计算理论第一次作业

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1.

(1) M 为 00010 时:

$q_0 00010B \Rightarrow 0q_0 010B \Rightarrow 00q_0 010B \Rightarrow 000q_0 10B \Rightarrow 0000q_1 B \Rightarrow 00000q_2$

(2) M 为 001000 时:

$q_0 001000B \Rightarrow 0q_0 01000B \Rightarrow 00q_0 1000B \Rightarrow 000q_1 000B \Rightarrow 0000q_1 00B \Rightarrow 00000q_1 0B \Rightarrow 000000q_1 B \Rightarrow 0000000q_2$

(3) M 为 0010001 时:

$q_0 0010001B \Rightarrow 0q_0 010001B \Rightarrow 00q_0 10001B \Rightarrow 000q_1 0001B \Rightarrow 0000q_1 001B \Rightarrow 00000q_1 01B \Rightarrow 000000q_1 1B$

没有对应转移函数，停机

2.

设图灵机 $M = \{Q, \Sigma, \Gamma, \delta, q_0, B, F\}$

其中 $Q = \{q_0, q_1, q_2, q_3\}$，$\Sigma = \{0, 1\}$，$\Gamma = \{0, 1, B\}$，$F = \{q_3\}$

$\delta(q_0, 0) = (q_0, 0, R)$

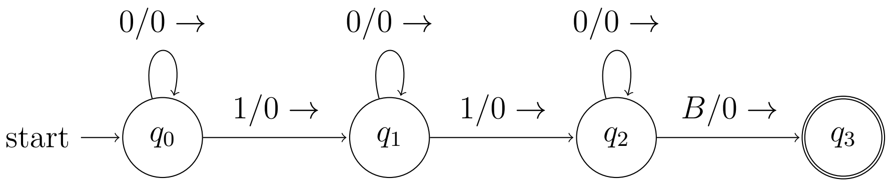
$\delta(q_0, 1) = (q_1, 0, R)$

$\delta(q_1, 0) = (q_1, 0, R)$

$\delta(q_1, 1) = (q_2, 1, R)$

$\delta(q_2, 0) = (q_2, 0, R)$

$\delta(q_2, B) = (q_3, 0, R)$



3.

(1) M 为 0011 时:

$q_0 0011B \Rightarrow Xq_1 011B \Rightarrow X0q_1 11B \Rightarrow Xq_2 0Y1B \Rightarrow q_2 X0Y1B \Rightarrow Xq_0 0Y1B \Rightarrow XXq_1 Y1B \Rightarrow XXYq_1 1B \Rightarrow XXq_2 YYB \Rightarrow Xq_2 XYYB \Rightarrow XXq_0 YYB \Rightarrow XXYq_3 YB \Rightarrow XXYYq_3 B \Rightarrow XXYYBq_4$

(2) M 为 0101 时:

$q_0 0101B \Rightarrow Xq_0 101B \Rightarrow q_2 XY01B \Rightarrow Xq_0 Y01B \Rightarrow XYq_3 01B$

没有对应转移函数，停机

(3) M 为 00111 时:

$q_0 00111B \Rightarrow Xq_1 0111B \Rightarrow X0q_1 111B \Rightarrow Xq_2 0Y11B \Rightarrow q_2 X0Y11B \Rightarrow Xq_0 0Y11B \Rightarrow XXq_1 Y11B \Rightarrow XXYq_1 11B \Rightarrow XXq_2 YY1B \Rightarrow Xq_2 XYY1B \Rightarrow XXq_0 YY1B \Rightarrow XXYq_3 Y1B \Rightarrow XXYYq_3 1B$

没有对应转移函数，停机

4.

(1) 设图灵机 $M = \{Q, \Sigma, \Gamma, \delta, q_H, B, F\}$

其中 $Q = \{q_H, q_e, q_{l1}, q_{l2}, q_o, q_{else}, q_{acc}\}$，$\Sigma = \Gamma - \{B\}$，$F = \{q_{acc}\}$, Γ 为全部符号的集合（包括空白符号 B）

令 $* = \Gamma$，　$\neg B = \Sigma$ ，则:

