The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Properties of Stock Options

Topic 42

Exam Focus

Stock options have several properties relating both to their value and to the factors that affect their price. Six factors affect option prices: the current value of the stock; the strike price; the time to expiration; the volatility of the stock price; the risk-free rate; and dividends. The value of stock options have upper and lower pricing bounds. Be familiar with these pricing bounds as well as the relationships that exist between the value of European and American options.

Six Factors That Affect Option Prices

LO 42.1: Identify the six factors that affect an option's price and describe how these six factors affect the price for both European and American options.

The following six factors will impact the value of an option:

1. S_0 = current stock price.
2. X = strike price of the option.
3. T = time to expiration of the option.
4. r = short-term risk-free interest rate over *T*.
5. D = present value of the dividend of the underlying stock.
6. σ = expected volatility of stock prices over *T*.

When evaluating a change in any one of the factors, hold the other factors constant.

Current Price of the Stock

For call options, as *S* increases (decreases), the value of the call increases (decreases). For put options, as *S* increases (decreases), the value of the put decreases (increases). This simply states that as an option becomes closer to or more in-the-money, its value increases.

Strike Price of the Option

The effect of strike prices on option values will be exactly the opposite of the effect of the current price of the stock. For call options, as *X* increases (decreases), the value of the call decreases (increases). For put options, as *X* increases (decreases), the value of the put increases (decreases). This is the same as the logic for the current price of the stock: the option's value will increase as it becomes closer to or more in-the-money.

The Time to Expiration

For American-style options, increasing time to expiration will increase the option value. With more time, the likelihood of being in-the-money increases. A general statement cannot be made for European-style options. Suppose we have a 1-month and 3-month call option on the same underlying with the same exercise price. Also suppose a large dividend is expected to be paid in two months. Because the stock price and 3-month option price will fall when the dividend is paid in two months, the 1-month option may be worth more than the 3-month option.

The Risk-Free Rate Over the Life of the Option

As the risk-free rate increases, the value of the call (put) will increase (decrease). The intuition behind this property involves arbitrage arguments that require the use of synthetic securities.

Dividends

The option owner does not have access to the cash flows of the underlying stock, and the stock price decreases when a dividend is paid. Thus, as the dividend increases, the value of the call (put) will decrease (increase).

Volatility of the Stock Price Over the Life of the Option

Volatility is the friend of all options. As volatility increases, option values increase. This is due to the asymmetric payoff of options. Since long option positions have a maximum loss equal to the premium paid, increased volatility only increases the chances that the option will expire in-the-money. Many consider volatility to be the most important factor for option valuation.

Figure 1 summarizes the factors' effects on option prices: "+" indicates a positive effect on option price from an increase in the factor, and "–" is a negative effect on option price.

Figure 1: Summary of Effects of Increasing a Factor on the Price of an Option

Factor	*European Call*	*European Put*	*American Call*	*American Put*
S	+	–	+	–
X	–	+	–	+
T	?	?	+	+
σ	+	+	+	+
r	+	–	+	–
D	–	+	–	+

Upper and Lower Pricing Bounds

LO 42.2: Identify and compute upper and lower bounds for option prices on non-dividend and dividend paying stocks.

In addition to those previously introduced, consider the following variables:

- c = value of a European call option.
- C = value of an American call option.
- p = value of a European put option.
- P = value of an American put option.
- S_T = value of the stock at expiration.

Also, assume in the following examples that there are no transaction costs, all profits are taxed at the same rate, and borrowing and lending can be done at the risk-free rate.

Upper Pricing Bounds for European and American Options

A call option gives the right to purchase one share of stock at a certain price. Under no circumstance can the option be worth more than the stock. If it were, everyone would sell the option and buy the stock and realize an arbitrage profit. We express this as:

$$c \leq S_0 \text{ and } C \leq S_0$$

Similarly, a put option gives the right to sell one share of stock at a certain price. Under no circumstance can the put be worth more than the sale or strike price. If it were, everyone would sell the option and invest the proceeds at the risk-free rate over the life of the option. We express this as:

$$p \leq X \text{ and } P \leq X$$

For a European put option, we can further reduce the upper bound. Since it cannot be exercised early, it can never be worth more than the present value of the strike price:

$$p \leq Xe^{-rT}$$

Lower Pricing Bounds for European Calls on Nondividend-Paying Stocks

Consider the following two portfolios:

- Portfolio P_1: one European call, *c*, with exercise price *X* plus a zero-coupon risk-free bond that pays *X* at *T*.
- Portfolio P_2: one share of the underlying stock, *S*.

At expiration, *T*, Portfolio P_1 will always be the greater of *X* (when the option expires out-of-the-money) or S_T (when the option expires in-the-money). Portfolio P_2, on the other hand, will always be worth S_T. Therefore, P_1 is always worth at least as much as P_2 at

expiration. If we know that at T, $P_1 \geq P_2$, then it always has to be true because if it were not, arbitrage would be possible. Therefore, we can state the following:

$$c + Xe^{-rT} \geq S_0$$

Since the value of a call option cannot be negative (if the option expires out-of-the-money, its value will be zero), the lower bound for a European call on a nondividend-paying stock is:

$$c \geq \max (S_0 - Xe^{-rT}, 0)$$

Lower Pricing Bounds for European Puts on Nondividend-Paying Stocks

Consider the following two portfolios:

- Portfolio P_3: one European put, p, plus one share of the underlying stock, S.
- Portfolio P_4: zero-coupon risk-free bond that pays X at T.

At expiration, T, Portfolio P_3 will always be the greater of X (when the option expires in-the-money) or S_T (when the option expires out-of-the-money). Portfolio P_4, on the other hand, will always be worth X. Therefore, P_3 is always worth at least as much as P_4 at expiration. If we know that at T, $P_3 \geq P_4$, it has to be true always because if it were not, arbitrage would be possible. Therefore, we can state the following:

$$p + S_0 \geq Xe^{-rT}$$

Since the value of a put option cannot be negative (if the option expires out-of-the-money, its value will be zero), the lower bound for a European put on a nondividend-paying stock is:

$$p \geq \max (Xe^{-rT} - S_0, 0)$$

Computing Option Values Using Put-Call Parity

LO 42.3: Explain put-call parity and apply it to the valuation of European and American stock options.

The derivation of **put-call parity** is based on the payoffs of two portfolio combinations, a fiduciary call and a protective put.

A *fiduciary call* is a combination of a pure-discount (i.e., zero coupon), riskless bond that pays X at maturity and a call with exercise price X. The payoff for a fiduciary call at expiration is X when the call is out of the money, and $X + (S - X) = S$ when the call is in the money.

A *protective put* is a share of stock together with a put option on the stock. The expiration date payoff for a protective put is (X – S) + S = X when the put is in the money, and *S* when the put is out of the money.

Professor's Note: When working with put-call parity, it is important to note that the exercise prices on the put and the call and the face value of the riskless bond are all equal to X.

When the put is in the money, the call is out of the money, and both portfolios pay *X* at expiration.

Similarly, when the put is out of the money and the call is in the money, both portfolios pay *S* at expiration.

Put-call parity holds that portfolios with identical payoffs must sell for the same price to prevent arbitrage. We can express the put-call parity relationship as:

$$c + Xe^{-rT} = S + p$$

Equivalencies for each of the individual securities in the put-call parity relationship can be expressed as:

$$S = c - p + Xe^{-rT}$$
$$p = c - S + Xe^{-rT}$$
$$c = S + p - Xe^{-rT}$$
$$Xe^{-rT} = S + p - c$$

The single securities on the left-hand side of the equations all have exactly the same payoffs as the portfolios on the right-hand side. The portfolios on the right-hand side are the "synthetic" equivalents of the securities on the left. Note that the options must be European-style and the puts and calls must have the same exercise price for these relations to hold.

For example, to synthetically produce the payoff for a long position in a share of stock, you use the relationship:

$$S = c - p + Xe^{-rT}$$

This means that the payoff on a long stock can be synthetically created with a long call, a short put, and a long position in a risk-free discount bond.

The other securities in the put-call parity relationship can be constructed in a similar manner.

Professor's Note: After expressing the put-call parity relationship in terms of the security you want to synthetically create, the sign on the individual securities will indicate whether you need a long position (+ sign) or a short position (– sign) in the respective securities.

Example: Call option valuation using put-call parity

Suppose that the current stock price is $52 and the risk-free rate is 5%. You have found a quote for a 3-month put option with an exercise price of $50. The put price is $1.50, but due to light trading in the call options, there was not a listed quote for the 3-month, $50 call. **Estimate** the price of the 3-month call option.

Answer:

Rearranging put-call parity, we find that the call price is:

$$\text{call} = \text{put} + \text{stock} - Xe^{-rT}$$

$$\text{call} = \$1.50 + \$52 - \$50e^{-0.0125} = \$4.12$$

This means that if a 3-month, $50 call is available, it should be priced at $4.12 per share.

Lower Pricing Bounds for an American Call Option on a Nondividend-Paying Stock

LO 42.4: Explain the early exercise features of American call and put options.

Recall the following equation from our earlier discussion of the lower pricing bounds for a *European* call option:

$$c \geq \max(S_0 - Xe^{-rT}, 0)$$

Since the only difference between an American option and a European option is that the American option can be exercised early, American options can always be used to replicate their corresponding European options simply by choosing not to exercise them until expiration. Therefore, it follows that:

$$C \geq c \geq \max(S_0 - Xe^{-rT}, 0)$$

Note that when an American call is exercised, it is only worth $S_0 - X$. Since this value is never larger than $S_0 - Xe^{-rT}$ for any r and $T > 0$, it is never optimal to exercise early. In other words, the investor can keep the cash equal to X, which would be used to exercise the option early, and invest that cash to earn interest until expiration. Since exercising the American call early means that the investor would have to forgo this interest, it is never optimal to exercise an American call on a nondividend-paying stock before the expiration date (i.e., $c = C$).

Lower Pricing Bounds for an American Put Option on a Nondividend-Paying Stock

While it is never optimal to exercise an American call on a nondividend-paying stock, American puts are optimally exercised early if they are sufficiently in-the-money. If an option is sufficiently in-the-money, it can be exercised, and the payoff $(X - S_0)$ can be invested to earn interest. In the extreme case when S_0 is close to zero, the future value of the exercised cash value, Xe^{rT}, is always worth more than a later exercise, *X*. We know that:

$P \geq p \geq \max (Xe^{-rT} - S_0, 0)$ for the same reasons that $C \geq c$

However, we can place an even stronger bound on an American put since it can always be exercised early:

$P \geq \max (X - S_0, 0)$

Figure 2 summarizes what we now know regarding the boundary prices for American and European options.

Figure 2: Lower and Upper Bounds for Options

Option	*Minimum Value*	*Maximum Value*
European call	$c \geq \max (0, S_0 - Xe^{-rT})$	S_0
American call	$C \geq \max (0, S_0 - Xe^{-rT})$	S_0
European put	$p \geq \max (0, Xe^{-rT} - S_0)$	Xe^{-rT}
American put	$P \geq \max (0, X - S_0)$	X

Professor's Note: For the exam, know the price limits in Figure 2. You will not be asked to derive them, but you may be expected to use them.

Example: Minimum prices for American vs. European puts

Compute the lowest possible price for 4-month American and European 65 puts on a stock that is trading at 63 when the risk-free rate is 5%.

Answer:

$P \geq \max (0, X - S_0) = \max (0, 2) = \$2$

$p \geq \max (0, Xe^{-rT} - S_0) = \max (0, 65e^{-0.0167} - 63) = \$0.92$

Example: Minimum prices for American vs. European calls

Compute the lowest possible price for 3-month American and European 65 calls on a stock that is trading at 68 when the risk-free rate is 5%.

Answer:

$$C \geq \max(0, S_0 - Xe^{-rT}) = \max(0, 68 - 65e^{-0.0125}) = \$3.81$$

$$c \geq \max(0, S_0 - Xe^{-rT}) = \max(0, 68 - 65e^{-0.0125}) = \$3.81$$

Relationship Between American Call Options and Put Options

Put-call parity only holds for European options. For American options, we have an inequality. This inequality places upper and lower bounds on the difference between the American call and put options.

$$S_0 - X \leq C - P \leq S_0 - Xe^{-rT}$$

Example: American put option bounds

Consider an American call and put option on stock XYZ. Both options have the same 1-year expiration and a strike price of $20. The stock is currently priced at $22, and the annual interest rate is 6%. What are the upper and lower bounds on the American put option if the American call option is priced at $4?

Answer:

The upper and lower bounds on the difference between the American call and American put options are:

$$S_0 - X \leq C - P \leq S_0 - Xe^{-rT}$$

$$S_0 - X = 22 - 20 = \$2$$

$$S_0 - Xe^{-rT} = 22 - 20e^{-0.06(1)} = 22 - 18.84 = \$3.16$$

$$\$2 \leq C - P \leq \$3.16$$

or

$$-\$2 \geq P - C \geq -\$3.16$$

Therefore, when the American call is valued at $4, the upper and lower bounds on the American put option will be:

$$\$2 \geq P \geq \$0.84$$

The Impact of Dividends on Option Pricing Bounds

Since most stock options have an expiration of less than a year, dividends can be estimated fairly accurately. Recall that to prevent arbitrage, when a stock pays a dividend, its value must decrease by the amount of the dividend. This increases the value of a put option and decreases the value of a call option.

Consider the following portfolios:

- Portfolio P_6: one European call option, *c*, plus cash equal to $D + Xe^{-rT}$.
- Portfolio P_7: one share of the underlying stock, *S*.

Similar to the development of the $c \geq \max (S_0 - Xe^{-rT}, 0)$ equation, Portfolio P_6 is always at least as large as P_7, or:

$$c \geq S_0 - D - Xe^{-rT}$$

All else equal, the payment of a dividend will reduce the lower pricing bound for a call option.

For put options:

- Portfolio P_8: one European put, *p*, plus one share of the underlying stock, *S*.
- Portfolio P_9: cash equal to $D + Xe^{-rT}$.

Using the same development as the $p \geq \max (Xe^{-rT} - S_0, 0)$ equation:

$$p \geq D + Xe^{-rT} - S_0$$

All else equal, the payment of a dividend will increase the lower pricing bound for a put option.

Impact of Dividends on Early Exercise for American Calls and Put-Call Parity

When the dividend is large enough, American calls might be optimally exercised early. This will be the case if the amount of the dividend exceeds the amount of interest that is forgone as a result of the early exercise. Note that if a large dividend makes early exercise optimal, exercise should take place immediately before the ex-dividend date. Put-call parity is adjusted for dividends in the following manner:

$$p + S_0 = c + D + Xe^{-rT}$$

This equation is verified using the same development as was used to derive the $p + S_0 = c + Xe^{-rT}$ equation. The $S_0 - X \leq C - P \leq S_0 - Xe^{-rT}$ equation that we used to show the relationship between American call and put options is modified as follows:

$$S_0 - X - D \leq C - P \leq S_0 - Xe^{-rT}$$

Key Concepts

LO 42.1

Six factors influence the value of an option: current value of the underlying asset (stock); the strike price; the time to expiration of the option; the volatility of the stock price; the risk-free rate; and dividends.

With the exception of time to expiration, all of these factors affect European- and American-style options in the same way.

LO 42.2

Call options cannot be worth more than the underlying security, and put options cannot be worth more than the strike price.

When the stock does not pay a dividend, European call options cannot be worth less than the difference between the current stock price and the present value of the strike price. European put options cannot be worth less than the difference between the present value of the strike price and the current stock price.

LO 42.3

Put-call parity is a no-arbitrage relationship for European-style options with the same characteristics. It states that a portfolio consisting of a call option and a zero-coupon bond with a face value equal to the strike must have the same value as a portfolio consisting of the corresponding put option and the stock:

$$p + S_0 = c + Xe^{-rT}$$

LO 42.4

It is never optimal to exercise an American call option on nondividend-paying stock prior to expiration.

American put options on nondividend-paying stocks can be optimally exercised prior to expiration if the put is sufficiently in-the-money.

Call options are always worth more than corresponding put options prior to expiration when both are at-the-money.

The difference between prices of an American call and corresponding put is bounded below by the difference between the current stock price and strike price, and above by the difference between the current stock price and the present value of the strike price.

Concept Checkers

1. Which of the following will not cause a decrease in the value of a European call option position on XYZ stock?
   A. XYZ declares a 3-for-1 stock split.
   B. XYZ raises its quarterly dividend from $0.15 per share to $0.17 per share.
   C. The Federal Reserve lowers interest rates by 0.25% in an effort to stimulate the economy.
   D. Investors believe the volatility of XYZ stock has declined.

2. Consider a European put option on a stock trading at $50. The put option has an expiration of six months, a strike price of $40, and a risk-free rate of 5%. The lower bound and upper bound on the put are:
   A. $10, $40.00.
   B. $10, $39.01.
   C. $0, $40.00.
   D. $0, $39.01.

3. Consider a 1-year European put option that is currently valued at $5 on a $25 stock and a strike of $27.50. The 1-year risk-free rate is 6%. Which of the following is closest to the value of the corresponding call option?
   A. $0.00.
   B. $3.89.
   C. $4.10.
   D. $5.00.

4. Consider an American call and put option on the same stock. Both options have the same 1-year expiration and a strike price of $45. The stock is currently priced at $50, and the annual interest rate is 10%. Which of the following could be the difference in the two option values?
   A. $4.95.
   B. $7.95.
   C. $9.35.
   D. $12.50.

5. According to put-call parity for European options, purchasing a put option on ABC stock would be equivalent to:
   A. buying a call, buying ABC stock, and buying a zero-coupon bond.
   B. buying a call, selling ABC stock, and buying a zero-coupon bond.
   C. selling a call, selling ABC stock, and buying a zero-coupon bond.
   D. buying a call, selling ABC stock, and selling a zero-coupon bond.

Concept Checker Answers

1. **A** After a stock split, both the price of the stock and the strike price of the option will be adjusted, so the value of the option position will be the same. An increase in the dividend, a lower risk-free interest rate, and lower volatility of the price of the underlying stock, will all decrease the value of a European call option.

2. **D** The upper bound is the present value of the exercise price: $\$40 \times e^{-0.05 \times 0.5} = \$39.01$. Since the put is out-of-the-money, the lower bound is zero.

3. **C** $c = p - Xe^{-rT} + S_0 = \$5 - \$27.50e^{-0.06 \times 1} + \$25 = \$4.10$

4. **B** The upper and lower bounds are: $S_0 - X \leq C - P \leq S_0 - Xe^{-rT}$ or $\$5 \leq C - P \leq \$9.28$. Only $7.95 falls within the bounds.

5. **B** The formula for put-call parity is $p + S_0 = c + Xe^{-rT}$. Rearranging to solve for the price of a put, we have $p = c - S_0 + Xe^{-rT}$.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Trading Strategies Involving Options

Topic 43

Exam Focus

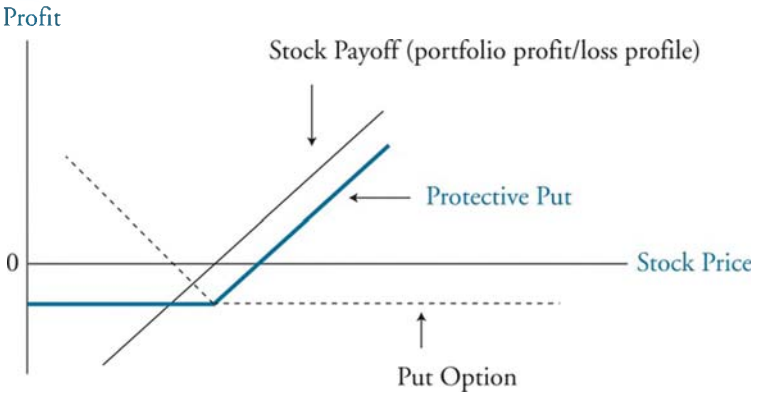
Traders and investors use option-based trading strategies to create an extraordinary spectrum of payoff profiles. This enables investors to take positions based on almost any possible expectation of the underlying stock over the life of the options. This topic describes the common option trading strategies and implementation. For the exam, know the general payoff graphs for each strategy discussed. In addition, know how to calculate the payoff for some of the more popular strategies including protective put, covered call, bull call spread, butterfly spread, and straddle.

Covered Calls and Protective Puts

LO 43.1: Explain the motivation to initiate a covered call or a protective put strategy.

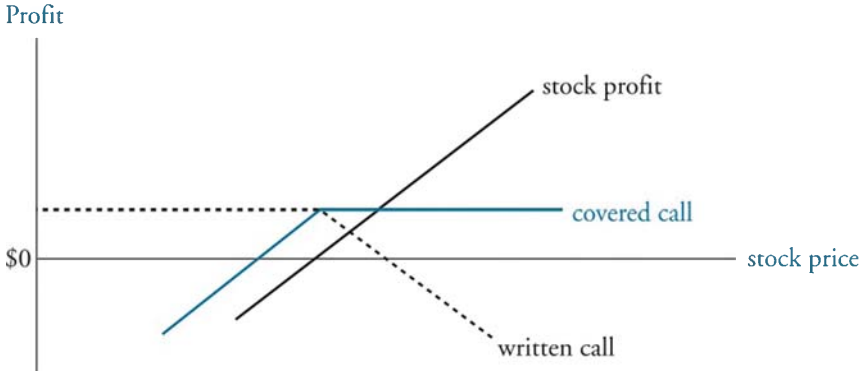
When an at-the-money long put position is combined with the underlying stock, we have created a **protective put** strategy. A protective put (also called *portfolio insurance* or a *hedged portfolio*) is constructed by holding a long position in the underlying security and buying a put option. You can use a protective put to limit the downside risk at the cost of the put premium, P_0. You will see by the diagram in Figure 1 that the investor will still be able to benefit from increases in the stock's price, but it will be lower by the amount paid for the put, P_0. Notice that the combined strategy looks very much like a call option. This should not be surprising since put-call parity requires that $p + S_0$ be the same as $c + Xe^{-rT}$. Figure 1 illustrates this property.

Figure 1: Protective Put Strategy



Another common strategy is to sell a call option on a stock that is owned by the option writer. This is called a **covered call** position. By writing an out-of-the-money call option, the combined position caps the upside potential at the strike price. In return for giving up any potential gain beyond the strike price, the writer receives the option premium. This strategy is used to generate cash on a stock that is not expected to increase above the exercise price over the life of the option.

Figure 2: Profit Profile for a Covered Call



Spread Strategies

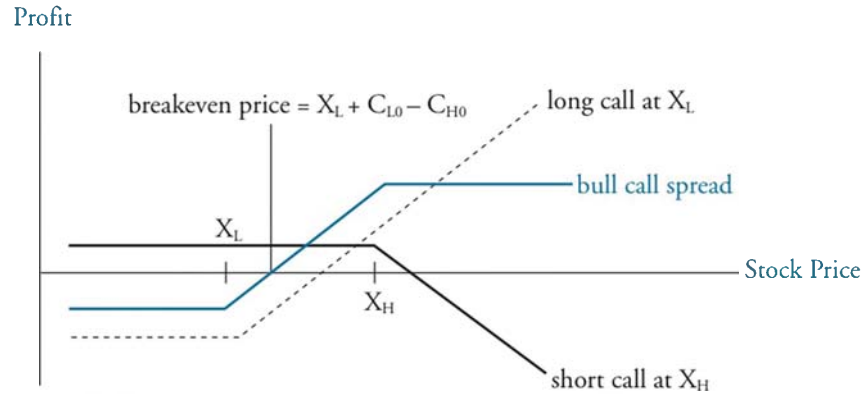
LO 43.2: Describe the use and calculate the payoffs of various spread strategies.

Several spread strategies exist. These strategies combine options positions to create a desired payoff profile. The differences between the options are either the strike prices and/or the time to expiration. We will discuss bull and bear spreads, butterfly spreads, calendar spreads, and diagonal spreads.

Bull and Bear Spreads

In a *bull call spread,* the buyer of the spread purchases a call option with a low exercise price, X_L, and subsidizes the purchase price of the call by selling a call with a higher exercise price, X_H. The buyer of a bull call spread expects the stock price to rise and the purchased call to finish in-the-money. However, the buyer does not believe that the price of the stock will rise above the exercise price for the out-of-the-money written call.

Figure 3: Bull Call Spread



Example: Bull call spread

An investor purchases a call for C_{L0} = \$3.00 with a strike of X = \$40 and sells a call for C_{H0} = \$1.00 with a strike price of \$50. **Compute** the payoff of a bull call spread strategy when the price of the stock is at \$45.

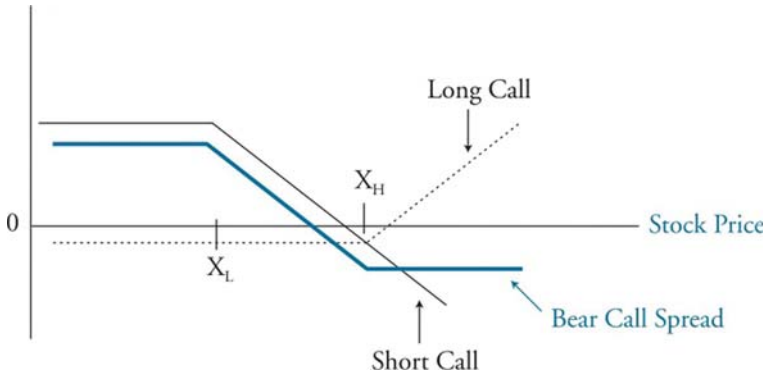
Answer:

$$\text{profit} = \max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0}$$

$$\text{profit} = \max(0, 45 - 40) - \max(0, 45 - 50) - 3 + 1 = \$3.00$$

A *bear call spread* is the sale of a bull spread. That is, the bear spread trader will purchase the call with the higher exercise price and sell the call with the lower exercise price. This strategy is designed to profit from falling stock prices (i.e., a "bear" strategy). As stock prices fall, the investor keeps the premium from the written call, net of the long call's cost. The purpose of the long call is to protect from sharp increases in stock prices. The payoff is the opposite (mirror image) of the bull call spread and is shown in Figure 4.

Figure 4: Bear Call Spread



Puts can also be used to replicate the payoffs for both a bull call spread and a bear call spread. In a *bear put spread* the investor buys a put with a higher exercise price and sells a put with a lower exercise price.

Example: Bear put spread

An investor sells a put for P_{L0} = \$3.00 with a strike of X = \$20 and purchases a put for P_{H0} = \$4.50 with a strike price of \$40. **Compute** the payoff of a bear put spread strategy when the price of the stock is at \$35.

Answer:

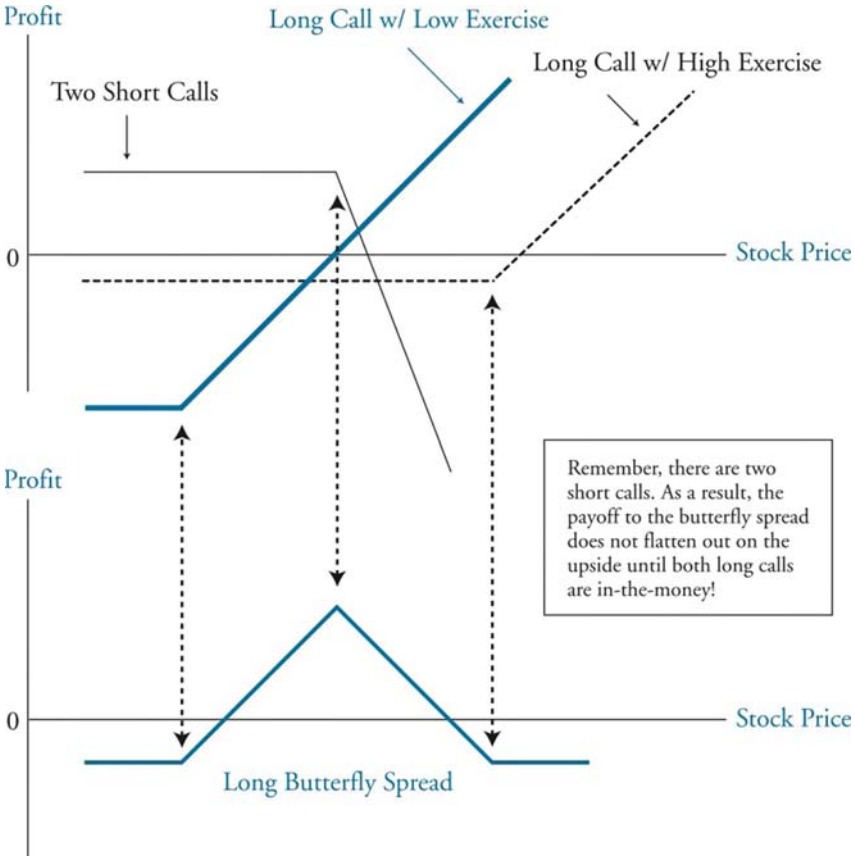
$$\text{profit} = \max(0, X_H - S_T) - \max(0, X_L - S_T) - P_{H0} + P_{L0}$$

$$\text{profit} = \max(0, 40 - 35) - \max(0, 20 - 35) - 4.50 + 3 = \$3.50$$

Butterfly Spreads

A *butterfly spread* involves the purchase or sale of *three* different call options. Here, the investor buys one call with a low exercise price, buys another call with a high exercise price, and sells *two* calls with an exercise price in between. The buyer of a butterfly spread is essentially betting that the stock price will stay near the strike price of the written calls. However, the loss that the butterfly spread buyer sustains if the stock price strays from this level is limited. The two graphs in Figure 5 illustrate the construction and payoffs of a butterfly spread.

Figure 5: Butterfly Spread Construction and Behavior



Example: Butterfly spread with calls

An investor makes the following transactions in calls on a stock:

- Buys one call defined by C_{L0} = \$ 7.00 and X_L = \$55.
- Buys one call defined by C_{H0} = \$2.00 and X_H = \$65.
- Sell two calls defined by C_{M0} = \$4.00 and X_M = \$60.

Compute the payoff of a butterfly spread strategy with calls when the stock is at \$60.

Answer:

$$\text{profit} = \max(0, S_T - X_L) - 2\max(0, S_T - X_M) + \max(0, S_T - X_H) - C_{L0} + 2C_{M0} - C_{H0}$$

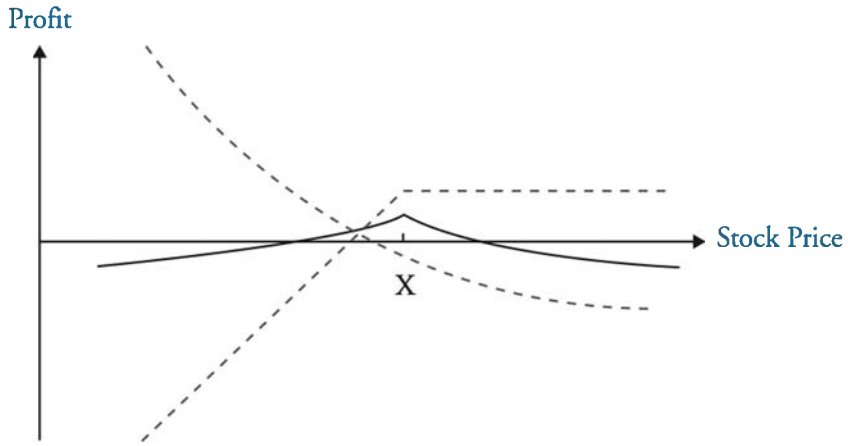
$$\text{profit} = \max(0, 60 - 55) - 2\max(0, 60 - 60) + \max(0, 60 - 65) - 7 + 2(4) - 2 = \$4.00$$

To create a butterfly spread with put options, the investor would buy a low and high strike put option and sell two puts with an intermediate strike price. Again, the combined position is constructed by summing the payoffs of the individual options at each stock price.

Calendar Spreads

A *calendar spread* is created by transacting in two options that have the same strike price but different expirations. Figure 6 shows a calendar spread using put options. The strategy sells the short-dated option and buys the long-dated option. Notice that the payoff here is similar to the butterfly spread. The investor profits only if the stock remains in a narrow range, but losses are limited. In this case, the losses are not symmetrical as they are in the butterfly spread. A calendar spread based on calls is created in similar fashion.

Figure 6: Calendar Spread (Using Two Put Options)



Calendar spreads are categorized differently depending on the relationship between the strike price and the current stock price. The strategy is referred to as a **neutral calendar spread** if the strike price is close to the current stock price. A **bullish calendar spread** has a strike price above the current stock price, and a **bearish calendar spread** has a strike price below the current stock price.

A **reverse calendar spread** produces a payoff that is opposite of the graph shown in Figure 6. Instead of selling a short-dated option and buying a long-dated option, the investor of a reverse calendar spread will buy a short-dated option and sell a long-dated option. The investor will profit when the stock is well above or below the strike price and will suffer a loss if the stock is near the strike price.

Diagonal Spreads

A *diagonal spread* is similar to a calendar spread except that instead of using options with the same strike price and different expirations, the options in a diagonal spread can have different strike prices in addition to different expirations.

