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A Comparison of Cointegration and Tracking Error Models for Mutual Funds and Hedge Funds

ISMA Centre Discussion Papers in Finance 2004-04
Preliminary version: March 2004

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Abstract

We present a detailed study of portfolio optimisation based on cointegration, a statistical tool that here exploits a long-run equilibrium relationship between stock prices and an index price. We compare the theoretical and empirical properties of cointegration optimal equity portfolios with those of portfolios optimised on the tracking error variance. From an eleven year out of sample performance analysis we find that for simple index tracking the additional feature of cointegration between the tracking portfolio and the index has no clear advantages or disadvantages relative to the tracking error variance (TEV) minimization model. However ensuring a cointegration relationship does pay off when the tracking task becomes more difficult. Cointegration optimal portfolios clearly dominate the TEV equivalents for all of the statistical arbitrage strategies based on enhanced indexation, in all market circumstances.

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JEL classification: C32, C51, G11, G23

Keywords: cointegration, tracking error, index tracking, statistical arbitrage

Introduction

Following the seminal work of Markowitz (1959), Sharpe (1964), Lintner (1965) and Black (1972), the fundamental statistical tool for traditional portfolio optimisation is the correlation analysis of asset returns. In particular, optimisation models for benchmark replication focus on minimising the variance of the tracking error, with additional constraints concerning the correlation of the portfolio returns with the benchmark returns, or the transaction costs involved in re-balancing the portfolio.

However, optimisation models based on tracking error or on correlation measures have a number of drawbacks, especially when applied to a passive investment framework. First, minimising tracking error with respect to an index which, being a linear combination of stock prices, contains a significant amount of noise may result in a portfolio that is very sample specific and unstable in volatile market circumstances. Additional limitations are generated by the very nature of correlation as a measure of dependency: it is only applicable to stationary variables, such as stock returns, which requires prior de-trending of level variables (i.e. stock prices) and has the disadvantage of loosing valuable information (i.e. the common trends in prices); as such, it is only a short-term statistic and it lacks stability; finally, depending on the model used to estimate it, correlation can be very sensitive to the presence of outliers, non-stationarity or volatility clustering, which limit the use of a long data history and can lead to erroneous conclusions about the nature of long term dependencies.

Given these limitations of correlation it is not surprising that most applied financial econometric analyses employ a different tool for modelling dependencies between time series. Amongst economists, *cointegration* has gained far wider acceptance than correlation. The influential work of Clive Granger (1966) has engendered a vast academic literature in this area[1] and earned him the Nobel Prize in 2003. Cointegration is an extremely powerful statistical tool that, in a sense, generalises correlation to non-stationary time series. Cointegration allows simple estimation methods such as least squares regression or maximum likelihood to capture dependencies between non-stationary series such as stock prices, while still encompassing the dynamic correlation of the associated stationary series such as stock returns.

[1] See Alexander (1999) for a useful survey of this literature.

The fundamental remark that justifies the application of cointegration to a set of stock prices is that they can share a *common stochastic trend* (Stock and Watson, 1991).[2] In this case, cointegration exists when there is at least one stationary linear combination of their prices, or simply put, there is mean reversion in their price spreads. The finding that the spread in a system of prices is mean reverting does not provide any information for forecasting the individual prices in the system, or the position of the system at some point in the future, but it does provide the valuable information that, irrespective to the position of the system, the prices will stay together on a long-run basis.

The cointegration approach to portfolio modelling, pioneered by Alexander (1999), enables the use of the entire information set comprised in a set of stock prices. Since prices are long memory processes (Granger and Terasvirta, 1993) cointegration is able to explain their long run behaviour. The rationale for constructing portfolios based on a cointegration relationship with a benchmark rests on two main features: first, the price difference between the benchmark and the portfolio is, by construction, stationary and this implies that the tracking portfolio will be 'tied' to the benchmark in the long run; secondly, the stock weights, being based on a long history of prices, will have an enhanced stability. These properties are the result of making full use of the information contained in stock prices prior to their de-trending. By contrast, correlation based tracking strategies are based on only partial information and there is no mechanism ensuring the reversion of the portfolio to the benchmark over the longer term. If the tracking error follows, for example, a random walk process, the portfolio can diverge significantly from its benchmark, unless frequently rebalanced. Thus hedging strategies based on cointegration, which focus on common trends only, may be more effective on the long run.

Considering the important comparative advantages of using cointegration rather than correlation to optimise equity portfolios, it should be possible to exploit, if found, a long-run relationship between equity prices and the market index price to construct trading strategies. Such evidence is provided by Alexander *et al*. (2001) who investigate the performance of different long-short strategies developed in the SP100 stock universe. Their application implies an extensive search over a large number of portfolios constructed on cointegration relationships and optimised on different model parameters (such as training period, targeted tracking error and the number of assets in the portfolio) for the best long-short combination. Their results, even if based on a black box selection algorithm, indicate that cointegration-

[2] According to Beveridge and Nelson (1981), a variable has a stochastic trend and is integrated of order one, if its first difference has a stationary invertible ARMA(p,q) representation plus a deterministic component.

based optimisation can ensure stable alpha, with low volatility and uncorrelated with market returns.

In another application of cointegration analysis to investment management that is particularly relevant to our line of research, Lucas (1997) considers the optimal asset allocation problem in the presence of cointegrated time series. Using a stylised asset allocation problem with a risk adverse investment manager, Lucas shows that cointegrating combinations of time series have lower long-term volatility than their non-cointegrated counterparts. From a short term or tactical asset allocation perspective, cointegration implies that the price series have an error-correcting behaviour, allowing the anticipation of future developments. According to Lucas' results, the presence of cointegration relations also has important consequences for the short-term predictability of asset prices.

Clearly, the presence of cointegration relationships generates a number of significant advantages for a trading strategy. However, even if advocated for some time and becoming a popular tool with practitioners, to our knowledge there are no rigorous academic studies that compare the theoretical and empirical properties of cointegration based portfolios with traditional optimisation models, such as the tracking error variance minimisation. We aim to fill this gap by investigating the properties of several trading strategies based on cointegration, such as a classic index tracking strategy, a long-short equity market neutral strategy and a number of strategies combining index tracking and long-short market neutral. When applied to constructing trading strategies in the Dow Jones Industrial stock universe, the cointegration technique produced good out-of-sample results over a very long period. The tracking portfolios have a strong relationship with the benchmark and the cointegration optimal statistical arbitrage portfolios clearly dominate their traditional equivalents.

The remainder of this paper is organised as follows: section 1 introduces the cointegration model, explains how it relates to traditional tracking error variance minimisation models and discusses the properties of cointegration optimal portfolios for different trading strategies; section 2 describes the data, the performance criteria and the dynamic out-of-sample testing procedure; section 3 presents and discusses the results of applying simple index tracking strategies; section 4 analyses cointegration optimal portfolios that implement different statistical arbitrage strategies; and finally, section 5 summarises and concludes.

1. Cointegration model for index tracking and statistical arbitrage

The interest in developing optimisation models for active and passive investments has been driving a large body of academic and practitioner research. Optimisation models for active investments are generally more diverse and more sophisticated than those for passive investments, the two categories having generally little in common. However the approach taken in this paper starts with index replication, a traditional passive investment strategy. Assuming we can find an appropriate replication model, the passive strategy is then extended into active management by constructing portfolios to track artificial indices, such as index plus and index minus, and trading on their spread. This is a standard statistical arbitrage strategy based on enhanced indexation.