$\delta(q_H, *) = (q_e, H, R)$

$\delta(q_e, *) = (q_{l1}, e, R)$

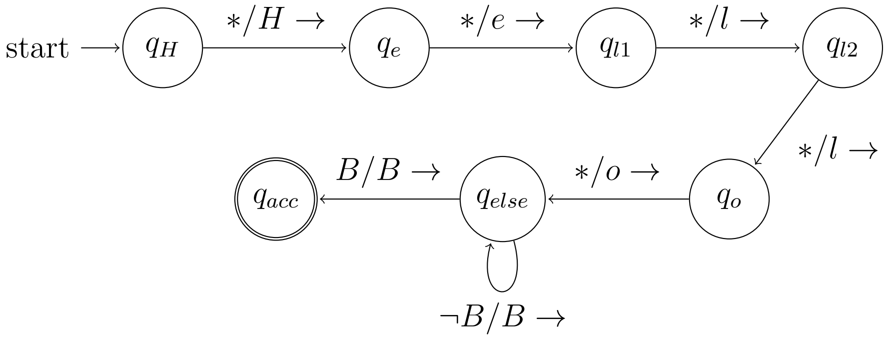
$\delta(q_{l1}, *) = (q_{l2}, l, R)$

$\delta(q_{l2}, *) = (q_o, l, R)$

$\delta(q_o, *) = (q_{acc}, o, R)$

$\delta(q_{else}, \neg B) = (q_{else}, B, R)$

$\delta(q_{else}, B) = (q_{acc}, B, R)$



(2) 设图灵机 $M = \{Q, \Sigma, \Gamma, \delta, q_{Bye-B}, B_0, F\}$

其中 $Q = \{q_{Bye-B}, q_{Bye-y}, q_{Bye-e}, q_{Hello-H}, q_{Hello-e}, q_{Hello-l1}, q_{Hello-l2}, q_{Hello-o}, q_{scan}, q_{acc}\}$,

$\Sigma = \Gamma - \{B_0\}$，$F = \{q_{acc}\}$, Γ 为全部符号的集合（包括空白符号 B_0）

令 $* = \Gamma$，　$\neg B_0 = \Sigma$ ，则：

$\delta(q_{Bye-B}, B) = (q_{Bye-y}, B, R)$

$\delta(q_{Bye-y}, y) = (q_{Bye-e}, y, R)$

$\delta(q_{Bye-e}, e) = (q_{scan}, e, R)$

$\delta(q_{scan}, B_0) = (q_{Hello-H}, B_0, L)$

$\delta(q_{Hello-H}, \neg B_0) = (q_{Hello-H}, B_0, L)$

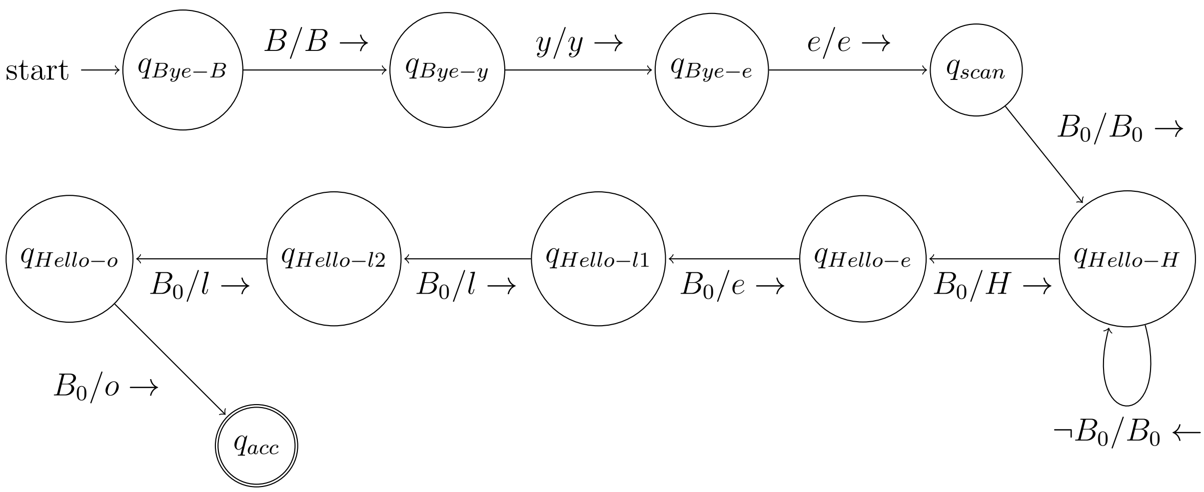
$\delta(q_{Hello-H}, B_0) = (q_{Hello-e}, H, R)$

$\delta(q_{Hello-e}, B_0) = (q_{Hello-l1}, e, R)$

$\delta(q_{Hello-l1}, B_0) = (q_{Hello-l2}, l, R)$

$\delta(q_{Hello-l2}, B_0) = (q_{Hello-o}, l, R)$

$\delta(q_{Hello-o}, B_0) = (q_{acc}, o, R)$



5.