Box Spreads

A *box spread* is a combination of a bull call spread and a bear put spread on the same asset. This strategy will produce a constant payoff that is equal to the high exercise price (X_H) minus the low exercise price (X_L). Under a no arbitrage assumption, the present value of the payoff will equal the net premium paid (i.e., profit will equal zero).

When the profit from this strategy is different than zero, an investor can capitalize on the arbitrage opportunity by either buying or selling the box. If the profit is positive, the investor will create a long box spread by buying a call at X_L, selling a call at X_H, buying a put at X_H, and selling a put at X_L. If the profit is negative, the investor will create a short box spread by buying a call at X_H, selling a call at X_L, buying a put at X_L, and selling a put at X_H. Note that box spread arbitrage is only successful with European options.

Combination Strategies

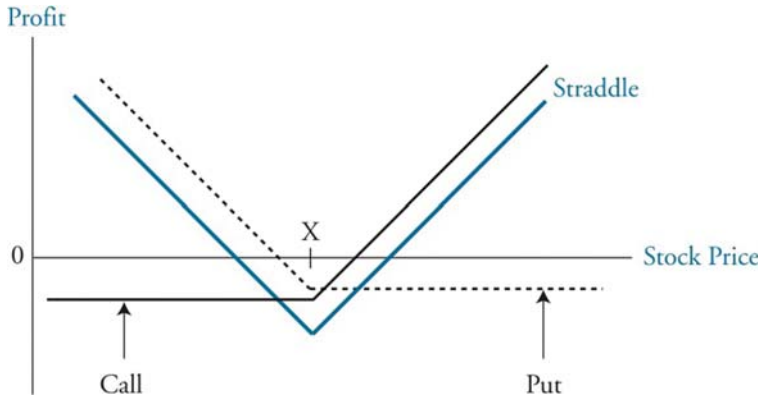
LO 43.3: Describe the use and explain the payoff functions of combination strategies.

Combinations are option strategies involving both puts and calls. We will discuss straddles, strangles, strips, and straps.

Straddle

A long *straddle* (bottom straddle or straddle purchase) is created by purchasing a call and a put with the same strike price and expiration. Figure 7 illustrates the payoff for a long straddle position. Both options have the same exercise price and expiration. Note that this strategy is profitable when the stock price moves strongly in either direction. This strategy bets on volatility. A short straddle (top straddle or straddle write) sells both options and bets on little movement in the stock. A short straddle bets on the same thing as the butterfly spread or the calendar spread, except the losses are not limited. It is a bet that will profit more if correct but also lose more if it is incorrect. Straddles are symmetric around the strike price.

Figure 7: Long Straddle Profit/Loss



Example: Straddle

An investor purchases a call on a stock, with an exercise price of $45 and a premium of $3, and purchases a put option with the same maturity that has an exercise price of $45 and a premium of $2. **Compute** the payoff of a straddle strategy if the stock is at $35.

Answer:

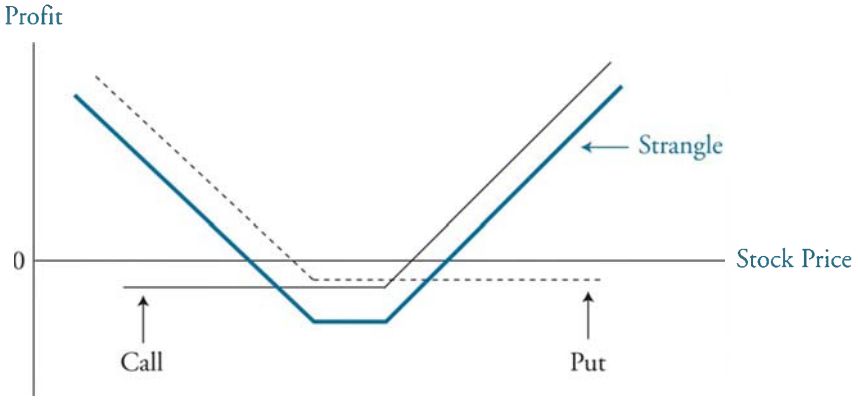
$$\text{profit} = \max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0$$

$$\text{profit} = \max(0, 35 - 45) + \max(0, 45 - 35) - 3 - 2 = \$5$$

Strangle

A *strangle* (or bottom vertical combination) is similar to a straddle except that the options purchased are slightly out-of-the-money, so it is cheaper to implement than the straddle. The payoff is similar to the straddle except for a flat section between the strike prices, as shown in Figure 8. Because it is cheaper, the stock will have to move more relative to the straddle before the strangle pays off. Strangles are also symmetric around the strikes.

Figure 8: Long Strangle Profit/Loss



Example: Strangle

An investor purchases a call on a stock, with an exercise price of $50 and a premium of $1.50, and purchases a put option with the same maturity that has an exercise price of $45 and a premium of $2. **Compute** the payoff of a strangle strategy if the stock is at $40.

Answer:

$$\text{profit} = \max(0, S_T - X_H) + \max(0, X_L - S_T) - C_0 - P_0$$

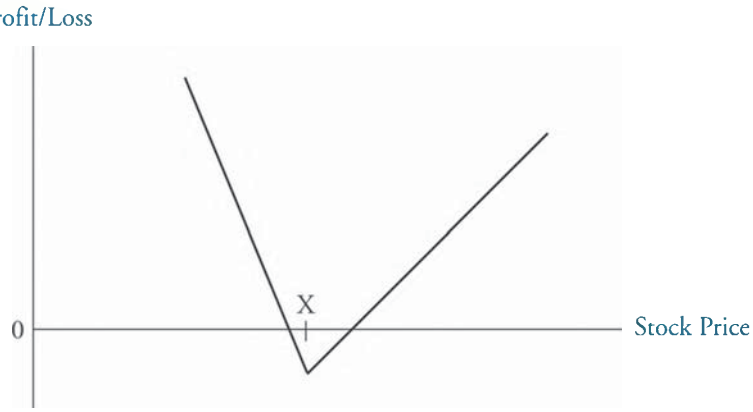
$$\text{profit} = \max(0, 40 - \$50) + \max(0, 45 - 40) - 1.50 - 2 = \$1.50$$

A short strangle (or a top vertical combination) is similar to the short straddle.

Strips and Straps

A *strip* involves purchasing two puts and one call with the same strike price and expiration. Figure 9 illustrates a strip. Notice the asymmetry of the payoff. A strip is betting on volatility but is more bearish since it pays off more on the downside.

Figure 9: Strip Profit/Loss

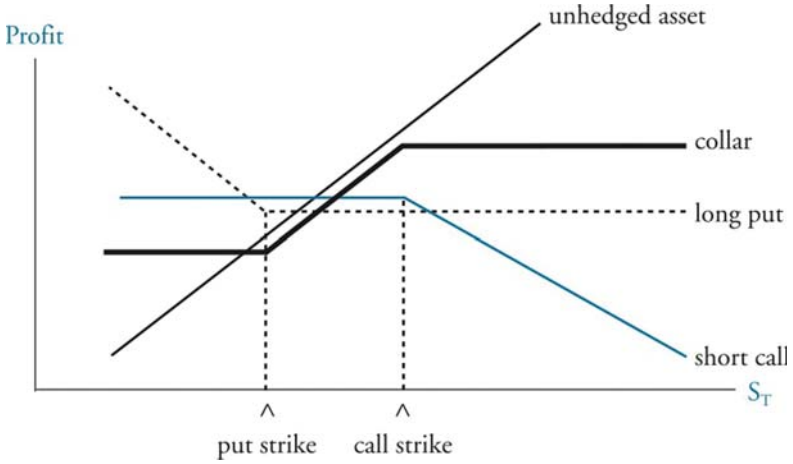


A *strap* involves purchasing two calls and one put with the same strike price and expiration. A strap is betting on volatility but is more bullish since it pays off more on the upside.

Collar

A *collar* is the combination of a protective put and covered call. The usual goal is for the owner of the underlying asset to buy a protective put and then sell a call to pay for the put. If the premiums of the two are equal, it is called a **zero-cost collar**.

Figure 10: Collar



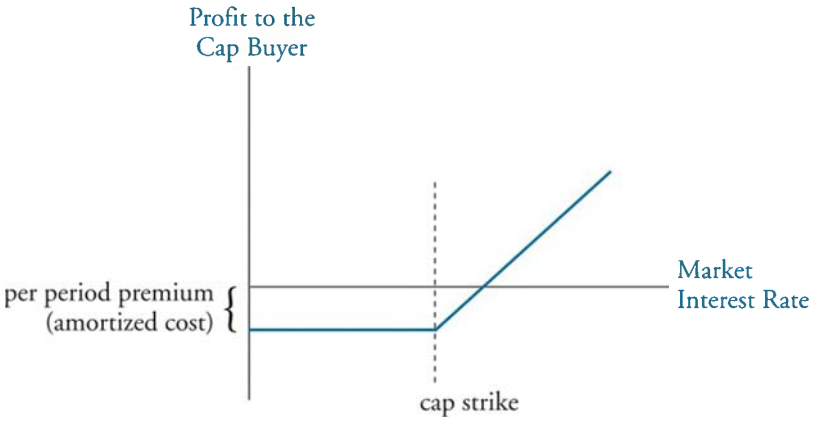
Interest Rate Caps and Floors

An **interest rate cap** is an agreement in which one party agrees to pay the other at regular intervals over a certain period of time when the benchmark interest rate (e.g., LIBOR) exceeds the strike rate specified in the contract. This strike rate is called the **cap rate**. For example, the seller of a cap might agree to pay the buyer at the end of any quarter over the next two years if LIBOR is greater than a cap rate of 6%.

The buyer of a cap has a position similar to that of a buyer of a call on LIBOR, both of whom benefit when interest rates rise. Because an interest rate cap is a multi-period agreement, a cap is actually a portfolio of call options on LIBOR called **caplets**. For example, the 2-year cap discussed above is actually a portfolio of eight interest rate options with different maturity dates.

The cap buyer pays a premium to the seller and exercises the cap if the market rate of interest rises above the cap strike. The diagram in Figure 11 illustrates the profits of an interest rate cap at the end of one particular settlement period. It has the familiar shape of a long position in a call option.

Figure 11: Profit to a Long Cap

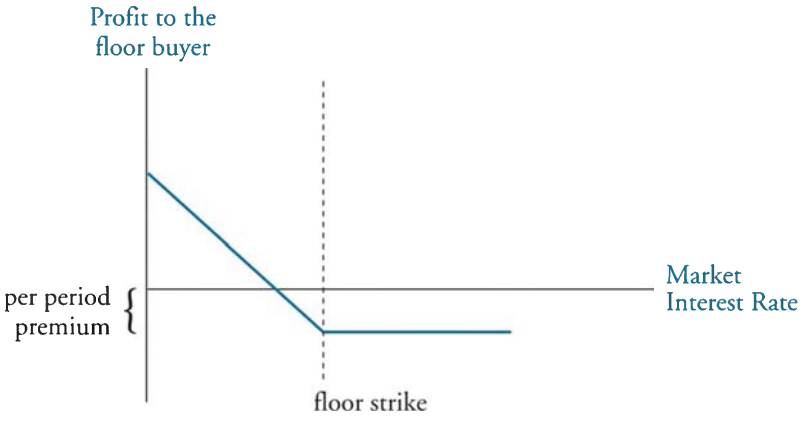


An **interest rate floor** is an agreement in which one party agrees to pay the other at regular intervals over a certain time period when the benchmark interest rate (e.g., LIBOR) falls below the strike rate specified in the contract. This strike rate is called the **floor rate**. For example, the seller of a floor might agree to pay the buyer at the end of any quarter over the next two years if LIBOR is less than a floor rate of 4%.

The buyer of a floor benefits from an interest rate decrease and, therefore, has a position that is similar to that of a buyer of a put on LIBOR, who benefits when interest rates fall and the price of the instrument rises. Once again, because a floor is a multi-period agreement, a floor is actually a portfolio of put options on LIBOR called **floorlets**.

The floor buyer pays a premium and exercises the floor if the market rate of interest falls below the floor strike. The diagram in Figure 12 illustrates the profits of an interest rate floor at the end of one particular settlement period. It has the same shape as a long put option.

Figure 12: Profit to a Long Floor



Options are traded both on *interest rates* and on *prices* of fixed-income securities. So far we've talked about options on interest rates. The values of comparable options on rates and prices respond differently to changes in interest rates because of the inverse relationship

between bond yields and bond prices. Figure 13 outlines how each type of option responds to changes in yields and bond prices.

Figure 13: Options on Rate vs. Options on Prices

Option	*If Rates* Increase *and Bond Prices* Decrease	*If Rates* Decrease *and Bond Prices* Increase
Value of call on LIBOR	Increases	Decreases
Value of call on bond price	Decreases	Increases
Value of put on LIBOR	Decreases	Increases
Value of put on bond price	Increases	Decreases

We can also interpret caps and floors in terms of options on the prices of fixed-income securities:

- A long cap is equivalent to a portfolio of long put options on fixed-income security prices.
- A long floor is equivalent to a portfolio of long call options on fixed-income security prices.

An **interest rate collar** is a simultaneous position in a floor and a cap on the same benchmark rate over the same period with the same settlement dates. There are two types of collars:

- The first type of collar is to purchase a cap and sell a floor. For example, an investor with a LIBOR-based liability could purchase a cap on LIBOR at 8% and simultaneously sell a floor on LIBOR at 4% over the next year. The investor has now hedged the liability so that the borrowing costs will stay within the "collar" of 4% to 8%. If the cap and floor rates are set so that the premium paid from buying the cap is exactly offset by the premium received from selling the floor, the collar is called a "zero-cost" collar.
- The second type of collar is to purchase a floor and sell a cap. For example, an investor with a LIBOR-based asset could purchase a floor on LIBOR at 3% and simultaneously sell a cap at 7% over the next year. The investor has now hedged the asset so the returns will stay within the collar of 3% to 7%. The investor can create a zero-cost collar by choosing the cap and floor rates so that the premium paid on the floor offsets the premium received on the cap.

Key Concepts

LO 43.1

Stock options can be combined with their underlying stock to generate various payoff profiles. A protective put combines an at-the-money long put position with the underlying stock. A covered call involves selling a call option on a stock that is owned by the option writer.

LO 43.2

Spread strategies combine options in the same option class to generate various payoff profiles.

The buyer of a bull call spread expects the stock price to rise and the purchased call to finish in-the-money. However, the buyer does not believe that the price of the stock will rise above the exercise price for the out-of-the-money written call.

The bear call spread trader will purchase the call with the higher exercise price and sell the call with the lower exercise price. This strategy is designed to profit from falling stock prices (i.e., a "bear" strategy). As stock prices fall, the investor keeps the premium from the written call, net of the long call's cost.

A box spread is an extreme method of locking in value. The dollar return for a box spread is fixed. It is a combination of a bull call spread and a bear put spread.

A calendar spread is created by transacting in two options that have the same strike price but different expirations.

The buyer of a butterfly spread is essentially betting that the stock price will stay near the strike price of the written calls. However, the loss that the butterfly spread buyer sustains if the stock price strays from this level is not large.

In a diagonal spread, options can have different strike prices and different expirations.

Bull call spread payoff:

$$\text{profit} = \max(0,S_T - X_L) - \max(0,S_T - X_H) - C_{L0} + C_{H0}$$

Bear put spread payoff:

$$\text{profit} = \max(0,X_H - S_T) - \max(0,X_L - S_T) - P_{H0} + P_{L0}$$

Butterfly spread payoff:

$$\text{profit} = \max(0,S_T - X_L) - 2\max(0,S_T - X_M) + \max(0,S_T - X_H) - C_{L0} + 2C_{M0} - C_{H0}$$

LO 43.3

Combination strategies combine puts and calls to generate various payoff strategies.

A long straddle (bottom straddle or straddle purchase) is created by purchasing a call and a put with the same strike price and expiration. Note that this strategy only pays off when the stock moves in either direction.

A strangle (or bottom vertical combination) is similar to a straddle except that the option purchased is slightly out-of-the-money, so it is cheaper to implement than the straddle.

A strip is betting on volatility but is more bearish since it pays off more on the down side.

A strap is betting on volatility but is more bullish since it pays off more on the up side.

Straddle payoff:

$$\text{profit} = \max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0$$

Strangle payoff:

$$\text{profit} = \max(0, S_T - X_H) + \max(0, X_L - S_T) - C_0 - P_0$$

Concept Checkers

1. An investor is very confident that a stock will change significantly over the next few months; however, the direction of the price change is unknown. Which strategies will most likely produce a profit if the stock price moves as expected?

   I. Short butterfly spread.
   II. Bearish calendar spread.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

2. Which of the following will create a bear spread?
   A. Buy a call with a strike price of X = 45 and sell a call with a strike price of X = 50.
   B. Buy a call with a strike price of X = 50 and buy a put with a strike price of X = 55.
   C. Buy a put with a strike price of X = 45 and sell a put with a strike price of X = 50.
   D. Buy a call with a strike price of X = 50 and sell a call with a strike price of X = 45.

3. An investor believes that a stock will either increase or decrease greatly in value over the next few months, but believes a down move is more likely. Which of the following strategies will be the best for this investor?
   A. A protective put.
   B. An at-the-money strip.
   C. An at-the-money strap.
   D. A top vertical combination.

4. An investor constructs a long straddle by buying an April $30 call for $4 and buying an April $30 put for $3. If the price of the underlying shares is $27 at expiration, what is the profit on the position?
   A. –$4.
   B. –$2.
   C. $2.
   D. $3.

5. Consider an option strategy where an investor buys one call option with an exercise price of $55 for $7, sells two call options with an exercise price of $60 for $4, and buys one call option with an exercise price of $65 for $2. If the stock price declines to $25, what will be the profit or loss on the strategy?
   A. –$3.
   B. –$1.
   C. $1.
   D. $2.

Concept Checker Answers

1. **A** A short butterfly spread will produce a modest profit if there is a large amount of volatility in the price of the stock. A bearish calendar spread is a play using options with different expiration dates.

2. **D** Spread strategies involve purchasing and selling an option of the same type. A bear spread with calls involves buying a call with a high strike price and selling a call with a low strike price. The investor profits if stock prices fall by keeping the premium from the written call, net of the premium from the purchased call. Note that a bear spread can also be constructed with put options by buying a put with a high strike price and selling a put with a low strike price. With a bear put spread, if the stock price declines and both puts are exercised, the investor receives the difference between the strike prices less the net premium paid.

3. **B** An at-the-money strip bets on volatility but is more bearish since it pays off more on the downside.

4. **A** The sum of the premiums paid for the position is $7. With the underlying stock at $27, the put will be worth $3, while the call option will be worthless. The value of the position is (–$7 + $3) = –$4.

5. **B** The strategy described is a butterfly spread where the investor buys a call with a low exercise price, buys another call with a high exercise price, and sell two calls with a price in between. In this case, if the option moves to $25, none of the call options will be in the money, so the profit is equal to the net premium paid, which is –$7 + (2 × $4) – $2 = –$1.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Exotic Options

Topic 44

Exam Focus

In this topic, we define and discuss the important characteristics of a variety of exotic options. The difference between exotic options and more traditional exchange-traded instruments is also highlighted. Be familiar with the payoff structures for the various exotic options discussed.

Evaluating Exotic Options

LO 44.1: Define and contrast exotic derivatives and plain vanilla derivatives.

LO 44.2: Describe some of the factors that drive the development of exotic products.

Plain vanilla derivatives include listed futures contracts and commonly used forwards and other over-the-counter (OTC) derivatives that are traded in fairly liquid markets. Exotic derivatives are customized to fit a specific firm need for hedging that cannot be met by plain vanilla derivatives. With plain vanilla derivatives, there is little uncertainty about the cost, the current market value, when they will pay, how much they will pay, and the cost of exiting the position. With exotic derivatives, some or all of these may be in question.

Exotic derivatives are developed for several reasons. The main purpose is to provide a unique hedge for a firm's underlying assets. Other reasons include addressing tax and regulatory concerns as well as speculating on the expected future direction of market prices.

Four questions that should be considered when evaluating exotic derivative strategies are:

- Will the strategy pay in the right circumstances to provide an effective hedge? Problems with understanding the payoff of the exotic derivative and credit risk of the derivative strategy can lead to a difference between the payoff the user expects and the actual payoff received.
- What is the cost of the exotic derivative hedging strategy?
- Is a pricing model needed, and does the user have the appropriate pricing model to estimate dealer cost and monitor the value of non-traded derivatives over time?
- How is a derivative position reversed? Note that the costs of exiting a position or strategy may involve penalties and large bid-ask spreads or require a pricing model to evaluate alternatives.

Using Packages to Formulate a Zero-Cost Product

LO 44.3: Explain how any derivative can be converted into a zero-cost product.

A package is defined as some combination of standard European options, forwards, cash, and the underlying asset. Bull, bear, and calendar spreads, as well as straddles and strangles, are examples of packages. Packages usually consist of selling one instrument with certain characteristics and buying another with somewhat different characteristics. Because packages often consist of a long position and a short position, they can be constructed so that the initial cost to the investor is zero.

For example, consider a zero-cost short collar. A short collar combines a long standard put option with an exercise price X_L and a short standard call option with exercise price X_H (where $X_L < X_H$). If the premium the investor pays for the put option is exactly offset by the premium the investor receives for the short call position, the investor's net cost for implementing the short collar strategy is zero. In any case where the investor's cash outflows from long positions are offset by cash inflows from short positions, the investor can use a package to create a zero-cost product.

Transforming Standard American Options Into Nonstandard American Options

LO 44.4: Describe how standard American options can be transformed into nonstandard American options.

Recall that standard exchange-traded American options can be exercised at any time prior to expiration. If some of the available expiration periods are restricted, or changes are made to other standard features, standard options become what we refer to as **nonstandard options**. Nonstandard options are common in the over-the-counter (OTC) market.

There are three common features that transform standard American options into nonstandard options:

- The most common transformation can be made to restrict early exercise to certain dates (e.g., a three month call option may only be exercised on the last day of each month.) This type of transformation results in a **Bermudan option.**
- Early exercise can be limited to a certain portion of the life of the option (e.g., there is a "lock out" period that does not allow a 6-month call option to be exercised in the first three months of the call's life).
- The option's strike price may change (e.g., the strike price of a 3-year call option with a strike price of 40 at initiation may rise to 44 in year 2 and 48 in year 3).

Exotic Option Payoff Structures

LO 44.5: Identify and describe the characteristics and pay-off structure of the following exotic options: gap, forward start, compound, chooser, barrier, binary, lookback, shout, Asian, exchange, rainbow, and basket options.

Gap Options

A gap option has two strike prices, X_1 and X_2. (X_2 is sometimes referred to as the trigger price.) If these two strike prices are equal, the gap option payoff will be the same as an ordinary option. If the two strike prices differ and the payoff for a gap option is non-zero, there will be a gap in the payoff graph that is either increased or decreased by the difference between the strike prices. Gap options can be valued with a slight modification to the Black-Scholes-Merton option pricing model, which will be discussed in Book 4.

For a *gap call option*, if X_2 is greater than X_1, and the stock price at maturity, S_T, is greater than the trigger price, X_2, then the payoff for the call option will be equal to $S_T - X_1$. If the stock price is less than or equal to X_2, the payoff will be zero. Note that a negative payoff can occur if the stock price is greater than X_2 and X_2 is less than X_1. In this case, the payoff will be reduced by $X_2 - X_1$.

For a *gap put option*, if X_2 is less than X_1, and the stock price at maturity, S_T, is less than the trigger price, X_2, then the payoff for the put option will be equal to $X_1 - S_T$. If the stock price is greater than or equal to X_2, the payoff will be zero. A negative payoff can occur if the stock price is less than X_2 and X_2 is greater than X_1. Like with a gap call option, if this is the case, the payoff will be reduced by $X_2 - X_1$.

Forward Start Options

Forward start options are options that begin their existence at some time in the future. For example, today an investor may purchase a 3-month call option that will not come into existence until six months from today. Employee incentive plans commonly incorporate forward start options in which at-the-money options will be created after some period of employment has passed. Note that when the underlying asset is a nondividend paying stock, the value of a forward start option will be identical to the value of a European at-the-money option with the same time to expiration as the forward start option.

Compound Options

Compound options are options on options. There are four key types of compound options:

- A *call on a call* gives the investor the right to buy a call option at a set price for a set period of time.
- A *call on a put* gives the investor the right to buy a put option at a set price for a set period of time.

- A *put on a call* gives the investor the right to sell a call option at a set price for a set period of time.
- A *put on a put* gives the investor the right to sell a put option at a set price for a set period of time.

Compound options have two levels of the underlying that determine their value—the value of the underlying option, which in turn is determined by the value of the underlying asset.

Compound options consist of two strike prices and two exercise dates. The first strike price and exercise date are used by the holder to evaluate whether to exercise the first option to receive the second option, where the second option is an option on the underlying asset, or just let the compound option expire. For example, a call on a call would be exercised if the price of the call on the underlying for the second call option were greater than the strike price of the initial option. The strike price and exercise date on the second call, however, are related to the value of the underlying asset.

Chooser Options

This interesting option allows the owner, after a certain period of time has elapsed, to choose whether the option is a call or a put. The option with the greater value after the requisite time has elapsed will determine whether the owner will choose the option to be a put or a call.

Barrier Options

Barrier options are options whose payoffs (and existence) depend on whether the underlying's asset price reaches a certain barrier level over the life of the option. These options are usually less expensive than standard options, and essentially come in either *knock-out* or *knock-in* flavors. Specific types of barrier options are:

- *Down-and-out call (put)*. A standard call (put) option that ceases to exist if the underlying asset price hits the barrier level, which is set below the current stock value.
- *Down-and-in call (put)*. A standard call (put) option that only comes into existence if the underlying asset price hits the barrier level, which is set below the current stock value.
- *Up-and-out call (put)*. A standard call (put) option that ceases to exist if the underlying asset price hits a barrier level, which is set above the current stock value.
- *Up-and-in call (put)*. A standard call (put) option that only comes into existence if the underlying asset price hits the above-current stock-price barrier level.

Barrier options have characteristics that can be very different from those of standard options. For example, vega, the sensitivity of an option's price to changes in volatility, is always positive for a standard option but may be negative for a barrier option. Increased volatility on a down-and-out option and an up-and-out option does not increase value because the closer the underlying gets to the barrier price, the greater the chance the option will expire.

Note that the value of a down-and-out call combined with the value of a down-and-in call is equal to the value of a standard call option. In other words, by knowing the value of two of these three options you can calculate the value of the remaining option (e.g., down-and-out call = standard call – down-and-in call). Similarly, the value of a standard put option is equal to the value of an up-and-out put plus the value of an up-and-in put.

Binary Options

Binary options generate discontinuous payoff profiles because they pay only one price at expiration if the asset value is above the strike price. The term binary means that the option payoff has one of two states: the option pays a set dollar amount at expiration if the option is above the strike price, or the option pays nothing if the price is below the strike price. Hence, a payoff discontinuity results from the fact that the payoff is only one value—it does not increase continuously with the price of the underlying asset as in the case of a traditional option.

In the case of a **cash-or-nothing call**, a fixed amount, *Q*, is paid if the asset ends up above the strike price. Since the Black-Scholes-Merton formula denotes $N(d_2)$ as the probability of the asset price being above the strike price, the value of a cash-or-nothing call is equal to $Qe^{-rT}N(d_2)$.

An **asset-or-nothing call** pays the value of the stock when the contract is initiated if the stock price ends up above the strike price at expiration. The corresponding value for this option is $S_0e^{-qT}N(d_1)$, where *q* is the continuous dividend yield.

Lookback Options

Lookback options are options whose payoffs depend on the maximum or minimum price of the underlying asset during the life of the option. A **floating lookback call** pays the difference between the expiration price and the minimum price of the stock over the horizon of the option. This essentially allows the owner to purchase the security at its lowest price over the option's life. On the other hand, a **floating lookback put** pays the difference between the expiration and maximum price of the stock over the time period of the option. This translates into allowing the owner of the option to sell the security at its highest price over the life of the option.

Lookback options can also be fixed when an exercise price is specified. A **fixed lookback call** has a payoff function that is identical to a European call option. However, for this exotic option, the final stock price (or expiration price) in the European call option payoff is replaced by the maximum price during the option's life. Similarly, a **fixed lookback put** has a payoff like a European put option but replaces the final stock price with the minimum price during the option's life.

Shout Options

A shout option allows the owner to pick a date when he "shouts" to the option seller, which then translates into an intrinsic value of the option at the time of the shout. At option expiration, the owner receives the maximum of the shout intrinsic value or the option expiration intrinsic value. In other words, for a shout call option, even if the price of the stock falls after the shout, the investor has locked in the difference between the price of the stock and the shout price. If the stock continues to rise, the shout option will have a payoff consistent with a standard call option. Note that most shout options allow for one "shout" during the option's life.

Asian Options

Asian options have payoff profiles based on the average price of the security over the life of the option. *Average price* calls and puts pay off the difference between the average stock price and the strike price. Note that the average price will be much less volatile than the actual price. This means that the price for an Asian average price option will be lower than the price of a comparable standard option. *Average strike* calls and average strike puts pay off the difference between the stock expiration price and average price, which essentially represents the strike price in a typical intrinsic value calculation. If the average price or strike price for an Asian option is based on a geometric average, then using an option pricing model is not a problem because a geometric average is lognormal. However, most Asian options base their average calculations on arithmetic averages, which complicates the pricing process. In this case, a lognormal distribution of prices is assumed, which provides an adequate approximation.

Exchange Options

A common use of an option to exchange one asset for another, often called an exchange option, is to exchange one currency with another. For example, consider a U.S. investor who holds an option to purchase euros with yen at a specified exchange rate. In this particular case, the option will be exercised if euros are more valuable to the U.S. investor than yen. Other applications, such as tender offers to exchange one stock for another, also arise in certain situations.

Basket Options

Basket options are simply options to purchase or sell baskets of securities. These baskets may be defined specifically for the individual investor and may be composed of specific stocks, indices, or currencies. Any exotic options that involve several different assets are more generally referred to as **rainbow options**.

Volatility and Variance Swaps

LO 44.6: Describe and contrast volatility and variance swaps.

A **volatility swap** involves the exchange of volatility based on a notional principal. One side of the swap pays based on a pre-specified fixed volatility while the other side pays based on realized volatility. Unlike the exotic options we have discussed thus far, volatility swaps are a bet on volatility alone as opposed to a bet on volatility and the price of the underlying asset.

Much like a volatility swap, a **variance swap** involves exchanging a pre-specified fixed variance rate for a realized variance rate. The variance rate being exchanged is simply the square of the volatility rate. However, unlike volatility swaps, variance swaps are easier to price and hedge since they can be replicated using a collection of call and put options.

Issues in Hedging Exotic Options

LO 44.7: Explain the basic premise of static option replication and how it can be applied to hedging exotic options.

The typical dynamic option-hedging situation uses option Greeks to measure sensitivity of the option value to changes in underlying asset characteristics (i.e., creating a delta-neutral portfolio). Hedging is simpler with some exotic options than it is with plain vanilla options. Asian options, for instance, depend on the average price of the underlying. Through time, the uncertainty of the average value gets smaller. Hence, the option begins to become less sensitive to changes in the value of the security because the payoff can be estimated more accurately.

Hedging positions in barrier and other exotic options are not so straightforward. This type of hedging requires the replication of a portfolio that is exactly opposite to the option position. When the replication portfolio requires frequent adjustments to the holdings in the underlying assets, the hedging procedure is referred to as dynamic options replication. **Dynamic options replication** requires frequent trading, which makes it costly to implement.

As an alternative, a **static options replication** approach may be used to hedge positions in exotic options. In this case, a short portfolio of actively traded options that approximates the option position to be hedged is constructed. This short replication options portfolio is created once, which drastically reduces the transaction costs associated with dynamic rebalancing.

Key Concepts

LO 44.1

Plain vanilla derivatives include listed futures contracts and commonly used forwards and other OTC derivatives that are traded in fairly liquid markets. Exotic derivatives are customized to fit a specific firm need.

LO 44.2

The main purpose for the development of exotic derivatives is to provide a unique hedge for a firm's underlying assets. Additional reasons include addressing tax and regulatory concerns as well as speculating on the expected future direction of market prices.

LO 44.3

Packages are portfolios of European options, forwards, cash, and the underlying asset. Given that packages often consist of a long position and a short position, they can be constructed so that the initial cost to the investor is zero.

LO 44.4

Restricting exercise dates and changing strike prices can transform standard options into nonstandard options.

LO 44.5

A gap option has two strike prices. If the two strike prices differ and the payoff is non-zero, there will be a gap in the payoff graph that is either increased or decreased by the difference between the strike prices.

Forward start options are options that commence in the future.

A compound option is defined as an option on another option.

Chooser options allow the owner to choose whether the option is a call or a put, after option initiation.

Barrier options are options whose payoffs (and existence) depend on whether the underlying's asset price reaches a certain barrier level over the life of the option.

Binary options either pay nothing (if price is below strike price) or a fixed amount at expiration.