Cointegration optimal index tracking

A cointegration optimal tracking portfolio[3] is constructed via a process that entails two stages: selecting the stocks to be included in the tracking portfolio and then determining the optimal portfolio holdings in each stock, according to a cointegration regression model.[4] The first stage, stock selection, can be the result of proprietary selection models, technical analysis or just the stock picking skills of a portfolio manager. The degree of cointegration and consequently the tracking performance will depend on the selection process. Conceivably, some stock selection criteria will be more consistent with the cointegration based allocation model than others. However critical, the selection process does not have special features in a cointegration based tracking technique, and identifying the most successful stock selection technique does not constitute the focus of this analysis. For illustrating the optimisation method we use a naive selection method based on the fact that in a price weighted index the higher the price of a stock the greater its influence on the index. Furthermore where possible we separate the effect of the stock selection criterion from the effect of the optimisation method on the portfolio performance.

The second stage of constructing cointegration optimal tracking portfolios is to determine the portfolio holdings in each of the selected stocks. This part is based on a standard *cointegration regression* between the market index and a number of *n* stocks from its components:

$$ln(index_t) = c_1 + \sum_{k=1}^{n} c_{k+1} ln(P_{k,t}) + e_t \tag{1}$$

where the stocks are selected so that the price spread between the portfolio and the index, ε_t, is stationary. Since equity indices are just linear combinations of stock prices, a stationary price

[3] To set up some terminology, we call 'cointegration optimal' the tracking portfolios that are constructed on a cointegration relationship with a benchmark. The price difference between the tracking portfolio value and the benchmark is called spread, which, in the case of a cointegration optimal portfolio, is stationary. The return difference between the tracking portfolio and the benchmark is denoted as 'tracking error' (this terminology is different from the one that has crept into common usage amongst practitioners, who define the tracking error as the standard deviation of the returns difference between the portfolio and the benchmark).

[4] The use of regression to determined optimal stock holdings for tracking a constant weighted index dates back to Hersom, Sutti, and Szego (1973). The novel idea in the cointegration regression is to determine the holdings using a prices on prices (or log prices on log prices) regression.

spread can be easily obtained provided that n is sufficiently large and, if the index is capitalisation weighted, the number of stocks in each issue is relatively stable.

The model specification (1) is not unique because the cointegration regression can also be estimated on level rather than log variables. However, the log variables specification[5] has the advantage that when taking the first difference of (1) the expected return on the index will equal the expected return on the tracking portfolio. Please note that the intercept term does not represent 'alpha' in the log prices regression, as it would represent in a returns regression.

The most general form of the model uses an ordinary least squares (OLS) criterion to estimate the stock coefficients. Note that the application of OLS to non-stationary dependent variables such as *ln*(*index*) is only valid in the special case of a cointegration relationship. The residuals in (1) are stationary if, and only if, the index and the tracking portfolio are cointegrated. If the residuals from the above regression are non-stationary the OLS coefficient estimates will not be consistent and no further inference will be valid. Therefore testing for cointegration is an essential step in constructing cointegration optimal tracking portfolios.

We use the Engle-Granger (1986) methodology for cointegration testing, which is particularly appealing for portfolio optimisation due to its intuitive and straightforward implementation.[6] The cointegration augmented Dickey-Fuller (ADF) regression estimated on the residuals of the cointegration regressions is:

$$\Delta \hat{e}_t = \gamma \hat{e}_{t-1} + \sum_{i=1}^{p} a_i \, \Delta \hat{e}_{t-i} + u_t \tag{2}$$

The null hypothesis tested is of no cointegration, i.e. $\gamma = 0$, against the alternative of $\gamma < 0$. The critical values for the t-statistic of γ are obtained using the response surfaces provided by MacKinnon (1991). If the null hypothesis of no cointegration is rejected, the cointegration optimal tracking portfolio based on OLS estimation of model (1) is expected to have very similar returns to the market index. That is, cointegration ensures that the price spread between the portfolio value and the benchmark is a mean-reverting process with minimum volatility.

[5] The log variables specification is possible because if the level variables are cointegrated, their logarithms will also be cointegrated (Hendry and Juselius, 2000).
[6] Its well-known limitations (small sample problems, asymmetry in treating the variables, at most one cointegration vector) are not effective in this case: the estimation sample is typically large, there is a strong economic background to treat the market index as the dependent variable, and identifying only one cointegration vector, i.e. one cointegration optimal portfolio, is sufficient for our purposes. Moreover, from all cointegrated vectors which can be identified through the maximum likelihood method of Johansen (1991), the OLS estimated coefficients ensure the smallest volatility of the spread between the portfolio value and the benchmark value.

The OLS coefficients in model (1), further normalised to sum up to one, provide the composition of the tracking portfolio. Instead of a two-stage process, i.e. normalising the coefficients after estimation, the unit-sum constraint is normally implemented directly using a constrained least squares method. Indeed this approach must be taken when additional constraints such as no-short sales or maximum exposures are imposed on stock weights.

Long-short cointegration optimal portfolios

A natural extension of the simple cointegration tracking strategy presented above is to exploit the tracking potential of cointegration, attempting to replicate enhanced benchmarks constructed by adding to/subtracting from the index returns an annual excess return of a%, uniformly distributed over daily returns. Then, self-financing statistical arbitrage portfolios can be set up as a difference between two portfolios tracking a 'plus' and a 'minus' benchmark. This statistical arbitrage is expected to generate returns according to the 'plus'/'minus' spread with a low volatility. Moreover, if each 'plus' and 'minus' portfolio is tracking its benchmark accurately and their tracking error is not correlated with the market returns, the statistical arbitrage will give a market neutral portfolio (the effect of netting similar betas).

The new cointegration regressions can be written as:

$$ln(index\_plus_t) = a_1 + \sum_{k=1}^{n} a_{k+1} ln(P_{k,t}) + e\_plus_t \tag{3}$$

$$ln(index\_minus_t) = b_1 + \sum_{k=1}^{n} b_{k+1} ln(P_{k,t}) + e\_minus_t \tag{4}$$

Naturally it will become more difficult to construct cointegrated portfolios as the deviation of the benchmark from the index increases. The cointegration relationship between the market index and its component stocks has a solid rationale, but this is not necessarily the case for portfolios tracking artificial benchmarks, which might conceivably be chosen to over-perform the market index by 50%. In this case, the difficulty in finding an appropriate cointegration relationship leads to an increased instability of the stock weights, higher transaction costs and higher volatility of returns. To avoid this, it is essential to ensure that all the portfolios tracking 'plus' or 'minus' benchmarks pass the cointegration test.

Note that stock weights need not be restricted to be positive in the tracking portfolios; in fact it is likely that we shall take some short positions in the portfolio tracking the 'minus' benchmark. The stock holdings in the cointegration optimal statistical arbitrage portfolio are obtained by netting their individual weights in the 'plus' and 'minus' portfolios.

