Seoul National University

M1522.000900 Data Structure

Spring 2025, Kang

Homework 4: Searching & Graphs

(Chapters 9 & 11)

Due: **23:59, June 8th (Sunday), 2025**

Reminders

- Lead TA: Minjun Kim (snuds.ta@gmail.com)
- The points of this homework add up to 100.
- All assignments must be done individually.
- **Type your answer in English** and submit your assignment **in PDF format**.
  - Answer written in Korean may get 0 points.
  - You will get a 10% deduction on your grade for this homework if your submission is either hand-written or non-PDF format.
- Name your file as “(studentID)-(name)-HW4.pdf”.
  (e.g., *202512345-GildongHong-HW4.pdf*)
- Whenever you are making an assumption, state it clearly.
- If you have any questions about the assignment, post them on eTL.

Submission

- Submit your assignment to **eTL**.
- The submission after the due date will be regarded as a late **submission, even if it is only one second late.** Late submissions are accepted **within only one week after the due date.**
- You do not need to specify whether to use the slip-days; they are automatically used.

Question 1 [10 points]

Jump search is a search algorithm which skips some of the items of an array.
Given n the number of items in the array, let the size $k = \sqrt{n}$.

For instance, consider a sorted array [0, 2, 5, 6, 8, 9, 12, 15, 16] ($n = 9$). If we search 15, we jump three times “5 → 9 → 16” and apply linear search for two elements “12 → 15”, thus the total number of comparisons is $3 + 2 = 5$. Similarly, if we search 10, we jump three times “5 → 9 → 16” and apply linear search for one element “12”, thus the total number of comparisons is $3 + 1 = 4$. Answer the following questions.

(1) Compute the expected number of comparisons by Jump Search when searching for an integer $i \sim \mathcal{U}(0, 10)$ in the following sorted array [5 points]

[1, 2, 3, 4, 5, 6, 7, 8, 9]

(2) Compute the expected number of comparisons by Jump Search when searching for an integer $i \sim \mathcal{U}(0, 18)$ in the following sorted array [5 points]

[1, 3, 5, 7, 9, 11, 13, 15, 17]

Question 2 [15 points]

Suppose that you have an empty closed hash table of 11 slots which are numbered 0 through 10, with the following hash and probe functions, respectively:

$$h(k) = k \; mod \; 11, \qquad P(k, i) = i \times \big((k-1) \; mod \; 3 + 1\big).$$

For each empty slot in the hash table after inserting the following integers (in the given order), compute the probability that it will be the next slot to be filled. Assume that the key value k to be inserted next is a random integer satisfying $100 \leq k \leq 199$.

20, 24, 10, 9, 33, 7

Question 3 [15 points]

Assume that you are hashing key K to a hash table with a total of n slots (indexed from 0 to $n-1$). For each of the following functions $h(K)$, is the function acceptable as a hash function, and if so, is it a good hash function? Let $Random(n)$ a random function which returns an integer between 0 and $n-1$.

(a) $h(k) = k/n$

(b) $h(k) = 1$

(c) $\big(k + Random(n)\big)\ mod\ n$

(d) $h(k) = k\ mod\ n$ (if n is a prime number)

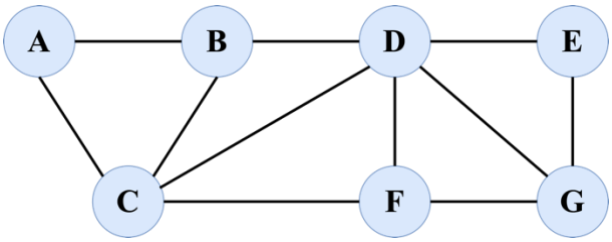
Question 4 [10 points]

Assume that the values A through H are stored in a self-organizing list, initially in ascending order. Consider the three self-organizing list heuristics: count, move-to-front, and transpose. For count, assume that the current item is moved ahead in the list passing over any other item whose count is smaller than that of the current item. For each heuristic, show the resulting list and the total number of comparisons required resulting from the following series of accesses:

D H H G H E G H G H E C E H G

Question 5 [10 points]

For the given graph below, answer the following questions.