Lookback options depend on the maximum or minimum value of the underlying asset during the life of the option.

Shout options allow the owner to receive either the intrinsic value of the option at the shout date or at expiration, whichever is greater.

Asian options have payoff profiles that depend on the average underlying asset price over the life of the option.

An exchange option is an option to exchange one asset for another.

Basket options allow the owner to buy or sell portfolios of assets. Exotic options that involve several different assets are more generally referred to as rainbow options.

LO 44.6

A volatility swap involves the exchange of volatility based on a notional principal. A variance swap involves exchanging a pre-specified fixed variance rate for a realized variance rate.

LO 44.7

Exotic options can be hedged in either a dynamic or static context, depending on the characteristics of the option.

Concept Checkers

1. A down-and-in call option is an option that comes into existence only when the underlying asset price:
   A. rises to a set barrier level.
   B. falls to a set barrier level.
   C. falls to a set average barrier level.
   D. rises to a set average barrier level.

2. A cash-or-nothing put option has a payout profile equivalent to zero or:
   A. the underlying asset price if the value of the asset ends below the strike price.
   B. the underlying asset price if the value of the asset ends above the strike price.
   C. a set amount if the value of the asset ends below the strike price.
   D. a set amount if the value of the asset ends above the strike price.

3. An Asian option can be hedged dynamically because the:
   A. average value of the underlying asset price decreases uncertainty the closer the option gets to expiration.
   B. average value of the underlying asset price increases uncertainty the closer the option gets to expiration.
   C. maximum value of the underlying asset price decreases uncertainty the closer the option gets to expiration.
   D. minimum value of the underlying asset price increases uncertainty the closer the option gets to expiration.

4. Which of the following options is most likely to have a negative vega?
   A. A chooser option close to expiration.
   B. A forward start put option before the start date.
   C. An Asian put option close to the beginning of the option's life.
   D. An up and out put when the stock price is close to the barrier.

5. Under which of the following circumstances would the value of an up and out call option be zero?
   A. The strike price is above the barrier price.
   B. The stock price is below the barrier price.
   C. The stock price is above the strike price.
   D. The stock price is below the strike price.

Concept Checker Answers

1. **B** Down-and-in call options are standard options that come into existence only if the asset price falls to a set barrier price level, which is set below the current stock price.

2. **C** Cash-or-nothing put options pay only a set amount if the stock price ends below the strike price. These options differ from standard put options because the payment is a set amount that does not continuously increase with the decrease in stock price.

3. **A** Dynamic hedging can be used to hedge Asian options because uncertainty in the expiration value is decreased the closer one gets to expiration. This occurs because the intrinsic value becomes "set" due to the averaging effect over the life of the option.

4. **D** Vega is the sensitivity of the price of an option to changes in volatility of the underlying stock. For most options, vega is always positive—as volatility of the underlying stock increases, the price of the option also increases. An exception would be a knockout barrier option when the stock price is close to the barrier. Higher volatility means the barrier is more likely to be reached and the option will cease to exist.

5. **A** With an up and out call, if the stock price rises beyond the barrier price, the option ceases to exist. It therefore follows that if the strike price is above the barrier price, the option will never come into the money because the option will cease to exist before the option will ever come into the money.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Commodity Forwards and Futures

Topic 45

Exam Focus

This topic focuses on the pricing relationships that exist when commodities have characteristics such as lease rates, storage costs, and/or convenience yields. Before you begin this topic, recall the no-arbitrage pricing relationships for futures contracts that were discussed in Topic 38 (Determination of Forward and Futures Prices). For the exam, you should understand the basic futures pricing equation and how it is adjusted for lease rates, storage costs, and/or convenience yields.

Pricing Commodity Forwards and Futures

LO 45.1: Apply commodity concepts such as storage costs, carry markets, lease rate, and convenience yield.

Professor's Note: LO 45.1 is addressed throughout this topic.

Commodity and financial forward contracts are similar in some regards. For example, the prices of both are logically based on expected spot prices. Some financial forwards (e.g., S&P 500 Index) are based on the expected future spot price minus dividends received during the holding period. The price of a commodity forward must also be based on expectations, but there are several factors to consider. For example, based on their physical qualities, some commodities are *storable* (e.g., metals) and the associated costs depend on the physical characteristics of the commodity. Also, due to their physical nature, others are not storable (e.g., electricity, perishable foods).

Some commodities are also appropriate for *leasing*. That is, an investor without a current need purchases the commodity and then lends it out to others who do have a current need. Just as with the loan of any asset the lender requires a return, so a **lease rate** (i.e., required return) is established. An investor that does not charge such rental fees would be in effect providing an interest-free loan of the money tied up in the purchase of the commodity. Later in this topic, we will discuss the lease rate in detail.

Since commodity forward prices are based on expected spot prices and expected spot prices are, in turn, dependent on expected supply and demand forces, forward prices for commodities need not be constant from period to period. There are factors such as weather that can affect expected supply. For example, severe weather might be expected to reduce future coffee supplies, so the forward coffee price might incorporate the expected shortage into an increased forward price. Demand for a commodity can also be subject to change. For example, demand for electricity is not constant during the day nor is it constant across

different seasons of the year or in different locations across the country. Estimating the expected spot price for a commodity, therefore, must utilize forecasts of all relevant factors.

For a given commodity on any trading day, several futures contracts will exist with varying maturity dates. The prices of the commodity futures contracts will differ with the different contract expiration dates. The set of futures prices for a given commodity is known as a **forward curve** or a **forward strip** on that particular day.

Forward Pricing

LO 45.2: Explain the basic equilibrium formula for pricing commodity forwards.

LO 45.13: Explain how to create a synthetic commodity position, and use it to explain the relationship between the forward price and the expected future spot price.

Assume that we do not know the forward price of the commodity and wish to derive it. $F_{0,T}$ is the forward price at time 0 for delivery at time T while S_t represents the spot price at time *t*.

A synthetic commodity forward price can be derived by combining a long position on a commodity forward, $F_{0,T}$, and a long zero-coupon bond that pays $F_{0,T}$ at time *T*.

The total cost at time 0 is equivalent to the cost of the bond, $e^{-rT}F_{0,T}$, where *r* represents the continuously compounded risk-free rate of return. The forward contract does not have any initial cash flows at time 0. The payoff at time *T* will be the payoff from the forward contract ($S_T - F_{0,T}$) plus the payoff from the bond ($F_{0,T}$):

$$S_T - F_{0,T} + F_{0,T} = S_T \text{ (same as payoff at time } T \text{ from a long position in the commodity)}$$

$F_{0,T}$ is the known (and certain) market price at time 0. Given that the forward price is certain, we can discount the forward price at the risk-free rate. However, the *expected spot price* at time *T* is uncertain and we cannot use the risk-free rate to discount it; we instead use a risk-adjusted discount rate (α). The higher the uncertainty in the future spot price, the higher the value of α.

The present value (at t = 0) of the expected spot price at time *T* is $E(S_T)e^{-\alpha T}$, where α represents the discount rate for the S_T cash flow at time *T*. This is equal to the present value of the forward price discounted at the risk-free rate:

$$e^{-rT}F_{0,T} = E(S_T)e^{-\alpha T}$$

Multiplying each side of the equation by e^{rT} allows us to express the commodity forward price as follows:

$$F_{0,T} = E(S_T)e^{(r-\alpha)T}$$

Thus, the forward price today is a biased estimate of the expected commodity spot price at time *T*. The bias is a function of the risk premium on the commodity, $r - \alpha$.

The net present value (NPV) of an investment in commodity would then be:

$$NPV = E(S_T)e^{-\alpha T} - S_0$$

Substituting $e^{-rT}F_{0,T} = E(S_T)e^{-\alpha T}$ and setting NPV = 0, we can see that:

$$e^{-rT}F_{0,T} = S_0 \text{ or}$$

$$F_{0,T} = S_0e^{rT}$$

Since we set NPV = 0 for the above pricing relationship, we call it the no-arbitrage price. The no-arbitrage forward price is, therefore, just the future value of the spot price—or the value of the commodity adjusted for the cost of carrying it (interest only).

Professor's Note: We have assumed that the risk-free and risk-adjusted rates are continuously compounded in our pricing relationships. We can easily adjust the formula if the rates were periodically compounded or in units of currency. Later in the topic, we will discuss leasing rates, storage costs, and convenience yields, which we will also continue to assume are continuously compounded rates. We will also consider situations where some variables are not continuously compounded.

Commodity Arbitrage

LO 45.3: Describe an arbitrage transaction in commodity forwards, and compute the potential arbitrage profit.

The forward price of a commodity reflects the cost of carrying the commodity until the futures expiration date. The cost of carry includes interest cost (as already discussed) as well as any storage costs. **Storage costs** are the cost of storing a commodity and includes incidental costs such as insurance and spoilage. The markets for those commodities that are storable are called as **carry markets**.

In carry markets, the forward price relationship is modified to include storage costs (λ) as follows:

$$F_{0,T} = S_0e^{(r + \lambda)T}$$

A **cash-and-carry arbitrage** consists of buying the commodity, storing/holding the commodity, and selling the commodity at the futures price when the contract expires. The steps in a cash-and-carry arbitrage are as follows:

At the initiation of the contract:

- Borrow money for the term of the contract at market interest rates.
- Buy the underlying commodity at the spot price.
- Sell a futures contract at the current futures price.

At contract expiration:

- Deliver the commodity and receive the futures contract price.
- Repay the loan plus interest.

If the futures contract is overpriced, this 5-step transaction will generate a riskless profit. The futures contract is overpriced if the actual market price is greater than the no-arbitrage price.

Example: Futures cash-and-carry arbitrage

Assume the spot price of gold is $900/oz., that the 1-year futures price is $975/oz., and that an investor can borrow or lend funds at 5%. Storage costs are 2% annually. **Calculate** the arbitrage profit.

Answer:

The futures price, according to the no-arbitrage principle, should be:

$$F_{0,T} = \$900e^{(0.05 + 0.02)1} = \$965$$

Instead, it's trading at $975. That means the futures contract is overpriced, so we should conduct cash and carry arbitrage by going short in the futures contract, buying gold in the spot market, and borrowing money to pay for the purchase. If we borrow $900 to fund the purchase of gold, we must repay $965 after 1 year (at maturity of the futures contract).

Today		*1 year from today*	
Transaction	*Cash flow*	*Transaction*	*Cash flow*
Short futures	$0	Settle short position by delivering gold	+$975
Buy gold in spot market	–$900		
Borrow at 5%	+$900	Repay loan	–$965
Total cash flow	$0	Total cash flow = arbitrage profit	+$10

The riskless profit is equal to the difference between the futures contract proceeds and the loan payoff, or $975 – $965 = $10. Notice that this profit is equal to the difference between the actual futures price of $975 and the no-arbitrage price of $965.

If the futures price is too low (which presents a profitable arbitrage opportunity), the opposite of each step should be executed to earn a riskless profit.

This is **reverse cash-and-carry arbitrage**. The steps in reverse cash-and-carry arbitrage are as follows.

At the initiation of the contract:

- Sell commodity short.
- Lend short sale proceeds at market interest rates.
- Buy futures contract at market price.

At contract expiration:

- Collect loan proceeds.
- Take delivery of the commodity for the futures price and cover the short sale commitment.

Example: Futures reverse cash-and-carry arbitrage

Assume again that the spot price of gold is $900/oz., and that the continuously compounded risk-free rate and storage costs are 5% and 2%, respectively as before. Also, assume that the one-year futures price is now $950/oz. **Calculate** any profits from arbitrage.

Answer:

The futures price, according to the no-arbitrage principle, is again, $965. Instead, it's trading at $950; the futures contract is underpriced. We should conduct reverse cash-and-carry arbitrage by going long the futures contract, shorting gold, and investing the short-sale proceeds as follows:

Today		*1 year from today*	
Transaction	*Cash flow*	*Transaction*	*Cash flow*
Long futures	$0	Settle long position by buying gold	–$950
Short gold	+$900	Deliver gold to close short position	$0
Invest short-sale proceeds at 5%	–$900	Receive investments proceeds	+$965
Total cash flow	$0	Total cash flow = arbitrage profit	+$15

The riskless profit is equal to the loan proceeds less the futures contract payment, or $965 – $950 = $15.

Professor's Note: It may help to remember "buy low, sell high." If the futures price is "too high," sell the futures and buy the spot. If the futures price is "too low," buy the futures and sell the spot.

Lease Rates

LO 45.4: Define the lease rate and explain how it determines the no-arbitrage values for commodity forwards and futures.

A **lease rate** is the amount of interest a lender of a commodity requires. The lease rate is defined as the amount of return the investor requires to buy and then lend a commodity. From the commodity borrower's perspective, the lease rate represents the cost of borrowing

the commodity. The lease rate and risk-free rate are important inputs to determine the commodity forward price.

The commodity forward price for time T with an active lease market is expressed as:

$$F_{0,T} = S_0 e^{(r - \delta)T}$$

where:
S_0 = current spot price
$r - \delta$ = risk-free rate less the lease rate

The lease rate, δ, is income earned only if the commodity is loaned out.

Example: Pricing a commodity forward with a lease payment

Calculate the 12-month forward price for a bushel of corn that has a spot price of $5 and an annual lease rate of 7%. The appropriate continuously compounding annual risk-free rate for the commodity is equal to 9%.

Answer:

We can determine the 12-month forward price as follows:

$$F_{0,T} = (S_0)e^{(r - \delta)T} = \$5 \times e^{(0.09 - 0.07)} = \$5.101$$

To further illustrate that this relationship must hold, consider the following no-arbitrage example.

Example: No-arbitrage for a commodity forward

Continuing with our previous example, suppose the 1-year futures price of corn is $5.08/bushel. **Calculate** the arbitrage profit.

Answer:

Since the futures price of $5.08 is less than $5.101 calculated in the previous example, an arbitrage profit can be earned by reverse cash-and-carry arbitrage: Long futures contract and short in the spot market. Figure 1 shows the calculations.

Figure 1: No-Arbitrage Opportunity on Bushel of Corn

Transaction	*Time = 0*	*Time = 1*
Borrow corn at lease rate of 7%	-	$(0.36)
Sell borrowed corn	$5.00	-
Invest proceeds at 9%	$(5.00)	$5.47
Long futures contract	$0	$(5.08)
Return borrowed corn	$0	-
Total	$0	$0.03

Arbitrage profit at T=1 is the same (except for rounding difference) as the difference between the no-arbitrage futures price and the futures price in the market.

Contango and Backwardation

An upward-sloping forward curve indicates that forward prices more distant in time are higher than nearer-term forward prices. The market is described as being in **contango** with an upward-sloping forward curve. A contango commodity market occurs when the lease rate is less than the risk-free rate. Based on the commodity forward formula, $F_{0,T} = S_0 e^{(r - \delta)T}$, if $r > \delta$, the forward price must be greater than the spot price.

The market is described as being in **backwardation** with a downward-sloping forward curve. A backwardation commodity market occurs when the lease rate is greater than the risk-free rate. Based on the commodity forward formula, $F_{0,T} = S_0 e^{(r - \delta)T}$, if $r < \delta$, the forward price must be less than the spot price.

Storage Costs and Convenience Yields

LO 45.5: Define carry markets, and illustrate the impact of storage costs and convenience yields on commodity forward prices and no-arbitrage bounds.

LO 45.6: Compute the forward price of a commodity with storage costs.

As we discussed earlier, existence of storage costs increases the forward price at least by the future value of the storage costs to compensate the seller for costs incurred while storing the commodity. When storage costs are discrete rather than continuously compounded, the forward price can be calculated as:

$$F_{0,T} \geq S_0 e^{rT} + \lambda(0,T)$$

where:
$\lambda(0,T)$ = FV (at time T) of storage costs

If storage costs are paid continuously and are proportional to the value of the commodity, the no-arbitrage forward price becomes:

$$F_{0,T} = S_0 e^{(r+\lambda)T}$$

where:
λ = continuous annual storage cost proportional to the value of the commodity

Example: Commodity forward pricing with storage costs

Calculate the 3-month forward price for a bushel of soybeans if the current spot price is \$3/bushel, the effective monthly interest rate is 1%, and the monthly storage costs are \$0.04/bushel.

Answer:

First, calculate the future cost of storage for three months, $\lambda(0,T)$, as follows:

$$\$0.04 + \$0.04(1.01) + \$0.04(1.01)^2 = \$0.1212$$

The amount of \$0.1212 represents the three months storage costs plus interest. Next, add the cost of storage to the spot price plus interest.

$$F_{0,T} = S_0 e^{rT} + \lambda(0,T) \approx 3.00(1.01^3) + \$0.1212 = \$3.0909 + \$0.1212 = \$3.2121$$

If the owners of the commodity need the commodity for their business, holding physical inventory of the commodity creates value. For example, assume a manufacturer requires a specific commodity as a raw material. To reduce the risk of running out of inventory and slowing down production, excess inventory is held by the manufacturer. This reduces the risk of idle machines and workers. In the event that the excess inventory is not needed, it can always be sold. The non-monetary benefit of holding excess inventory is referred to as the **convenience yield.** The convenience yield is only relevant when a commodity is stored

(i.e., in a carry market). A convenience yield *cannot* be earned by the average investor who does not have a business reason for holding the commodity.

The forward price including a convenience yield is calculated as follows:

$$F_{0,T} \geq S_0 e^{(r + \lambda - c)T}$$

where:
c = continuously compounded convenience yield

For the investor who does not earn a convenience yield, cash-and-carry arbitrage implies that:

$$F_{0,T} \leq S_0 e^{(r + \lambda)T}$$

Example: Impact of convenience yield on the no-arbitrage cash-and-carry commodity forward pricing range

Suppose the owner of a commodity decides to lend out the commodity. The commodity has a continuously compounded convenience yield of *c*, proportional to the value of the commodity.

Determine the range of forward prices that must exist to prevent arbitrage.

Answer:

The minimum forward price that the owner of a commodity (i.e., the short) would accept is the current spot price plus interest and storage costs offset by any convenience yield (otherwise there would be the possibility of a reverse cash-and-carry arbitrage):

$$F_{0,T} \geq S_0 e^{(r + \lambda - c)T}$$

The long party in a forward contract agrees to take delivery on some future date at a fixed forward price agreed on today. A long party can alternatively buy the commodity in the spot market and pay for storage until the delivery date. Therefore, the maximum forward price that the long party would be willing to pay is the spot price plus interest and storage cost:

$$F_{0,T} \leq S_0 e^{(r + \lambda)T}$$

The two inequalities provide the range of arbitrage-free forward prices as follows:

$$S_0 e^{(r + \lambda - c)T} \leq F_{0,T} \leq S_0 e^{(r + \lambda)T}$$

The upper bound of the forward price depends on storage costs, but not on the convenience yield. The lower bound adjusts for the convenience yield and, therefore, explains why forward prices may appear lower than pure spot plus storage costs at times.

Reverse cash-and-carry arbitrage is generally difficult because it depends on lease rates, which we will consider in the next section.

Comparing Lease Rates, Storage Costs, and Convenience Yields

LO 45.7: Compare the lease rate with the convenience yield.

Here is a handy guide for relating forward and spot commodity prices on the exam. Start with the basic expression relating forward and spot prices:

$$F_{0,T} = S_0 e^{rT}$$

This expression says that if there are no costs or benefits associated with buying and holding the commodity, the forward price is just the spot price compounded at the risk-free rate over the holding period.

If there are benefits (e.g., lease rates or convenience yield) to buying the commodity today, the holder is willing to accept a lower forward price. The forward price is reduced by the benefit, either the lease rate or convenience yield:

$$F_{0,T} = S_0 e^{(r - c)T} < S_0 e^{rT}$$

where c = the convenience yield, or

$$F_{0,T} = S_0 e^{(r - \delta)T} < S_0 e^{rT}$$

where δ = the lease rate

If there are costs, such as storage costs, associated with purchasing the commodity today, the forward price is increased by the cost:

$$F_{0,T} = S_0 e^{(r + \lambda)T} > S_0 e^{rT}$$

where λ = the storage costs

Of course, there can be combinations of costs and benefits, so be sure to increase the exponent for costs and reduce it for benefits:

$$F_{0,T} = S_0 e^{(r + \lambda - c)T}$$

Note that in the previous equation, the lease rate is equal to the convenience yield minus storage costs. Hence, lease rates are negatively related to storage costs. This explains why sometimes when storage costs are high, lease rates can be negative!

Arbitrage-free conditions dictate that (annual) lease rates should be equal to either:

- The (annual) risk-adjusted required rate of return on commodity investment minus the expected (annualized) price appreciation of the commodity:

$$\delta = \alpha - \frac{1}{T}\left[E(S_T) / S_0\right]$$

or

- The (annual) risk-free rate minus the (annualized) forward premium on the commodity:

$$\delta = r - \frac{1}{T}\left[F_{0,T} / S_0\right]$$

The first formula indicates that the maximum lease rate an owner may be able to charge is the risk-adjusted required rate of return on the commodity less the expected price appreciation (which the lender will enjoy upon return of the loaned commodity). The second formula starts with risk-free rate and subtracts the guaranteed forward premium (which the owner would receive if the commodity is sold using a forward contract).

Commodity Characteristics

LO 45.8: Identify factors that impact gold, corn, electricity, natural gas, and oil forward prices.

Certain commodities exhibit unique properties that impact their forward price. For example, gold, corn, electricity, natural gas, and oil are all commodities with characteristics that differ with respect to storage costs, the ability to store, production costs, and demand seasonality. These differences are reflected in lease rates, storage costs, and convenience yields that influence the commodity forward prices and the shape of the forward curves.

Gold Forward Prices

Gold forward prices indicate presence of positive lease rates; holders of gold (e.g., central banks) do lend it out for a fee. Hence the forward prices are lower than pure spot prices plus interest and storage costs. It would make sense, therefore, for an investor to obtain exposure to gold via long forward contracts (synthetic gold) rather than physically buying and holding gold (and not earning a lease payment). Sometimes, when the storage costs are very high, lease rates may turn negative (holders of gold are willing to pay someone to store rather than pay high storage costs themselves).

The value of gold is also influenced by the cost of production. The value of gold production of a mine can be calculated as the present value of the forward prices minus the cost of production computed at the risk-free rate of return.

Corn Forward Prices

Corn production is seasonal while its demand is relatively even throughout the year. This timing mismatch between production and consumption means that corn must be stored. Thus, storage (and interest) costs are highly relevant in corn forward prices. Corn forward

prices tend to fall around harvest and subsequently rise (until the next harvest) to reflect the cost of storage. Thus, the corn forward curve increases until harvest time, drops sharply and then slopes upward again after harvest time is over.

Example: Corn commodity pricing with storage costs

Suppose the spot price today for a bushel of corn is $2.25, the continuously compounded interest rate is 5.5%, and the storage cost is 2.0% per month. **Calculate** the 6-month forward price.

Answer:

$$F_{0,0.5} = \$2.25 \times e^{(0.00458 + 0.02)6} = \$2.25 \times 1.15893 = \$2.61$$

Professor's Note: The 0.458% used for the monthly interest rate is the annual rate divided by 12.

Electricity Forward Prices

Electricity is not a storable commodity; once produced, it must be used or it will likely go to waste. The demand for electricity is not constant and will vary with time of day, day of the week, and with season. Given the non-storability characteristic of electricity, its price is set by demand and supply at a given point in time. Since arbitrage opportunities do not exist with electricity (i.e., the inability to buy electricity during one season and sell it in another season) futures prices on electricity will vary much more during the trading day than financial futures. Electricity futures prices are primarily driven by expected spot prices in the future.

Natural Gas Forward Prices

Natural gas is an example of a commodity with constant production but seasonal demand. Natural gas is expensive to store, and demand in the United States peaks during high periods of use in the winter months. In addition, natural gas prices vary geographically due to high international transportation costs. Storage is at its peak in the fall just prior to the peak demand. Therefore, the natural gas forward curve rises steadily in the fall.

Example: Calculation of natural gas forward price with storage costs

Calculate the natural gas implied storage cost for the month of October if the October spot price is 4.071, the annual risk-free rate of interest is 6%, and the November forward price is 4.157.

Answer:

$$\$4.157 = \$4.071e^{0.005} + \lambda_{October}$$

$$\$4.157 = \$4.091 + \lambda_{October}$$

$$\$4.157 - \$4.091 = \lambda_{October}$$

$$\$0.066 = \lambda_{October}$$

In the southern hemisphere, the natural gas forward curve peaks in June and July rather than December and January. In tropical areas, the forward curve is relatively flat due to flat demand and supply patterns.

Oil Forward Prices

The physical characteristics of oil make it is easier to transport than natural gas. Therefore, the price of oil is comparable worldwide. In addition, demand is high in one hemisphere when it is low in the other. Lower transportation costs and more constant worldwide demand cause the long-run forward price to be more stable.

Commodity Spread

LO 45.9: Compute a commodity spread.

A **commodity spread** results from a commodity that is an input in the production process of other commodities. For example, soybeans are used in the production of soybean meal and soybean oil. A trader creates a **crush spread** by holding a long (short) position in soybeans and a short (long) position in soybean meal and soybean oil.

Similarly, oil can be refined to produce different types of petroleum products (distillates) such as heating oil, kerosene, or gasoline. This process is known as **cracking**, and thus the difference in prices of crude oil, heating oil, and gasoline is known as a **crack spread**. For example, seven gallons of crude oil may be used to produce four gallons of gasoline and three gallons of heating oil. Commodity traders would refer to this crack spread as 7-4-3, reflecting the seven gallons of crude oil, four gallons of gasoline, and three gallons of heating oil. Thus, an oil refiner could lock in the price of the crude oil input and the finished good outputs by an appropriate crack spread reflecting the refining process. However, this is not a perfect hedge because there are other distillates that can also be produced (e.g., jet fuel).

Example: Pricing a crack (commodity) spread

Suppose we plan on buying crude oil in one month to produce gasoline and kerosene for sale in two months. The 1-month futures price for crude oil is currently $30/barrel. The 2-month futures prices for gasoline and heating oil are $41/barrel and $31.50/barrel, respectively. **Calculate** the 5-3-2 crack (commodity) spread.

Answer:

The 5-3-2 spread tells us the amount of gross margin that can be locked in by buying five barrels of oil and producing three barrels of gasoline and two barrels of heating oil.

gross margin for a 5-3-2 spread =
(3 × $41) + (2 × $31.50) – (5 × $30) = $123 + $63 – $150 = $36 for five barrels, or $7.20/barrel

Professor's Note: There is no calculation for interest adjustment in this example.

Basis Risk

LO 45.10: Explain how basis risk can occur when hedging commodity price exposure.

As you may recall, **basis** is the difference between the spot price (or rate) and the futures price (or rate). While hedging using futures, if the values of both move together perfectly, an investor long or short the asset can lock in a return or value by selling or buying futures, respectively.

Professor's Note: When you expect to receive the commodity in the future, we say you are long the commodity and you will hedge the value of the expected commodity by selling the corresponding futures contracts. If you will deliver the commodity in the future without first owning the commodity, you are short, and you will hedge by taking a long position in the corresponding futures contracts.

Any time spot and futures prices do not move together perfectly, the hedger faces **basis risk.** An example with financial futures is using a basket currency futures contract to hedge the value of a transaction in an emerging market. Since the hedged asset (i.e., the emerging market currency) and the underlying in the futures contract are not identical, there is risk associated with changes in their relative prices. Also, if the financial futures contract must be rolled over, or if it matures after the delivery date, this mismatch adds to the basis risk.

As with financial futures, every commodity futures contract specifies a delivery amount and a delivery date. In addition, however, every commodity futures contract specifies a delivery *location* and the deliverable *grade* (i.e., quality). For example, an investor planning to receive oil in New York City might use NYMEX futures, which specify delivery in Oklahoma. At the producer level, an Iowa corn farmer might use CBOT corn futures, which specify

delivery in Chicago. For commodities, as opposed to financial contracts, these specifications introduce storage and transportation cost complexities for hedgers.

Strip Hedge vs. Stack Hedge

LO 45.11: Evaluate the differences between a strip hedge and a stack hedge and explain how these differences impact risk management.

An oil producer may enter into a long-term contract with a refinery to supply a fixed number of barrels of oil per month at a fixed price. If the market prices of oil subsequently rise, the producer would lose out on the price appreciation because of the locked-in price. A natural way for the producer to hedge this fixed price contract is to enter into several futures contracts as a long party, matching the maturities and quantities with their obligations under the fixed price agreement. If the price rises, the gains on the futures contract offset the lost gains on the fixed price agreement. This type of hedging arrangement is known as a **strip hedge**. However, if the price of oil falls, the producers would take a loss on the futures contract (effectively reducing their selling price).

To reduce transaction costs associated with several futures contracts, the oil producer might instead utilize a **stack hedge**. To form a stack hedge, the oil producer would enter into a one-month futures contract equaling the total value of the year's promised deliveries. As transaction costs are less for nearer-term (e.g., one-month) contracts, the total cost of implementing this strategy is less than for a comparable strip hedge. At the end of the first month, the producer rolls into another one-month contract, and so forth, each month setting the total amount of the contract equal to the remaining promised deliveries. This strategy of continually rolling into the next near-term contract is referred to as **stack and roll**.

A stack hedge has the advantage when near-term contracts are more liquid (and hence the bid-ask spread is lower) and, therefore, lower transaction costs. In addition, an oil producer may prefer a stack hedge in order to speculate on the shape of the forward curve. For example, assume the forward curve looks unusually steep. A hedger needing a long hedge position would then enter into a stacked hedge with a large near-term contract. If the forward curve later flattens, the long hedger locks in all the oil at a relatively cheap near-term price compared to the more expensive futures using a strip strategy.

Example: Creation of a strip or stack hedge

Determine how an oil producer could hedge the risk of an agreement to supply 150,000 barrels of oil each month for a year at a fixed price.

Answer:

The oil producer could enter into a strip hedge by obtaining a long futures contract position for every month of the year for 150,000 barrels.

Alternatively, the oil producer could create a long position of a near-term futures contract for a little less than 1,800,000 barrels. At the end of the month, the oil producer would enter into a new near-term futures contract for a smaller amount representing the present value of future deliveries.

Cross Hedging

LO 45.12: Provide examples of cross-hedging, specifically the process of hedging jet fuel with crude oil and using weather derivatives.

In many cases, a futures contract with an underlying instrument that is exactly the same as the position to be hedged will not exist. For example, there are no contracts for jet fuel futures in the United States. Therefore, hedging a jet fuel exposure requires use of a **cross hedge**. In a cross hedge, a futures contract that is highly correlated with the underlying exposure is selected. Some firms hedge the cost of jet fuel with crude oil futures while others use a combination of crude oil and heating oil futures. Three factors are relevant when making a cross hedge decision:

- The liquidity of the futures contract (since physical delivery may not be an option).
- The correlation between the underlying for the futures contract and the asset(s) being hedged.
- The maturity of the futures contract.

Each of these factors has an impact on hedge effectiveness. The liquidity of the cross hedge is important in order for the portfolio manager to quickly unwind the futures obligation. Thus, the manager should try to choose among liquid instruments to find the futures contract whose maturity most closely matches that of the horizon of the hedged position.

To illustrate the concept of cross hedging, consider a firm that uses crude oil futures to hedge jet fuel prices. The payoff from this type of hedge will depend on both the change in jet fuel prices and the change in oil futures prices. Thus, the number of crude oil futures contracts required is estimated using regression analysis, where the change in jet fuel prices is dependent on the change in oil futures prices. The slope coefficient from the regression results will provide the portfolio manager with hedge ratio information regarding the degree to which changes in the crude price explain changes in the price of jet fuel.

A cross hedge is also applied when firms use **weather derivatives**. Weather risk is a business risk that is faced by many agricultural firms as well as by firms providing recreational services. A cross hedge is used to mitigate losses caused by changes in weather.

Utility companies use weather derivatives, which are based on "degree days," to hedge the cost of energy purchases. Much of the energy supplied by utilities is used for heating or cooling with variations in demand highly influenced by weather patterns.

Utilities can use derivatives with payoffs based on the weather experienced at weather stations that are representative of the areas that they serve. For example, a utility located in the northeast U.S. contracts for energy needs based on average weather experienced over prior years as well as on forecasts for the coming year. Unhedged, the utility would leave itself exposed to rising prices from energy producers in the event that the coming winter is far worse than predicted.

If hedging with weather derivatives (specifically weather options), and a harsher winter materialized (i.e., more heating degree days than the strike value of the contract), the utility would receive the contract specified payment. Otherwise, the contract would expire worthless. The actual measurements are from specified U.S. government sites in the areas specified by the contract.

The use of weather derivatives by other investors is growing, but one of the biggest problems is basis risk. That is, it is difficult to accurately match up the exposure of other assets to the weather with that specified by the contracts. Other than large-scale exposure, such as that experienced by utilities, many producers are much more susceptible to more local variations. For instance, a large farming operation has exposure to the rain falling on its own fields and may suffer losses from too much or too little rain. The rain on its fields may not have a high correlation with the rain experienced at a weather station 50 miles away.