Tracking error variance (TEV) minimisation model

We shall compare the performance of cointegration optimal portfolios with portfolios that are optimised using the standard tracking error variance (TEV) minimisation model of Roll (1992). Using a similar notation to the cointegration optimal tracking portfolio, the tracking error variance minimisation model can be written as:

$$r_{index,t} = \sum_{k=1}^{n} c_k r_{k,t} + e_t \qquad (5)$$

where $r_{index,t}$ is the log return on the index at time t, $r_{k,t}$ is the log return on the stock k at time t, and e_t is the tracking error. We note that the analytic solution of Roll (1992) is not applicable for portfolios containing only a subset of the stocks in the benchmark. Instead, the allocations are estimated using a numerical optimisation to minimise the variance of the tracking error subject to the constraints of zero expected tracking error and a unit sum on the coefficients.

The main difference between models (1) and (5) concerns the objective of index tracking strategy. In the first case, it is the (squared) price spread between the replica portfolio and benchmark that is minimised, while in the second case, it is the (squared) change in this spread (i.e. the return) that is minimised. This seemingly innocuous difference actually results in quite different tracking error characteristics: with cointegration the values of the index and the tracking portfolio are 'tied' together; with TEV optimisation the value difference can diverge significantly, before returning to zero.

To illustrate this, we use a three year daily data sample from the Dow Jones Industrial Average (DJIA) stocks. Figure 1 compares the in-sample price spread between the index and the tracking portfolio optimised according to models (1) and (5). By construction the spread between the cointegration tracking portfolio and the index must be both mean reverting and have low variance. However the TEV criterion targets the return spread, not the price spread. The price spread shown in Figure 1 (which is typical for TEV portfolios) diverges quite far from the benchmark before returning to zero. In-sample it must return to zero by definition of OLS but out-of-sample the price spread of the TEV optimal portfolio need not return to zero after a period of time. This may happen because there is nothing in the TEV model to ensure that the price spread is mean-reverting. This simple example shows that, in theory, cointegration based index tracking is more appropriate than standard TEV minimisation – or indeed any other traditional strategy that is based on returns optimisation rather than price optimization.

2. Dynamic performance testing methodology

We investigate the empirical properties of cointegration optimal portfolios using a database of daily close prices over the period Jan-90 to Dec-03 for the stocks included in the Dow Jones

Industrial Average (DJIA) index as of 31-Dec-03. As benchmark, we use a DJIA historical series reconstructed to match the last available membership of the index. By doing this, we compare the performance of portfolios comprising the stocks currently included in DJIA with a market index constructed from the same stocks. The use of the reconstructed index ensures consistency in the treatment of dividends and stock splits (both index and stock prices are adjusted for dividends and stock splits) and eliminates a potential survivorship bias.[7]

The thirty stock price series were downloaded from yahoo-financial.com and any missing observation was replaced by the last close price available for that particular stock. As expected, based on standard unit root tests such as (2) all stocks price series proved to have significant stochastic trends but the associated returns series were clearly stationary, thus satisfying the conditions for cointegration analysis. To test the performance of different cointegration portfolios we generated optimal portfolios based on different parameters: *number of stocks in the tracking portfolios* (20, 25 and 30 stocks, selected according to their price ranking, starting with the highest prices stocks); *calibration period* up to 5 years of daily data prior to the moment of the portfolio construction; *model specification* - with or without constraints on the portfolio weights; *rebalancing period* (every 2-weeks, monthly, every 3-months and every 6-months); and the *spread* between the benchmarks tracked (up to 30% per annum).

The first cointegration optimal tracking portfolios were constructed in Jan-93. All portfolios were re-balanced at the given frequency with the stock selection based on the new stock rankings and the optimal weights based on the new coefficients of the cointegration regression. At each re-balancing the cointegration regression was re-estimated over the rolling fixed-length calibration period preceding the re-balancing moment. The number of shares held in each stock was determined by the previous portfolio value, the current stock price and the stock weight in the cointegration regression. In between re-balancings, the portfolios were left unmanaged, i.e. the number of stocks was kept constant. We followed exactly the same process for the TEV optimal portfolios.

To account for the impact of the bid-ask spread and the brokerage fees on the portfolio returns a fixed amount of 20 basis points transaction costs on each trade value was assumed. This is in line with previous studies on the transaction costs incurred on NYSE (Chalmers, Edelen and Kadlec, 1999; NYSE research report, 2001). The repo costs are normally small so these were computed at 0.25% of the increase in the short position in a bull market for a particular

[7] A tracking portfolio comprising the stocks currently included in the index is likely to outperform the actual index, because it does not include the index 'losers', i.e. stocks which had a poor performance and were subsequently eliminated from the index during the sample period.

stock (defined as an increase in price over the last 10 trading days) and at 0.35% on the same amount in a bear market for that particular stock (defined as a decrease in price over the last 10 trading days). These cost levels are conservative, given the fact that DJIA stocks are known to be very liquid and their trading generates a low market impact. In any case we are primarily interested in the comparative effect of the transaction costs on different strategies, and on a relative basis the particular cost rate we employ is less important.

3. Index tracking

The first target of our empirical analysis is the out-of-sample performance of the two approaches to index tracking, using the cointegration model specification (1) and the TEV model specification (5). We start the analysis with a standard two-weeks rebalancing frequency and no constraints on the portfolio weights, and subsequently extend it to other rebalancing frequencies and introduce constraints.

Number of stocks and calibration period

We first note that the degree of cointegration between the cointegration optimal tracking portfolio and the benchmark increases with the number of stocks in the portfolio and marginally with the sample size. A number of portfolios with either too few stocks (less than 20 stocks) or weights based on a too short calibration period (one or two years) proved not to be sufficiently cointegrated with the market index so we excluded them from the following analysis. Secondly, note that *none* of the TEV optimal portfolios was cointegrated with the benchmark, however many stocks are included. They all have non-stationary in-sample price spreads, such as that illustrated in Figure 1.

Once the minimum calibration period (of about three years) for ensuring cointegration was used, increasing it had little impact of the tracking performance of the cointegration optimal portfolios. For reasons of space, we focus in the following on a calibration period of three years. To allow a *ceteris paribus* comparison we must use the same calibration period for the TEV model.

In an independent application of TEV to portfolios with relatively few stocks the calibration period used could be shorter than three years, so that the non-stationary price spread (such as that illustrated in Figure 1) becomes less of a problem. However, apart from a better in-sample model specification, shorter calibration periods result in TEV portfolios that are more sample specific, have a less stable structure and require more frequent re-balancing.

Table 1 reports summary statistics for the cointegration and TEV optimal tracking portfolios comprising different numbers of stocks and based on a calibration period of three years, over the period Jan-93 to Dec-03. For each tracking portfolio, we report the average annual tracking error (after transaction costs), its volatility, the correlation of the tracking portfolio returns with the benchmark returns as well as the correlation of the tracking error with the benchmark returns, the skewness and excess kurtosis of the tracking error, the in-sample ADF statistic for the cointegration regression, the average annual transaction costs, the Sharpe ratio of the tracking portfolios after transaction costs, the tracking portfolio beta and the empirical probability of observing a negative tracking error.