(1)

设图灵机 $M = \{Q, \Sigma, \Gamma, \delta, q_0, B, F\}$

其中 $Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$, $\Sigma = \{0, 1, 2\}$, $\Gamma = \{0, 1, 2, X, Y, Z, B\}$, $F = \{q_6\}$

$\delta(q_0, 0) = (q_1, X, R)$

$\delta(q_0, Y) = (q_4, Y, R)$

$\delta(q_1, 0) = (q_1, 0, R)$

$\delta(q_1, Y) = (q_1, Y, R)$

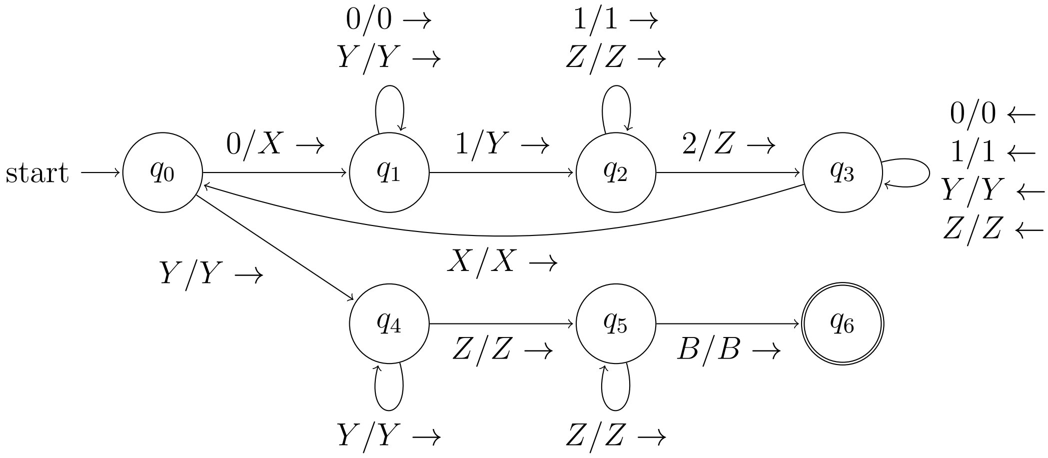
$\delta(q_1, 1) = (q_2, Y, R)$

$\delta(q_2, 1) = (q_2, 1, R)$

$\delta(q_2, Z) = (q_2, Z, R)$

$\delta(q_2, 2) = (q_3, Z, L)$

$\delta(q_3, 0) = (q_3, 0, L)$
$\delta(q_3, 1) = (q_3, 1, L)$
$\delta(q_3, Y) = (q_3, Y, L)$
$\delta(q_3, Z) = (q_3, Z, L)$
$\delta(q_3, X) = (q_0, X, R)$
$\delta(q_4, Y) = (q_4, Y, R)$
$\delta(q_4, Z) = (q_5, Z, R)$
$\delta(q_5, Z) = (q_5, Z, R)$
$\delta(q_5, B) = (q_6, B, R)$



(2)

假设 $L = \{0^n 1^n 2^n | n \geq 1\}$ 是上下文无关语言，则 L 满足泵引理
取 $z = 0^N 1^N 2^N \in L$，$z = uvwxy$
由泵引理，存在这样的 z 使得 $|vwx| \leq N$，则 vwx 最多包含两种字符，即 01 或 12，分类讨论：
i. 若 vwx 仅包含 0 或 1 或 2，以 0 为例，有：
　　取 uv^2wx^2y，显然 0 的个数多于 1 和 2，$uv^2wx^2y \notin L$，矛盾
ii. 若 vwx 仅包含 01 或 12，以 01 为例，有：
　　不妨设 $v = 0^t 1^s, w = 1^l$，取 uv^2wx^2y
　　$uv^2wx^2y = 0^{N+t}1^s w 1^{1l} y$，显然 0 的数量多于 2，$uv^2wx^2y \notin L$，矛盾
　　当 $x = 0^t 1^s$ 时，同理有 0 和 1 的数量都多于 2，矛盾
故 $L = \{0^n 1^n 2^n | n \geq 1\}$ 不是上下文无关语言

6.