Key Concepts

LO 45.1

When holding a commodity requires storage costs, the forward price must be greater than the spot price to compensate for the financial and physical storage costs.

The market in which a commodity is stored is referred to as a carry market.

A lease rate is the amount of rent a lender of a commodity requires.

Convenience yield is the non-monetary benefit earned from holding an excess inventory of a commodity.

LO 45.2

The commodity forward price today is defined as a biased estimate of the expected spot commodity price at time T as follows:

$$F_{0,T} = E(S_T)e^{(\text{risk-free rate} - \text{discount rate})T}$$

LO 45.3

The steps in a cash-and-carry arbitrage are as follows:

At the initiation of the contract:

Step 1: Borrow money for the term of the contract at market interest rates.
Step 2: Buy the underlying commodity at the spot price.
Step 3: Sell a futures contract at the current futures price.

At contract expiration:

Step 1: Deliver the commodity and receive the futures contract price.
Step 2: Repay the loan plus interest.

LO 45.4

The lease rate is defined as the amount of return the investor requires to buy and then lend a commodity. If an active lease market exists for a commodity, a commodity lender can earn the lease rate by buying a commodity and immediately selling it forward.

Markets characterized by upward-sloping forward curves (i.e., contango) occur when the lease rate is less than the risk-free rate. When lease rates exceed the risk-free rate, backwardation occurs.

LO 45.5

Convenience yield is the non-monetary benefit enjoyed by producers who hold excess inventory of raw material inputs. This excess inventory precludes disruptions in the production process caused by temporary shortages of the input in the market. Accounting for existence of a convenience yield, the forward price is calculated as:

$$F_{0,T} \geq S_0 e^{(r+\lambda-c)T}$$

Accordingly, the arbitrage-free range of the forward price is:

$$S_0 e^{(r+\lambda-c)T} \leq F_{0,T} \leq S_0 e^{(r+\lambda)T}$$

LO 45.6

A commodity owner will only store the commodity if the forward price is greater than or equal to the spot price plus the future storage costs as follows:

$F_{0,T} \geq S_0 e^{rT} + \lambda(0,T)$, where $\lambda(0,T)$ represents the future value of storage costs.

If storage costs are continuously compounded, the no-arbitrage forward price becomes:

$$F_{0,T} = S_0 e^{(r+\lambda)T}$$

LO 45.7

Lease rates and convenience yields both reduce the futures price. The lease rate is the monetary benefit that the holder of a commodity can earn by lending the commodity. The convenience yield is the non-monetary benefit enjoyed by the user of the commodity by having an excess inventory of the commodity, reducing potential production disruptions caused by temporary shortages of that input.

LO 45.8

Since gold can earn a return by being loaned out, strategies for holding synthetic gold offer a higher return than holding just the physical gold without lending it out.

Corn is an example of a commodity with seasonal production and constant demand.

Electricity is not a storable commodity. In addition, demand for electricity is not constant and will vary with time of day, day of the week, and season.

Natural gas is an example of a commodity with constant production but seasonal demand.

Oil is easier to transport than natural gas. Therefore, the price of oil is comparable worldwide. Supply and demand adjust to price changes in the long run.

LO 45.9

A commodity spread results from a commodity that is an input in the production process of other commodities. For example, a 7-4-3 crack spread refers to the profit from holding four gasoline futures plus three heating oil futures less seven crude oil futures.

LO 45.10

Basis risk results from the inability to create a perfect hedge due to differences in commodities with respect to timing, grade, storage costs, and/or transportation costs.

LO 45.11

A strip hedge is created by buying futures contracts that match the maturity and quantity for every month of the obligation. A stack hedge is created by buying a futures contract with a single maturity based on the present value of the future obligations. Advantages of the stack hedge are the availability and liquidity of near-term contracts and narrower bid-ask spreads for near-term contracts.

LO 45.12

There are no contracts for jet fuel futures in the United States. Therefore, hedging jet fuel costs requires a cross hedge (e.g., hedge with crude oil futures). A cross hedge is also applied when firms use weather derivatives.

LO 45.13

A synthetic commodity forward price can be derived by combining a long position on a commodity forward, $F_{0,T}$, and a long zero-coupon bond that pays $F_{0,T}$ at time T.

Concept Checkers

1. Which of the following statements regarding lease rates is(are) true? The lease rate is:

   I. the amount of return the investor requires to lend a commodity.
   II. very similar to the dividend yield in an equity forward contract.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

2. Suppose there is an active lending market for a bushel of soybeans (which has a current spot price of $4/bushel). If the annual lease rate is equal to 7% and the effective annual risk-free rate is equal to 7%, how could an investor create an arbitrage opportunity (assuming the forward contract is overpriced)? An individual could:
   A. borrow money at 7% and purchase a bushel of soybeans and sell it forward.
   B. borrow a bushel of soybeans and sell at the spot price and long the forward.
   C. sell a bushel of soybeans at the forward price and lend the money at the risk-free rate.
   D. go long in soybean forward contracts, short in soybean spot prices, and lend the excess proceeds at the risk-free rate.

3. What is the 3-month forward price for a bushel of corn if the current spot price for corn is $3/bushel, the effective monthly interest rate is 1.5%, and the monthly storage costs are $0.03/bushel?
   A. $3.18.
   B. $3.23.
   C. $3.29.
   D. $3.31.

4. Suppose we plan on buying crude oil in one month to produce gasoline and heating oil for sale in two months. The 1-month futures price for crude oil is currently $42.5/barrel. The 2-month futures prices for gasoline and heating oil are $45/barrel and $43.50/barrel, respectively. Ignoring any interest cost, what is the 7-5-2 crack (commodity) spread?
   A. $2.07/barrel.
   B. $6.00/barrel.
   C. $14.50/barrel.
   D. $22.09/barrel.

5. Which of the following statement(s) is/are example(s) of basis risk?
   A. Long oil futures with contract specified delivery in a different geographical region.
   B. Hedging a short position in a commodity with a desired distant delivery with long near-term contracts.
   C. Using a eurodollar contract due to lack of commodity futures.
   D. All of the above.

Concept Checker Answers

1. **C** A *lease rate* is the amount of interest a lender of a commodity requires. From the borrower's perspective, the lease rate represents the cost of borrowing the commodity. The lease rate in the pricing of a commodity future is very similar to the dividend payment in a financial forward.

2. **A** An individual could borrow money at the risk-free rate of 7% to purchase a bushel of soybeans and sell it forward. The individual immediately lends the bushel of soybeans out at a lease rate of 7%. At the end of the lease period, T_1, the individual would pay back the loan with interest at $4.28, sell the soybeans at forward price (>$4.00), and receive the lease payment of $0.28. In order for a no-arbitrage position to exist, the forward price, $F_{0,1}$, must be equal to $4.00.

No-Arbitrage Opportunity on Bushel of Soybeans

Transaction	*Time* = T_0	*Time* = T_1
Borrow @ 7%	$4.00	($4.28)
Buy a bushel of soybeans	($4.00)	–
Lend bushel of soybeans	$0	$0.28
Short forward	$0	$F_{0,1}$
Total	$0	$F_{0,1} - $4.00

3. **B** First calculate the future cost of storage for three months, $\lambda(0,T)$, as follows:

$\$0.03 + \$0.03(1.015) + \$0.03(1.015)^2 = \$0.0914$

The amount of $0.0914 represents the 3-month storage costs plus interest. Next, add the cost of storage to the spot price plus interest.

$F_{0,T} = S_0e^{rT} + \lambda(0,T) = \$3.00(1.015^3) + \$0.0914 = \$3.1370 + \$0.0914 = \$3.23$

4. **A** The 7-5-2 spread tells us the amount of profit that can be locked in by buying seven barrels of oil and producing five barrels of gasoline and two barrels of heating oil.

Profit for a 7-5-2 spread =

(5 × $45) + (2 × $43.50) – (7 × $42.5) = $225 + $87 – $297.5 = $14.50 for seven barrels, or $14.5 / 7 barrels = $2.07/barrel.

5. **D** All choices are examples of basis risk—imperfect hedges due to mismatches between the underlying exposure and the hedge contract specifications. Differences due to timing, grade, storage costs, and/or transportation costs can create a basis risk.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Exchanges, OTC Derivatives, DPCs and SPVs

Topic 46

Exam Focus

In this topic, we look at the role of exchanges and the differences between exchange-traded derivatives and over-the-counter (OTC) derivatives trading. We then examine the three forms of clearing and the role of the central counterparty (CCP) in clearing and mitigating counterparty risk. For the exam, be able to compare and contrast exchange-traded and OTC derivatives. Also, be familiar with the development of central clearing, including the various mechanisms that exist to manage risks, including special purpose vehicles (SPVs), derivatives product companies (DPCs), monolines, and credit derivative product companies (CDPCs). In addition, be able to contrast these mechanisms with CCPs.

Exchange Functions

LO 46.1: Describe how exchanges can be used to alleviate counterparty risk.

Market participants can trade derivatives bilaterally or through exchanges. An **exchange** is a central market where standardized futures, options, and other derivatives contracts can be traded. Exchanges have a long history dating back to at least the 19th century. They have evolved from simple trading forums without risk management functions to sophisticated financial centers with settlement and counterparty risk management functions.

Exchange functions fall into three primary categories: product standardization, trading venue, and reporting services.

- *Product standardization.* Exchanges set the terms of traded, standardized products. Terms include maturity dates, trading price increments, and delivery grades and locations.
- *Trading venue.* Exchanges may be physical locations or electronic platforms that provide a central location for trading, which then facilities price discovery. Entities trading on an exchange must accept the exchange's rules and conditions.
- *Reporting services.* Exchanges report transaction prices to various entities, including trading participants, vendors, and subscribers.

Forms of Clearing

LO 46.2: Explain the developments in clearing that reduce risk.

Clearing is the process of reconciling and matching contracts between counterparties from the time the commitments are made until settlement. Clearing, along with the mechanisms of margining and netting, are important counterparty risk mitigants. *Margining* involves

posting both initial and variation margins. *Initial margin* represents upfront funds posted to mitigate against counterparty default, while *variation margin* represents the daily transfer of funds (cash or other assets) to cover position gains and losses. *Netting* refers to consolidating multiple offsetting positions between counterparties into a single payment.

The three forms of clearing that we look at in this section include: direct clearing, clearing rings, and complete clearing (i.e., central clearing).

Direct clearing is a mechanism for bilaterally reconciling commitments between two counterparties. For example, consider a scenario where counterparty X has an agreement to sell 10 contracts at $50 to counterparty Y, and Y has an agreement to sell 10 contracts to X at $55. Instead of exchanging the full 10 contracts and associated payments of $500 and $550, under a direct netting scenario only the net payment of the $50 difference is paid by X to Y. This type of direct clearing for OTC derivatives is typically called netting, or payment of difference.

A **clearing ring** is a mechanism to reduce counterparty exposure between three or more exchange members. A clearing ring is voluntary for exchange members. Once members join, however, they must accept the rules of the exchange and must accept each other's contracts and allow for counterparties to be substituted. For example, if counterparty X is long a contract with counterparty Y, and Y is long the same number of contracts with Z, then Y may be removed from the ring and the two separate obligations would be replaced with a single obligation between X and Z. Clearing rings are designed to mitigate counterparty risk, improve liquidity and facilitate the close-out process. Not all exchange members would benefit from joining a clearing ring. Members that have a single position with another counterparty would not benefit from the ring.

Complete clearing refers to clearing through a central counterparty (CCP). The CCP, which can be either operated directly by an exchange or provided as a service by an independent third party, assumes the contractual obligations of clearing exchange members and acts as a buyer to sellers and a seller to buyers. By doing so, the CCP reduces counterparty risk and facilitates both clearing and settlement. Complete clearing can be seen as an improvement to a clearing ring since it reduces the risk of member failure and any resulting contagion effect.

CCPs also use margining rules to help protect against counterparty risk. Initial margin involves members pledging upfront funds to offset closeout costs in a member default scenario. Variation margin involves settlement of daily profit and loss of derivatives contracts (mark-to-market process).

Exchange-Traded vs. OTC Derivatives

LO 46.3: Compare exchange-traded and OTC markets and describe their uses.

Exchange-traded derivatives are standardized contracts with a liquid, active, and regulated market, with the exchange or CCP acting as the central counterparty to trades. In contrast, OTC derivatives are privately negotiated bilateral contracts transacted in a market with little or no regulation. OTC derivatives have historically been traded between an end user and a dealer. The terms, settlement, and documentation are bilaterally negotiated. This allows for

contracts to be tailored to the specific needs of counterparties and includes a high level of customization. Customization is beneficial for hedging since it reduces basis risk, or the risk of term mismatches, which is a concern when using standardized exchange-traded contracts.

Given that OTC derivatives are bilaterally transacted between two counterparties, counterparty credit risk is a concern. In addition, due to the highly customized nature of the contracts, unwinding a trade may only be done at unfavorable terms (with the original counterparty or a new one). Novation of contracts (i.e., replacement of contracts) may also be problematic given the lack of fungibility in the OTC markets. In other words, contracts typically cannot be easily closed out given a lack of substitution between contracts.

The following table provides a comparison of the differences between exchange-traded and OTC derivatives.

	Exchange-Traded Derivatives	*OTC Derivatives*
Terms	Standardized	Custom, negotiable
Maturity	Standardized	Negotiable, non-standard
Liquidity	Strong	Weak
Credit risk	Little (CCP guarantee)	High (bilateral)

The clearing process is also different for exchange-traded and OTC derivatives. Whereas exchange-traded derivatives are typically shorter term and are settled within a few days, OTC derivatives are longer term with later settlements. This makes clearing more challenging for OTC derivatives.

Clearing and settlement on exchanges are functions carried out centrally by the CCP. For OTC derivatives, clearing and settlement are done bilaterally. To benefit from central clearing, an OTC derivative does not have to become exchange traded, and instead could be cleared by CCPs (e.g., LCH.Clearnet's SwapClear service offers clearing of bilaterally traded OTC derivatives).

Classes of OTC Derivatives

LO 46.4: Identify the classes of derivative securities and explain the risk associated with them.

OTC derivatives comprise of five broad classes: interest rate, foreign exchange, equity, commodity, and credit derivatives. Interest rate derivatives dominate the five classes and comprise nearly three quarters of the total gross notional outstanding. As of June 2013, interest rate derivatives had a gross notional value outstanding of $561.3 trillion, which represented 84% of the total gross notional value of OTC derivatives of $668 trillion. The second and third largest categories were foreign exchange derivatives and credit default swaps, respectively, followed by equity and commodity derivatives. It is important to note, however, that although interest rate derivatives comprise the majority of the OTC derivatives market, counterparty risk is particularly a concern for certain foreign exchange derivatives (including cross-currency swaps), which typically have long-dated maturities and require the exchange of notional principal. Furthermore, credit default swaps tend to

be more volatile and can carry significant wrong-way risk (when the credit quality of the counterparty is inversely related to the level of exposure to the counterparty).

As mentioned, interest rate derivatives dominate the market by gross notional value outstanding of contracts. However, measuring OTC derivatives exposure through gross notional value can be misleading. A basic fixed-for-floating coupon interest rate swap, for example, does not have principal risk because only the coupon cash flows are exchanged at each settlement. Furthermore, even coupon risk is lower, because only the net cash flows are exchanged. When considering cash flows, the swap may have a negative value to a party when its counterparty defaults. As a result, gross market value is often seen as a more useful measure for OTC derivatives, including the ratio of gross market value to gross notional value. The ratio is typically relatively small, and was close to 3% (at June 2013) for interest rate, foreign exchange, and credit default swaps.

Mitigating Risks of OTC Derivatives

LO 46.5: Identify risks associated with OTC markets and explain how these risks can be mitigated.

The default of a large market participant can create a ripple effect leading to systemic risk through the failure of many counterparties. Various mitigants exist to contain or reduce the risk of the initial default, including capital requirements, regulation, netting, and margining. While these mitigants have been widely utilized, they can create additional complexity which in turn may increase risk in the market.

In addition to these mitigants, other mechanisms also exist for controlling counterparty risk. These include special purpose vehicles (SPVs), derivatives product companies (DPCs), monolines, and credit derivative product companies (CDPCs). While these mechanisms have largely lost their relevance today, they provide important historical lessons for managing counterparty risk. Ultimately, rather than relying on these credit mitigants, systemic risk may be best managed by central clearing through CCPs, which act to manage systemic failure in a controlled way. Losses are spread across all exchange members through a loss sharing mechanism, thereby containing systemic risk.

Special Purpose Vehicles (SPVs)

SPVs are bankruptcy remote legal entities set up by a parent firm to shield the SPV from any financial distress of the firm. The firm transfers assets to the SPV, which in turn issues structured products to investors to finance a particular project. The primary benefit of using an SPV is to obtain a strong credit rating, typically AAA. The SPV's rating is therefore stronger than the firm's credit rating. As a result, issuing securities through the SPV is more beneficial (i.e., lower cost of funding) than if the firm issued securities directly in the market.

One of the main benefits of SPVs is that they alter bankruptcy rules and transform counterparty risk into legal risk. The specific legal risk is that the courts may view the SPV and the originating firm as a single legal entity. This is referred to as *consolidation,* and it would effectively negate the intended benefits of the SPV as a separate, bankruptcy remote

legal entity. In the United States, there is a history of consolidation rulings by courts, whereas United Kingdom courts have been more reluctant to issue consolidation rulings. Differences in U.S. and U.K. court rulings were particularly evident during the bankruptcy of Lehman Brothers, which used SPVs to protect investors from its own counterparty risk. Legal risk is therefore an important consideration, especially in central clearing where a CCP must be certain of its legal authorities.

Derivatives Product Companies (DPCs)

DPCs are set up by firms as bankruptcy remote subsidiaries to originate derivatives products and sell them to investors. Unlike SPVs, however, in order to receive a strong (e.g., AAA) rating they are separately capitalized and have restrictions on their activities and margin. DPCs calculate their internal quantitative risk assessment to quantify credit risk and to make sure they are benchmarked similarly relative to the desired AAA ratings criteria.

A DPC's AAA rating depends on three criteria: (1) market risk minimization through participating on both sides of the market, (2) parent support, with the bankruptcy remote status shielding against the parent's potential distress, and (3) credit risk and operational risk management through restrictions like limits, margin, and daily mark to market.

DPCs used defined triggers for their own failure through a "pre-packaged bankruptcy" process, which lays out the bankruptcy process and is intended to provide a simpler alternative to the standard bankruptcy process. Once a firm enters bankruptcy, DPCs could either continue on as part of another firm, or be terminated.

The advent of alternative AAA-rated entities, the perception that DPCs were inextricably linked to their parents and the loss of credibility in their AAA rating following the global financial crisis essentially rendered DPCs obsolete mechanisms.

Monolines and Credit Derivative Product Companies (CDPCs)

Monolines are highly-rated insurance companies that provide financial guarantees, called "credit wraps" to investors. **CDPCs** are similar to the DPCs, discussed earlier, but have a business model more similar to that of a monoline.

Monolines and CDPCs are well-capitalized entities with their AAA ratings supported by capitalization requirements based on possible losses and related to the assets for which they provided guarantees. They are generally highly leveraged entities that do not have to post margin. During the recent global financial crisis, several monolines failed (including XL Financial Assurance Ltd., AMBAC Insurance Corporation, MBNA Insurance Corporation), and both monolines and CDPCs, which are considered similar to monolines, fell out of favor.

Lessons Learned from Risk Mitigation

The history of SPVs, DPCs, monolines, and CDPCs provide the following valuable lessons for CCPs in a central clearing setting:

1. CCPs give priority to OTC derivatives counterparties to the detriment of other parties, including bondholders. This increases the risk in other markets.

2. Relying on a solid legal framework exposes CCPs and exchange members to legal risk. For example, as seen in the case of SPVs and DPCs, courts may change the priority of claims in a bankruptcy scenario, or courts in different jurisdictions may rule in contradictory ways.

3. Although CCPs share similarities with monolines and CDPCs in that they are highly-rated entities set up to manage counterparty risk, CCPs do not take residual risk in the market given that they maintain a matched book of trades. This is in contrast to monolines and CDPCs, which typically have one-way market exposures.

4. In contrast to monolines and CDPCs, which post no variation margin and often no initial margin, CCPs require members to post both initial and variation margin.

Key Concepts

LO 46.1

Trading derivatives can be done bilaterally or through exchanges. An exchange is a central market where standardized contracts can be traded. Exchanges perform three primary functions: product standardization, trading venue, and reporting services.

LO 46.2

Clearing, margining, and netting are important counterparty risk mitigants. Clearing is the process of reconciling and matching contracts between counterparties. Margining represents both upfront funds posted to mitigate against counterparty default (initial margin), and daily transfer of funds to cover position gains and losses (variation margin). Netting refers to consolidating multiple offsetting positions between counterparties into a single payment.

The three forms of clearing include direct clearing, clearing rings, and complete clearing (i.e., central clearing). Direct clearing is a mechanism for bilaterally reconciling commitments between two counterparties. A clearing ring is a mechanism to reduce counterparty exposure between members by allowing for counterparty substitution. Complete clearing is clearing through a CCP, where the CCP assumes the obligations of clearing exchange members.

LO 46.3

The main benefits of OTC derivatives include customization of terms, settlement, and documentation, which are negotiated bilaterally between two parties. Customization can be beneficial since it reduces basis risk (i.e., risk of term mismatches).

Disadvantages of OTC derivatives include counterparty risk, difficulty in unwinding trades, and novation of contracts.

Clearing is more challenging for OTC derivatives compared to exchange-traded derivatives given the generally longer maturities. OTC derivatives trades could be cleared by CCPs.

LO 46.4

OTC derivatives comprise of five broad classes of derivatives: interest rate, foreign exchange, equity, commodity, and credit derivatives. Interest rate derivatives comprise the largest class, followed by foreign exchange derivatives and credit derivatives.

LO 46.5

Mechanisms for controlling counterparty risk include: special purpose vehicles (SPVs), derivatives product companies (DPCs), monolines, and credit derivative product companies (CDPCs).

SPVs are bankruptcy remote legal entities set up by a parent firm to shield the SPV from any financial distress of the firm. SPVs essentially alter bankruptcy rules and transform counterparty risk into legal risk. The legal risk is consolidation, or the risk that the courts view the SPV and the originating firm as the same legal entity.

DPCs are bankruptcy remote subsidiaries of firms set up to originate derivatives products sold to investors. DPCs are separately capitalized and have restrictions on their activities and margin. They are generally AAA rated where the rating depends on three criteria: (1) market risk minimization (2) parent support, and (3) credit risk and operational risk management.

Monolines are highly-rated insurance companies that provide financial guarantees, or "credit wraps" to investors. CDPCs are akin to DPCs, but with a business model that is closer to that of a monoline.

Concept Checkers

1. Which of the following functions is least likely performed by an exchange?
   A. Derivatives contract design and specifying contract terms.
   B. Price negotiation through a bilateral process.
   C. Limiting access to approved firms and individuals.
   D. Reporting transaction prices to trading participants and data vendors.

2. Consider counterparties *A*, *B*, and *C*, which are members of a derivatives exchange. *A* is short a derivatives position with *B*, and *B* is short the same derivatives position with *C*. Replacing these two positions with a single position between *A* and *C* is an example of:
   A. direct clearing.
   B. bilateral clearing.
   C. complete clearing.
   D. clearing ring.

3. When contrasting exchange-traded derivatives and over-the-counter (OTC) derivatives, basis risk and credit risk are generally a concern for:

	Basis Risk	Credit Risk
A.	exchange-traded derivatives	exchange-traded derivatives
B.	exchange-traded derivatives	OTC derivatives
C.	OTC derivatives	exchange-traded derivatives
D.	OTC derivatives	OTC derivatives

4. A credit default swap (CDS) most likely has higher counterparty risk than an interest rate swap (IRS) given that it has:

	Wrong-Way Risk	Reduced Volatility
A.	Yes	No
B.	Yes	Yes
C.	No	No
D.	No	Yes

5. Which of the following entities best describe a mechanism that transforms counterparty risk into legal risk?
   A. Derivatives product companies (DPCs).
   B. Credit derivative product companies (CDPCs).
   C. Special purpose vehicles (SPVs).
   D. Monolines.

Concept Checker Answers

1. **B** Exchanges set specific prices and standardize contracts. They do not negotiate prices bilaterally. Price negotiation through a bilateral process is a feature of the OTC derivatives market.

2. **D** Clearing rings allow members to substitute counterparties and replace multiple positions with a single netted position. They are designed to mitigate counterparty risk, improve liquidity, and facilitate the close-out process.

3. **B** Basis risk refers to the risk of term mismatches, and is a concern when using standardized exchange-traded contracts that may not precisely match the term or size of position to be hedged. Basis risk is a less of a concern with OTC derivatives, which can be customized to meet specific investor hedging needs.

   Counterparty (credit) risk refers to the risk of counterparty default or non-payment. It is a risk specific to OTC derivatives trading, and is less of a concern when trading with central counterparties through exchanges.

4. **A** CDSs typically have *higher* volatility and exhibit considerable *wrong-way risk* (a risk that arises when the exposure to a counterparty is inversely related to the credit quality of the counterparty).

5. **C** SPVs are bankruptcy remote legal entities set up by a firm to shield the SPV from financial distress of the parent firm. SPVs essentially alter bankruptcy rules and transform counterparty risk into legal risk.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Basic Principles of Central Clearing

Topic 47

Exam Focus

This topic covers the principles of central clearing, including the functions and mechanics of a central counterparty (CCP). For the exam, be able to describe these functions and understand the key related terminology. Clearing, novation, netting, and offset are also important concepts for the exam. In addition, understand the advantages and disadvantages of CCPs, and be able to discuss common terms, including offsetting, loss mutualization, moral hazard, adverse selection, and procyclicality.

The Role of a Central Counterparty

LO 47.1: Provide examples of the mechanics of a central counterparty (CCP).

Clearing and Settlement

A **central counterparty** (CCP) plays an important role in the clearing and settlement of transactions following the initial trade execution. **Clearing** refers to the processes (including margining and netting) between the period from trade execution until settlement. This period is typically short (a few days or months) for classically cleared non over-the-counter (OTC) derivatives. In contrast, for OTC derivatives this time period could extend to years or even decades. **Settlement** of a trade occurs when the trade is completed and all payments have been made and legal obligations satisfied.

A CCP's primary function is to simplify the operational processes and reduce counterparty risk that exists in the bilateral market. When a CCP interjects itself as the central counterparty between OTC trades and acts as the seller to each buyer and the buyer to each seller, it reduces the interconnectedness of trades and of participants, and reduces the risk of default or non-payment by a counterparty. At the same time, the process improves trade liquidity and transparency.

Auctions and Loss Mutualization

Key functions of a CCP related to the clearing process include: margining, novation, netting, managing the auction process, and loss mutualization. Margining will be discussed in LO 47.3, and novation and netting will be discussed in LO 47.4.

When a central clearing member defaults, rather than closing out the trades at market value, the CCP typically auctions off the trades to the surviving members through an **auctioning** process. Participating in the auctioning process is in the best interest of the members in order to minimize their losses through lower market prices or through default funds.

Loss mutualization is a form of insurance and refers to members' contributions to a default fund to cover future losses from member defaults. Since all members must contribute to the fund, the potential losses from the default of any given member are contained. When a member does default, any amounts that cannot be covered from the member's own resources are covered from the fund. Given that losses are spread among surviving members, it is possible that a member will suffer losses even if it never traded with the defaulting counterparty or had no positions with the CCP.

Other Mechanics of a CCP

Products

The OTC derivatives markets include a wide range of products ranging from standard to non-standard and exotic derivatives. There are currently four categories of OTC derivatives according to their stages of central clearing history:

1. Products with a long history of central clearing (e.g., interest rate swaps).
2. Products with a short history of central clearing (e.g., index credit default swaps).
3. Products that may soon be centrally cleared (e.g., interest rate swaptions, credit default swaps).
4. Products that are not suitable for central clearing (e.g., exotic derivatives including Asian options, and derivatives with illiquid reference assets).

The following conditions are important for a product to be centrally cleared:

- *Standardization*: Legal and economic terms should be standard.
- *Complexity*: Transactions need to be easily valued for trading and margin purposes, therefore only less complex (i.e., vanilla, not exotic) trades can be cleared.
- *Liquidity*: Cleared products are typically more liquid than OTC products. Liquidity is important for determining market price for initial margin and default fund contributions, and for the auctioning process. CCPs are also reluctant to develop clearing capability for products that could not be properly cleared due to thin trading. Liquidity also allows for easier close out of trades in a default scenario.

Participants

Transacting with CCPs is restricted to clearing members only. Becoming a member includes a number of requirements, including:

- *Admission criteria*: CCPs set different criteria for admission, including restrictions on credit quality (e.g., investment grade only) and size (e.g., minimum $50 million).
- *Financial commitment*: The primary financial commitment by a member is to contribute to the CCP's default fund.
- *Operational criteria*: Members' operational requirements include posting margin, and participating in "fire drills" to simulate member default and in auctions if default does occur.

These criteria can be onerous, and as a result, only large banks or global financial institutions typically become clearing members. Smaller entities including small banks and financial institutions and some non-financial end users would likely not participate as direct clearing members, but would participate in the clearing process through transacting with a member on a principal-to-principal basis, or on an agency basis. These players would therefore have a bilateral relationship with the clearing member but not the CCP. This clearing process may be similar to the clearing between the member and the CCP, with some differences, including no default fund commitment by the non-member players.

Number of CCPs

A single, large CCP may be optimal given the benefits of offsetting trades and cost minimization through economies of scale. However, it is generally not feasible to have a single CCP for the following reasons:

- *Regional differences*: Regional CCPs may be beneficial to centrally clear trades in the region's currency and under the laws and regulations of the region.
- *Product types*: CCPs often specialize in clearing certain derivatives products.
- *Regulatory reasons*: Regulations may dictate that products be cleared by local CCPs. However, CCPs need not operate in isolation, and CCPs working together may be necessary. It is important to recognize, however, that this may increase systemic risk and the risk of contagion during stress times.

Types of CCPs

Arguments exist for both a utility-driven CCP and a profit-driven CCP. A utility-driven CCP would be focused on long-term stability rather than short-term profits. A profit-driven CCP would be focused on the bottom line in order to attract personnel and build the best systems. Currently, there are stronger arguments in favor of profit-driven CCPs.

Failure of a CCP

While it is an infrequent event, CCPs do fail. However, the failure of a large and systemically important CCP could lead to potentially catastrophic events. As a result, CCPs must maintain sufficient loss absorption methods to withhold large member defaults. The financial trouble of a CCP may ultimately result in liquidity support from a central bank.

Central Clearing

LO 47.2: Describe advantages and disadvantages of central clearing of OTC derivatives.

The following table illustrates the primary differences between the OTC derivatives markets and CCPs and exchanges. It should be noted that while the OTC market provides the greater breadth of participants and products and offers customization, the CCP/exchange-based market offers stronger margining and loss buffers and reduced counterparty and systemic risk.

	OTC Derivatives	*CCP/Exchanges*
Trading	Bilateral	Bilateral / Centralized
Counterparty	Original trade counterparty	CCP (replaces counterparty)
Participants	All	Clearing members (dealers)
Products	All (including non-standard, exotic)	Standard, vanilla
Margining	Bilateral, custom	Full margining set by CCP (initial, variation)
Loss buffers	Margin, regulatory capital	Initial margin, default fund, CCP capital

Advantages of Central Clearing

Central clearing through CCPs has the following advantages:

- *Transparency*: In OTC markets, parties typically do not see all outstanding trades between the various counterparties. CCPs have a consolidated view of trading positions and can therefore better react to extreme events.
- *Offsetting*: By transacting through a CCP, duplicate bilateral contracts can be offset, which improves flexibility for new transactions and reduces costs.
- *Loss mutualization*: A member's losses are distributed among all surviving members, which spread the impact of losses, reduce costs, and minimize market impact and systemic risk.
- *Legal and operational efficiency*: The centralized role of CCPs in the clearing (margining, netting) and settlement process improves operational efficiency while reducing costs.
- *Liquidity*: The daily margining of products in a centrally-cleared market ensures greater transparency in product valuation, which increases product liquidity.
- *Default management (counterparty risk)*: CCPs act as the counterparty to each trade, which reduces counterparty risk. Member defaults are centrally managed through the auction process which minimizes price disruptions.

Disadvantages of Central Clearing

While we noted loss mutualization as an advantage of the central clearing process, it can lead to potential problems, including moral hazard and adverse selection.

- *Moral hazard*: Moral hazard is the risk that one party will take on higher risk knowing that another party bears the costs of this risk. In central clearing, the risk is that members will have less incentive to monitor risk knowing that the CCP takes on most of the risks.
- *Adverse selection*: Adverse selection is the risk that participants with a better understanding of product risks and pricing will trade more products whose risks the CCP underprices, and will trade fewer products whose risks the CCP overprices.
- *Bifurcation*: The separation of trading into cleared and non-cleared products can increase cash flow volatility even for hedged products.
- *Procyclicality*: Procyclicality essentially reflects the downside of margining. It reflects a scenario where a CCP increases margin requirements (initial margin) in volatile markets or during a crisis, which may aggravate systemic risk.