Optimisation alpha

The first observation is that, for both models, the tracking portfolios comprising 20 and 25 stocks under-perform the benchmark, even before transaction costs, while the 30-stocks portfolios consistently over-perform it. Figure 2 plots the cumulative returns of the tracking portfolios comprising 30-stocks. Whereas the over-performance of the 30-stocks portfolios is clearly generated by the optimisation models, the under-performance of the 20 and 25-stocks portfolios can either be caused by the stock selection method or by the optimisation algorithm. In order to distinguish between these we construct a simple price weighted 'index20' portfolio comprising the same 20 (respectively 30) stocks that are selected for the tracking portfolios. Then the difference between the index20 (respectively index 25) portfolio and the corresponding tracking portfolio is only due to the optimisation of allocations. We find that during the calibration period – that is, *in*-sample – the index20 portfolio over-perform the 'index30' portfolio (i.e. the price weighted portfolio of all stocks, the 'reconstructed' DJIA) on average by 1% in annual terms, over the period 1993-2003. Also, the 'index25' portfolio over-performs the 'index30' portfolio by 0.7% p.a., on average. However, *out*-of-sample, over a ten-days no trading horizon the index20 and index25 portfolios significantly under-perform the index30 'reconstructed' DJIA, by 3.3% (respectively 2.0%) on average in annual terms.

On the one hand this is evidence of mean-reversion in stock returns.[8] On the other hand for the purpose of our analysis we can conclude that the under-performance of 20 (respectively 25) stock tracking portfolios is *not* due to the optimisation models, but is a result of the stock selection criterion used. After transaction costs the negative tracking error of the optimised

[8] If we assume that the highest prices stocks had an above-the-average performance prior to the stock selection moment, their below-the-average performance over the next two weeks following the portfolio construction moment indicates mean reversion in stock returns. This phenomenon has been extensively studied (e.g. De Bondt and Thaler, 1985; Lo and MacKinlay, 1988; Poterba and Summers, 1988; and Jagadeesh and Titman, 1993), and behavioural explanations have been provided for it (e.g. Odean, 1999; De Long, Shleifer, Summers and Waldmann, 1990a; Lakonishok, Shleifer and Vishny, 1994; and Shleifer and Vishny, 1997).

portfolios is smaller, in absolute terms, than the under-performance of the index20 (respectively index25) portfolio.

Thus, both cointegration and TEV optimisation models are enhancing the returns of tracking portfolios relative to the price-weighted portfolio of the identical stocks. For instance, the cointegration and the TEV optimal portfolios containing all thirty stocks over-perform the 'reconstructed' DJIA index in our out-of-sample tests between Jan-93 and Dec-03. However, as shown in Figure 3, this over-performance is not uniformly accumulated over the sample. With the exception of the year 1994, over-performance occurred during the main market crises within the data sample: the Asian crisis, the Russian crisis and the technology market crash. Why should this be so? The answer is that the tracking portfolio weights are constructed on a relatively long calibration periods so they tend to ignore short-term movements in stock prices, although these are immediately accounted for in the price-weighted index structure. This results in tracking error, which nevertheless mean reverts when prices mean revert. However, if in addition to mean reversion in prices there is a marked asymmetry (in that prices fall faster than they rise) the gains in the price spread will out-weigh the losses.

If we take the example of a stable trending market in which the prices of certain stocks increase well above their historic average, then the tracking portfolio weights lag behind the index weights and consequently generate relative losses for the tracking portfolios for as long as the trend continues. However, when the prices of these stocks revert to their historical equilibrium levels, the tracking portfolios, being still under-weighted on them, realise relative gains. In some cases (i.e. during market crash periods) the losses are less significant than the gains. This is due to the asymmetry of stock markets: it is a well-documented stylised fact that stock prices usually fall more rapidly than they rise.[9] When prices increase *slowly*, they allow the tracking portfolios to adjust to the new information, and to track the benchmark reasonably well. When these prices fall *suddenly*, the tracking portfolios, with weights that are lagging the market weights, realise large gains.

It is notable that the cointegration optimal tracking portfolio consistently under-performed the benchmark during the last three years; we will discuss this later when we examine the statistical arbitrage performance of the two strategies.

[9] This is the result of the leverage effect (Black, 1976; Christie, 1982; French, Schwert and Stambaugh, 1987) and the presence of positive feed-back: an initial sell reaction to some bad news will be followed by more selling, driving the prices faster below their fundamental levels (De Long, Shleifer, Summers and Waldmann, 1990a).

Comparison between cointegration and TEV tracking portfolios

Having set forth in section one the conceptual difference between the cointegration-based tracking and the tracking error variance minimisation models, we now investigate the difference in their empirical performance. Table 1(a) shows that the out-of-sample performance of the cointegration optimal and TEV tracking portfolios is very similar. Despite the fact that in-sample the cointegration portfolio is 'tied' to the benchmark whilst the TEV is not, both models produce returns that are very highly correlated with the benchmark out-of-sample. The tracking errors are uncorrelated with the market and the tracking portfolios betas are very close to one. Both portfolios generate a positive alpha when the effect of the stock selection is neutralised (illustrated by Figure 3) and throughout most of the sample period the cumulative tracking error from the cointegration optimal model is well above that of the TEV model, although the opposite happens during the last three years of the sample. As one would expect, since the TEV portfolios are specifically constructed to minimise the variance of the tracking error, the TEV portfolios have lower tracking error volatility.

The TEV portfolios generate marginally lower transaction costs, and they also have slightly better Sharpe ratios. However the tracking error has a distribution closer to normality, with less excess kurtosis, in the case of the cointegration optimal portfolios. Also, the probability of under-performing the benchmark is, in general, marginally smaller for the cointegration optimal portfolios. Clearly, the extra feature of cointegration with the benchmark can be achieved at no significant costs for the tracking portfolio. In this simple index tracking exercise with no weights constraints, neither advantages nor limitations of having a cointegration relationship with the benchmark are empirically evident.

Re-balancing period and weight constraints

The investigation of the transaction costs in Table 1(a) is essential for understanding the characteristics of optimised tracking portfolios. For both models, the transaction costs significantly decrease as the number of stocks in the tracking portfolios increases. In fact, this is simply the stock selection effect. Since at each re-balancing we select the first k stocks according to their price and then optimise the portfolio to replicate the index, as the composition of the first k stocks group changes, the portfolio weights will also change significantly.

One way of reducing the transaction costs is to reduce the re-balancing frequency. Given the length of the calibration period used to optimise the tracking portfolios reducing the re-balancing frequency should not deteriorate dramatically the tracking performance. Indeed, the results in Table 1(a) show that, as the rebalancing frequency is reduced to as low as twice per year, the tracking portfolios constructed with both models continue to have a good performance. The correlation of the tracking portfolios with the benchmark continues to be very high, while the Sharpe ratios of the tracking portfolios are maintained. In fact, the average tracking error gross of transaction costs declines slightly when the rebalancing frequency is reduced. But the transaction costs are significantly lowered by reducing the rebalancing frequency, as it is the volatility of the tracking error.

It is important to note that robustness to reducing the re-balancing frequency arises for different reasons: in the cointegration portfolio it is driven by the stability of the cointegration relationship; for the TEV model it is the result of using a large calibration period. With a shorter calibration period, as one would use in an independent application of TEV, we have found that the portfolio structure is much less stable and the overall performance deteriorates when the rebalancing frequency is reduced.

