rather than technological ones does not necessarily mean lack of conciseness, and should not be penalized. This does not mean that repetitive calculations are condoned.

			Evidence – page no	Evidence – page no
	Introduction			
1	A clear concise title	☐		
2	Introduction including aim (what you will answer/investigate) and	☐		
3	Introduction including rationale (why it was chosen)	☐		
4	Establish personal interest	☐		
	Graphs	☐		
5	Have you introduced your graph/table or diagram	☐		
6	Have you drawn conclusions from your graph/table or diagram?	☐		
7	Are your Graphs, tables and diagrams are clearly titled	☐		
8	Are your graphs referred to clearly in the text i.e. Graph 1, Graph 2, Table 1 etc.	☐		
9	Are your graph, table and diagram on the same page where possible as the text referring to it.	☐		

10	Any table contains just the relevant information needed, extra values needed for the calculation are included in the appendix (If it does not fit on one page consider it being put in appendix.	☐		
	Coherence and organisation	☐		
11	Topic sentences (what the paragraph is about) at the beginning of each paragraph	☐		
12	Are Exploration specific vocabulary defined	☐		
13	All the mathematical steps can easily follow as all necessary working out in calculations is included	☐		
14	Proof read by peer – give evidence	☐		
15	Can the explanation in the text be understood and followed by a peer	☐		
16	There are no values that just "appear" with no explanation of where they come from	☐		
17	No repetition in work	☐		
18	Each paragraph/section follows in a logical order i.e. the reader does not need to "turn back" to understand	☐		
19	There is clear evidence in the exploration that I have researched my topic well.	☐		
20	Are the pages numbers	☐		
21	References	☐		
22	A bibliography is included	☐		
	Conclusion	☐		
23	Have you fulfilled/addressed the aims of your exploration	☐		
24	Do you conclude throughout the exploration and reflect?	☐		
25	The last paragraphs draw all the conclusions together and reflect on them	☐		

Avoid over describing mathematical expressions/ methods using words

- Mathematics is itself a language

Avoid repeating yourself to make a point or a calculation with only the numbers changed
Avoid big leaps in the mathematical steps.

What is the difference between Criterion A (Presentation) and B (Mathematical communication)?

Whereas, mathematical communication focuses on the appropriateness of the mathematics. An exploration that is logically set out in terms of its overall structure could score well in criterion A despite using inappropriate mathematics. Conversely, an exploration that uses appropriate diagrams and technology to develop the ideas could score well in criterion B but poorly in criterion A because it lacked a clear aim or conclusion, for example. Presentation is focusing on the overall organization and coherence of the exploration.

Criterion B – Mathematical communication

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	The exploration contains some relevant mathematical communication which is partially appropriate.
2	The exploration contains some relevant appropriate mathematical communication.
3	The mathematical communication is relevant, appropriate and is mostly consistent.
4	The mathematical communication is relevant, appropriate and consistent throughout.

The "mathematical communication" criterion assesses to what extent the student has:

- used appropriate mathematical language (**notation, symbols, terminology**). Calculator and computer notation is acceptable only if it is software generated. Otherwise it is expected that students use appropriate mathematical notation in their work
- defined **key terms** and variables, where required
- used **multiple forms of mathematical representation**, such as formulae, diagrams, tables, charts, graphs and models, where appropriate
- used a **deductive method** and set out proofs logically where appropriate

Examples of level 1 can include graphs not being labelled, consistent use of computer notation with no other forms of correct mathematical communication.

Level 4 can be achieved by using only one form of mathematical representation as long as this is appropriate to the topic being explored. For level 4, any *minor* errors that do not impair clear communication should not be penalizsed.

			Evidence – page no	Evidence – page no
1	Did you define all variable appearing in formula charts and graphs	☐		
2	Did you use different presentation e.g. graph, chart, formula, photograph, diagram, screenshot etc	☐		
	Graphs Tables and Charts	☐		
3	Do your graphs have correct labels, scales and units on the axes if appropriate (handwritten is acceptable)	☐		
4	Do your tables have appropriate headings and linked to the discussion in the appropriate place	☐		
		☐		
5	Are your results to an appropriate degree of accuracy and explained why	☐		
6	Check that you do not have computer notation 2^x not 2^x, use not * and use 0.028 and not 2.8E-2	☐		

7	Are variable defined?	☐		
8	To enhance mathematical communication use of appropriate ICT tools such as:			
	Graphical Display Calculator	☐		
	Mathematical Software – Graphed, Geogebra, Autograph	☐		
	Spreadsheets	☐		
	Data bases	☐		
	Drawing and Word Processing Software	☐		

Criterion C – Personal Engagement

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	There is evidence of some personal engagement.
2	There is evidence of significant personal engagement.
3	There is evidence of outstanding personal engagement.

The "personal engagement" criterion assesses the extent to which the student engages with the topic by exploring the mathematics and making it their own. It is not a measure of effort.