Margining

LO 47.3: Compare margin requirements in centrally cleared and bilateral markets, and explain how margin can mitigate risk.

One of the risk mitigation tools employed by CCPs to minimize counterparty and market risk is margining. **Margining** by CCPs is stricter than in the OTC derivatives markets and it involves posting cash or marketable security collateral for initial margin and variation margin requirements. Initial margin represents cash or liquid assets transferred by a member at trade inception to cover a worst-case loss in the event of a member default. Variation margin is typically cash posted by a member to cover the daily net change of the member's position.

CCPs normally set margin requirements based only on the risks of the members' transactions. For initial margin, the credit quality of the member is typically not a consideration and therefore members with different credit risk may be posting the same amount of initial margin.

Novation and Netting

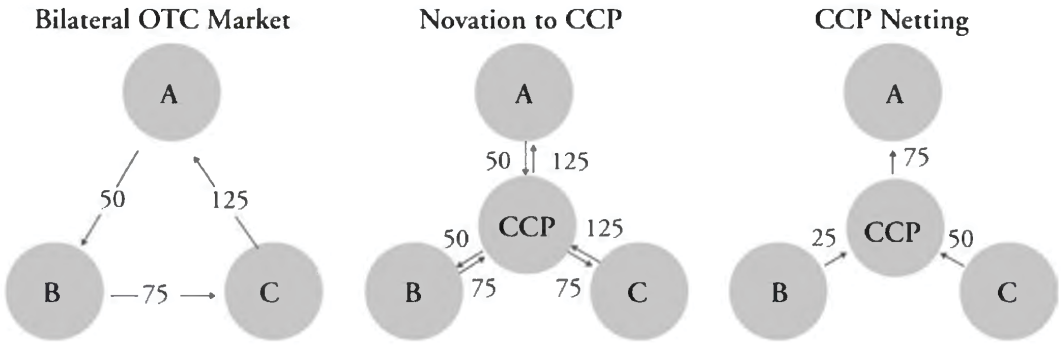
LO 47.4: Compare and contrast bilateral markets to the use of novation and netting.

The legal process of interposing the CCP between the seller and the buyer is called **novation**. Through novation, one contract (the bilateral contract between OTC participants) is replaced with another contract (or contracts) with the CCP. This is important because novation transforms the process from bilateral trading to trading with a CCP, where the CCP is the insurer of counterparty risk. Assuming the legal enforceability of novation, the old contracts cease to exist and the original bilateral parties have no further obligations to each other. At the same time, because all trades are centralized, the CCP maintains a "matched book" of trades with no net market risk. The CCP does have conditional credit risk from a member's potential default.

Market participants often prefer to offset rather than to terminate trades, which creates redundant trades. When trades are novated to a CCP, these redundant trades become a single net obligation between each participant and the CCP. This process is called **multilateral offsetting**, or **netting**. Netting reduces total risk and minimizes the potential of a domino effect stemming from the default of a participant.

The process of moving from bilateral to novation to netted positions is illustrated in Figure 1.

Figure 1: Multilateral Offsetting



Impact of Central Clearing

LO 47.5: Assess the impact of central clearing on the broader financial markets.

By now it should be evident that central clearing through a CCP has significant advantages, but it is not without its challenges. When a CCP is included in the clearing process, systemic risk in the financial markets is reduced, but can be increased at the same time. Systemic risk is reduced because CCPs reduce counterparty risk by offsetting positions (novation and netting), they provide transparency for the market, and improve liquidity. However, the potential requirement that members post higher initial margin during times of increased market volatility could increase systemic risk. In addition, concentrating all trades in a single place exposes the market to the risk of CCP failure and heightened systemic risk.

It is worth noting that the protection that CCPs offer for OTC derivatives may come at the expense of other groups. For example, netting and margining protects OTC derivatives participants, but may not benefit creditors. In addition, the long-term maturity of OTC derivatives contracts, often years or decades, also poses challenges for CCPs. It is not yet evident that they are effective in clearing these long-dated, more complex and illiquid trades. Central clearing of exotic or non-standard derivatives may also be problematic.

Key Concepts

LO 47.1

CCPs play important roles in the clearing and settlement of transactions. Clearing refers to the processes between the period from trade execution until settlement. Settlement refers to the satisfaction of legal obligations and trade completion.

Functions of a CCP include novation, netting, margining, managing the auction process, and loss mutualization. Auctioning refers to selling off the defaulted member's trades to the surviving members through an auctioning process. Loss mutualization refers to members' contributions to a default fund to cover future losses from member defaults.

Other aspects and mechanics of CCPs include:

- *Categories of OTC derivatives products*: (1) long history of central clearing (e.g., interest rate swaps), (2) short history of central clearing (e.g., index credit default swaps), (3) soon to be centrally cleared (e.g., interest rate swaptions, credit default swaps), and (4) not suitable for central clearing (e.g., exotic derivatives).
- *Conditions needed for central clearing*: product standardization, lower complexity, and high liquidity.
- *Participants*: Transacting with CCPs is restricted to clearing members only. Member criteria include admission criteria, financial commitment, and operational criteria.
- *Number of CCPs*: It is generally not feasible to have a single CCP due to regional differences in trades and requirements, differences in product types, and regulatory reasons.
- *Types of CCPs*: CCPs could be utility-driven CCP (i.e., focused on long-term stability) or profit-driven CCP (i.e., focused on bottom line). Arguments generally support profit-driven CCPs.
- *Failure of a CCP*: The potential failure of a large CCP could create a catastrophic event. CCPs must therefore ensure sufficient loss absorption capacity.

LO 47.2

Advantages of CCPs include: transparency, offsetting, loss mutualization, legal and operational efficiency, liquidity, and default management.

Disadvantages of CCPs include: moral hazard, adverse selection, separation of cleared and non-cleared products, and procyclicality of margin requirements.

LO 47.3

Margining includes posting both initial margin and variation margin. Margining tends to be more stringent in central clearing than in OTC markets. CCPs set margin requirements based only on the risks of the members' transactions, and the credit quality of the member is typically not a consideration for initial margin.

LO 47.4

Novation refers to replacing a bilateral OTC contract with another contract (or contracts) with the CCP, where the CCP is the insurer of counterparty risk. The CCP maintains a "matched book" of trades with no net market risk.

Multilateral offset, or netting, refers to creating a single net obligation between each participant and the CCP from the various bilateral OTC trades (which typically include redundant trades). Netting reduces total risk and minimizes contagion from a member default.

LO 47.5

By including a CCP in the clearing process, systemic risk can be both reduced and increased. Systemic risk is reduced because counterparty risk is reduced, and transparency and liquidity improve. Systemic risk is increased because higher initial margin during times of stress would heighten market risk, and the failure of a CCP may lead to a catastrophic event.

Concept Checkers

1. Which of the following statements on central clearing is accurate?
   I. The composition of clearing members typically includes a combination of large global banks and smaller banks and non-financial institutions.
   II. In the auction process, a CCP normally does not close out trades at their market value.
   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

2. Alex Dell, a derivatives trader, has some reservations about the central clearing of OTC derivatives with a central counterparty (CCP). Specifically, he is worried that clearing members' willingness to monitor credit risk may decline since the CCP assumes most of the risks, and that CCPs may increase margin requirements during a period of market stress. Which of the following concepts best describe Dell's reservations?

	Decline in Willingness	Higher Margin Requirements
A.	Moral hazard	Procyclicality
B.	Adverse selection	Offsetting
C.	Moral hazard	Offsetting
D.	Adverse selection	Procyclicality

3. In a recently released report to management, a credit analyst indicates that the level of initial margin set by a central counterparty (CCP) is dependent on the risk of the member that is required to post it, and on the risks of the specific derivatives transactions. The analyst is correct with respect to:

	Risk of Member	Risk of Transactions
A.	Yes	Yes
B.	Yes	No
C.	No	Yes
D.	No	No

4. Alpha Bank recently noted that its bilateral over-the-counter (OTC) trade obligations with Beta Bank ceased to exist and the bank now directly faces a central counterparty (CCP) for its trade obligations. Which of the following concepts best identify this scenario?
   A. Netting.
   B. Novation.
   C. Margining.
   D. Multilateral offsetting.

5. Erin Parker and Nate James are analysts at a large financial institution. During one of their recent discussions on OTC derivatives and central clearing with a central counterparty (CCP), Parker states that: "CCPs are beneficial because they convert operational and legal risk into counterparty risk." James adds to that statement by suggesting: "When requiring higher margin in turbulent times, CCPs reduce systemic risk."

With respect to the statements made:

A. Only Parker is correct.
B. Only James is correct.
C. Both Parker and James are correct.
D. Neither Parker nor James is correct.

Concept Checker Answers

1. **B** Clearing members typically include large players only, including large banks and global financial institutions. In the auction process, a CCP normally does not close out trades at their market value. Instead, trades are auctioned to existing members.

2. **A** Dell's reservations describe moral hazard and procyclicality, respectively. In central clearing, moral hazard is the risk that members have less incentive to monitor risk knowing that the CCP assumes most of the risks of the transactions. Procyclicality describes a scenario where a CCP increases margin requirements (initial margin) in volatile markets or during a crisis, which may aggravate systemic risk.

   Offsetting describes the elimination of duplicate bilateral contracts by transacting through a CCP, which improves flexibility and reduces costs. Adverse selection is the risk that participants with a better understanding of product risks and pricing will trade more products whose risks the CCP underprices, and fewer products whose risks the CCP overprices.

3. **C** CCPs set initial margin requirements based on the risk of the transactions, but not on the risk of the members.

4. **B** Novation describes the process where one contract (the bilateral contract between OTC participants) is replaced with another contract (or contracts) with the CCP. As a result, counterparties' bilateral obligations with each other cease to exist.

   Multilateral offsetting, also called netting, refers to creating, from the various bilateral OTC trades, a single net obligation between each participant and the CCP.

   Margining is the process of posting some form of collateral, typically cash or marketable securities, to cover member defaults (initial margin) or security mark-to-market movements (variation margin).

5. **D** Neither Parker nor James is correct. CCPs convert counterparty risk into operational and legal risk. Also, when CCPs require higher margin in turbulent times, CCPs can increase systemic risk. This risk is known as procyclicality.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Risks Caused by CCPs

Topic 48

Exam Focus

This short and qualitative topic deals with the many risks specific to central counterparties (CCPs). Some of the types of risks such as default, liquidity, and operational risks are common to most entities while others such as settlement and payment risk, and custody risk are more specific to CCPs. For the exam, it is important to have an understanding of the full range of risks faced by CCPs as well as the methods to prevent CCP failures.

Risks Faced by Central Counterparties

LO 48.1: Identify and explain the types of risks faced by CCPs.

Default Risk

The default of a clearing member and its flow through effects is the most significant risk for a CCP. Because of a default, there may be the default or distress of other clearing members given that default correlation is likely to be high among over-the-counter (OTC) derivatives market participants.

In the event of a failed auction or an insufficient number of bids, the CCP will be required to pass on the defaulting member's losses through rights of assessment, loss allocation methods, or both. Passing on losses to other clearing members may result in defaults by those members. The loss allocation methods may be considered unfair because some of them, such as variation margin gains haircutting (VMGH) and tear-ups, impose losses on "winning positions." With VMGH, members whose positions increased in value (i.e., they are owed variation margin) will likely not receive the full amount for their gains (i.e., haircutting). Members who instead owe money to the CCP will still be required to pay the full margin amount to the CCP. In a tear-up, the CCP terminates the unmatched position, and may balance resources by drawing from both the defaulter's initial margin and the default fund.

Some clearing members may resign from the CCP after the default of another clearing member. In such instances, the applicable initial margins and default funds must be returned to the resigning clearing member. The initial resignation may result in a negative reputational impact to the CCP as witnessed by further resignations of clearing members.

Model Risk

OTC derivatives are not priced by the market but are instead priced using valuation models that perform the mark-to-market function, which subjects CCPs to model risk. Especially

sensitive to model risk would be a CCP's determination of initial margins. In that context, model risk could arise due to errors pertaining to volatility, tail risk, complex dependencies, and wrong-way risk.

Many models are linear in nature, which means that an initial margin will be adjusted in proportion to the increase in the size of the position. However, for large or concentrated positions, the margin may be too low. The use of a supplement to the computation, such as a margin multiplier, may assist in sufficient coverage of the risk.

Liquidity Risk

There are large amounts of cash inflows and outflows flowing through the CCP due to initial margins and margin calls. As a result, CCPs are exposed to liquidity risk. The CCP attempts to earn the greatest return possible on the funds it holds without incurring too much credit or liquidity risk, thereby most commonly investing in short-term deposits, repos, and reverse repos. Should there be a default by one or more members, the CCP is still required to meet the obligations of the other members.

There is the risk that the CCP's investments are not always quickly and easily convertible to cash, which may require some liquidity support from a central bank. In this regard, a CCP is required to have sufficient liquid resources to meet its obligations in the event of the failure of one or two of its largest clearing members. The Basel III leverage ratio (calculated as the bank's tier 1 capital divided by its exposure) requirements serve to minimize the amount of risk taking. Exposure would include the gross notional amount of centrally cleared OTC derivatives transactions. Overall, regulations have attempted to address a CCP's potential liquidity risks; however, they may reduce the availability of clearing services.

Operational Risk

Due to the centralization of some functions within a CCP to increase efficiency, additional risks arise that affect counterparties due to concentration at the CCP. CCPs face operational risks that are common to all entities such as business interruption due to information systems failures and internal or external fraud. However, a systems failure within a CCP could have a disastrous impact on many counterparties, especially if they hold large positions.

Legal Risk

Legal risks in the form of litigation or claims may arise due to differing laws in different jurisdictions or laws that are inconsistent with the CCP's regulations. A good example would involve the segregation and movement of margin and positions (i.e., netting) through a CCP.

Other Risks

Investment risk refers to the risk of losses of margin funds resulting from investment actions performed within or outside of the stated investment policy.

Settlement and payment risk refers to the risk that a bank no longer provides cash settlement services between a CCP and its members.

Foreign exchange risk refers to the risk of mismatches between margin payments and cash inflows or outflows in different currencies.

Custody risk refers to the risk of loss of securities, margins, or both by a custodian due to its failure, fraud, or negligence.

Concentration risk refers to the risk of clearing members, margins, or both that are located in a single geographic area. Essentially, it is a lack of diversification.

Sovereign risk refers to the risk that a foreign government could default on its debt obligations, thereby causing members to fail. It also refers to any potential loss in the value of sovereign bonds held as margin.

Wrong-way risk refers to the risk that exposure to a counterparty is negatively correlated with the credit quality of the counterparty. In other words, it occurs when credit exposure to a counterparty and the default risk of the counterparty increase together.

Overall, it is probable that the various loss events will be correlated and will impact the CCP at the same time. In the case of a default, there will probably be a major market impact that increases the probability of operational and investment issues. Additionally, in a default scenario, there is usually a wide spread between gain and loss positions that increase legal and fraud risks.

Risks to Clearing Members and Non-Members

LO 48.2: Identify and distinguish between the risks to clearing members as well as non-members.

Non-members face exposure from CCPs, clearing members, and other non-members. If a CCP fails, a non-member may be able to avoid losses so long as its counterparty (a clearing member) is solvent. Unlike clearing members, non-members are not required to contribute to default funds so, therefore, non-members are not exposed to losses that result from CCP failures.

Furthermore, the extent of non-members' losses due to defaults of CCPs and clearing members lies with the initial margins and whether they are segregated, guaranteed, or both. In addition, non-members face the risk of not being able to port their trades should the counterparty member default. As a result, such trades may have to be closed out at a loss.

Finally, one has to consider non-members' liability with respect to CCP loss allocation rules. It is possible that clearing members are able to pass on losses to non-members through VMGH or tear-up, which would reduce the gains of non-members. Clearing members are unable to pass on losses resulting from default fund utilization, rights of assessment, and forced allocation.

Lessons Learned from CCP Failures

LO 48.3: Identify and evaluate lessons learned from prior CCP failures.

There are five key lessons learned from prior CCP failures:

1. Operational risk must be controlled to the maximum extent possible. For example, information systems should be updated sufficiently to be robust enough to handle unusually high trading volumes and to detect significant price changes.

2. Variation margins should be recalculated often and collected quickly (i.e., multiple times a day in certain cases). Having an information system that allows for automated payments could assist in preventing liquidity shortfalls. In addition, having cross-margining linkage arrangements (offsetting of hedged positions) between CCPs may avoid liquidity problems due to the hedging activities of the various CCPs.

3. Initial margins and default funds should be sufficiently large in order to withstand significant negative asset value declines as well as increased return correlations during a crises. The assumptions behind the initial margin computations need to be amended to account for significant changes in the market.

4. CCPs must actively monitor positions, penalize overly concentrated positions, and promptly liquidate or hedge extremely large positions.

5. CCPs must have one or more external sources of liquidity to avoid default due to illiquidity (even though it is still solvent).

Key Concepts

LO 48.1

CCPs face five major risks: default risk, model risk, liquidity risk, operational risk, and legal risk. Other risks they may face include investment risk, settlement and payment risk, foreign exchange risk, custody risk, concentration risk, sovereign risk, and wrong-way risk.

The default of a clearing member and its flow through effects is the most significant risk for a CCP. Because of a default, there may be the default or distress of other clearing members given that default correlation is likely to be high among OTC derivatives market participants.

LO 48.2

Non-members face exposure from CCPs, clearing members, and other non-members.

If a CCP fails, a non-member may be able to avoid losses so long as its counterparty is solvent. Non-members are not required to contribute to default funds so they are not exposed to losses that result from CCP failures. The extent of non-members' losses lies with the initial margins and whether they are segregated, guaranteed, or both. Non-members face the risk of not being able to port their trades should the counterparty member default.

LO 48.3

Lessons learned from prior CCP failures include:

- Operational risk must be controlled to the maximum extent possible.
- Variation margins should be recalculated often and collected quickly.
- CCPs should have an information system that allows for automated payments.
- There should be cross-margining linkage arrangements between CCPs.
- Initial margins and default funds should be sufficiently large.
- CCPs must actively monitor positions.
- CCPs must have one or more external sources of liquidity.

Concept Checkers

1. Which of the following risks facing a central counterparty (CCP) is most likely to be introduced during a market crisis?
   A. Default risk.
   B. Liquidity risk.
   C. Operational risk.
   D. Settlement and payment risk.

2. Which of the following statements regarding risks facing a CCP is correct?
   A. A good example of legal risk would involve netting arrangements.
   B. Default correlations tend to be low among OTC derivatives market participants.
   C. Many models for pricing OTC derivatives are linear in nature, which may result in excessive margins for large positions.
   D. Investment risk refers to the risk of losses of margin funds resulting from investment actions performed outside of the stated investment policy.

3. A non-clearing member would face exposure from defaults by which of the following parties?
   I. Clearing members.
   II. Other non-clearing members.
   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

4. Which of the following losses may be borne by non-members of CCPs?
   A. Rights of assessment.
   B. Tear-up.
   C. Default fund utilization.
   D. Forced allocation.

5. Which of the following statements regarding lessons learned from prior CCP failures is correct?
   A. CCPs face the risk of default through insolvency even though they may be liquid.
   B. In extreme cases, variation margins should be recalculated and collected on a daily basis.
   C. Initial margins must be sufficient enough to withstand situations of extreme dependency.
   D. CCPs should actively monitor positions but they need not penalize unintended concentrated positions.

Concept Checker Answers

1. **D** Settlement and payment risk refers to the risk that a bank no longer provides cash settlement services between a CCP and its members. Such risk is not likely to be present during normal periods but is much more likely to be present during crisis periods.

   The other risks mentioned are present in both normal and crisis periods. For example, liquidity and default risks are always present but would be exacerbated during a crisis period. Operational risks such as inadequate information systems that give rise to business interruption could be present in a normal period.

2. **A** Legal risks in the form of litigation or claims may arise due to differing laws in different jurisdictions or laws that are inconsistent with the CCP's regulations. A good example would involve the segregation and movement of margin and positions (i.e., netting) through a CCP.

   Response B is not correct, because default correlation is likely to be *high* among OTC derivatives market participants. Response C is not correct, because many models are linear in nature, which means that an initial margin will be adjusted in proportion to the increase in the size of the position. However, for large and/or concentrated positions, the margin may be too low. Response D is not correct, because investment risk refers to the risk of losses of margin funds resulting from investment actions performed *within or outside* of the stated investment policy.

3. **C** Non-clearing members face exposure from CCPs, clearing members, and other non-clearing members.

4. **B** Clearing members may be able to pass on losses to non-members through a "tear-up," which would reduce the gains of non-members. Clearing members are unable to pass on losses resulting from default fund utilization, rights of assessment, and forced allocation.

5. **C** An example of extreme dependency would be the increase in correlation of returns during a market crisis.

   Response A is not correct, because CCPs face the risk of default through *illiquidity* even though they may be *solvent*. Response B is not correct, because in extreme cases, variation margins should be recalculated and collected up to several times a day. Response D is not correct, because CCPs should actively monitor positions and penalize overly concentrated positions regardless of whether they were intended or unintended.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Foreign Exchange Risk

Topic 49

Exam Focus

Exposure to foreign exchange risks is a natural result of the globalization of financial institutions. These risks arise when foreign currency trading and/or foreign asset-liability positions are mismatched in individual currencies. Unexpected volatility can generate significant losses for the firm, which could, in turn, threaten profitability or even solvency. These risks can be mitigated by direct hedging through matching foreign asset-liability books of business, hedging through forward contracts, and through foreign asset and liability portfolio diversification.

Sources of Foreign Exchange Risk

LO 49.1: Calculate a financial institution's overall foreign exchange exposure.

Large financial institutions (banks) frequently take significant positions in foreign currency assets and liabilities as a result of their foreign exchange trading activities. When looking at such financial institutions' currency trading activities, the aggregate position size in a particular currency may look extremely large; however, since buys and sells will offset one another in terms of exposure, the net exposure to the currency may actually be quite small.

A bank's actual exposure to any given currency can be measured by the **net position exposure**. Net exposure is the extent to which a bank is net long (or *positive*) or net short (or *negative*) in a given currency. For example, a bank's net euro (EUR) exposure would be:

$$\text{net EUR exposure} = (\text{EUR assets} - \text{EUR liabilities}) + (\text{EUR bought} - \text{EUR sold})$$

$$\text{net EUR exposure} = \text{net EUR assets} + \text{net EUR bought}$$

The first component of the equation represents the bank's net asset position in the EUR, or the difference between the EUR denominated assets (e.g., loans) and its EUR denominated liabilities (e.g., deposits). If this component is positive, the bank has more EUR denominated assets than liabilities. The second component represents its net position in its trading book; if it is positive, the bank has purchased more EURs than it has sold.

A **positive net exposure** position means that the bank is *net long in a currency*. In this instance, the financial institution faces the risk that the foreign currency will *fall* in value against the domestic currency.

A **negative net exposure** position means that the bank is *net short in a currency*. The financial institution faces the risk that the foreign currency will *rise* in value against the domestic currency.

Therefore, if a financial institution does not maintain a balanced position in a currency, the institution will be exposed to variations in the foreign exchange (FX) rate of that currency.

The more volatile the FX rate, the more potential impact a net exposure (either long or short) will have on the value of a bank's foreign currency portfolio.

LO 49.2: Explain how a financial institution could alter its net position exposure to reduce foreign exchange risk.

A financial institution could reduce its foreign exchange exposure by altering either or both components of the net position exposure equation. For example, to reduce its exposure to zero, it could:

- Match its foreign currency assets to its liabilities on the balance sheet (the first component would be equal to zero) and match its long and short trading positions in the currency (the second component would be zero).
- Be long the currency in one component and short the currency in the other component so the two positions offset.

Financial holding companies could further reduce FX exposure by aggregating and netting individual exposures across their operating units, so that positive exposures in some parts of the business are offset by negative exposures in other units.

LO 49.3: Calculate a financial institution's potential dollar gain or loss exposure to a particular currency.

The potential gain/loss exposure to a foreign currency (FC) is a function of the size of the position and the potential change in the value of the foreign currency:

dollar gain/loss in EUR = net EUR exposure (measured in $) × % change in the $/FC rate

Note that a positive exposure and a positive change in the FC yield a gain; a negative exposure and negative change in the FC also yield a gain. Similarly, a positive exposure and negative change or a negative exposure and a positive change yield a loss.

Example: Computing potential currency gain/loss

A financial institution has the following positions in the British pound:

- Assets: £100,000
- Liabilities: £200,000
- Bought: £40,000
- Sold: £20,000

Suppose the value of the pound is expected to change from 1.3 to 1.4 $/£. **Calculate** the potential gain or loss exposure to the pound.

Answer:

The financial institution's net position exposure in the pound is (£100,000 – £200,000) + (£40,000 – £20,000) = –£80,000, or –£80,000 × 1.3 = –$104,000.

The value of the pound is expected to change by: (1.4 – 1.3) / 1.3 = 7.69%.

The potential dollar gain/loss exposure to the pound = –$104,000 × 0.0769 = –$7,998 (loss).

Foreign Trading Activities

LO 49.4: Identify and describe the different types of foreign exchange trading activities.

A financial institution's buying and selling of foreign currencies, and hence the institution's position in the FX market, reflects four key trading activities:

1. Enabling customers to participate in international commercial business transactions.
2. Enabling customers to take positions in real or financial foreign investments. Note that a financial institution may also transact in foreign currencies to take positions in real or financial foreign investments for its own portfolio.
3. Offsetting exposure in a given currency for hedging purposes.
4. Speculating on foreign currencies in search of profit by forecasting and/or anticipating futures FX rate movements.

When a bank is buying or selling a foreign currency for the purpose of either allowing its customers to participate in international commercial business transactions or investing in real or financial foreign investments, the bank typically serves as an agent for the customers (receives a fee) and does not assume the FX risk itself.

When a bank is buying or selling a currency for hedging purposes, this will reduce FX exposure.

The fourth activity, trading foreign currencies with the intent to profit by anticipating future foreign currency rate movements, relates to open positions that are taken for speculative purposes and represents an unhedged position in a given currency. These speculative trades are usually made directly with other financial institutions or arranged through FX specialist brokers.

Sources of Profits and Losses on Foreign Exchange Trading

LO 49.5: Identify the sources of foreign exchange trading gains and losses.

Most returns on FX trading arise from speculation in currencies or taking an unhedged position in a particular currency. Financial institutions also earn fees as a secondary source of revenues. These revenues are earned from market-making activities, acting as agents for retail or wholesale customers, or a combination of both.

LO 49.6: Calculate the potential gain or loss from a foreign currency denominated investment.

Mismatched Foreign Asset and Liability Positions

A financial institution can also have foreign exchange exposure due to mismatches between foreign financial asset and liability portfolios. The following example shows the exposure resulting from such a mismatch.

Example: Foreign investment returns

Figure 1: Balance Sheet

Assets	*Liabilities*
USD50 million U.S. loans, 1-year maturity, in USD, yielding 8% USD50 million equivalent Swiss loans, 1-year maturity, made in CHF, yielding 13%	USD100 million U.S. CDs, 1-year maturity, in USD, yielding 6%

This firm has matched the duration of its assets and liabilities ($D_A = D_L = 1$ year) but has mismatched the currency composition of its portfolio. Note that the firm would earn a positive spread of 2% (8% – 6%) from investing domestically. In order to invest in Switzerland, this firm decides to take 50% of its $100 million and make 1-year Swiss loans while keeping 50% to make U.S. dollar loans. What transactions must the firm undertake to make the CHF-denominated loan (assuming the FX position is not hedged)?

Answer:

1. Sell USD50 million for CHF on the spot currency markets at the beginning of the year. If the exchange rate is USD1.70 to 1 CHF, this yields USD50,000,000 / 1.7 = CHF29,411,765.
2. Use the CHF29,411,765 to make 1-year Swiss loans at a 13% interest rate.
3. At the end of the one year, CHF revenue from these loans will be CHF29,411,765(1.13) = CHF33,235,294 (assuming no default).
4. At the end of the year, repatriate these funds back to the United States. In other words, the U.S. bank will sell CHF33,235,294 in the FX market at the spot exchange rate that exists at the end of the year.

In this example, we assume the spot FX rate has not changed over the 1-year period and remains at USD1.70/CHF. The dollar proceeds from the Swiss investment would be:

$$\text{CHF33,235,294} \times \text{USD1.70 / CHF} = \text{USD56,500,000, for a return of:}$$

$$\frac{\text{USD56,500,000} - \text{USD50,000,000}}{\text{USD50,000,000}} = 13.0\%$$

Thus, the weighted return on this portfolio will be:

$$(0.5)(0.08) + (0.5)(0.13) = 0.105 \text{ or } 10.5\%$$

This exceeds the cost of the bank CDs by 4.5% (=10.5% – 6.0%).

Example, continued:

Now, suppose that at the end of the year, the Swiss franc has *fallen* in value relative to the U.S. dollar. If the exchange rate is now USD1.55/CHF, **compute** what the Swiss loan revenues would be at the end of Year 1.

Answer:

The Swiss loan revenues at the end of one year equal:

$$\text{CHF33,235,294} \times \text{USD1.55 / CHF} = \text{USD51,514,706, for a return of:}$$

$$\frac{\text{USD51,514,706} - \text{USD50,000,000}}{\text{USD50,000,000}} = 3.03\%$$

Thus, the weighted return on this portfolio will be:

$$(0.5)(0.08) + (0.5)(0.0303) = 0.0552 \text{ or } 5.52\%$$

Under this scenario, the bank would actually have a negative interest margin on its balance sheet investments of –0.48% since its cost of funds (COFs) is 6.0%. The loss occurs because the Swiss franc has depreciated over the term of the loan and the borrowers, in effect, have paid back the loan with "cheaper" Swiss francs.

Example, continued:

If the Swiss franc had *appreciated* against the dollar over the year, the bank would have generated a double benefit: (1) from the appreciation of the franc, and (2) from the higher yield on the domestic Swiss loans. If the exchange rate is now USD1.82/CHF, **compute** what the Swiss loan revenues would be at the end of Year 1.

Answer:

$$\text{CHF33,235,294} \times \text{USD1.82 / USD} = \text{USD60,488,235, for a return of:}$$

$$\frac{\text{USD60,488,235} - \text{USD50,000,000}}{\text{USD50,000,000}} = 20.98\%$$

The previous example illustrates an important concept. As with any investment, returns for the bank's portfolio are derived from differences between income and costs. However, foreign investing provides the additional dynamic of having profits or losses affected by changes in foreign exchange rates. There are two principle methods available to control the scale of FX exposure: on-balance-sheet hedging and off-balance-sheet hedging.

Balance Sheet Hedging

LO 49.7: Explain balance-sheet hedging with forwards.

On-Balance-Sheet Hedging

On-balance-sheet hedging is achieved when a financial institution has a matched maturity and currency foreign asset-liability book.

Figure 2: Balance Sheet

Assets	*Liabilities*
USD50 million U.S. loans, 1-year maturity, in USD, yield 8%	USD50 million U.S. CDs, 1-year maturity, in USD, yielding 6%
USD50 million equivalent Swiss loans, 1-year maturity, made in CHF, yielding 13%	USD50 million Swiss CDs, 1-year maturity, raised in CHF, yielding 10%

Using the data in Figure 2, we can examine the effects of the franc depreciating by the same amount as in the previous example. Recall that in the previous example when the Swiss franc depreciated, the bank earned a negative interest margin.

1. The bank borrows USD50 million equivalent in Swiss francs for one year at an interest rate of 10%. At the exchange rate of USD1.70/CHF, this equates to USD50,000,000 / 1.70 = CHF29,411,765.
2. At the end of one year, the bank must pay back the Swiss franc CD holders their principal and interest: CHF29,411,765(1.10) = CHF32,352,941.
3. If the franc *depreciated* to USD1.55/CHF in the period, repayment in dollar terms would be CHF32,352,941 × USD1.55/CHF = USD 50,147,059, or a dollar cost of funds of 0.3%.
4. The bank makes CHF29,411,765 in loans at 13% for one year.
5. At the end of one year, the loans are repaid with interest. CHF29,411,765(1.13) = CHF33,235,294, but at USD1.55/CHF, this equals only USD51,514,706 for a return of 3.03%.

At the end of the year, we would have the following.

Average return on assets:

$(0.5)(0.08)+(0.5)(0.0303)=0.0552$ or 5.52%

U.S. asset return + CHF asset return = overall return

Average cost of funds:

$(0.5)(0.06)+(0.5)(0.003)=0.0315$ or 3.15%

U.S. cost of funds + CHF cost of funds = overall cost

Net return:

$5.52\%-3.15\%=2.37\%$

average return on assets − average cost of funds

By directly matching foreign assets and liabilities, we can lock in a positive return or profit spread if exchange rates move in either direction over the investment period.