Finally, we investigate the effect of imposing no short sale and concentration constraints on the tracking portfolio weights. Such constraints are widely used in practice by mutual funds and other institutional investors. Our first observation is that in the presence of such constraints we still find cointegration relationships between the tracking portfolios and the benchmark (the price spread for the TEV portfolios remains nonstationary). Indeed given the results in Table 1(b), the alpha tends to increase in the presence of constraints for both models, at the expense of slightly higher tracking error volatility. The correlation of the tracking portfolios with the benchmark remains very high, and overall the Sharpe ratios of the tracking portfolios are slightly improved. However, the main benefit of imposing the constraints turns out to be in terms of the excess kurtosis, which is significantly reduced. Considering the findings of Jagannathan and Ma (2003) on the effectiveness of imposing constraints to reduce measurement errors, we can infer that the excess kurtosis displayed by the unconstrained tracking portfolios is due to the presence of outliers.

To conclude, the empirical performance of the cointegration tracking model is very similar to that of the traditional TEV minimising model. When the impact of the selection criterion is neutralised, both models enhance the performance of the benchmark. Having measured performance relative to a 'reconstructed' index of the same stocks and thus having excluded the possibility that survivorship bias or dividend effects have influenced our results, both models produce a significant positive tracking error out-of-sample and better Sharpe ratios than the benchmark, even after transaction costs. The periods responsible for largest part of the positive tracking error coincide with the main market crises during our data sample: the Asian and Russian crises and the burst of the technology bubble. The tracking performance was shown to be robust to imposing weight constraints and reducing the rebalancing frequency. These results are thus of great relevance for institutional investors such as mutual index funds.

4. Statistical arbitrage strategies

To examine the performance of the cointegration optimal statistical arbitrage model described in section one, we use similar principles to those in section three. For reasons of space, we neutralise the effect of the stock selection criterion and use all thirty stocks in each portfolio. We also impose no constraints on the portfolio weights. Statistical arbitrage implies very dynamic portfolio management so we assume the highest rebalancing frequency from the tracking simulations, i.e. every 10-trading days. Also, we only consider portfolios based on a 3-years estimation period. However, the reader should keep in mind that, as shown in the previous section, fine-tuning these model parameters can improve the strategy performance significantly.

As a basis for tracking we create six ‘plus’/’minus’ benchmarks by adding/subtracting annual returns of 5%, 10% and 15% from the reconstructed DJIA returns, uniformly distributed. Then using both the cointegration and the TEV models we construct portfolios to track these artificial indices. Despite the fact that the artificial benchmarks diverge significantly from the actual index values (up to plus/minus 15% p.a.), we are still able to find portfolios that are cointegrated with them. The price spreads between the cointegration optimal tracking portfolios and their artificial benchmarks prove to be stationary. Finally, we set up statistical arbitrage strategies that are long on the ‘plus’ tracking portfolios and short on the ‘minus’ tracking portfolios.

The summary out-of-sample performance results for the two statistical arbitrage strategies over the period Jan-93 to Dec-03 are presented in Table 2(a). For both models, the best performance is produced by strategies tracking narrow spreads, such as plus 5% hedged with the portfolio tracking the actual benchmark. As the spread between the benchmarks tracked increases, the volatility of the strategy returns increases significantly, without being compensated by additional returns. Table 2(a) actually shows a negative relationship between the long-short portfolio returns and the spread between the benchmarks tracked. This is the combined effect of higher transaction costs and increased volatility. Portfolios tracking large spreads tend to assume more aggressive positions. Before transaction and repo costs the arbitrage returns are substantially higher when tracking large ‘plus’ or ‘minus’ spreads. However, as the degree of cointegration decreases, the stock weights become more unstable, which results in higher transaction and repo costs. Another feature noticeable in Table 2(a) is that the 'minus' portfolios tend to be more volatile than their 'plus' equivalents.

Both cointegration and TEV models exhibit low unconditional correlation with the market returns and close to normal return distributions, with negative but not significant skewness and excess kurtosis in the range of 1 to 2, much lower than that of the market index.

The most important finding in Table 2(a) is that the cointegration optimal portfolios are clearly dominating the TEV statistical arbitrage portfolios. The TEV portfolios generally have lower volatility, but they also have much lower returns, which results in lower Sharpe ratios. Most of the cointegration based statistical arbitrage portfolios have positive average returns, but the large majority of TEV statistical arbitrage portfolios actually generate negative average returns. Also, the TEV portfolios display slightly higher correlation with the market returns.

Why does the cointegration based statistical arbitrage perform better than the TEV statistical arbitrage? Given the similarity of the empirical performance of the two optimisation strategies for index tracking already documented in section three, this is a surprising result. To answer this question we emphasise the practical difference between tracking an index and tracking an enhanced index. In the first case one aims to identify the portfolio that stays closest to a real index. Most portfolios comprising a sufficient number of stocks are likely to stay close to their market index, irrespective of the existence of cointegration. In these circumstances, one cannot really observe the advantage of having a cointegration relationship between the tracking portfolio and the benchmark. However, the replication task becomes significantly more difficult when one aims to identify portfolios tracking artificial indices that are designed to under-perform or over-perform the actual index. In this case, ensuring a stationary spread between the portfolio value and the index starts to pay off and enhances the out-of-sample performance of the statistical arbitrage. In fact, the mean reversion of returns prevents the TEV based statistical arbitrage from generating consistent out-of-sample returns. Simply identifying the over-performers or under-performers in-sample does not guarantee that they will continue to do so out-of-sample. By contrast, the fact that the cointegration optimal tracking portfolio is 'tied' to the artificial benchmark gives a more reliable basis for statistical arbitrage.

Despite the more attractive features of the cointegration optimal statistical arbitrage, the average performance of both models over the entire eleven years period from 1993-2003 is not very encouraging. In terms of Sharpe ratios, the best statistical arbitrage over-performed the market index (Sharpe ratio of 0.37, as compared to 0.3), but most strategies had lower ratios. So, given the time variability identified in the simple tracking performance, and the fact that this seemed to deteriorate after year 2000, we have split the sample in two: 1993-1999 and 2000-2003 and reported the results in Table 2(b) over the two sub-samples.

Indeed, there is a large difference in the performance of both statistical arbitrage models over the period 1993-1999, as compared to 2000-2003. Still, the consistent result over both sub-samples is the dominance of the cointegration optimal statistical arbitrage over the TEV strategy. Over the first sub-sample, both models produce positive average returns with relatively low volatility and distributions close to normality. The average Sharpe ratio produced by the best cointegration statistical arbitrage was 1.12, while the highest average ratio produced by the TEV model was 0.97. The arbitrage returns tend to increase with the spread between the benchmarks tracked, but so does their volatility. Thus the best performance continues to be achieved with portfolios tracking narrow spreads around the benchmark.

Over the second sub-sample, 2000-2003, all statistical arbitrage strategies generated negative returns with higher volatility. These losses are put in perspective if we consider that the market index lost over the same period an annual average of 4.8%, with an average volatility of 21%. The statistical arbitrage returns remained close to market neutrality during this period, so for the cause of their ineffectiveness one has to look beyond the general market decline during the period analysed. In fact, the reason for this poor performance lies with the features of the calibration period. The three years preceding 2000 and the years 2000-2002 have been marked by several market crises on a general background of increased volatility. The long run equilibrium relationships between sectors and industries have been affected. The calibration of statistical arbitrage portfolios on such eventful samples is very difficult – it is the quality of the calibration data that is responsible for their poor out-of-sample performance.