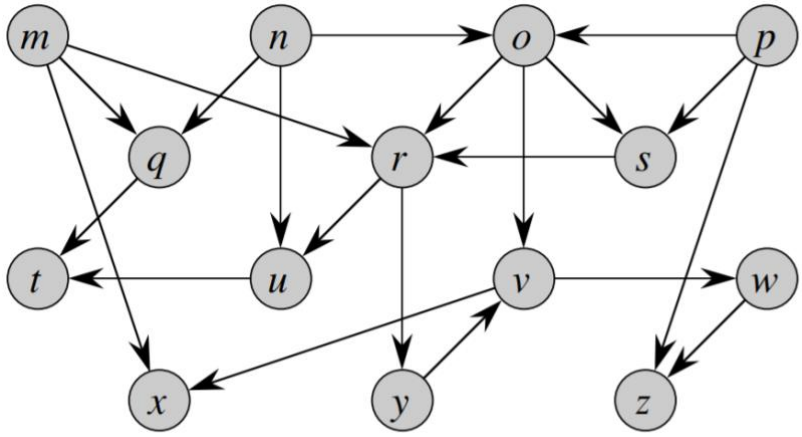


(1) Draw the adjacency matrix of the given graph [5 points]

(2) Identify the total number of simple paths and simple cycles in the graph. [5 points]

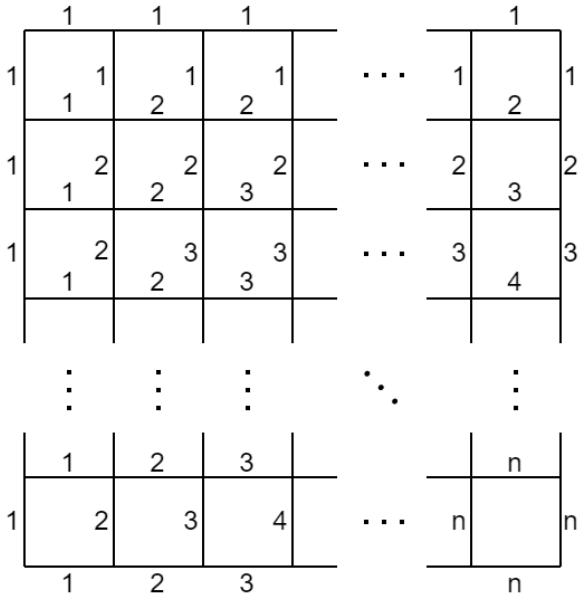
Question 6 [10 points]

Produced by topological sort with DFS, write down the topological sort sequence of the DAG below:



Question 7 [15 points]

Find the total weight of an MST of the following undirected graph in terms of $n \geq 1$:



Question 8 [15 points]

Answer the following questions.

(1) Consider an undirected weighted graph $\boldsymbol{\mathcal{G}} = (\boldsymbol{\mathcal{V}}, \mathcal{E})$ with vertices $\mathcal{V} = \{a, b, c, d, e, f\}$ and adjacency matrix $\boldsymbol{A}$ (rows and columns ordered as a through f), use Dijkstra's algorithm to compute the shortest-path distance from vertex a to vertex e (provide a step-by-step solution). [5 points]

$$\boldsymbol{A} = \begin{bmatrix} 0 & 7 & 9 & \infty & \infty & 14 \\ 7 & 0 & 10 & 15 & \infty & \infty \\ 9 & 10 & 0 & 11 & \infty & 2 \\ \infty & 15 & 11 & 0 & 6 & \infty \\ \infty & \infty & \infty & 6 & 0 & 9 \\ 14 & \infty & 2 & \infty & 9 & 0 \end{bmatrix}.$$

(2) Given the graph $\mathcal{G}$ in question (1), run the Prim's algorithm starting from vertex a and report the total weight of the resulting MST. [5 points]

(3) Let K_8 be the complete undirected graph on eight vertices with all edge weights distinct and positive. What is the maximum number of edges that must be examined before Kruskal's algorithm finishes building the MST? [5 points]