Off-Balance-Sheet Hedging

Rather than matching foreign assets with foreign liabilities, we may choose to remain unhedged on the balance sheet. If we do, we could hedge off-balance-sheet by taking a position in the forward market. This hedge would appear as a contingent off-balance-sheet claim and as a gain or loss below the net income line on the income statement.

Referring to the previous example, the function of the forward FX contract is to offset the uncertainty of the future spot rate on the CHF at the end of the investment horizon. A forward foreign exchange agreement involves the exchange of a foreign currency at some point in the future at an exchange rate that is determined today. Rather than repatriating CHF and exchanging them for USD at the end of the period at an unknown rate, the bank can enter into a contract to sell forward the *expected* principal and interest on the loan at the current known **forward exchange rate** for USD/CHF, with the delivery of Swiss francs to the buyer of the forward contract taking place at the end of the investment horizon. This method effectively removes the future spot exchange rate uncertainty that is related to investment returns on the Swiss loan. By using the data in Figure 2, we can illustrate how this technique would work.

Example: Hedging with forward contracts

Outline the transactions necessary for the financial institution to use an off-balance-sheet hedge for the asset-liability position described in Figure 2.

Answer:

The following transactions create the off-balance-sheet hedge.

1. The U.S. bank sells USD50 million for Swiss francs at the *spot* exchange rate *today* and receives USD50,000,000 / USD1.7/CHF = CHF29,411,765.

2. Immediately after the sale, the bank lends the CHF29,411,765 to a Swiss customer at 13% for one year.

3. In addition, the bank sells the expected principal and interest proceeds from the franc loan forward for U.S. dollars at today's forward rate (say, USD1.65/CHF) for 1-year delivery at a forward discount of 2.96%: (USD1.65 – USD1.70) / USD1.70 = –2.94%.

   The forward buyer of the francs will pay USD54,838,235 to the seller when the bank delivers the CHF33,235,294 proceeds of the loan to the financial institution seller.

$$\begin{aligned}\text{CHF}29,411,765(1.13)\times\text{USD}1.65/\text{CHF} &= \text{CHF}33,235,294\times\text{USD}1.65/\text{CHF}\\ &= \text{USD}54,838,235\end{aligned}$$

4. At the end of one year, the Swiss borrower repays the loan to the bank plus interest in Swiss francs (CHF33,235,294).

5. The bank gives the CHF33,235,294 to the buyer of the 1-year forward contract and receives USD54,838,235.

By using this method, the bank knows it has locked in a guaranteed return of 9.68% on the Swiss franc (assuming, of course, the loan will not default and the forward buyer does not renege on the forward contract).

$$\frac{\text{USD}54,838,235-\text{USD}50,000,000}{\text{USD}50,000,000}=0.0968=9.68\%$$

The overall expected return on the bank's asset portfolio would then be:

$$(0.5)(0.08)+(0.5)(0.0968)=8.84\%$$

Regardless of spot exchange rate fluctuations over the year, the bank has locked in a risk-free return spread of 2.84% (8.84% return – 6% cost of funds) over the cost of funds for the bank's CDs.

Interest Rate Parity

LO 49.8: Describe how a non-arbitrage assumption in the foreign exchange markets leads to the interest rate parity theorem, and use this theorem to calculate forward foreign exchange rates.

Because the hedged Swiss loans offer a higher return than the U.S. loans, it makes sense for the bank to focus its activities on making hedged Swiss loans. However, as more is invested in Swiss loans, the bank must buy more Swiss francs. This will continually reduce the forward rate spread until no additional profits could be made by making the forward contract-hedged investments.

As the bank moves into more Swiss loans, the spot exchange rate for buying francs will rise. In equilibrium, the forward exchange rate would have to fall to completely eliminate the attractiveness of the Swiss investments.

This relationship is called **interest rate parity** (IRP) since the discounted spread between domestic and foreign interest rates equals the percentage spread between forward and spot exchange rates. In other words, the hedged dollar return on foreign investments should be equal to the return on domestic investments. IRP implies that in a competitive market, a firm should not be able to make excess profits from foreign investments (i.e., a higher domestic currency return from lending in a foreign currency and locking in the forward rate of exchange).

For the exam, you should know that the exact IRP equation using direct quotes is:

$$\text{forward} = \text{spot}\left[\frac{(1+r_{DC})}{(1+r_{FC})}\right]^{T}$$

where:
r_{DC} = domestic currency rate
r_{FC} = foreign currency rate

If this equality does not hold, an arbitrage opportunity exists. To remember this formula, note that when the forward and spot rates are expressed as direct quotes (DC/FC), the right-hand side of the equation also has the domestic (interest rate) in the numerator and the foreign (interest rate) in the denominator.

If we expressed the forward and spot rates as indirect quotes (FC/DC), then the right-hand side of the equation would have the foreign (interest rate) in the numerator and the domestic (interest rate) in the denominator. So it's either domestic over foreign for everything, or foreign over domestic for everything.

IRP can also be stated using continuously compounded rates as follows:

$$\text{forward} = \text{spot} \times e^{(r_{DC}-r_{FC})T}$$

Example: Interest rate parity

Suppose you can invest in the New Zealand dollar (NZD) at 5.127%, or you can invest in Swiss francs at 5.5%. You are a resident of New Zealand, and the current spot rate is 0.79005 NZD/CHF. **Calculate** the 1-year forward rate expressed in NZD/CHF.

Answer:

$$\text{forward(DC / FC)} = \text{spot(DC / FC)}\left[\frac{(1+r_{DC})}{(1+r_{FC})}\right] = 0.79005\left(\frac{1.05127}{1.055}\right) = 0.78726$$

Professor's Note: Notice here that the NZD/CHF rate fell from 0.79005 to 0.78726. This implies that it now takes fewer NZD to buy one CHF. So, in other words, the New Zealand dollar has appreciated relative to the Swiss franc. Consequently, the Swiss franc has depreciated relative to the New Zealand dollar.

Diversification in Multicurrency Foreign Asset-Liability Positions

LO 49.9: Explain why diversification in multicurrency asset-liability positions could reduce portfolio risk.

LO 49.10: Describe the relationship between nominal and real interest rates.

Our previous examples have used matched and mismatched asset-liability portfolios that involve only one foreign currency. In reality, most financial institutions hold positions in many different currencies in their asset-liability portfolios. Since currencies may be less than perfectly correlated, diversification across several asset and liability markets can potentially reduce portfolio risk as well as the cost of funds. Domestic and foreign interest rates and stock returns generally do not move together perfectly over time. This means that the risks from mismatching one-currency positions may be offset by potential gains from asset-liability portfolio diversification.

Each domestic and foreign nominal interest rate consists of two components. The first component is the **real interest rate,** which reflects a given currency's real demand and supply for its funds. Differences in real interest rates will cause a flow of capital into those countries with the highest available *real* rates of interest. Therefore, there will be an increased demand for those currencies, and they will appreciate relative to the currencies of countries whose available real rate of return is low.

The second component is the **expected inflation rate,** which reflects the amount of compensation required by investors to offset the expected erosion of real value over time due to inflation. Differences in inflation rates will cause the residents of the country with the highest inflation rate to demand more imported (cheaper) goods. For example, if prices in the United States are rising twice as fast as in Australia, U.S. citizens will increase their

demand for Australian goods (because Australian goods are now cheaper relative to domestic goods). If a country's inflation rate is higher than its trading partners', the demand for the country's currency will be low, and the currency will depreciate.

The **nominal interest rate**, *r*, is the compounded sum of the real interest rate, *real r*, and the expected rate of inflation, *E*(i), over an estimation horizon. This relationship is often called the Fisher equation:

$$\text{exact methodology: } (1 + r) = (1 + \text{real } r)[1 + E(i)]$$

$$\text{linear approximation: } r \approx \text{real} + E(i)$$

We can use the Fisher equation to explain why we might expect the changes in value of foreign currencies to be less than perfectly positively correlated, and consequently why diversification in multicurrency portfolios might reduce foreign exchange risk.

If real interest rates and expected inflation are perfectly correlated across countries (which is unlikely), then global markets are perfectly economically integrated, and there would be no diversification benefit of holding multicurrency portfolios. However, to the extent global markets aren't perfectly integrated, foreign exchange rates are positively but not perfectly positively correlated, and there are risk-reduction benefits from holding multicurrency portfolios.

Key Concepts

LO 49.1

Net exposure in a foreign currency measures the extent to which a bank is net long or net short a foreign currency. A financial institution's net currency exposure is calculated as:

net currency exposure = (currency assets – currency liabilities)
+ (currency bought – currency sold)

A net long (short) position in a currency means that a bank faces the risk that the FX rate will fall (rise) in value versus the domestic currency.

LO 49.2

A financial institution could reduce its foreign exchange exposure by altering either or both components of the net position exposure equation. For example, to reduce its exposure to zero, it could:

- Match its foreign currency assets to its liabilities on the balance sheet (the first component would be equal to zero) and match its long and short trading positions in the currency (the second component would be zero).
- Be long the currency in one component and short the currency in the other component so the two positions offset.

LO 49.3

The potential gain/loss exposure to a foreign currency is a function of the size of the position and the potential change in the value of the foreign currency:

dollar gain/loss in EUR = net EUR exposure (measured in $) × % change in the $/FC rate

If a financial institution fails to maintain a balanced position, the institution will be exposed to variations in the FX rate. The more volatile the FX rate, the more potential impact a net exposure (either long or short) will have on the value of a bank's foreign currency portfolio.

LO 49.4

A financial institution's buying and selling of foreign currencies, and hence the institution's position in the FX market, reflects four key trading activities:

- Enabling customers to participate in international commercial business transactions.
- Enabling customers (or the financial institution itself) to take positions in real and financial foreign investments.
- Offsetting exposure in a given currency for hedging purposes.
- Speculating on future FX rate movements.

LO 49.5

Most of the profits and losses on FX come from speculation or open position taking. A secondary source of revenue comes from market-making activities and/or agency fees.

LO 49.6

Returns for the bank's portfolio are derived from differences between income and costs. However, there is an extra dimension of return and risk from adding foreign currency assets and liabilities to a portfolio.

LO 49.7

There are two principle methods of better controlling the impact of FX exposure:

- On-balance-sheet hedging is achieved when a financial institution has a matched maturity and foreign currency balance sheet.
- Off-balance-sheet hedging occurs through the purchase of forwards for institutions that choose to remain unhedged on the balance sheet.

LO 49.8

Interest rate parity (IRP) suggests that the discounted spread between domestic and foreign interest rates equals the percentage spread between forward and spot exchange rates. IRP can be stated using continuously compounded rates as follows:

$$\text{forward rate} = \text{spot rate} \times e^{(r_{DC} - r_{FC})T}$$

LO 49.9

Since domestic and foreign interest rates and stock returns are not perfectly positively correlated, opportunities for potential gains from asset-liability portfolio diversification can offset currency risk.

LO 49.10

The real interest rate reflects a given currency's real demand and supply for its funds. The nominal interest rate is the compounded sum of the real interest rate and the expected rate of inflation over an estimation horizon. To the extent global markets aren't perfectly integrated, foreign exchange rates are positively but not perfectly positively correlated, and there are risk-reduction benefits from holding multicurrency portfolios.

Concept Checkers

1. A U.S. financial institution is short $5 million in euros on its trading book, has euro-denominated assets of $17 million and euro-denominated liabilities of $8 million. Note: all positions are valued in dollars. If the dollar is expected to appreciate versus the euro by 1.5%, the institution's potential dollar gain or loss exposure to the euro is closest to:
   A. $210,000 loss.
   B. $60,000 loss.
   C. $60,000 gain.
   D. $210,000 gain.

2. A U.S. financial institution is short €5 million on its trading book, has euro-denominated assets of €17 million and euro-denominated liabilities of €8 million. In order to eliminate its exposure to the euro, it would most likely:
   A. take a €5 million long position on its trading book.
   B. purchase €9 million of euro-denominated deposits.
   C. sell €9 million of euro-denominated loans.
   D. sell €4 million of euro-denominated loans.

3. The majority of a bank's foreign currency exposure and gains and losses from that exposure come from:
   A. enabling customers to participate in international commercial business transactions.
   B. enabling customers (or the financial institution itself) to take positions in real and financial foreign investments.
   C. offsetting exposure to a given currency for hedging purposes.
   D. speculating on future FX rate movements.

4. The annual interest rate in the United States is 3% and 7% in Mexico. The spot rate for the Mexican peso is MXN20/USD. The 6-month arbitrage-free forward rate is closest to:
   A. MXN19.25/USD.
   B. MXN19.62/USD.
   C. MXN20.38/USD.
   D. MXN20.78/USD.

5. The real interest rate in Zambonia is 3% and expected inflation is 50%. The nominal interest rate is closest to:
   A. 47%.
   B. 53%.
   C. 55%.
   D. 95%.

Concept Checker Answers

1. **B** Net EUR exposure (measured in dollars) = ($17 – $8) + (–$5) = $4 million. If the dollar is expected to appreciate by 1.5%, the euro is expected to depreciate by approximately 1.5%; therefore, the potential loss exposure is $4 million × –0.015 = –$60,000.

2. **D** Net EUR exposure = (€17 – €8) + (–€5) = €4 million; to reduce the exposure to zero, sell €4 of euro-denominated loans so loans total €13 million, and exposure = (€13 – €8) + (–€5) = €0.

3. **D** Most of the profits and losses on FX come from speculation or open position taking. A secondary source of revenue comes from market-making activities and/or agency fees.

4. **C** F(6-month) = MXN20/USD$(1.07/1.03)^{0.5}$ = MXN20.38/USD

5. **C** Nominal interest rate = [(1.03)(1.50) – 1] = 54.5%

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Corporate Bonds

Topic 50

Exam Focus

The term "bond" refers to a variety of assets which offer a wide range of interest rate payments from fixed cash payments, to accruals without cash, to payments in the form of additional securities. In this topic, we will provide an overview of major fixed-income instruments and their payment structures. We will also address the impact of credit risk and event risk on bond ratings and features. For the exam, be familiar with the types of bonds discussed and the methods for retiring bonds. Also, know the terminology associated with high-yield issues.

Bond Indenture and Role of Corporate Trustee

LO 50.1: Describe a bond indenture and explain the role of the corporate trustee in a bond indenture.

The **bond indenture** is a document that sets forth the obligation of the issuer and the rights of the investors in the bonds (i.e., the bondholders). It is usually a detailed document filled with legal language. One of the roles of the **corporate trustee** is to interpret this language and represent the interests of the bondholders. Banks or trust companies most often serve as corporate trustees, and the position requires that they act in a fiduciary capacity on behalf of the bondholders. The trustee would authenticate the issue, which includes keeping track of the amount of bonds issued and making sure the number does not exceed the limit specified in the indenture. The trustee would monitor the corporation's activities to make sure the issuer abides by the indenture's covenants (e.g., maintaining key ratios below a given number).

All corporate bond offerings over $5 million and sold in interstate commerce must have a corporate trustee as set forth in the **Trust Indenture Act**. The corporate trustees must be competent and financially responsible and should also not have any conflicts of interest, (e.g., being a creditor of the issuer). The indenture would specify how the trustee would make reports to bondholders and what to do if the issuer fails to pay interest or principal. As mentioned earlier, the basic goal of the trustee is to protect the rights of bondholders.

Maturity Date

LO 50.2: Explain a bond's maturity date and how it impacts bond retirements.

The maturity date of a bond is when the bond issuer's obligations are fulfilled. At maturity, the issuer pays the principal and any accrued interest or premium. The contract, as set forth by the indenture, may terminate prior to the maturity date if, for example, the corporation chooses to retire the bonds early. The longer the maturity of the bond, the more time a company has to retire the bond issue.

Interest Payment Classifications

LO 50.3: Describe the main types of interest payment classifications.

LO 50.4: Describe zero-coupon bonds and explain the relationship between original-issue discount and reinvestment risk.

The main types of bond interest payment classifications are: straight-coupon bonds, zero-coupon bonds, and floating-rate bonds. The interest rate on a bond is often called the **coupon**. However, bonds today technically no longer have coupons attached directly to them. Now, bonds are registered and represented by a certificate, or they are kept in book-entry form where one master or global certificate is issued and held by a central securities depository that issues receipts. This method is considered a safer way to make payments.

Straight-coupon bonds, also called fixed-rate bonds, have a fixed interest rate set for the entire life of the issue. In the United States, fixed-rate bonds typically pay interest every six months. In Europe and some other countries, bonds make annual interest rate payments. A bond issued in the United States with an 8% interest rate and a $1,000 par value on March 1, 2009 will pay $40 of interest each September 1 and each March 1 until its maturity date or until the bond is retired, at which time the issuer would pay both the final interest payment and the $1,000 principal back to the bondholder.

In addition to just paying a fixed dollar interest, bonds in the United States have been issued that pay in foreign currency. Two other variations are a participating bond and an income bond. **Participating bonds** pay at least the specified interest rate but may pay more if the company's profits increase. **Income bonds** pay at most the specified interest, but they may pay less if the company's income is not sufficient. In both cases, the conditions for paying more or less than the specified coupon would be set forth in the indenture.

Floating-rate bonds are also known as variable rate bonds. The interest paid is generally linked to some widely used reference rate such as LIBOR or the Federal Funds rate.

Zero-coupon bonds pay the face value or principal at maturity. There is not a cash interest payment; instead, the bondholder earns a return by purchasing the bond at a discount to face value and receiving the full face value at maturity. Variations of the zero-coupon bond include the **deferred-interest bond** (DIB) and the **payment-in-kind bond** (PIK). The DIB will not pay cash interest for some number of years early in the life of the bond. That period is the deferred-interest period. During this period, cash interest accrues and is then paid semiannually until maturity or when redeemed. PIK bonds pay interest with additional bonds for the initial period, and then cash interest after that period ends.

Most zero-coupons issued today share a host of other features such as being convertible, callable, and putable. A zero-coupon bond's interest rate is determined by the original-issue discount (OID):

$$\text{original-issue discount (OID)} = \text{face value} - \text{offering price}$$

The value of the bond grows each year and thus pays implicit interest, which is a function of the OID and the term-to-maturity. In other words, the rate of return depends on the amount of the discount and the period over which it grows.

One advantage of zero-coupon bonds is zero **reinvestment risk**. The bondholder does not have to make an effort to reinvest cash interest payments or worry about the available rates at which to reinvest them. A disadvantage is that the bondholder must pay taxes each year on the accrued interest even though no cash is received from the bond issuer.

If the issuer goes into bankruptcy prior to the maturity of a zero-coupon bond, the bondholders are only entitled to the accrued interest up to that date and not the full face value of the bond. In other words, the zero-coupon bond creditor can only claim the original offering price plus accrued and unpaid interest up to the date of the bankruptcy filing. The bond issuer faces a huge liability with a zero-coupon bond because of the large balloon payment at maturity.

Bond Types

LO 50.5: Distinguish among the following security types relevant for corporate bonds: mortgage bonds, collateral trust bonds, equipment trust certificates, subordinated and convertible debenture bonds, and guaranteed bonds.

Corporate bonds can have a security, such as real property, underlying the issue. Those who own mortgage bonds have a first-mortgage lien on the properties of the issuer. This security allows the issuer to pay a lower rate of return than it would have to pay on unsecured bonds, which are known as debentures. The lien gives the bondholders the right to sell the mortgaged property to satisfy unpaid obligations to bondholders. In practice, this right is usually used for bargaining purposes only, and the bankruptcy takes the form of reorganization as opposed to liquidation.

Mortgage bonds can be issued in a series in a blanket arrangement. In this case, one group of bonds is issued under the mortgage, and then others are issued later. When earlier issues mature, additional bonds are then issued in their place.

Collateral trust bonds are backed by stocks, notes, bonds, or other similar obligations that the company owns. The underlying assets are called the collateral or personal property. The issuers are holding companies, and the collateral consists of claims on their subsidiaries.

The trustee holds the collateral for the benefit of the bondholders; however, the issuer retains voting rights for stock used as collateral, so they retain control over their subsidiaries. The indenture may have provisions covering what to do if the value of the collateral falls below the value of the loan. If the collateral falls in value, the issuer may have to contribute additional securities to back the bonds. The issuer may be able to withdraw collateral if the value rises in order to exceed the loan value. Like mortgage bonds, collateral trust bonds may be issued in series.

Equipment trust certificates (ETCs) are a variation of a mortgage bond where a particular piece of equipment underlies the bond. The usual arrangement is that the borrower does not actually purchase the equipment. Instead, the trustee purchases the equipment and leases it to the user of the equipment (the effective borrower), who pays rent on the equipment, and that rent is passed through to the holders of the ETCs. The payments to the creditors are called dividends. The trustee pays for the equipment with the money raised from the issuance of the ETCs, usually about 80% of the value of the equipment, and what is effectively a down payment from the user of the equipment. This provides more security to the creditors than that of a mortgage bond. It is especially attractive if the equipment is standardized, as in the case of railroad cars, which provides for easy sale or lease of the equipment in the case the user of the equipment defaults. ETCs are generally considered the most secure type of bond since the underlying assets are actually owned by the trustee and rented to the borrower.

As noted earlier, **debentures** are unsecured bonds (i.e., they do not have any assets underlying the issue). Most corporate bonds are debentures and usually pay a higher interest rate for that reason. However, if the company is highly rated and has not issued any secured bonds, then debentures are almost the equivalent of mortgage bonds in that they have a claim on all the assets of the issuer along with the general creditors. If the issuer has issued secured debt along with debentures, the debenture holders have a claim on the assets that are not backing the secured debt. Typically the issuer is restricted to one issue of debentures if there is already secured debt. If there is no secured debt, and the company issues debentures, there is often a negative-pledge clause that says that the debentures will be secured equally with any secured bonds that may be issued in the future.

Subordinated debenture bonds have a claim that is at the bottom of the list of creditors if the issuer goes into default. They are bonds that are unsecured and have another unsecured bond with a higher claim above them. This means that the issuer has to offer a higher interest rate on the subordinated debentures.

Issuers may choose to issue **convertible debentures**, which give the bondholder the right to convert the bond into common stock. This feature will lower the interest rate paid. The cost to the issuer, however, is the possibility of increased dilution of the stock. A variation of convertible debentures is **exchangeable debentures** that are convertible into the common stock of a corporation other than that of the issuer.

Bonds issued by one company may also be guaranteed by other companies. These bonds are known as **guaranteed bonds**. A guarantee does not ensure that the issue will be free of default risk since the risk will depend on the ability of the guarantor(s) to satisfy all obligations.

Methods for Retiring Bonds

LO 50.6: Describe the mechanisms by which corporate bonds can be retired before maturity.

There are a variety of methods for retiring debt, and some are included in the bond's indenture while others are not included. The indenture would include the call and refunding provisions, sinking funds, maintenance and replacement funds, and redemption through sale of assets. The indenture would not include fixed-spread tender offers.

Call and refunding provisions are essentially call options on the bonds that the issuer owns and give the issuer the right to purchase at a fixed price either in whole or in part prior to maturity. These provisions allow a firm to call back debt that has a high coupon and reissue debt with a lower coupon. Other reasons for exercising these options are to eliminate restrictive covenants, alter capital structure, increase shareholder value, or improve financial/managerial flexibility. A **call provision** can either be a fixed-price call or a make-whole call.

- **Fixed-price call.** The firm can call back the bonds at specific prices that can vary over the life of the bonds as specified in the indenture. They generally start out high and decline toward par. Also, for most bonds, the bonds are not callable during the first few years of the issue's life.
- **Make-whole call.** In this case, market rates determine the call price, which is the present value of the bond's remaining cash flows subject to a floor price equal to par value. A discount rate based on the yield of comparable-maturity Treasury securities (usually the rate plus a premium) determines the present value and the bond's price. The redemption price is the greater of that present value or the par value plus accrued interest.

A **sinking fund provision** generally means the issuing firm retires a specified portion of the debt each year as outlined in the indenture. The bonds can either be retired by use of a lottery where the owners of the selected bonds must redeem them, or the bonds are purchased in the open market. The purchase of some sufficient amount of equipment in excess of the value of the amount of the bonds to be retired is another action that may satisfy a sinking fund provision.

The lottery approach to satisfying the sinking fund is very similar to a call provision in that the bondholders must sell back their bonds at a specified price. Unlike the call provision, there may be advantages to the bondholders. First, the retirement of bonds improves the financial health of the firm. Second, the redemption price may exceed the market price. However, the indenture may give the issuer some flexibility as to how much of the bonds to call back each period, which would give the firm some latitude to call back more bonds when the market conditions are favorable to do so. One example is an accelerated sinking-fund provision, which allows the firm to call back more bonds in early years, which the firm would do if interest rates fall in those early years.

A **maintenance and replacement fund** (M&R) has the same goal as a sinking fund provision, which is to maintain the credibility of the property backing the bonds. The provisions differ in that the M&R provision is more complex since it requires valuation formulas for the underlying assets. The main point is that the provision specifies that the fund must keep up the value of the underlying assets much like a home mortgage specifies the home buyer must keep up the value of the home. One way to satisfy the provision is

to acquire sufficient cash to maintain the health of the firm. That cash can then be used to retire debt.

Tender offers are usually a means for retiring debt for most firms. The firm openly indicates an interest in buying back a certain dollar amount of bonds or, more often, all of the bonds at a set price. The goal is to eliminate restrictive covenants or to use excess cash. If the first tender offer price does not get sufficient interest, the firm can increase its offer price. Firms can also announce that they will buy back bonds based on the price as determined by a certain market interest rate (e.g., the yield to maturity on a comparable-maturity Treasury plus a spread). This lowers interest rate risk for both the bondholders and the bond issuer.

As a final note, the issuing firm may be able to call back bonds if it is necessary to sell assets associated with the bond issue. For example, if the government requires a firm to sell property, but that property is being used as collateral for the bonds, the firm would sell the property and call back the bonds.

Credit Risk

LO 50.7: Differentiate between credit default risk and credit spread risk.

Credit risk includes credit default risk and credit spread risk. **Credit default risk** is the uncertainty concerning the issuer making timely payments of interest and principal as prescribed by the bond's indenture. The most widely-used indicators of this risk are bond ratings that major rating agencies assign when those agencies perform credit analysis of a firm. Fitch Ratings, Moody's, and Standard & Poor's are the main rating agencies in the United States. The agencies assign a symbol associated with the rating (e.g., AAA or Aaa for the corporate debt with the least credit default risk). The rating can be interpreted as a probability of default within some time period.

Credit spread risk focuses on the difference between a corporate bond's yield and the yield on a comparable-maturity benchmark Treasury security. This difference is known as the **credit spread**. It should be noted that other factors such as embedded options and liquidity factors can affect this spread; therefore, it is not only a function of credit risk.

The risk here is from possible changes in this spread from changes in investor risk aversion, which will change the value of the associated bond. Other factors affecting credit spreads are macroeconomic forces such as the level and slope of the Treasury yield curve, the business cycle, and issue-specific factors such as the corporation's financial position and the future prospects of the firm and its industry.

A method commonly used to evaluate credit spread risk is **spread duration**. The duration of the spread is the approximate percentage change in a bond's price for a 100 basis point change in the credit spread assuming that the Treasury rate is constant. If a bond has a spread duration of 4, for example, a 50 basis point change in the spread will change the value of the bond by 2%.

Event Risk

LO 50.8: Describe event risk and explain what may cause it in corporate bonds.

Event risk addresses the adverse consequences from possible events such as mergers, recapitalizations, restructurings, acquisitions, leveraged buyouts, and share repurchases, which may escape being included in the indenture. Such events can drastically change the firm's capital structure and reduce the creditworthiness of the bonds and their value. In order to protect shareholders, a company may include in the indenture a **poison put**, which can require the company to repurchase the debt at or above par value in the event of a takeover not approved by the board of directors (i.e., a hostile takeover). The purpose of this feature is to protect bondholders, but its effectiveness toward this goal can be misleading in that the acquiring firm may offer a sufficiently high price for the stock so that the hostile takeover becomes friendly. As a result, the poison puts would not be exercised.

Investors can lobby for clauses in the indenture to activate a put option for a variety of reasons including a change in the bond's rating. Some of the debt rating services issue commentary on the indenture's protective features, which could include the possibility of the firm being able to circumvent the features through careful legal moves (e.g., turning a hostile takeover into a friendly takeover). It should be noted that event risk can change on a market level. During times of increased merger activity, for example, the event risk increases for most bonds.

High-Yield Bonds

LO 50.9: Define high-yield bonds, and describe types of high-yield bond issuers and some of the payment features unique to high yield bonds.

High-yield bonds (a.k.a. junk bonds) are those bonds rated below investment grade by ratings agencies. This includes a broad range of ratings below the cutoff, (e.g., BB to default). **Businessman's risk** refers to bonds with a rating at the bottom rung of the investment-grade category (Baa and BBB) or at the top end of the speculative-grade category (Ba and BB). Over long periods of time, high-yield bonds should offer higher average returns. However, over shorter periods, the returns will be volatile where large losses are possible.

There are many types of high-yield bonds. One type includes companies who issue bonds with a non-investment grade rating. Such issuers include young and growing companies that do not have strong financial statements but who have promising prospects. Firms may issue such bonds to raise venture capital, and their prospects are tied to a particular project or story, which gives them the name "story bonds."

Established firms that have had a deteriorating financial situation may need to raise debt capital as well, and they would issue bonds that reflect their situation. Also, an established firm who already has unsecured debt issued with an investment-grade credit rating may be able to issue subordinated debt, but that debt would be non-investment grade.

Fallen angels are another type of high-yield bond. They are bonds that were issued with an investment-grade rating, but then events led to the ratings agencies lowering the rating to below investment grade. If the issuers are in or near bankruptcy, they are often called "special situations," which could either pay off if the company recovers or lead to big losses.

Restructurings and **leveraged buyouts** may increase the credit risk of a company to the point where the bonds become non-investment grade. The new management may pay high dividends, deplete the acquired firm's cash, and lower the rating of the existing bonds. In this process, the firm may issue non-investment grade debt to pay off the bridge loans taken to finance the acquisition.

High-yield bonds can have several types of coupon structures. There are **reset bonds,** where designated investment banks periodically reset the coupon to reflect market rates and the creditworthiness of the issuer. There are also **deferred-coupon structures,** which include three types: (1) deferred-interest bonds, (2) step-up bonds, and (3) payment-in-kind bonds. Deferred-interest bonds sell at a deep discount and do not pay interest in the early years of the issue, say, for three to seven years. Step-up bonds pay a low coupon in the early years and then a higher coupon in later years. Payment-in-kind bonds allow the issuer to pay interest in the form of additional bonds over the initial period.

Default Rate

LO 50.10: Define and differentiate between an issuer default rate and a dollar default rate.

A default occurs if there are any missed or delayed disbursements of interest and/or principal. It has been proven that lower credit ratings indicate a higher probability of default, but there are two ways to measure default: by the raw number of issuers that defaulted or the dollar amount of issues that defaulted. For each approach in measuring default rates, there are different formulas, which can lead to researchers reporting different default rates for the same data set.

The **issuer default rate** is the number of issuers that defaulted over a year divided by the total number of issuers at the beginning of the year. It is only a proportion of the number of issuers who do fulfill their obligations and does not include a measure of the dollar amount involved.

The **dollar default rate** is the par value of all bonds that defaulted in a given calendar year divided by the total par value of all bonds outstanding during the year. Over a multi-year period, often-used measures are ratios of cumulative dollar value of all defaulted bonds divided by some weighted-average measure of all bonds issued. One such measure attempts to weight the bonds outstanding by the number of years they are in the market:

$$\frac{\text{cumulative dollar value of all defaulted bonds}}{(\text{cumulative dollar value of all issuance}) \times (\text{weighted average \# of years outstanding})}$$

Another measure simply takes a raw total as shown in the following equation:

$$\frac{\text{cumulative dollar value of all defaulted bonds}}{\text{cumulative dollar value of all issuance}}$$

Recovery Rate

LO 50.11: Define recovery rates and describe the relationship between recovery rates and seniority.

The recovery rate is the amount received as a proportion of the total obligation after a bond defaults. Measuring this can be complicated because the value of the total obligation requires computing the present value of the remaining cash flows at the time of the default. Furthermore, some of the amount that the investor recovers may be in the form of securities (e.g., stock in the company). A study by Moody's estimated that the recovery rate for bonds has been about 38%. Bonds with higher seniority will obviously have higher recovery rates.

Key Concepts

LO 50.1

A bond indenture sets forth the obligations of the issuer. The trustee interprets the legal language of the indenture and works to make sure the issuer fulfills obligations to bondholders.

LO 50.2

The bond issuer's obligations are fulfilled on the maturity date or before. Bonds can be retired before that date.

LO 50.3

The main types of interest payment classifications are straight-coupon bonds, zero-coupon bonds, and floating-rate bonds. Straight-coupon bonds pay a fixed cash coupon periodically. Floating-rate bonds pay a cash amount that varies with market rates. Zero-coupon bonds increase in value over the life of the issue.

There are many variations of the main types of bond structures. For example, deferred-interest bonds are a mix of zero-coupon and coupon bonds in that they do not pay cash interest in early years and pay a cash coupon in later years. Some bonds have principal in one currency and pay coupons in another currency.