To summarise the results in this section, both strategies yield returns according to the spread between the benchmarks tracked, have lower volatility than the market, low market correlation and near to normal returns distributions. Nevertheless we have found that these results do, however, depend on the quality of the calibration data and that optimising portfolios on stressful samples is risky. Targeting larger spreads is penalised in terms of volatility and transaction costs, thus the net returns are not linearly related to the arbitrage spread. We have shown that the benefit of ensuring a cointegration relationship with the benchmark pays off for the statistical arbitrage strategies, which clearly dominate their TEV equivalents. However these results only illustrate the raw performance of the statistical arbitrage strategies. There is considerable scope for enhancement, by using stock selection methods, appropriate calibration periods and rebalancing frequencies, or by imposing portfolio constraints.

5. Concluding remarks

Given the increasing popularity of cointegration based strategies for all types of investors, we aimed to conduct a thorough investigation of trading strategies based on cointegration in a realistic, out of sample framework. The theoretical benefits of having a cointegration relationship between a tracking portfolio and its benchmark are clear: the two are 'tied' together in the long run, their price spread has minimum volatility, and the model makes full use of the information contained in stock prices, including that in their common trends. When testing the empirical performance of cointegration-based portfolio optimisation models, we have shown that their out-of-sample performance is, on average, very similar to that of the traditional TEV minimising model when applied to simple index tracking. Ensuring a cointegration relationship between a tracking portfolio and a benchmark does not seem to bring any obvious advantage, or cost. This is merely because if a sufficient number of stocks is included in the portfolio, most models will return a reasonably good index tracking performance, irrespective of the existence of any cointegration relationship.

Yet our comparison of the cointegration optimal and TEV statistical arbitrage strategies has revealed some interesting results. Depending on the characteristics of the calibration data for statistical arbitrage portfolios, both strategies yielded returns according to the spread between the benchmarks tracked, had lower volatility than the market, low market correlation and near to normal returns distributions. But we found that the cointegration optimal statistical arbitrage strategies dominate their TEV equivalents over an eleven year out-of-sample performance analysis. Thus the benefit of cointegration relationships appears to be that they are more robust, out-of-sample, than relationships that are identified on returns. This ensures a reliable foundation for statistical arbitrage, reducing the risk of over-hedging and the associated trading costs.

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Table 1 (a) Tracking performance for cointegration optimal and TEV portfolios based on a calibration period of three years (unconstrained weights)

	2 weeks rebalancing			monthly rebalancing			3 months rebalancing			6 months rebalancing		
Cointegration	20 stocks	25 stocks	30 stocks	20 stocks	25 stocks	30 stocks	20 stocks	25 stocks	30 stocks	20 stocks	25 stocks	30 stocks
Average annual TE	-2.76%	-1.36%	0.68%	-2.32%	-0.61%	0.71%	-2.29%	-0.92%	0.38%	-2.10%	-1.09%	-0.02%
TE volatility	4.63%	3.59%	2.77%	4.46%	3.48%	2.83%	4.33%	3.32%	2.72%	4.35%	3.32%	2.75%
Correlation TP/DJIA	0.97	0.98	0.99	0.97	0.98	0.99	0.97	0.98	0.99	0.97	0.98	0.99
Correlation TE/DJIA	0.19	0.20	0.12	0.09	0.11	0.07	0.03	0.04	0.02	-0.02	-0.01	-0.03
TE skew	0.08	0.10	0.01	-0.09	-0.04	-0.17	-0.14	-0.10	-0.08	-0.20	-0.29	-0.24
TE excess kurtosis	3.57	3.02	2.68	2.40	2.63	4.09	1.81	1.39	1.94	2.16	1.86	2.45
ADF statistic	-6.41	-6.72	-7.01	-6.40	-6.75	-7.00	-6.39	-6.71	-7.00	-6.51	-6.74	-6.93
Average trans costs	0.97%	0.69%	0.28%	0.58%	0.42%	0.21%	0.31%	0.24%	0.14%	0.18%	0.14%	0.10%
Sharpe ratio TP	0.18	0.26	0.38	0.20	0.29	0.37	0.18	0.26	0.34	0.21	0.27	0.33
Beta TP	1.05	1.04	1.02	1.02	1.02	1.01	1.01	1.01	1.00	1.00	1.00	1.00
Prob(TE<0)	52.37%	51.72%	49.69%	51.47%	51.36%	49.27%	52.21%	51.22%	49.05%	51.20%	50.61%	48.80%
TEV	20 stocks	25 stocks	30 stocks	20 stocks	25 stocks	30 stocks	20 stocks	25 stocks	30 stocks	20 stocks	25 stocks	30 stocks
Average annual TE	-2.37%	-1.02%	0.90%	-1.90%	-0.65%	0.88%	-2.28%	-0.75%	0.64%	-2.31%	-0.84%	0.37%
TE volatility	3.51%	2.74%	2.21%	3.46%	2.70%	2.24%	3.26%	2.52%	2.06%	3.30%	2.53%	2.04%
Correlation TP/DJIA	0.98	0.99	0.99	0.98	0.99	0.99	0.98	0.99	0.99	0.98	0.99	0.99
Correlation TE/DJIA	0.12	0.14	0.13	0.03	0.05	0.06	-0.03	-0.01	-0.01	-0.08	-0.07	-0.05
TE skew	-0.02	0.24	0.44	-0.14	-0.01	-0.06	-0.08	0.05	0.12	-0.24	-0.03	-0.01
TE excess kurtosis	4.71	4.32	5.27	4.64	5.53	7.75	3.66	2.94	2.51	3.42	4.39	3.72
ADF statistic	-2.00	-1.85	-1.58	-1.92	-1.85	-1.53	-1.86	-1.90	-1.56	-1.82	-1.81	-1.55
Average trans costs	0.55%	0.41%	0.19%	0.33%	0.24%	0.12%	0.18%	0.12%	0.07%	0.11%	0.07%	0.05%
Sharpe ratio TP	0.21	0.29	0.40	0.22	0.30	0.38	0.19	0.27	0.35	0.20	0.29	0.36
Beta TP	1.02	1.02	1.02	1.01	1.01	1.01	1.00	1.00	1.00	0.98	0.99	0.99
Prob(TE<0)	52.84%	51.43%	49.47%	51.69%	50.70%	49.38%	52.40%	50.19%	49.66%	51.72%	50.06%	49.76%

Table 1 (b) Tracking performance for cointegration optimal and TEV portfolios based on a calibration period of three years (constrained weights)