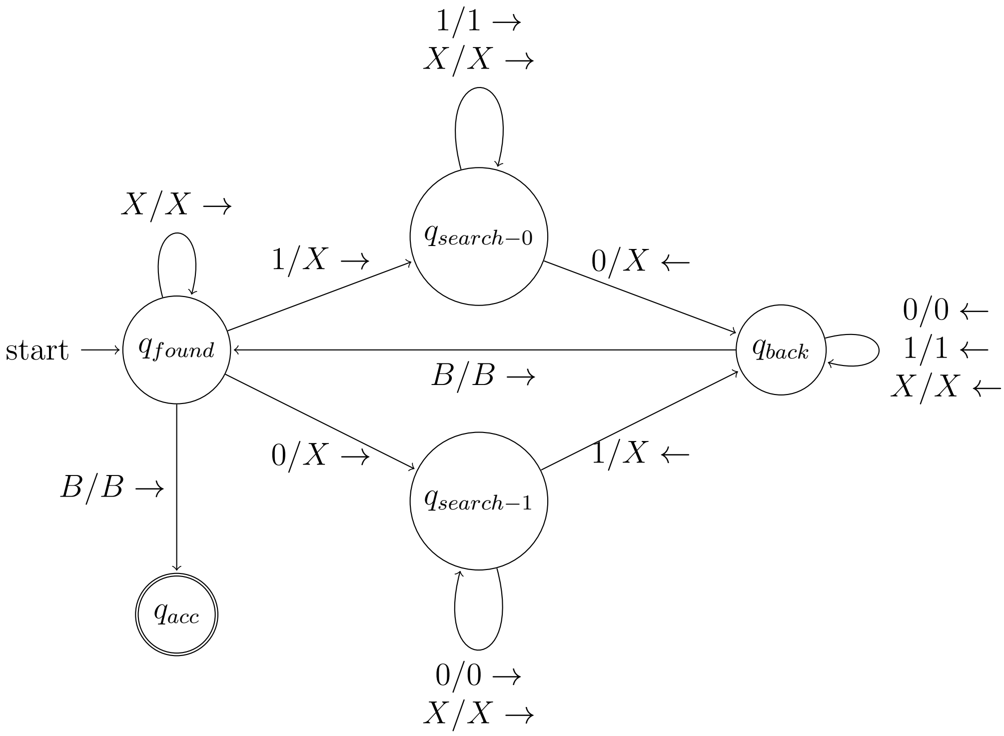
思路：

向右扫描遇到第一个字符 a，向左回到 B，向右找到 1-a 并改成 X，向左回到 B，向右扫描遇到第一个非 X 的字符 b，改成 X，向左回到起点，向右找字符 1 - b 改成 X $\cdots$

解：

设图灵机 $M = \{Q, \Sigma, \Gamma, \delta, q_{found}, B, F\}$
其中 $Q = \{q_{found}, q_{search-0}, q_{search-1}, q_{back}, q_{acc}\}$，$\Sigma = \{0, 1\}$，$\Gamma = \{0, 1, X, B\}$，$F = \{q_{acc}\}$
$\delta(q_{found}, 0) = (q_{search-1}, X, L)$
$\delta(q_{found}, 1) = (q_{search-0}, X, L)$
$\delta(q_{found}, X) = (q_{found}, X, R)$
$\delta(q_{found}, B) = (q_{acc}, B, R)$
$\delta(q_{search-0}, 0) = (q_{back}, X, L)$
$\delta(q_{search-0}, 1) = (q_{search-0}, 1, R)$
$\delta(q_{search-0}, X) = (q_{search-0}, X, R)$

$\delta(q_{search-1}, 1) = (q_{back}, X, L)$
$\delta(q_{search-1}, 0) = (q_{search-1}, 0, R)$
$\delta(q_{search-1}, X) = (q_{search-1}, X, R)$
$\delta(q_{back}, 0) = (q_{back}, 0, L)$
$\delta(q_{back}, 1) = (q_{back}, 1, L)$
$\delta(q_{back}, X) = (q_{back}, X, L)$
$\delta(q_{back}, B) = (q_{back}, q_{found}, R)$



7.