Personal engagement may be recognized in different ways. These include thinking independently or creatively, presenting mathematical ideas in their own way, exploring the topic from different perspectives, making and testing predictions. Further (but not exhaustive) examples of personal engagement at different levels are given in the teacher support material (TSM).

There must be evidence of personal engagement demonstrated in the student's work. It is not sufficient that a teacher comments that a student was highly engaged.

Textbook style explorations or reproduction of readily available mathematics without the candidate's own perspective are unlikely to achieve the higher levels.

Significant: The student demonstrates authentic personal engagement in the exploration on a few occasions and it is evident that these drive the exploration forward and help the reader to better understand the writer's intentions.

Outstanding: The student demonstrates authentic personal engagement in the exploration in numerous instances and they are of a high quality. It is evident that these drive the exploration forward in a creative way. It leaves the impression that the student has developed, through their approach, a complete understanding of the context of the exploration topic and the reader better understands the writer's intentions.

			Evidence – page no	Evidence – page no
1	Have you included some of your own examples to present the mathematical ideas in your own way?	☐		
2	Have you explained your personal interest in the topic?	☐		

3	Have you mentioned your interest or what you consider to be the significance for any findings that have been made in your topic?			
4	• You can demonstrate personal engagement by using the following attributes and skills	☐		
5	• Thinking and working independently	☐		
6	• Thinking Creatively	☐		
7	• Addressing your personal interests	☐		
8	• Presenting mathematical ideas in your own way	☐		
9	• Asking questions, making conjectures and investigating mathematical ideas	☐		
10	• Looking for and creating mathematical models for real-world situations	☐		
	• Considering historical and global perspectives	☐		
12	• Exploring unfamiliar mathematics	☐		

Criterion D – Reflection

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	There is evidence of limited reflection.
2	There is evidence of meaningful reflection.
3	There is substantial evidence of critical reflection.

The "reflection" criterion assesses how the student reviews, analyses and evaluates the exploration. Although reflection may be seen in the conclusion to the exploration, it may also be found throughout the exploration.

Simply describing results represents **limited reflection**. Further consideration is required to achieve the higher levels.

Some ways of showing **meaningful reflection** are: linking to the aims of the exploration, commenting on what they have learned, considering some limitation or comparing different mathematical approaches.

Critical reflection is reflection that is crucial, deciding or deeply insightful. It will often develop the exploration by addressing the mathematical results and their impact on the student's understanding of the topic. Some ways of showing critical reflection are: considering what next, discussing implications of results, discussing strengths and weaknesses of approaches, and considering different perspectives.

Substantial evidence means that the critical reflection is present throughout the exploration. If it appears at the end of the exploration it must be of high quality and demonstrate how it developed the exploration in order to achieve a level 3.

Further (but not exhaustive) examples of reflection at different levels are given in the teacher support material (TSM).

			Evidence – page no	Evidence – page no
1	Reflect upon the validity of results (including accuracy)	☐		

2	Links between different Mathematical ideas	☐		
3	Explain why a particular course of action/method has been chosen.	☐		
4	Contextualise results with respect to the task.	☐		
5	Reflect upon quality of examples throughout the task	☐		
6	Conclude with what has been learned.	☐		

Criterion E – Use of Mathematics SL

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	Some relevant mathematics is used.
2	Some relevant mathematics is used. Limited understanding is demonstrated.
3	Relevant mathematics commensurate with the level of the course is used. Limited understanding is demonstrated.
4	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is partially correct. Some knowledge and understanding are demonstrated.
5	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is mostly correct. Good knowledge and understanding are demonstrated.
6	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Thorough knowledge and understanding are demonstrated.

The "Use of mathematics" SL criterion assesses to what extent students use mathematics that is **relevant** to the exploration.

Relevant refers to mathematics that supports the development of the exploration towards the completion of its aim. Overly complicated mathematics where simple mathematics would suffice is not relevant.

Students are expected to produce work that is **commensurate with the level** of the course, which means it should not be completely based on mathematics listed in the prior learning. The mathematics explored should either be part of the syllabus, or at a similar level.

A key word in the descriptor is **demonstrated**. The command term demonstrate means "to make clear by reasoning or evidence, illustrating with examples or practical application". Obtaining the correct answer is not sufficient to demonstrate understanding (even some understanding) in order to achieve level 2 or higher.

For knowledge and understanding to be **thorough** it must be demonstrated throughout.

The mathematics can be regarded as **correct** even if there are occasional minor errors as long as they do not detract from the flow of the mathematics or lead to an unreasonable outcome.

Students are encouraged to use technology to obtain results where appropriate, but **understanding must be demonstrated** in order for the student to achieve higher than level 1, for example merely substituting values into a formula does not necessarily demonstrate understanding of the results.

The mathematics only needs to be what is required to support the development of the exploration. This could be a few small elements of mathematics or even a single topic (or sub-topic) from the syllabus. It is better to do a few things well than a lot of things not so well. If the mathematics used is relevant to the topic being explored, commensurate with the level of the course and understood by the student, then it can achieve a high level in this criterion.