	2 weeks rebalancing			monthly rebalancing			3 months rebalancing			6 months rebalancing		
Cointegration	20 stocks	25 stocks	30 stocks	20 stocks	25 stocks	30 stocks	20 stocks	25 stocks	30 stocks	20 stocks	25 stocks	30 stocks
Average annual TE	-1.96%	-0.72%	1.46%	-1.40%	0.13%	1.48%	-1.61%	-0.18%	1.21%	-1.67%	-0.36%	0.90%
TE volatility	4.72%	3.86%	3.09%	4.64%	3.80%	3.17%	4.62%	3.64%	3.11%	4.66%	3.68%	3.15%
Correlation TP/DJIA	0.97	0.98	0.98	0.97	0.98	0.98	0.97	0.98	0.98	0.97	0.98	0.98
Correlation TE/DJIA	0.14	0.13	0.06	0.02	0.04	0.01	-0.04	-0.02	-0.04	-0.07	-0.06	-0.07
TE skew	0.12	0.15	0.05	0.07	0.04	-0.10	0.08	-0.04	-0.08	-0.07	-0.15	-0.13
TE excess kurtosis	2.88	2.07	1.83	2.90	1.98	2.64	2.67	0.86	1.54	1.53	1.32	1.83
ADF statistic	-5.64	-6.25	-6.48	-5.62	-6.28	-6.46	-5.61	-6.25	-6.40	-5.58	-6.26	-6.29
Average trans costs	0.88%	0.63%	0.26%	0.52%	0.38%	0.20%	0.31%	0.21%	0.14%	0.17%	0.13%	0.09%
Sharpe ratio TP	0.23	0.30	0.43	0.25	0.34	0.42	0.22	0.30	0.38	0.24	0.31	0.39
Beta TP	1.04	1.03	1.01	1.01	1.01	1.00	0.99	1.00	0.99	0.98	0.99	0.99
Prob(TE<0)	52.73%	51.65%	48.79%	51.58%	51.10%	48.13%	53.05%	51.29%	48.17%	52.28%	51.02%	49.35%
TEV	20 stocks	25 stocks	30 stocks	20 stocks	25 stocks	30 stocks	20 stocks	25 stocks	30 stocks	20 stocks	25 stocks	30 stocks
Average annual TE	-1.81%	-0.43%	1.54%	-1.31%	-0.07%	1.52%	-1.74%	-0.21%	1.18%	-1.78%	-0.33%	0.91%
TE volatility	3.72%	2.95%	2.48%	3.69%	2.94%	2.52%	3.57%	2.79%	2.37%	3.65%	2.83%	2.38%
Correlation TP/DJIA	0.98	0.99	0.99	0.98	0.99	0.99	0.98	0.99	0.99	0.98	0.99	0.99
Correlation TE/DJIA	0.07	0.07	0.06	-0.02	-0.01	-0.01	-0.08	-0.07	-0.08	-0.13	-0.12	-0.12
TE skew	-0.02	0.17	0.30	-0.13	-0.03	-0.06	-0.13	-0.03	0.03	-0.23	-0.10	-0.05
TE excess kurtosis	3.57	2.70	3.23	3.33	3.69	4.82	2.91	1.72	1.83	2.44	2.58	2.47
ADF statistic	-2.05	-1.92	-1.70	-2.00	-1.91	-1.65	-1.92	-1.93	-1.68	-1.90	-1.90	-1.66
Average trans costs	0.61%	0.43%	0.20%	0.37%	0.25%	0.13%	0.20%	0.13%	0.08%	0.12%	0.08%	0.05%
Sharpe ratio TP	0.24	0.32	0.44	0.26	0.33	0.42	0.22	0.31	0.39	0.24	0.32	0.39
Beta TP	1.02	1.01	1.01	1.00	1.00	1.00	0.98	0.99	0.99	0.97	0.98	0.98
Prob(TE<0)	51.87%	49.73%	48.68%	50.81%	49.63%	48.64%	52.13%	49.09%	48.59%	52.20%	48.98%	49.24%

Table 2 (a) Statistical arbitrage performance for cointegration optimal and TEV portfolios with 30 stocks, based on a calibration period of three years and rebalanced every two weeks

Sample: Jan 1993 – Dec 2003		coint				TEV			
		plus 0%	**plus 5%**	**plus 10%**	**plus 15%**	**plus 0%**	**plus 5%**	**plus 10%**	**plus 15%**
minus 0%	Annual return	N/A	1.03%	0.93%	0.42%	N/A	0.57%	0.12%	-0.59%
	Annual vol	N/A	2.82%	5.15%	7.94%	N/A	1.93%	3.88%	6.24%
	Skewness	N/A	-0.14	-0.13	-0.16	N/A	0.09	-0.01	-0.06
	Xs kurtosis	N/A	1.98	1.34	1.58	N/A	3.88	1.42	1.39
	Correlation	N/A	0.20	0.20	0.19	N/A	0.32	0.33	0.30
	Sharpe ratio	N/A	0.37	0.18	0.05		0.29	0.03	-0.09
minus 5%	Annual return	-0.92%	0.11%	0.01%	-0.50%	-1.64%	-1.07%	-1.52%	-2.23%
	Annual vol	4.77%	5.84%	8.81%	11.75%	4.00%	4.73%	7.19%	9.70%
	Skewness	-0.17	-0.12	-0.14	-0.18	-0.34	-0.15	-0.17	-0.17
	Xs kurtosis	1.65	1.56	1.63	1.75	2.28	1.32	1.34	1.45
	Correlation	0.16	0.23	0.20	0.20	0.19	0.29	0.29	0.27
	Sharpe ratio	-0.19	0.02	0.00	-0.04	-0.41	-0.23	-0.21	-0.23
minus 10%	Annual return	-0.62%	0.41%	0.30%	-0.20%	-2.07%	-1.50%	-1.95%	-2.66%
	Annual vol	7.27%	8.55%	11.52%	14.44%	6.10%	6.97%	9.41%	11.92%
	Skewness	-0.15	-0.11	-0.13	-0.16	-0.26	-0.14	-0.16	-0.16
	Xs kurtosis	1.57	1.51	1.57	1.68	1.72	1.27	1.30	1.38
	Correlation	0.18	0.21	0.20	0.20	0.22	0.28	0.28	0.27
	Sharpe ratio	-0.09	0.05	0.03	-0.01	-0.34	-0.21	-0.21	-0.22
minus 15%	Annual return	-0.06%	0.98%	0.87%	0.37%	-2.41%	-1.84%	-2.29%	-3.00%
	Annual vol	9.72%	11.10%	14.04%	16.97%	8.22%	9.15%	11.59%	14.08%
	Skewness	-0.10	-0.08	-0.11	-0.13	-0.21	-0.13	-0.15	-0.15
	Xs kurtosis	1.63	1.52	1.56	1.65	1.51	1.23	1.26	1.33
	Correlation	0.18	0.21	0.20	0.20	0.23	0.28	0.28	0.27
	Sharpe ratio	-0.01	0.09	0.06	0.02	-0.29	-0.20	-0.20	-0.21

Table 2 (b) Statistical arbitrage performance for cointegration optimal and TEV portfolios with 30 stocks, based on a calibration period of three years and rebalanced every two weeks (subsamples)