思路:

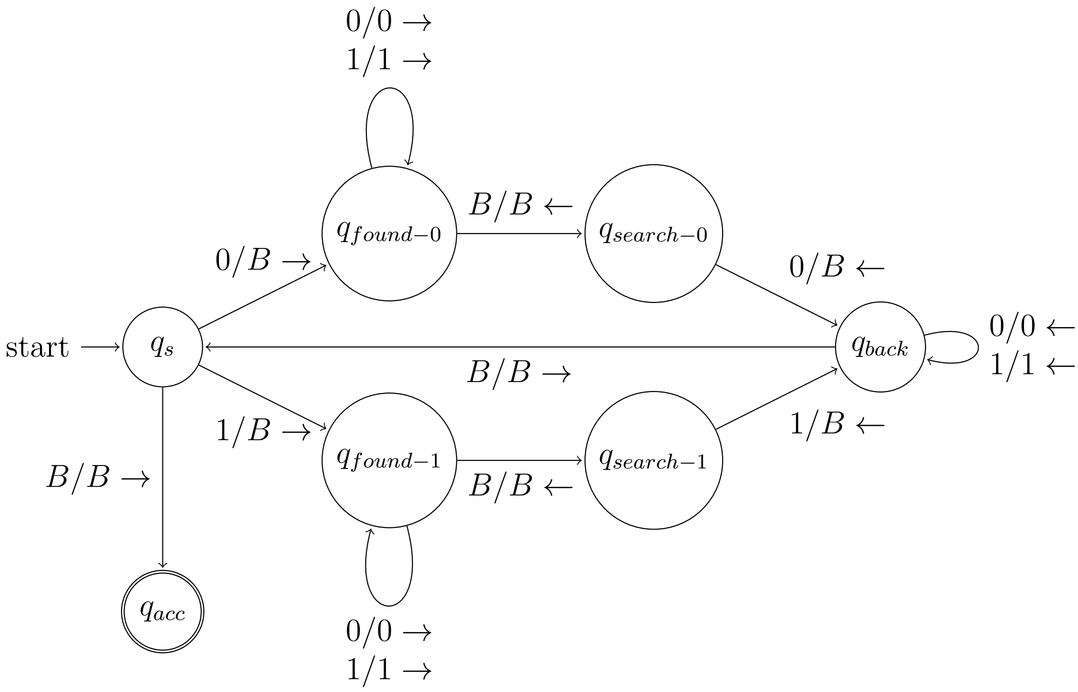
扫描遇到第一个字符，用状态记录这个字符，找到字符串末尾，和记录对比，相同则改为 B，走到最左，进行下一轮 $\cdots$

解:

设图灵机 $M = \{Q, \Sigma, \Gamma, \delta, q_{found}, B, F\}$

其中 $Q = \{q_s, q_{found-0}, q_{found-1}, q_{search-0}, q_{search-1}, q_{back}, q_{acc}\}$, $\Sigma = \{0, 1\}$, $\Gamma = \{0, 1, B\}$, $F = \{q_{acc}\}$

$\delta(q_s, 0) = (q_{found-0}, B, R)$
$\delta(q_s, 1) = (q_{found-1}, B, R)$
$\delta(q_s, B) = (q_{acc}, B, R)$
$\delta(q_{found-0}, 0) = (q_{found-0}, 0, R)$
$\delta(q_{found-0}, 1) = (q_{found-0}, 1, R)$
$\delta(q_{found-0}, B) = (q_{search-0}, B, L)$
$\delta(q_{found-1}, 0) = (q_{found-1}, 0, R)$
$\delta(q_{found-1}, 1) = (q_{found-1}, 1, R)$
$\delta(q_{found-1}, B) = (q_{search-1}, B, L)$
$\delta(q_{search-0}, 0) = (q_{back}, B, L)$
$\delta(q_{search-1}, 1) = (q_{back}, B, L)$
$\delta(q_{back}, 0) = (q_{back}, 0, L)$
$\delta(q_{back}, 1) = (q_{back}, 1, L)$
$\delta(q_{back}, B) = (q_s, B, R)$


0/0 →
1/1 →
$q_{found-0}$
B/B ←
$q_{search-0}$
0/B →
0/B ←
start
q_s
q_{back}
0/0 ←
1/1 ←
B/B →
1/B →
1/B ←
B/B →
$q_{found-1}$
$q_{search-1}$
B/B ←
q_{acc}
0/0 →
1/1 →