LO 50.4

Zero-coupon bonds have low reinvestment risk. The interest is based on the time-to-maturity at issuance and the original-issue discount, which is the difference between the face value and the offering price. In the case of bankruptcy, the bondholder has a claim only equal to the issue price plus accrued interest to that date, and not the full face value.

LO 50.5

The holder of a mortgage bond has the first lien on real property owned by the issuer.

Collateral trust bonds are backed by stocks and bonds that represent claims against the subsidiaries of the issuer. The collateral is also called personal property.

Equipment trust certificates are a form of mortgage bond where the trustee actually owns the property and rents it to the bond issuer. The property is often in the form of standardized equipment (e.g., rail cars) that is easily sold.

Debentures are unsecured debt. Owners of debentures have a claim on the company's assets not backing outstanding secured debt.

LO 50.6

Call provisions allow the firm to retire debt early at a given price. Sinking-fund provisions require the firm to buy back portions of debt each year. Call provisions are generally considered detrimental to bondholders, but sinking-fund provisions may be beneficial to bondholders.

A maintenance and replacement fund helps maintain the financial health of the firm. Cash in the fund can be used to retire debt.

Bond issuers can retire debt through a tender offer. The offer price may either be a fixed price or a price that varies with a market rate such as that on comparable Treasury securities.

LO 50.7

Credit default risk is the possibility that the issuer does not make the payments specified in the indenture. Credit spread risk is the price risk from changes in the spread of a bond's interest rate over the corresponding Treasury rate.

LO 50.8

Event risk is the possibility that a merger, restructuring, acquisition, et cetera, increases the risk of the bond by changing the ability of the firm to pay off the bonds. The indenture can try to address some of these events, but some can be omitted and lawyers can find loopholes around those included.

LO 50.9

High-yield bonds can either be issued by growing, risky firms or established firms with senior debt outstanding. High-yield bonds may also be fallen angels (i.e., one-time investment grade bonds).

High-yield bonds may have coupon structures which allow the firm to conserve cash in early years, such as: (1) deferred-interest bonds, (2) step-up bonds, and (3) payment-in-kind bonds.

LO 50.10

The issuer default rate is a proportion based on the number of issues that default as a proportion of all issues. The dollar default rate estimates the dollar amount of defaulted bonds compared to the dollar amount of the corresponding population of bonds outstanding.

LO 50.11

In the event of default, the recovery rate refers to the amount a bondholder receives as a proportion of the amount owed. Bonds with higher seniority usually have higher recovery rates.

Concept Checkers

1. Which of the following responsibilities is least likely to be part of the role of a corporate trustee in a bond issue?
   A. Interpret the language of the indenture.
   B. Determine the interest rate on a reset bond.
   C. Keep track of the amount of bonds issued by the corporation.
   D. Monitor the corporation's activities to make sure the corporation abides by the indenture's covenants.

2. In bankruptcy, the holder of a zero-coupon bond obligation of the bankrupt corporation would have a claim equal to:
   A. the face value of the bond.
   B. the issuing price of the bond only.
   C. the issuing price plus accrued interest.
   D. nothing, since zeros are always unsecured.

3. All other things being equal, which of the following types of bond instruments would have the lowest interest rate?
   A. Equipment trust certificates.
   B. Mortgage bonds.
   C. Junior debentures.
   D. Senior debentures.

4. Which of the following methods for retiring bonds before maturity is generally considered the most detrimental for the bondholders?
   A. Tender offers.
   B. Call provision.
   C. Sinking fund provision.
   D. Maintenance and replacement funds.

5. With respect to default risk and credit spread risk, the ratings of bond-rating agencies such as Moody's provide information concerning:
   A. default risk only.
   B. credit spread risk only.
   C. both default risk and credit spread risk.
   D. neither default risk nor credit spread risk.

Concept Checker Answers

1. **B** Investment banks other than the trustee set the rate on a reset bond.

2. **C** The claim equals the value at that point in time as implied by the issuing price, the original-issue discount, and accrued interest.

3. **A** ETCs, or equipment trust certificates, are generally the most secure because the underlying assets are actually owned by the trustee and rented to the borrower. Also, the assets are usually standardized for easy resale.

4. **B** The call provision gives the issuer the right to purchase the bonds at a given price, which the issuer would not do unless that price was below the market price. Sinking fund provisions can benefit bondholders because the issuer is obligated to purchase bonds, which improves the creditworthiness of the issue, and the issuer may have to do so at a price higher than the market price. There are no features in M&R funds or tender offers that would be detrimental to bondholders.

5. **A** Bond rating agencies issue ratings based on the default risk of the issue. Credit spread risk is determined by spread duration.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Mortgages and Mortgage-Backed Securities

Topic 51

Exam Focus

Mortgage-backed securities (MBSs) are debt securities backed by a pool of residential loans, which serve to transform mortgages from an illiquid asset into a liquid asset. Because the underlying mortgages can be prepaid, prepayment risk is a major concern for MBS investors. Monte Carlo simulation is the most common methodology used for valuing MBSs because it is able to account for prepayment risk. Alternate interest rate paths are assumed in the model to generate an option-adjusted spread (OAS). For the exam, be able to calculate the payments for a fixed-rate, level paying mortgage. Also, be familiar with the factors that affect prepayment rates and how to measure prepayment speeds with a conditional prepayment rate (CPR). Finally, be prepared to discuss the steps involved in valuing an MBS using the Monte Carlo methodology and understand the advantages and disadvantages of using an OAS.

Residential Mortgage Products

LO 51.1: Describe the various types of residential mortgage products.

A **mortgage** is a loan that is collateralized with a specific piece of real property. Before the 1970s, mortgages existed solely in the **primary market** where banks that issued the mortgage loans collected all interest and principal payments from the borrower. Within the past few decades, it is more common for mortgage lenders to sell the loans in the **secondary market** through a process known as **securitization**. The secondary market has allowed more banks to issue mortgage loans.

In the secondary market, mortgages are pooled together and packaged to investors in the form of a **mortgage-backed security** (MBS). The payments of an MBS can follow a **pass-through structure** where the interest and principal collected from the borrower pass through the banks and ultimately end up with the MBS investor. Because default risk is present in mortgage lending, banks will often guarantee the borrower's payments when mortgages are securitized.

Lien Status

Whether the mortgage is a first lien, a second lien, or a subsequent lien will greatly impact the lender's ability to recover the balance owed in the event of default. For example, a first lien would give the lender the first right to receive proceeds on liquidation, so from a seniority perspective, a first lien is more desirable than a second lien.

Original Loan Term

Mortgage terms of 10 to 30 years are common, with the most popular being 30 years (long term). However, medium terms in the 10- to 20-year range are starting to become more common, given the desire of many individuals to pay off their mortgages as soon as possible.

Credit Classification

Classifying loans between prime and subprime is determined mainly by credit score (i.e., Fair Isaac Corporation or FICO model).

Prime (A-grade) loans constitute most of the outstanding loans. They have low rates of delinquency and default as a result of low **loan-to-value (LTV) ratios** (i.e., far less than 95%), borrowers with stable and sufficient income (i.e., *front income ratio* of no more than 28% of monthly income to service payments relating to the home and *back income ratio* of no more than 36% for those payments plus other debt payments), and a strong history of repayments (e.g., FICO score of 660 or greater). Home payments include interest, principal, property taxes, and homeowners insurance.

Subprime (B-grade) loans have higher rates of delinquency and default compared to prime loans. They could be associated with high LTV ratios (i.e., 95% or above), borrowers with lower income levels, and borrowers with marginal or poor credit histories (e.g., FICO score below 660). High LTV ratios suggest a higher risk of default. Upon issuance, subprime loans are carefully scrutinized by the servicer to ensure timely payments.

Alternative-A loans are the loans in between prime and subprime. Although they are essentially prime loans, certain characteristics of Alternative-A loans make them riskier than prime loans. For example, the loan value may be unusually high, the LTV ratio may be high, or there may be less documentation available (e.g., income verification, down payment source).

Interest Rate Type

Fixed-rate mortgages have a set rate of interest for the term of the mortgage. Payments are constant for the term and consist of blended amounts of interest and principal.

Adjustable-rate mortgages (ARMs) have rate changes throughout the term of the mortgage. The rate is usually based on a base rate (e.g., prime rate, LIBOR) plus a spread. Rates can usually change on a monthly, semiannual, or annual basis. The risk of default is high, especially if there are large rate increases after the first year, thereby significantly increasing the total payment amount (due to the increase in interest).

Prepayments and Prepayment Penalties

Prepayments reduce the mortgage balance and amortization period. They can occur because of the following reasons:

- Home is sold, which requires the mortgage balance to be paid off.
- Refinancing due to lower rates or more attractive loan features elsewhere.
- Partial prepayments by the borrower during the term.

To counteract the negative effects of prepayments, many loans contain prepayment penalties. They are amounts payable to the servicer for prepayments within a certain time and/or over a certain amount. Soft penalties are those that may be waived on the sale of the home; hard penalties may not be waived.

Credit Guarantees

The ability to create mortgage-backed securities requires loans that have credit guarantees.

Government loans are those that are backed by federal government agencies (e.g., Government National Mortgage Association or GNMA).

Conventional loans could be securitized by either government-sponsored enterprises (GSEs): Federal Home Loan Mortgage Corporation (FHLMC) or Federal National Mortgage Association (FNMA). For a guarantee fee, these GSEs will guarantee payment of principal and interest to the investors.

Agency (or conforming) MBSs are those that are guaranteed by any of three government-sponsored entities (GSEs): GNMA, FNMA, and FHLMC. Most of the MBSs are issued by these GSEs.

Also known as **private label** securitizations, non-agency (or non-conforming) MBSs grew along with U.S. home prices over time up to the 2007 credit crisis. The GSEs have restrictions on what mortgages they can guarantee/securitize [e.g., dollar value limit, loan-to-value (LTV) ratio limit], which opened up the private label market for those participants willing to take on the risks inherent in nonconventional loans—**jumbo loans** (mortgage principal balance over the limit) and/or loans with high LTVs. The rising prices of the underlying homes held as collateral provided some risk mitigation. Unfortunately, the falling prices of homes and the credit crisis beginning in 2007 caused a significant drop in MBS issuances in the non-agency segment because they did not have government guarantees.

With agency MBSs, the investor bears no credit risk because the GSEs have been paid a fee to guarantee the underlying mortgages. If there is a default with a mortgage, the GSE will pay the outstanding balance to the investors. With a non-agency MBS, there is some credit risk but that is mitigated through the process of subordination.

Fixed-Rate, Level-Payment Mortgages

LO 51.2: Calculate a fixed rate mortgage payment, and its principal and interest components.

A **conventional mortgage** is the most common residential mortgage. The loan is based on the creditworthiness of the borrower and is collateralized by the residential real estate that it is used to purchase. If a borrower's credit quality is questionable or the borrower is lacking a sufficient down payment, the mortgage lender may require mortgage insurance to guarantee the loan. Mortgage insurance is made available by both government agencies and private insurers. The cost of the insurance is borne by the borrower and effectively raises the interest rate on the mortgage loan.

There are a wide variety of mortgage designs that specify the rates, terms, amortization, and repayment methods. All of the concepts associated with risk analysis and valuation, however, can be understood through an examination of **fixed-rate, level payment, fully amortized mortgage loans**. This common type of mortgage loan requires equal payments (usually monthly) over the life of the mortgage. Each of these payments consists of an interest component and a principal component.

There are four important features of fixed-rate, level payment, fully amortized mortgage loans to remember when we move on to mortgage-backed securities (MBS):

1. The amount of the principal payment increases as time passes.

2. The amount of interest decreases as time passes.

3. The servicing fee also declines as time passes.

4. The ability of the borrower to repay results in **prepayment risk**. Prepayments and curtailments reduce the amount of interest the lender receives over the life of the mortgage and cause the principal to be repaid sooner.

Example: Calculating a mortgage payment

Consider a 30-year, $500,000 level payment, fully amortized mortgage with a fixed rate of 12%. **Calculate** the monthly payment and **prepare** an amortization schedule for the first three months.

Answer:

The monthly payment is $5,143.06:

N = 360; I/Y = 1.0 (12/12); PV = –500,000; FV = 0; CPT → PMT = 5,143.06

With reference to the partial amortization schedule in the figure below, the portion of the first payment that represents interest is $5,000.00 (0.01 × $500,000). The remainder of the payment, $143.06 ($5,143.06 – $5,000.00), goes toward the reduction of principal. The portion of the second payment that represents interest is $4,998.57 (0.01 × $499,856.94). The remaining $144.49 ($5,143.06 – $4,998.57) goes toward the further reduction of principal.

Monthly Amortization Schedule for a 30-Year, $500,000 Mortgage Loan at 12%

Payment Number	*Initial Principal*	*Monthly Payment*	*Interest Component*	*Reduction of Principal*	*Outstanding Principal*
1	$500,000.00	$5,143.06	$5,000.00	$143.06	$499,856.94
2	499,856.94	5,143.06	4,998.57	144.49	499,712.45
3	499,712.45	5,143.06	4,997.12	145.94	499,566.51

Notice that the monthly interest charge is based on the beginning-of-period outstanding principal. As time passes, the proportion of the monthly payment that represents interest decreases, and, because the payment is level, the proportion that goes toward the repayment of principal increases. This process continues until the outstanding principal reaches zero and the loan is paid in full.

The incremental reduction of outstanding principal is referred to as scheduled amortization (or scheduled principal repayment). The previous figure is a portion of what is commonly called an **amortization schedule**. Amortization schedules are easily constructed using an electronic spreadsheet.

The collection of payments and all of the other administrative activities associated with mortgage loans are paid for via a servicing fee, also known as the servicing spread, because it is usually built into the mortgage rate.

For example, if the mortgage rate is 10.5% and the servicing fee is 35 basis points, the provider of the mortgage funds will receive 10.15%. This amount is called the net interest or net coupon. The dollar amount of the servicing fee is based on the outstanding loan balance; thus, it declines as the mortgage is amortized. Keep in mind that the reduction in principal associated with each payment is based on the mortgage rate and is unaffected by the servicing fee.

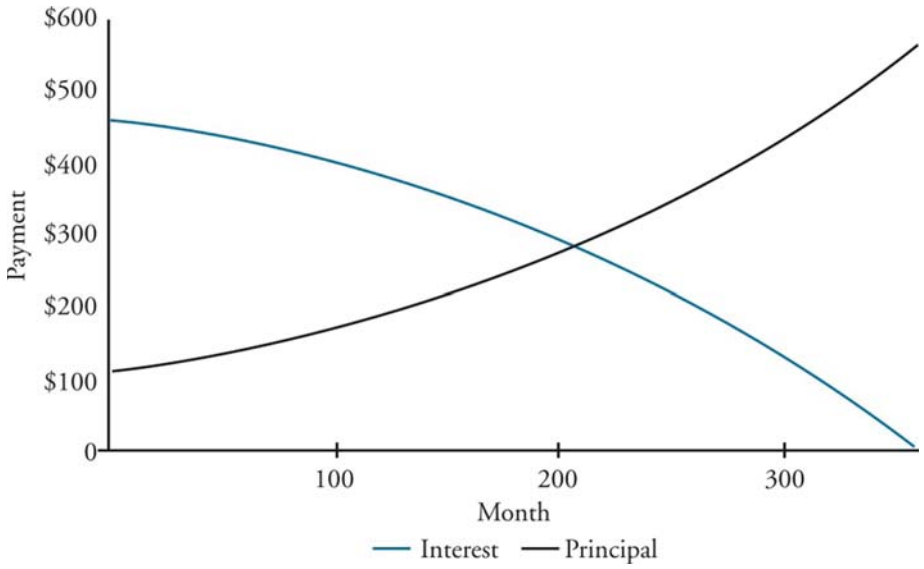
Allocation Between Principal and Interest

Fully amortizing fixed-rate mortgage:

- The mortgage payment consists primarily of interest in the early years.
- Interest is calculated on a declining principal balance so the interest payable will gradually decrease over time. As a result, more of the fixed mortgage payment will be applied toward reducing the principal amount.
- The crossover point is the point in the mortgage where principal and interest allocation amounts are the same. After that point, relatively more amounts will be allocated to principal.
- Mortgages with shorter amortization periods result in less interest paid and more of the payment applied toward reducing the principal balance sooner. In other words, equity buildup occurs at a quicker rate when the amortization period is shorter.

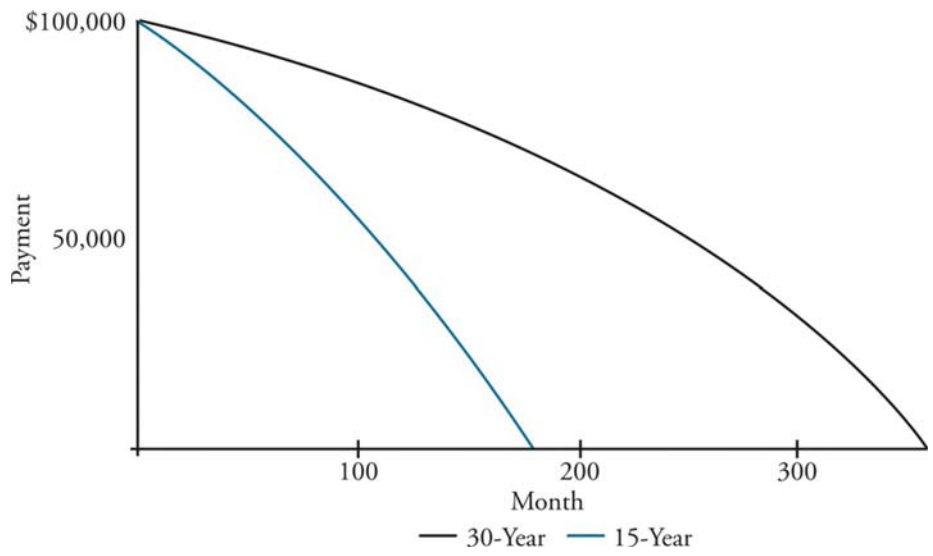
Figure 1 illustrates the relationship of interest and principal over the term of the loan.

Figure 1: Interest and Principal Over Time



As indicated previously, the reason principal payments will increase over time is due to the reduction in the outstanding loan balance. Figure 2 illustrates the relationship between loan balance and time for a $100,000 loan.

Figure 2: Loan Balance Over Time



Prepayment

LO 51.3: Describe the mortgage prepayment option and the factors that influence prepayments.

In the previous example, it was assumed that the borrower paid the exact amount of the monthly payment, and the interest and principal followed the amortization schedule. However, it is possible for a borrower to pay an amount in excess of the required payment or even to pay off the loan entirely. The option to prepay a mortgage is essentially a call option for the borrower. The borrower is in a position that is very similar to the issuer of a callable bond. A prepayment will effectively free the borrower of the mortgage obligation.

Mortgage Prepayment Option

Mortgage prepayments come in two forms: (1) increasing the frequency or amount of payments (where permitted) and (2) repaying/refinancing the entire outstanding balance. Prepayments are much more likely to occur when market interest rates fall and borrowers wish to refinance their existing mortgages at a new and lower rate. For the lender, prepayments represent a loss for two reasons: (1) they stop receiving interest income at the high rate and (2) they have to reinvest the proceeds received from prepayment at the prevailing lower market rates. Therefore, the pricing of the initial mortgage rate should be somewhat higher to take into account the possibility of prepayment. With agency MBSs, prepayments and defaults have the same impact on investors. Prepayments result in the investors actually receiving cash from the borrowers, whereas with defaults, the borrower does not pay the outstanding mortgage balances, but the GSE does, thereby causing a prepayment.

Other Factors That Influence Prepayments

Seasonality. The summertime is a popular time for individuals to move (and mortgages must be paid out prior to the sale of a home), so it is the period of time with the greatest prepayment risk. Given some time lags, the prepayments often start to appear in the late summer and early fall.

Age of mortgage pool. Refinancing often involves penalties and administrative charges, so borrowers tend not to do so until several years into the mortgage. Also, it takes some time for borrowers to build up equity and savings to make prepayments and/or attempt to refinance. As a result, the lower the age of the mortgage pool, the less likely the risk of prepayment.

Personal. Marital breakdown, loss of employment, family emergencies, and destruction of property are commonly cited reasons for prepayments based on personal reasons. It is difficult to assess this type of prepayment risk.

Housing prices. Property value increases may spur an increase in prepayments caused by borrowers wanting to take out some of the increased equity for personal use. Property value decreases reduce the value of collateral, reduce the ability to refinance, and, therefore, decrease the risk of prepayment. The increasingly popular use of home equity lines of credit where the mortgage balance is revolving (i.e., mortgage balance can be drawn up to a certain limit and paid down to zero at any time) reduces refinancing and prepayment risk due to the nature of the loan.

Refinancing burnout. To the extent that there has been a significant amount of prepayment or refinancing activity in the mortgage pool in the past, the risk of prepayment in the future decreases. That is because presumably the only borrowers remaining in the pool are those who were unable to refinance earlier (e.g., due to poor credit history or insufficient property value), and those who did refinance have been removed from the pool already. Also, those who made only large prepayments (instead of fully refinancing) in the past would have exhausted their savings to make the prepayment and would require quite some time to do so again in the future.

Securitization

LO 51.4: Summarize the securitization process of mortgage backed securities (MBS), particularly formation of mortgage pools including specific pools and TBAs.

LO 51.5: Calculate weighted average coupon, weighted average maturity, and conditional prepayment rate (CPR) for a mortgage pool.

To reduce the risk from holding a potentially undiversified portfolio of mortgage loans, a number of financial institutions (i.e., originators) will work together to pool residential mortgage loans with similar characteristics into a more diversified portfolio. They will then sell the loans to a separate entity, called a **special purpose vehicle** (SPV), in exchange for

cash. An issuer will purchase those mortgage assets in the SPV and then use the SPV to issue MBSs to investors; the securities are backed by the mortgage loans as collateral.

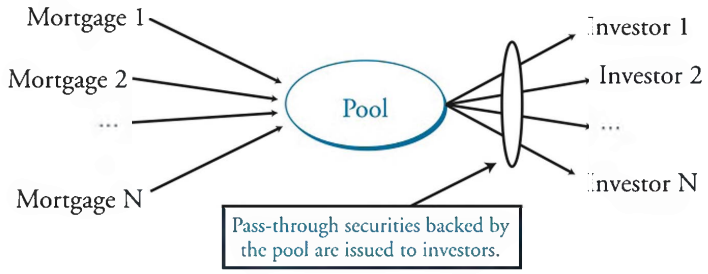
As of now, the securitization process has become a crucial part of the U.S. credit system. Financial institutions expect to originate mortgage loans and sell them through securitization. The lack of a securitization market for mortgages would lead to the downfall of mortgage lending because financial institutions would not want to retain the risks.

Pass-Through Securities

A **mortgage pass-through security** represents a claim against a pool of mortgages. Any number of mortgages may be used to form the pool and any mortgage included in the pool is referred to as a **securitized mortgage**. The mortgages in the pool have different maturities and different mortgage rates. The **weighted average maturity** (WAM) of the pool is equal to the weighted average of all mortgage ages in the pool, each weighted by the relative outstanding mortgage balance to the value of the entire pool. The **weighted average coupon** (WAC) of the pool is the weighted average of the mortgage rates in the pool. The investment characteristics of a mortgage pass-through are a function of its cash flow features and the strength of its government guarantee.

As illustrated in Figure 3, pass-through security investors receive the monthly cash flows generated by the underlying pool of mortgages, less any servicing and guarantee/insurance fees. The fees account for the fact that **pass-through rates** (i.e., the coupon rate on the pass-through) are less than the average coupon rate of the underlying mortgages in the pool.

Figure 3: Mortgage Pass-through Cash Flow



Because pass-through securities may be traded in the secondary market, they effectively convert illiquid mortgages into liquid securities (as mentioned, this process is called **securitization**). More than one class of pass-through securities may be issued against a single mortgage pool.

The timing of the cash flows to pass-through security holders does not exactly coincide with the cash flows generated by the pool. This is due to the delay between the time the mortgage service provider receives the mortgage payments and the time the cash flows are "passed through" to the security holders.

The most important characteristic of pass-through securities is their prepayment risk; because the mortgages used as collateral for the pass-through can be prepaid, the pass-throughs themselves have significant prepayment risk.

Measuring Prepayment Speeds

Prepayments cause the timing and amount of cash flows from mortgage loans and MBSs to be uncertain; they speed up principal repayments and reduce the amount of interest paid over the life of the mortgage. Thus, it is necessary to make specific assumptions about the rate at which prepayment of the pooled mortgages occurs when valuing pass-through securities. Two industry conventions have been adopted as benchmarks for prepayment rates: the **conditional prepayment rate** (CPR) and the **Public Securities Association** (PSA) prepayment benchmark.

The *CPR* is the annual rate at which a mortgage pool balance is assumed to be prepaid during the life of the pool. A mortgage pool's CPR is a function of past prepayment rates and expected future economic conditions.

We can convert the CPR into a monthly prepayment rate called the **single monthly mortality rate** (SMM) (also referred to as constant maturity mortality) using the following formula:

$$SMM = 1 - (1 - CPR)^{1/12}$$

If given the SMM rate, you can annualize the rate to solve for the CPR using the following formula:

$$CPR = 1 - (1 - SMM)^{12}$$

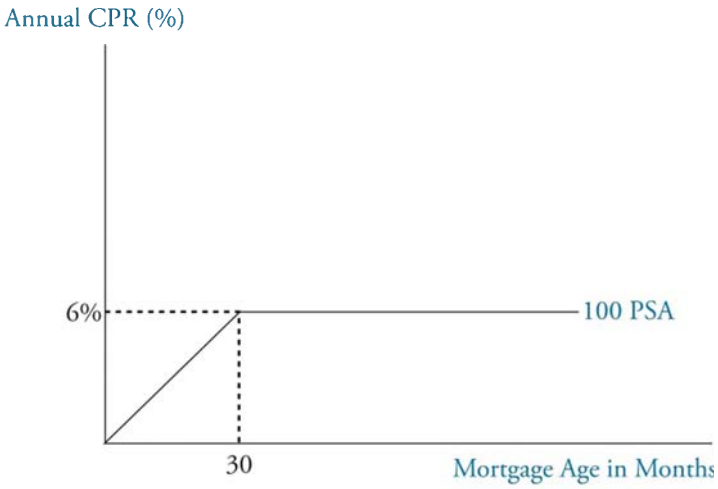
An SMM of 10% implies that 10% of a pool's beginning-of-month outstanding balance, less scheduled payments, will be prepaid during the month.

The *PSA prepayment benchmark* assumes that the monthly prepayment rate for a mortgage pool increases as it ages or becomes seasoned. The PSA benchmark is expressed as a monthly series of CPRs.

The PSA standard benchmark is referred to as 100% PSA (or just 100 PSA). 100 PSA (see Figure 4) assumes the following graduated CPRs for 30-year mortgages:

- CPR = 0.2% for the first month after origination, increasing by 0.2% per month up to 30 months. For example, the CPR in month 14 is 14(0.2%) = 2.8%.
- CPR = 6% for months 30 to 360.

Figure 4: 100 PSA



Remember that the CPRs are expressed as annual rates.

A particular pool of mortgages may exhibit prepayment rates faster or slower than 100% PSA, depending on the current level of interest rates and the coupon rate of the issue. A 50% PSA refers to one-half of the CPR prescribed by 100% PSA, and 200% PSA refers to two times the CPR called for by 100% PSA.

Example: Computing the SMM

Compute the CPR and SMM for the 5th and 25th months, assuming 100 PSA and 150 PSA.

Answer:

Assuming 100 PSA:

CPR(month 5) = 5 × 0.2% = 1%
100 PSA = 1 × 0.01 = 0.01
SMM = $1 - (1 - 0.01)^{1/12} = 0.000837$

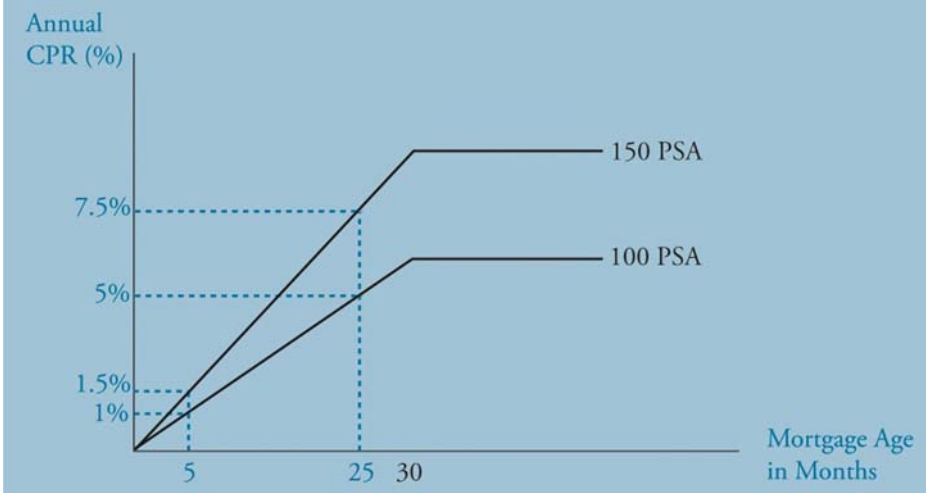
CPR (month 25) = 25 × 0.2% = 5%
100 PSA = 1 × 0.05 = 0.05
SMM = $1 - (1 - 0.05)^{1/12} = 0.004265$

Assuming 150 PSA:

CPR(month 5) = 5 × 0.2% = 1%
150 PSA = 1.5 × 0.01 = 0.015
SMM = $1 - (1 - 0.015)^{1/12} = 0.001259$

CPR (month 25) = 25 × 0.2% = 5%
150 PSA = 1.5 × 0.05 = 0.075
SMM = $1 - (1 - 0.075)^{1/12} = 0.006476$

Figure 5: Prepayment Speeds for 5th and 25th Months at 100 and 150 PSA



It is important for you to recognize that the nonlinear relationship between CPR and SMM implies that the SMM for 150% PSA does *not* equal 1.5 times the SMM for 100% PSA. Also, keep in mind that the PSA standard benchmark is nothing more than a market convention. It is not a model for predicting prepayment rates for MBS. In fact, empirical studies have shown that actual CPRs differ substantially from those assumed by the PSA benchmark.

Trading Pass-Through Securities

Trade settlements occur every month on a predetermined basis; delivery dates during a month are specified. In addition, prices are usually quoted for three settlements months, however, trades could be done for a longer period into the future.

Fixed rate pass-through securities (i.e., agency mortgage pools) trade in one of the following ways:

- Specified pools.
- To Be Announced (TBA).

The **specified pools** market identifies the number and balances of the pools prior to a trade. As a result, the characteristics of a given pool will influence the price of a trade. For example, high loan-balance pools, which make better use of prepayment options, trade for relatively lower prices.

The **TBA** market, which is more liquid than specified pools, involves identifying the security and establishing the price in a forward market. However, there is a pool allocation process whereby the actual pools are not revealed to the seller until immediately before settlement. The characteristics of the pools that can be used for TBA trades are regulated to ensure reasonable consistency.

Dollar Roll Transaction

LO 51.6: Describe a dollar roll transaction and how to value a dollar roll.

MBS trading requires the same securities to be priced for different settlement dates. A **dollar roll transaction** occurs when an MBS market maker buys positions for one settlement month and, at the same time, sells those same positions for another month.

How to Value a Dollar Roll

The process involves assessing the income and the expenses related over the holding period. Income is determined by coupon payments, reinvested interest, and principal payments. Expenses are determined by financing costs [i.e., repurchase (repo) market]. One could purchase the security in the earlier (front) month, hold it, and then dispose of it in the later (back) month at settlement.

The back month price of a dollar roll should take both income and expenses into account so the net cash flows are equivalent to simply purchasing the security in the back month for settlement at that time. However, empirical evidence suggests that the most likely outcome is that a price drop between the two settlement dates makes purchasing the security in the back month more attractive. In other words, purchasing a position for back month settlement results in financing at an implied repo rate lower than that of the repurchase market.

Factors that impact dollar roll valuations:

- The security's coupon, age, and WAC.
- Holding period (period between the two settlement dates).
- Assumed prepayment speed.
- Funding cost in the repo market.

Factors Causing a Dollar Roll to Trade Special

When the price difference/drop is large enough to result in financing at less than the implied cost of funds, then the dollar roll is trading *special*. It could be caused by:

- A decrease in the back month price (due to an increased number of sale/settlement transactions on the back month date by originators).

- An increase in the front month price (due to an increased demand in the front month for deal collateral).
- Shortages of certain securities in the market that require the dealer to suddenly purchase the security for delivery in the front month, thereby increasing the front month price.