Sample: Jan 1993 – Dec 1999		**coint** plus 0%	plus 5%	plus 10%	plus 15%	**TEV** plus 0%	plus 5%	plus 10%	plus 15%
minus 0%	Annual return	N/A	2.63%	3.56%	4.28%	N/A	1.58%	2.01%	2.30%
	Annual vol	N/A	2.35%	4.78%	7.51%	N/A	1.63%	3.69%	6.00%
	Skewness	N/A	-0.01	0.03	0.01	N/A	0.03	-0.07	-0.06
	Xs kurtosis	N/A	0.46	0.69	0.75	N/A	1.07	1.00	1.47
	Correlation	N/A	0.09	0.15	0.16	N/A	0.16	0.24	0.24
	Sharpe ratio	N/A	1.12	0.74	0.57	N/A	0.97	0.54	0.38
minus 5%	Annual return	-0.88%	1.75%	2.68%	3.41%	-0.99%	0.59%	1.01%	1.30%
	Annual vol	4.63%	5.80%	8.70%	11.48%	3.63%	4.62%	6.96%	9.32%
	Skewness	-0.03	0.05	0.03	0.01	-0.16	-0.09	-0.12	-0.11
	Xs kurtosis	0.48	0.54	0.54	0.59	1.40	1.23	1.25	1.40
	Correlation	0.21	0.21	0.20	0.19	0.30	0.29	0.28	0.27
	Sharpe ratio	-0.19	0.30	0.31	0.30	-0.27	0.13	0.15	0.14
minus 10%	Annual return	0.40%	3.03%	3.96%	4.68%	-0.58%	1.00%	1.43%	1.72%
	Annual vol	7.25%	8.56%	11.45%	14.23%	5.79%	6.85%	9.17%	11.53%
	Skewness	0.01	0.05	0.04	0.02	-0.14	-0.09	-0.12	-0.11
	Xs kurtosis	0.45	0.56	0.55	0.59	1.25	1.17	1.19	1.31
	Correlation	0.22	0.21	0.20	0.20	0.30	0.29	0.29	0.28
	Sharpe ratio	0.05	0.35	0.35	0.33	-0.10	0.15	0.16	0.15
minus 15%	Annual return	2.19%	4.82%	5.75%	6.47%	-0.08%	1.50%	1.93%	2.22%
	Annual vol	9.79%	11.15%	14.01%	16.79%	7.97%	9.06%	11.38%	13.72%
	Skewness	0.08	0.09	0.07	0.05	-0.12	-0.09	-0.11	-0.11
	Xs kurtosis	0.65	0.69	0.64	0.66	1.17	1.12	1.15	1.25
	Correlation	0.22	0.21	0.20	0.20	0.30	0.29	0.29	0.28
	Sharpe ratio	0.22	0.43	0.41	0.39	-0.01	0.17	0.17	0.16

Sample: Jan 2000 – Dec 2003		**coint** plus 0%	plus 5%	plus 10%	plus 15%	**TEV** plus 0%	plus 5%	plus 10%	plus 15%
minus 0%	Annual return	N/A	-1.80%	-3.76%	-6.44%	N/A	-1.23%	-3.23%	-5.73%
	Annual vol	N/A	3.49%	5.73%	8.64%	N/A	2.38%	4.19%	6.64%
	Skewness	N/A	-0.12	-0.24	-0.33	N/A	0.20	0.08	-0.04
	Xs kurtosis	N/A	1.59	1.61	2.19	N/A	3.84	1.77	1.23
	Correlation	N/A	0.28	0.25	0.23	N/A	0.45	0.44	0.37
	Sharpe ratio	N/A	-0.52	-0.66	-0.75	N/A	-0.52	-0.77	-0.86
minus 5%	Annual return	-1.00%	-2.80%	-4.76%	-7.44%	-2.78%	-4.01%	-6.02%	-8.51%
	Annual vol	5.00%	5.90%	9.00%	12.21%	4.58%	4.91%	7.57%	10.33%
	Skewness	-0.38	-0.40	-0.42	-0.44	-0.46	-0.22	-0.21	-0.23
	Xs kurtosis	3.14	3.22	3.27	3.28	2.43	1.40	1.40	1.40
	Correlation	0.10	0.25	0.22	0.20	0.09	0.30	0.29	0.28
	Sharpe ratio	-0.20	-0.48	-0.53	-0.61	-0.61	-0.82	-0.79	-0.82
minus 10%	Annual return	-2.44%	-4.24%	-6.19%	-8.88%	-4.71%	-5.94%	-7.94%	-10.44%
	Annual vol	7.30%	8.54%	11.62%	14.81%	6.62%	7.16%	9.82%	12.57%
	Skewness	-0.43	-0.40	-0.42	-0.44	-0.37	-0.21	-0.20	-0.22
	Xs kurtosis	3.51	3.18	3.24	3.27	2.02	1.40	1.40	1.39
	Correlation	0.13	0.23	0.21	0.20	0.14	0.28	0.28	0.27
	Sharpe ratio	-0.33	-0.50	-0.53	-0.60	-0.71	-0.83	-0.81	-0.83
minus 15%	Annual return	-4.05%	-5.85%	-7.81%	-10.49%	-6.56%	-7.79%	-9.79%	-12.29%
	Annual vol	9.59%	11.01%	14.08%	17.26%	8.64%	9.31%	11.96%	14.70%
	Skewness	-0.44	-0.39	-0.41	-0.43	-0.32	-0.19	-0.19	-0.20
	Xs kurtosis	3.49	3.02	3.11	3.17	1.86	1.40	1.40	1.39
	Correlation	0.15	0.22	0.21	0.20	0.16	0.27	0.27	0.26
	Sharpe ratio	-0.42	-0.53	-0.55	-0.61	-0.76	-0.84	-0.82	-0.84

Figure 1 In-sample spread (cumulative return difference) between the tracking portfolios and the benchmark

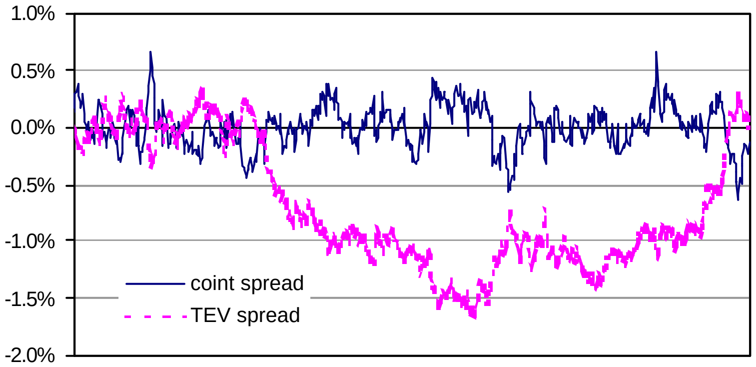

Figure 2 Cumulative out-of-sample returns on DJIA and cointegration optimal, respectively TEV 30 stocks tracking portfolios, calibrated on 3 years sample period and rebalanced every 10-trading days

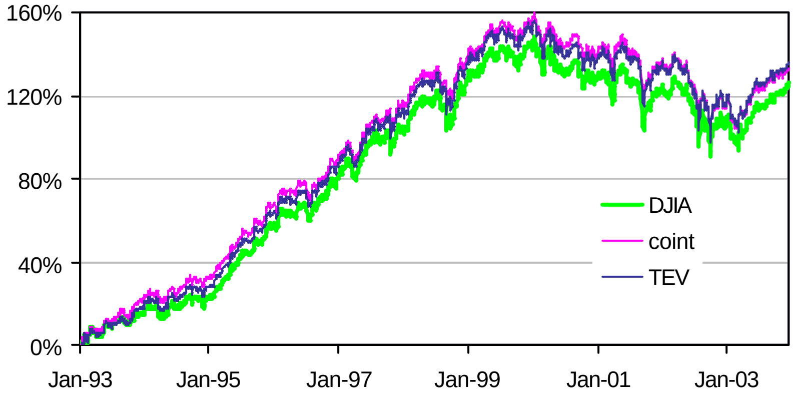

Figure 3 Cumulative out-of-sample tracking error of the 30-stocks cointegration optimal, respectively TEV tracking portfolios