**M1522.000900 Data Structure**

**Spring 2025, Kang**

**Homework 1 Answer Sheet [1/2]**

2023-12753 EunSu Yeo

* Write your answers on the “your answer” columns.
* Do **NOT** write anything on “score” columns.
* Before submission, delete all the blue-colored texts, and convert the file into **PDF-format**.
* Write your proof on the solution sheet and leave the “your answer” column blank.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Question | | Your Answer | Points | Score |
| 1 | (1) | Satisfy: Antisymmetric, Transitive.  Doesn’t satisfy: Reflexive, Symmetric | 2.5 |  |
| (2) | Satisfy: Reflexive, Symmetric, Transitive.  Doesn’t satisfy: Antisymmetric | 2.5 |  |
| (3) | Satisfy: Antisymmetric, Transitive.  Doesn’t satisfy: Reflexive, Symmetric | 2.5 |  |
| (4) | Satisfy: Reflexive  Doesn’t satisfy: Symmetric, Antisymmetric, Transitive. | 2.5 |  |
| 2 | (1) |  | 2 |  |
| (2) |  | 2 |  |
| (3) |  | 2 |  |
| (4) |  | 2 |  |
| (5) |  | 2 |  |
| 3 | (1) |  | 5 |  |
| (2) |  | 5 |  |
| (3) |  | 5 |  |
| 4 | (1) |  | 5 |  |
| (2) |  | 5 |  |
| (3) |  | 5 |  |

**M1522.000900 Data Structure**

**Spring 2025, Kang**

**Homework 1 Answer Sheet [2/2]**

2023-12753 EunSu Yeo

* Write your answers on the “your answer” columns.
* Do **NOT** write anything on “score” columns.
* Before submission, delete all the blue-colored texts, and convert the file into **PDF-format**.
* Write your proof on the solution sheet and leave the “your answer” column blank.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Question | | Your Answer | Points | Score |
| 5 | | is efficient for ,  when ,  when ,  when | 10 |  |
| 6 | (1) |  | 2.5 |  |
| (2) |  | 2.5 |  |
| (3) |  | 2.5 |  |
| (4) |  | 2.5 |  |
| 7 | (1) | Yes, program B improves in time complexity over program A. | 7.5 |  |
| (2) | X is times faster than Y | 7.5 |  |
| 8 | (1) |  | 5 |  |
| (2) |  | 10 |  |
| Total | | | 100 |  |

**Homework 1  
Solution Sheet**

2023-12753

EunSu Yeo

1. **Question 1 [10 points]**
   1. **Q1(1) [2.5 points]**

**Solution.**

Reflexive:

A person can’t be a senior to himself.

So is impossible.

Due to the definition of reflexive, doesn’t satisfy reflexive.

Symmetric:

Let’s assume a person a is a senior to a person b. Then b can’t be a senior to a

So is impossible.

Due to the definition of symmetric, doesn’t satisfy symmetric.

Antisymmetric:

The condition of definition of antisymmetric is . but as I described in Symmetric, it is impossible. As the condition of definition is false the proposition is true.

So satisfy antisymmetric.

Transitive:

If a is senior to b and b is senior to c then a is also senior to c. Due to the definition of Transitive, satisfy Transitive.

* 1. **Q1(2) [2.5 points]**

**Solution.**

Reflexive:

is possible. Due to the definition of reflexive, satisfy reflexive.

Symmetric:

is possible. Due to the definition of symmetric, satisfy symmetric.

Antisymmetric:

is not true.

When a and b are different people.

It satisfies the given condition. So due to the definition of Antisymmetric, doesn’t satisfy antisymmetric.

Transitive:

is possible. Due to the definition of Transitive, satisfy Transitive.

* 1. **Q1(3) [2.5 points]**

**Solution.**

Reflexive:

is impossible due to . So doesn’t satisfy reflexive.

Symmetric:

is impossible due to . Due to the definition of symmetric, doen’t satisfy symmetric.

Antisymmetric:

It’s impossible to satisfy the given if condition due to . As the condition of definition is false the proposition is true. So satisfy antisymmetric.

Transitive:

is possible. Due to the definition of Transitive, satisfy Transitive.

* 1. **Q1(4) [2.5 points]**

**Solution.**

Reflexive:

is true due to . So satisfy reflexive.

Symmetric:

is false due to . In case when a and b only satisfy the relation ,the next condition doesn’t satisfy. So this is the error case. Due to the definition of symmetric, doesn’t satisfy symmetric.

Antisymmetric:

If a and b is in same apartment and they are different people. Still it satisfies the condition. So we can not assert a=b. So doesn’t satisfy antisymmetric.