			Evidence – page no	Evidence – page no
1	Did you use mathematical operations to reach a conclusion?	☐		
2	Did you outline a mathematical strategy? (Describing what operations or methods you will use to reach your aim.)	☐		
3	Did you use mathematical in addition to descriptive language?	☐		
4	Did you use any mathematical ideas and methods that you learned in your Mathematics SL class?	☐		
5	Did you represent some of your results as algebraic expressions? Or as simplified geometric representations?	☐		
6	Did you explain at least one of your conclusions using mathematics?	☐		
7	Did any of your calculations support you in reaching your aim?	☐		
8	Did you explain most of your conclusions mathematically?	☐		
9	Did you consider special cases and limitations of the mathematical methods you used? (Such as what if certain parameters are 0 or cannot be calculated at all.)	☐		
10	Did you identify and name the mathematical concepts and methods that you used?	☐		
11	Did you explain the links between the methods you used? (Such as showing a pattern as a function or a sequence or linking the algebra to shapes used.)	☐		
12	Did you define all your variables?	☐		
13	Did you show good understanding by rephrasing the mathematics used in context of your exploration?	☐		
14	Did you verify your calculations?	☐		

Criterion E – Use of Mathematics HL

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	Some relevant mathematics is used. Limited understanding is demonstrated.
2	Some relevant mathematics is used. The mathematics explored is partially correct. Some knowledge and understanding is demonstrated.
3	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Some knowledge and understanding are demonstrated.
4	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Good knowledge and understanding are demonstrated.
5	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct and demonstrates sophistication or rigour. Thorough knowledge and understanding are demonstrated.
6	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is precise and demonstrates sophistication and rigour. Thorough knowledge and understanding are demonstrated.

The "Use of mathematics" HL criterion assesses to what extent students use **relevant** mathematics in the exploration.

Students are expected to produce work that is **commensurate with the level** of the course, which means it should not be completely based on mathematics listed in the prior learning. The mathematics explored should either be part of the syllabus, at a similar level or slightly beyond. However, mathematics of a level slightly beyond the syllabus is **not** required to achieve the highest levels.

A key word in the descriptor is **demonstrated**. The command term demonstrate means to make clear by reasoning or evidence, illustrating with examples or practical application. Obtaining the correct answer is not sufficient to demonstrate understanding (even some understanding) in order to achieve level 2 or higher.

For knowledge and understanding to be **thorough** it must be demonstrated throughout. Lines of reasoning must be shown to justify steps in the mathematical development of the exploration.

Relevant refers to mathematics that supports the development of the exploration towards the completion of its aim. Overly complicated mathematics where simple mathematics would suffice is not relevant.

The mathematics can be regarded as **correct** even if there are occasional minor errors as long as they do not detract from the flow of the mathematics or lead to an unreasonable outcome. **Precise** mathematics is error-free and uses an appropriate level of accuracy at all times.

Sophistication: To be considered as sophisticated the mathematics used should be commensurate with the HL syllabus or, if contained in the SL syllabus, the mathematics has been used in a complex way that is beyond what could reasonably be expected of an SL student. Sophistication in mathematics may include understanding and using challenging mathematical concepts, looking at a problem from different perspectives and seeing underlying structures to link different areas of mathematics.

Rigour involves clarity of logic and language when making mathematical arguments and calculations. Mathematical claims relevant to the development of the exploration must be justified or proven.

Students are encouraged to use technology to obtain results where appropriate, but **understanding must be demonstrated** in order for the student to achieve level 1 or higher, for example merely substituting values into a formula does not necessarily demonstrate understanding of the results.

The mathematics only needs to be what is required to support the development of the exploration. This could be a few small elements of mathematics or even a single topic (or sub-topic) from the syllabus. It is better to do a few things well than a lot of things not so well. If the mathematics used is relevant to the topic being explored, commensurate with the level of the course and understood by the student, then it can achieve a high level in this criterion.

What is the difference between precise and correct?

As outlined in criterion E (use of mathematics), "precise" mathematics requires absolute accuracy with appropriate use of notation. "Correct" mathematics may contain the occasional error as long as it does not seriously interfere with the flow of the work or give rise to conclusions or answers that are clearly wrong.

			Evidence – page no	Evidence – page no
1	The mathematics use is commensurate with HL Mathematics	☐		
2	The mathematics is part of the course/a similar level or beyond	☐		
3	My Mathematical arguments and calculations involve clarity of logic and language	☐		
4	An appropriate degree of accuracy is used at all times	☐		
5	The mathematics is error-free and precise	☐		
6	I have demonstrated knowledge and understanding	☐		
7	I have applied mathematics in different contexts	☐		
8	I have applied problem-solving techniques	☐		
9	Where appropriate, I have recognized and explained patterns	☐		
10	I have generalized and justified conclusions	☐		