Transitive:

In case a is senior to b and b and c is living in the same department it doesn’t satisfies the condition. Due to the definition of Transitive, doesn’t satisfy Transitive.

1. **Question 2 [10 points]**
   1. **Q2(1) [2 points]**

**Solution.**

Assume a triangle has two right angles. This means that two of its angles are 90 degrees each. The sum of angles in any triangle must be 180 degrees. However, if two angles are already 90 degrees, their sum is 180 degrees, leaving no degree for a third angle. This contradicts the definition of a triangle, which must have three angles. Therefore, a triangle cannot have more than one right angle

* 1. **Q2(2) [2 points]**

**Solution.**

Assume is rational. Then it can be written as p/q, in which p and q are integers with no common factor without 1. Then let’s square both sides.

Then we can know that p is divided by 3. Then lets put p=3k(k is integer that is not 0)

Then we can know that q is also divided by 3. This contradicts out initial assumption. So is irrational.

* 1. **Q2(3) [2 points]**

**Solution.**

Assume there are only finite prime numbers. Which are .

Then the number is not divided by any of prime numbers. So this number is prime number, which is contradicting the assumption. So, there must be infinitely many prime numbers.

* 1. **Q2(4) [2 points]**

**Solution.**

Assume both a and b are odd, square of an odd number is always 1 when modulo 4

Then , a square of c can only be 0 when modulo 4. So a and b can’t be both odd numbers.

* 1. **Q2(5) [2 points]**

**Solution.**

Assume there is no hash collision, meaning h is an one-to-one function.

Then each element in X maps unique elements in Y each. Since |X|>|Y|, there are more elements in X than in Y. This contradicts the assumption due to the pigeonhole principle. So hash collision must occur.

1. **Question 3 [15 points]**
   1. **Q3(1) [5 points]**

**Solution.**

T(n)=2T(n-1)+1

T(1)=1

T(2)=3

T(3)=7

T(4)=15

T(5)=31

…

T(10)=1023

* 1. **Q3(2) [5 points]**

**Solution.**

T(n)=2\*T(n-1)+1

* 1. **Q3(3) [5 points]**

**Solution.**

T(n)+1=2(T(n-1)+1)

Set T(n)+1=S(n)

S(n)=2S(n-1)

1. **Question 4 [15 points]**
   1. **Q4(1) [5 points]**

**Solution.**

Catalan number recurrence relation.

* 1. **Q4(2) [5 points]**

**Solution.**

For n=1, using the answer in 4.1

So, the base case holds.

* 1. **Q4(3) [5 points]**

**Solution.**

Assume the closed-form solution holds for some n smaller or equal than k.

Then show C(k+1)

By calculation,

So

Then we can finally figure out that

And our answer is correct by mathematical induction.

1. **Question 5 [10 points]**

**Solution.**

we know that

So, for N that is sufficiently large might be the most efficient.

Based on this information and information in HW1, let’s compare the complexity of the four algorithms given.

Case1) is efficient for

Due to information #1 & #2

Case2) is efficient for

Due to information #1 & #5

Case3) is efficient for

Due to information #2 & #6

And also log(log2)=0. So the smallest value is 10log(logn) when n is very close to 2, which shows 10loglogn is the most efficient graph when n is close to 2.

Case4) is efficient for

Due to information #5 & #6

1. **Question 6 [10 points]**
   1. **Q6(1) [2.5 points]**

**Solution.**

While loop is based on s<n

And increases to s =

Which is a bit smaller than n

And we know that sigma is represented as form. So, k is form.

And the while loop loops k times.

So, the time complexity is

* 1. **Q6(2) [2.5 points]**

**Solution.**

For loop based on n and m so the time complexity is each.

is the answer.

* 1. **Q6(3) [2.5 points]**

**Solution.**

Double for loop is based on n so the time complexity is .

is the answer.

* 1. **Q6(4) [2.5 points]**

**Solution.**

1. **Question 7 [15 points]**
   1. **Q7(1) [7.5 points]**

**Solution.**

Yes, program B improves in time complexity over program A.

Proof)

Let’s assume

Then

Let’s assume

Then

**Q7(2) [7.5 points]**

**Solution.**

Due to Q7.1

Let’s assume

Cause program A in X and program B in Y has finished in same time, we can know that X is times faster than Y

1. **Question 8 [15 points]**
   1. **Q8(1) [7.5 points]**

**Solution.**

powerN is called n times to calculate powerN(n).

* 1. **Q8(2) [7.5 points]**

**Solution.**

Let’s set then