Other Products

Collateralized Mortgage Obligations

All investors have varying degrees of concern about exposure to prepayment risk. Some are primarily concerned with extension risk (the increase in the expected life of a mortgage pool due to rising interest rates and lower prepayment rates), while others want to minimize exposure to contraction risk (the decrease in the expected life of a mortgage pool due to falling interest rates and higher prepayment rates). Fortunately, all of the pass-through securities issued on a pool of mortgages do not have to be the same. The ability to partition and distribute the cash flows generated by a mortgage pool into different risk packages has led to the creation of **collateralized mortgage obligations** (CMOs).

CMOs are securities issued against pass-through securities (securities secured by other securities) for which the cash flows have been reallocated to different bond classes called *tranches.* Each tranche has a different claim against the cash flows of the mortgage pass-throughs or pool from which it was derived. Each CMO tranche represents a different mixture of contraction and extension risk. Hence, CMO securities can be more closely matched to the unique asset/liability needs of institutional investors and investment managers.

Planned Amortization Class Tranches

The most common type of CMO today is the **planned amortization class** (PAC). A PAC is a tranche that is amortized based on a sinking fund schedule that is established within a range of prepayment speeds called the *initial PAC collar* or *initial PAC bond.*

What makes a PAC bond work is that it is packaged with a *support*, or *companion*, tranche created from the original mortgage pool. Support tranches are included in a structure with PAC tranches specifically to provide prepayment protection for the PAC tranches (each tranche is, of course, priced according to the timing risk of the cash flows). If prepayment rates are faster than the upper repayment rate, the PAC tranche receives principal according to the PAC schedule, and the support tranche absorbs (i.e., receives) the excess. If prepayment speeds are below the lower repayment rate, the funds needed to keep the PAC on schedule come from the cash flows scheduled for the support tranche(s). It should be pointed out that the extent of prepayment risk protection provided by a support tranche increases as its par value increases relative to its associated PAC tranche.

There is an *inverse* relationship between the prepayment risk of PAC tranches and the prepayment risk associated with the support tranches. In other words, *the certainty of PAC bond cash flow comes at the expense of increased risk to the support tranches.*

To understand the relatively high prepayment risk for support tranches, consider the situation in which prepayments are slower than planned. Because the PAC tranches have

priority claim against the cash flows, principal payments to the support tranches must be deferred until the PAC repayment schedule is satisfied. Thus, the average life of the support tranche is extended. Similarly, when actual prepayments come at a rate that is faster than expected, the support tranches must absorb the amount that is in excess of that required to maintain the repayment schedule for the PAC. In this case, the average life of the support tranche is contracted. If these excesses continue to occur, the support tranches will eventually be paid off and the principal will then go to the PAC holders. When this happens, the PAC is referred to as a *broken* or *busted* PAC, and any further prepayments go directly to the PAC tranche. Essentially, the PAC tranche becomes an ordinary sequential-pay structure.

Notice that the prepayment risk protection provided by the support tranches causes their average lives to extend and contract. This relationship is such that as the prepayment risk protection for a PAC tranche increases, its average life variability decreases, and the average life variability of the support tranche increases.

Strips

A distinguishing characteristic of a traditional pass-through security is that the interest and principal payments generated by the underlying mortgage pool are allocated to the bondholders on a pro rata basis. This means that each pass-through certificate holder receives the same amount of interest and the same amount of principal. *Stripped MBSs* differ in that principal and interest are not allocated on a pro rata basis. The unequal allocation of principal and interest results in a price/yield relationship that is different from that of the underlying pass-through.

The two most common types of stripped MBSs are **principal-only strips** (PO strips) and **interest-only strips** (IO strips). PO strips are a class of securities that receive only the principal payment portion of each mortgage payment, while IO strips are a class that receive only the interest component of each payment.

PO strips are sold at a considerable discount to par. The PO cash flow stream starts out small and increases with the passage of time as the principal component of the mortgage payments grows. The investment performance of a PO is extremely sensitive to prepayment rates. Higher prepayment rates result in a faster-than-expected return of principal and, thus, a higher yield. Since prepayment rates increase as mortgage rates decline, PO prices increase when interest rates fall. The entire par value of a PO is ultimately paid to the PO investor. The only question is whether realized prepayment rates will cause it to be paid sooner or later than expected.

In contrast to PO strips, an IO strip cash flow starts out big and gets smaller over time. Thus, IOs have shorter effective lives than POs.

The major risk associated with IO strips is that the value of the cash flow investors receive over the life of the mortgage pool may be less than initially expected and possibly less than the amount originally invested. Why? The amount of interest produced by the pool depends on its beginning-of-month balance. If market rates fall, the mortgage pool will be paid off sooner than expected, leaving IO investors with no interest cash flow. Therefore, IO investors want prepayments to be slow.

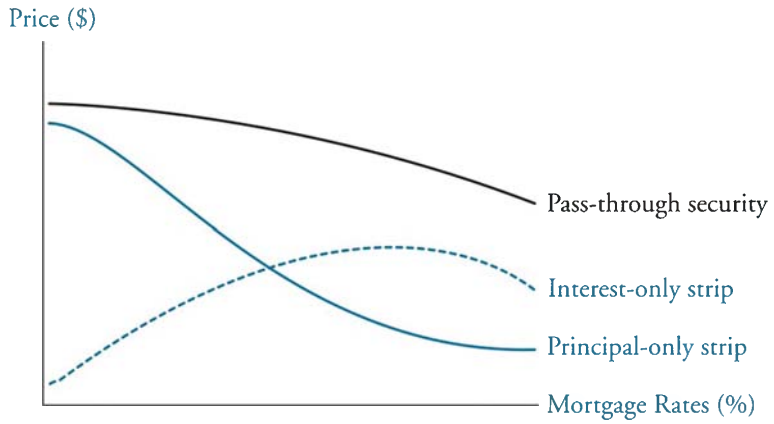
An interesting property of an IO is that its price has a tendency to move in the same direction as market rates. When market rates decline below the contract rate and prepayment rates increase, the diminished cash flow usually causes the IO price to decline, despite the fact that the cash flows are discounted at a lower rate. As interest rates rise above the contract rate, the expected cash flows improve. Even though the higher rate must be used to discount these improved cash flows, there is usually a range above the contract rate for which the price increases.

Both IOs and POs exhibit greater *price volatility* than the pass-through from which they were derived. This occurs because IO and PO returns are negatively correlated (their prices respond in opposite directions to changes in interest rates), but the combined price volatility of the two strips equals the price volatility of the pass-through.

The price/yield relationships for IO and PO securities are shown in Figure 6. Notice the following:

- The underlying pass-through security exhibits significant negative convexity.
- The PO exhibits some negative convexity at low rates.
- The IO price is positively related to mortgage rates at low current rates.
- The PO and IO prices are more volatile than the underlying pass-through.

Figure 6: Investment Characteristics of IOs and POs



Prepayment Modeling

LO 51.7: Explain prepayment modeling and its four components: refinancing, turnover, defaults, and curtailments.

Borrowers may prepay their mortgages due to the sale or destruction of the property or a desire to refinance at lower prevailing rates. In addition, prepayments may occur because the borrower has defaulted on the mortgage and the lender is forced to sell the property to cover the mortgage. Finally, many mortgages have partial prepayment privileges (curtailments) that may be used, especially when the borrower has excess cash available to do so.

Refinancing a mortgage involves using the proceeds of a new mortgage to pay off the principal from an existing mortgage. If a homeowner is holding a high interest rate mortgage and the current mortgage rates fall, the incentive to refinance is large (given that rates decline enough to cover the transaction costs of refinancing). Historically, if mortgage rates fall by more than 2%, refinancing activity increases dramatically. This is known as the *media effect* because large declines in rates will likely gain the attention of the media.

Extracting home equity is another motive for refinancing a mortgage. Given a substantial increase in property value, a borrower may take out a new mortgage with a higher balance that not only pays off the existing mortgage but also has extra cash for other purposes. Extracting home equity is also known as *cash-out refinancing*.

Incentive functions are used to model refinancing activity and are based on the term structure of mortgage rates. Past rates, also called *lagged rates*, can be included in the model to help explain refinancing behavior. Incentive functions essentially forecast the present value of any dollar gains given that a borrower will refinance.

The path that mortgage rates follow on their way to the current level affects prepayments through *refinancing burnout*. To better understand this phenomenon, consider a mortgage pool that was formed when rates were 12%, then interest rates dropped to 9%, rose to 12%, and then dropped again to 9%. Many homeowners will have refinanced when interest rates dipped the first time. On the second occurrence of 9% interest rates, most homeowners in the pool who were able to refinance would have already done so.

It is typically the case that the mortgage is due once the property is sold. This is referred to as *due on sale*. Because most borrowers sell their homes without regard for the path of mortgage rates, MBS investors will be subjected to a degree of **housing turnover** that does not correlate with the behavior of rates. One factor that slows the degree of housing turnover is known as the *lock-in effect*. This essentially means that borrowers may wish to avoid the costs of a new mortgage, which likely consists of a higher mortgage rate.

Modeling turnover typically starts with a base rate and then adjusts for seasonality (turnover is higher in the summer and lower in the winter). The turnover model may also include a *seasoning ramp*, which is partially based on improvements to creditworthiness over time, and, thus, the homeowner's increased ability to prepay the mortgage. As a group, housing turnover only accounts for 10% of overall prepayments.

When a borrower **defaults**, mortgage guarantors pay the interest and principal outstanding. These payments act as a source of prepayment. Modeling prepayments from default requires an analysis of loan-to-value (LTV) ratios and FICO scores, as well as an overall analysis of the housing market.

Partial payments by the borrower are referred to as **curtailments**. These partial payments tend to occur when a mortgage is older or has a relatively low balance. Thus, prepayment modeling due to curtailment typically takes into account the age of the mortgage.

Dynamic Valuation

LO 51.8: Describe the steps in valuing an MBS using Monte Carlo simulation.

Any discussed earlier, mortgage borrowers have an option to prepay the underlying securities. The value of MBSs with embedded options to prepay cannot be determined using traditional option valuation techniques. Therefore, the Monte Carlo valuation methodology is used to value MBSs and other fixed-income securities with embedded options.

The **binomial model** is only applicable for securities where the decision to exercise a call option is not dependent on how interest rates evolve over time. While the binomial model is useful for callable agency debentures and corporate bonds, it is not applicable to valuing an MBS. The historical evolution of interest rates over time impacts prepayments and makes the binomial model inappropriate for MBSs.

Prepayments on mortgage pass-through securities are interest rate path-dependent. This means that a given month's prepayment rate depends on whether there were prior opportunities to refinance since the origination of the underlying mortgages. For example, if mortgage rates trend downward over a period of time, prepayment rates will increase at the beginning of the trend as homeowners refinance their mortgages, but prepayments will slow as the trend continues because many of the homeowners who can refinance will have already done so. As mentioned earlier, this prepayment pattern is called refinancing burnout. Another problem of the path-dependency of MBSs is related to the nature of structured securities, such as collateralized mortgage obligations (CMOs). The amount a CMO tranche receives in the form of cash flows for a specific month depends on the outstanding balances of other tranches in the deal. These outstanding balances are impacted by earlier principal and interest prepayments.

The **Monte Carlo methodology** is a simulation approach for valuing MBSs. Monte Carlo is actually a process of steps rather than a specific model. It is extremely useful when there are numerous variables with multiple outcomes. Monte Carlo is used to provide a probability distribution of the value of an MBS. The valuation of an MBS is influenced by future interest rates, the shape of the yield curve, future interest rate volatility, prepayment rates, default rates, and recovery rates.

Each of these variables or parameters of the Monte Carlo model could have multiple outcomes with different probabilities associated for each outcome. One valuation approach in these circumstances is the **best guess approach** where the expected value of each variable is used to estimate the value of the MBS. Unfortunately, this method is highly inaccurate. For example, suppose the probability of the best guess occurring for each variable is 70%. Then, with six different variables, the probability that the best guess MBS value will occur is only 11.8% (= 0.70^6).

The Monte Carlo approach provides a range of possible outcomes with a probability distribution for the value of a mortgage security. The mean or average value of this range of outcomes is then taken as the estimated value of the MBS. The other information, such as the range of possible outcomes and percentile information, is useful in gauging the value of the security.

The following steps are required to value a mortgage security using the Monte Carlo methodology:

Step 1: Simulate the interest rate path and refinancing path.
Step 2: Project cash flows for each interest rate path.
Step 3: Calculate the present value of cash flows for each interest rate path.
Step 4: Calculate the theoretical value of the mortgage security.

Step 1: Simulate the interest rate path and refinancing path.

The first step in applying the Monte Carlo approach is to estimate monthly interest rates for the entire life of the mortgage security. For example, a 30-year mortgage security would require 360 monthly interest rates. In equations to follow, the total number of months on an interest rate path will be denoted by T. Also, the total number of interest rate paths or *trials* that are simulated will be denoted by N. Random interest rate paths are generated using the term structure of interest rates and a volatility assumption. The term structure of interest rates is created using the theoretical spot rate (zero-coupon) curve for the market on the pricing date. The simulations are adjusted to ensure the average simulated price of a zero-coupon Treasury bond is equal to the actual price corresponding to the pricing date. Some models use LIBOR or swap rates instead of Treasury rates.

The dispersion of future interest rates in the simulation is determined by the volatility assumption. It is common practice to use more than one level of volatility. For example, with a short/long yield volatility approach, the volatility is specified based on maturities. One volatility number is used for shorter maturities (short yield volatility) and a second yield volatility is specified for longer maturities (long yield volatility). Short yield volatility is typically assumed to be greater than long yield volatility. When yield volatility is assumed for each maturity, it is referred to as **term structure yield volatility**.

The derivatives market is used to construct an arbitrage-free term structure of future interest rates. Short-term interest rate paths are used to discount the cash flows in Step 3 of the Monte Carlo process. These interest rate paths are also used to create the prepayment paths or *vectors*, which are cash flows for each interest rate path. The prepayment vector is computed based on refinancing rates that are available each month. The mortgagor has an incentive to refinance if the refinancing rate is low relative to the mortgagor's original coupon rate. The relationship between refinancing rates and short-term interest rates is an important assumption of the model.

Step 2: Project cash flows for each interest rate path.

Cash flows for each month on each interest rate path are equal to the scheduled principal for the mortgage pool, the net interest, and prepayments. Scheduled principal payments are simply calculated based on the projected mortgage balance from the prior month. A prepayment model is used rather than a simple prepayment rate. A prepayment rate is specified for each month on a given interest rate path, and rates for a given month across all interest rate paths are not the same. In fact, there could actually be $T \times N$ different prepayment rates.

CMO deal structures dictate how principal and interest is to be paid. Therefore, it is necessary to reverse engineer the deal to determine the cash flows for a senior CMO. The cash flows for each month on an interest rate path are calculated using the scheduled principal, net interest, and prepayments for the collateral (i.e., the pool of agency pass-

throughs). The tranche's cash flows for each path are determined by the total principal and interest paid to the tranche, the interaction of the cash flow rules, and the prepayment model.

Step 3: Calculate the present value of cash flows for each interest rate path.

The present values of cash flows for each interest rate path are calculated by discounting the cash flows for each path by a discount rate. The discount rate is estimated using the simulated spot rates for each month on the interest rate path plus an appropriate spread. The simulated spot rates are determined from the simulated future monthly rates. The following equation quantifies the relationship that holds between the simulated spot rate, $z_T(n)$, for month T on path n, and the simulated future monthly rates, $f_j(n)$:

$$z_T(n) = \{[1 + f_1(n)][1 + f_2(n)] \ldots [1 + f_T(n)]\}^{1/T} - 1$$

where:
$z_T(n)$ = simulated spot rate for month T on path n
$f_j(n)$ = simulated future one-month rate for month j on path n

The interest rate paths for the simulated future one-month rates are converted to the interest rate paths for the simulated monthly spot rates. The present value of the cash flows for month T on interest rate path n discounted at the simulated spot rate for month T, $z_T(n)$, plus a spread, K, is:

$$PV[C_T(n)] = \frac{C_T(n)}{[1 + z_T(n) + K]^T}$$

where:
$PV[C_T(n)]$ = present value of cash flows for month T on path n
$C_T(n)$ = cash flow for month T on path n
$z_T(n)$ = spot rate for month T on path n
K = spread

The present value for path n is determined as the sum of the present values of the cash flows for each month on path n as follows:

$$PV[path(n)] = PV[C_1(n)] + PV[C_2(n)] + \ldots + PV[C_T(n)]$$

where:
$PV[path(n)]$ = present value of interest rate path n

Step 4: Calculate the theoretical value of the mortgage security.

The theoretical value for a specific interest rate path is thought of as the present value of all cash flows in that path, assuming that path was actually realized. The theoretical value of the mortgage security is calculated as the average present value of all theoretical values for each interest rate path as follows:

$$\text{theoretical value} = \frac{PV[path(1)] + PV[path(2)] + \ldots + PV[path(N)]}{N}$$

where:
N = number of interest rate paths

This average theoretical value is typically the only measurement that is evaluated when Monte Carlo simulations are used to value MBSs. It is unfortunate that other potentially valuable information, such as the distribution of the path present values, is usually ignored.

Option-Adjusted Spread

LO 51.9: Define Option Adjusted Spread (OAS), and explain its challenges and its uses.

The **option-adjusted spread** (OAS) is defined as the spread, *K*, that, when added to all the spot rates of all the interest rate paths, will make the average present value of the paths equal to the actual observed market price plus accrued interest. The OAS is mathematically determined by the following relationship:

$$\text{market price} = \frac{\text{PV[path(1)]} + \text{PV[path(2)]} + ... + \text{PV[path(N)]}}{\text{N}}$$

where:
N = number of interest rate paths

The left-hand side of the equation is the current market price of the MBS. The right-hand side of the equation is the Monte Carlo model's output of the average theoretical value of the MBS. The OAS is determined with an iterative process. If the average theoretical value determined by the model is higher (lower) than the MBS market value, the spread is increased (decreased).

The OAS can be interpreted as a measure of MBS returns that indicates the potential compensation after adjusting for prepayment risk. In other words, the OAS is *option adjusted* because the cash flows on the interest rate paths take into account the borrowers' option to prepay. An investor could estimate the value of a security using the OAS for comparable bonds to determine whether or not to invest in the security. A second approach is to compare the OAS generated at the market price to those available for comparable securities or an investment benchmark (such as a cost of funds).

Cash flows for MBSs are monthly annuity payments, while Treasury securities pay semiannual interest-only payments and a large bullet payment. The **zero-volatility spread** (*z*-spread) is a spread measure that an investor realizes over the entire Treasury spot rate curve, assuming the mortgage security is held to maturity. It is a more accurate measure because it compares an MBS to a portfolio of Treasury securities. The zero-volatility spread is the yield that equates the present value of the cash flows from the MBS to the price of the MBS discounted at the Treasury spot rate plus the spread. Thus, an iterative process is required to determine the zero-volatility spread.

The zero-volatility spread accounts for variations in MBS principal payments at a given prepayment rate or speed. However, it does not consider the impact that prepayment risk or changing prepayment rates have on the value of the MBS.

The option cost measures the prepayment (or option) risk. It is the implied cost of the option embedded in the MBS. The option cost is calculated as the difference between the OAS at the assumed volatility of interest rates and the zero-volatility spread as follows:

option cost = zero-volatility spread – OAS

Therefore, the option cost is a by-product of the Monte Carlo analysis and is not determined using traditional option value approaches. As volatility declines, the option cost decreases, and the previously described relationship suggests that OAS increases as volatility declines, all other things equal.

OAS Challenges

There are four important limitations to consider when using OAS:

- Modeling risk associated with Monte Carlo simulations.
- Required adjustments to interest rate paths.
- An underlying assumption of a constant OAS over time in the model.
- The dependency of the underlying prepayment model.

The OAS is generated through Monte Carlo simulations. Therefore, the OAS is subject to all modeling risks associated with the simulation. Interest rate paths must be adjusted to ensure securities or rates making up the benchmark curve are properly valued when using Monte Carlo methods. This process of adjusting interest rate paths is subject to modeling error. If there is a term structure to the OAS, then this is not reflected in the Monte Carlo process because the OAS methodology assumes a constant OAS.

The prepayment model is very complex, given the amount of uncertainty regarding important variables. The behavior of both borrowers and lenders changes over time. Thus, the greatest weakness of using OAS valuation estimates generated from the Monte Carlo simulation is the dependence on the prepayment model.

Additionally, both *z*-spreads and OAS measures assume the securities are held to maturity. Some investors may hold a security to maturity, but many investors will only hold a security over a finite horizon. Thus, the investor should analyze the securities in a manner that is consistent with the investor's asset management horizon.

Key Concepts

LO 51.1

Key attributes that define mortgages are lien status, original loan term, credit classification, interest rate type, prepayments/prepayment penalties, and credit guarantees.

Agency MBSs are those that are guaranteed by government-sponsored enterprises (GSEs). Most of the MBSs are issued by these GSEs.

The GSEs have restrictions on which mortgages they can guarantee/securitize, which opened up the private label market (non-agency MBSs) for those participants willing to take on the risks inherent in nonconventional loans—jumbo loans and/or loans with high loan-to-value ratios.

LO 51.2

A mortgage is a loan that is collateralized with a specific piece of real property, either residential or commercial. A level-payment, fixed-rate conventional mortgage has a fixed term, a fixed interest rate, and a fixed monthly payment. Even though the term, rate, and payment are fixed, the cash flows are not known with certainty because the borrower has the right to repay all or any part of the mortgage balance at any time.

LO 51.3

Mortgage prepayments come in two forms: (1) increasing the frequency or amount of payments and (2) repaying/refinancing the entire outstanding balance. Prepayments are much more likely to occur when market interest rates fall and borrowers wish to refinance their existing mortgages at a new and lower rate.

Other factors that influence prepayments include seasonality, age of mortgage pool, personal, housing prices, and refinancing burnout.

LO 51.4

To reduce the risk from holding a potentially undiversified portfolio of mortgage loans, a number of financial institutions (originators) will work together to pool residential mortgage loans with similar characteristics into a more diversified portfolio. They will then sell the loans to a separate entity, called a special purpose vehicle (SPV), in exchange for cash. An issuer will purchase those mortgage assets in the SPV and then use the SPV to issue mortgage-backed securities (MBSs) to investors; the securities are backed by the mortgage loans as collateral.

Fixed-rate pass-through securities trade in one of the following ways:

- The specified pools market.
- The To Be Announced (TBA) market.

LO 51.5

The value of an MBS is a function of:

- Weighted average maturity (WAM).
- Weighted average coupon (WAC).
- Speed of prepayments.

Regarding prepayment speeds, the single monthly mortality (SMM) rate is derived from the conditional prepayment rate and is used to estimate monthly prepayments for a mortgage pool:

$$SMM = 1 - (1 - CPR)^{1/12}$$

LO 51.6

A dollar roll transaction occurs when an MBS market maker is buying positions for one settlement month and, at the same time, selling those same positions for another month.

LO 51.7

Borrowers may prepay a mortgage due to the sale of the property or a desire to refinance at lower prevailing rates. In addition, prepayments may occur when the borrower has defaulted on the mortgage or when the borrower has cash available to make partial prepayments (curtailment).

LO 51.8

The Monte Carlo methodology is a simulation approach for valuing MBSs. The binomial model is not appropriate for valuing MBSs because MBSs have embedded prepayment options and the historical evolution of interest rates over time impacts prepayments.

A mortgage security is valued using the Monte Carlo methodology by simulating the interest rate path and refinancing path, projecting cash flows for each interest rate path, calculating the present value of cash flows for each interest rate path, and calculating the theoretical value of the mortgage security.

LO 51.9

The option-adjusted spread (OAS) is the spread that, when added to all the spot rates of all the interest rate paths, will make the average present value of the paths equal to the actual observed market price plus accrued interest. The zero-volatility spread (z-spread) is the spread that an investor realizes over the entire Treasury spot rate curve, assuming the mortgage security is held to maturity. The option cost is the implied cost of the embedded prepayment option and is calculated as the difference between the z-spread and OAS.

Four major limitations of OASs are related to: (1) modeling risk associated with Monte Carlo simulations, (2) required adjustments to interest rate paths, (3) model assumption of a constant OAS over time, and (4) dependency on the underlying prepayment model.

Concept Checkers

1. Which of the following factors is least likely to influence the level of residential mortgage prepayments?
   A. Seasonality.
   B. Inflation.
   C. Housing prices.
   D. Age of mortgage pool.

2. If the conditional prepayment rate (CPR) for a pool of mortgages is assumed to be 5% on an annual basis and the weighted average maturity of the underlying mortgages is 15 years, which of the following amounts is closest to the constant maturity mortality?
   A. 0.333%.
   B. 0.405%.
   C. 0.427%.
   D. 0.5%.

3. Which of the following factors would not cause a dollar roll to trade special?
   A. Decrease in the back month price.
   B. Increase in the front month price.
   C. Surplus of securities in the market used for settlement.
   D. Shortage of securities in the market used for settlement.

4. When using the Monte Carlo approach to estimate the value of mortgage-backed securities (MBSs), the model should:
   A. use one consistent volatility measure for all interest rate paths.
   B. use a short/long yield volatility approach.
   C. use annual interest rates over the entire life of the mortgage security.
   D. ignore the distribution of the interest rate paths used to determine the theoretical value.

5. All of the following describe limitations of using option-adjusted spreads (OASs) for valuing mortgage-backed securities (MBSs) except:
   A. modeling risk is associated with Monte Carlo simulations.
   B. model requires making adjustments to interest rate paths.
   C. model assumes a dynamic OAS over time.
   D. prepayment model influences the model valuation.

Concept Checker Answers

1. B Seasonality does impact the level of prepayments—they are noticeably higher in the summertime. Increases in housing prices may spur an increase in prepayments caused by refinancing mortgages stemming from borrowers wanting to take out some of the increased equity for personal use. The lower the age of the mortgage pool, the less likely the risk of prepayment.

2. C The constant maturity mortality (or single monthly mortality rate) is a monthly measure. Its relationship to CPR is as follows:

   $SMM = 1 - (1 - CPR)^{1/12} = 1 - (1 - 0.05)^{1/12} = 1 - 0.95^{1/12} = 0.43\%$

3. C When the drop is large enough to result in financing at less than the implied cost of funds, then the dollar roll is trading special. It could be caused by:
   - A decrease in the back month price (due to an increased number of sale/settlement transactions on the back month date by originators).
   - An increase in the front month price (due to an increased demand in the front month for deal collateral).
   - Shortages of certain securities in the market that require the dealer to suddenly purchase the security for delivery in the front month, which would increase the front month price.

4. B When using the Monte Carlo approach to estimate the value of MBSs, the model should use more than one volatility measure for all interest rate paths. It is very common to use a short/long yield volatility approach to estimate monthly rates. Although the information regarding the distributions of interest rate paths is oftentimes ignored, it contains valuable information and should be considered.

5. C When using OAS to value MBS, the model assumes a constant OAS over time. This is problematic if there is a term structure to the OAS because this is not reflected in the Monte Carlo process.

Self-Test: Financial Markets and Products

15 Questions: 36 Minutes

1. An investor enters a short position in a gold futures contract at $318.60. Each futures contract controls 100 troy ounces. The initial margin is $5,000 and the maintenance margin is $4,000. At the end of the first day the futures price rises to $329.22. Which of the following is the amount of the variation margin at the end of the first day?
   A. $0.
   B. $62.
   C. $1,000.
   D. $1,062.

2. A large-cap U.S. equity portfolio manager is concerned about near-term market conditions and wishes to reduce the systematic risk of her portfolio from 1.2 to 0.90. Her portfolio value is $56 million, and the S&P 500 futures index is currently trading at 1,050 and has a multiplier of 250. How can the portfolio manager's objective be achieved?
   A. Sell 47 contracts.
   B. Buy 47 contracts.
   C. Sell 64 contracts.
   D. Buy 64 contracts.

3. Suppose you observe a 1-year (zero-coupon) Treasury security trading at a yield to maturity of 5% (price of 95.2381% of par). You also observe a 2-year T-Note with a 6% coupon trading at a yield to maturity of 5.5% (price of 100.9232). And, finally, you observe a 3-year T-Note with a 7% coupon trading at a yield to maturity of 6.0% (price of 102.6730). Assume annual coupon payments and discrete compounding. Use the bootstrapping method to determine the 2-year and 3-year spot rates.

	2-year spot rate	3-year spot rate
A.	5.51%	5.92%
B.	5.46%	5.92%
C.	5.51%	6.05%
D.	5.46%	6.05%

4. Former Treasury Secretary Robert Rubin decided to stop issuing 30-year Treasury bonds in 2001 and to replace them by borrowing more with shorter-maturity Treasury bills and notes (although the U.S. Treasury has since resumed issuing 30-year bonds). Which of the following statements concerning this decision is most accurate?
   A. If the expectations theory of the term structure is correct, this decision will reduce the government's borrowing cost.
   B. If the liquidity theory of the term structure is correct, this decision will reduce the government's borrowing cost.
   C. If the liquidity theory of the term structure is correct, this decision will not change the government's borrowing cost.
   D. If the expectations theory of the term structure is correct, this decision will increase the government's borrowing cost.

5. A portfolio manager owns Macrogrow, Inc., which is currently trading at $35 per share. She plans to sell the stock in 120 days but is concerned about a possible price decline. She decides to take a short position in a 120-day forward contract on the stock. The stock will pay a $0.50 per share dividend in 35 days and $0.50 again in 125 days. The risk-free rate is 4%. The value of the trader's position in the forward contract in 45 days, assuming in 45 days the stock price is $27.50 and the risk-free rate has not changed, is closest to:
   A. $7.17.
   B. $7.50.
   C. $7.92.
   D. $7.00.

6. A 6-month futures contract on an equity index is currently priced at 1,276. The underlying index stocks are valued at 1,250 and pay dividends at a continuously compounded rate of 1.70%. The current continuously compounded risk-free rate is 5%. The potential arbitrage is closest to:
   A. 5.20.
   B. 8.32.
   C. 16.58.
   D. 26.00.

7. Company J and Company K enter into a 2-year plain vanilla interest rate swap. Company J agrees to pay Company K a periodic fixed rate on a notional principal over the swap's tenor. In exchange, Company K agrees to pay Company J a periodic floating rate on the same notional principal. Assume currency is the same, so the net payment will be exchanged. The exchanges will be made semi-annually. The reference rate is the 6-month LIBOR. The fixed rate of the swap is 1.1%, and the notional principal is $100 million. 6-month LIBOR rates are as follows:

Beginning of Period	*LIBOR*
1	0.5%
2	0.75%
3	1.00%
4	1.25%
5	1.50%

What is the net payment for the end of the first period?

A. Company J pays Company K $300,000.
B. Company J pays Company K $550,000.
C. Company K pays Company J $250,000.
D. Company K pays Company J $50,000.

Use the following information to answer Questions 8 and 9.

Stock ABC trades for $60 and has 1-year call and put options written on it with an exercise price of $60. The annual standard deviation estimate is 10%, and the continuously compounded risk-free rate is 5%. The value of the call is $4.09.

Chefron, Inc. common stock trades for $60 and has a 1-year call option written on it with an exercise price of $60. The annual standard deviation estimate is 10%, the continuous dividend yield is 1.4%, and the continuously compounded risk-free rate is 5%.

8. The value of the put on ABC stock is closest to:

A. $1.16.
B. $3.28.
C. $4.09.
D. $1.00.

9. The value of the call on Chefron stock is closest to:
   A. $3.51.
   B. $4.16.
   C. $5.61.
   D. $6.53.

10. One of your clients, Christopher Stachowski, realizes that the market prices of options must take into account the beliefs of the market participants. He thinks he will be able to make significant profits because he believes that there will be a large movement in the direction of stock prices but is unsure which direction. Such a belief is completely different from the other market participants. As a result, Christopher would like you to implement an options trading strategy to generate him those profits. Which of the following combination option strategies is likely to benefit the least amount from a large positive or negative movement in the price of the underlying?
   A. Strip.
   B. Strap.
   C. Collar.
   D. Long strangle.

11. Consider a bearish option strategy of buying one $50 put for $7, selling two $42 puts for $4 each, and buying one $37 put for $2. All the options have the same maturity. Calculate the final profit per share of the strategy if the underlying is trading at $33 at expiration.
   A. $1 per share.
   B. $2 per share.
   C. $3 per share.
   D. $4 per share.

12. You believe that a stock will increase in price and would like to buy a call option. You would like to choose the date during the option's term when the option payoff is determined. However, if the option payoff is greater at the option's maturity, you want to be paid this value. What type of option should you buy?
   A. Chooser option.
   B. Compound option.
   C. Shout option.
   D. Asian option.

13. Suppose the spot rate is 0.7102 USD/CHF. Swiss and U.S. interest rates are 7.6% and 5.2%, respectively. If the 1-year forward rate is 0.7200 USD/CHF, an investor could:
   A. not earn arbitrage profits.
   B. earn arbitrage profits by investing in USD.
   C. earn arbitrage profits by investing in CHF.
   D. only earn arbitrage profits by investing in a third currency.

14. Consider a U.K.-based company that exports goods to the EU. The U.K. company expects to receive payment on a shipment of goods in 60 days. Because the payment will be in euros, the U.K. company wants to hedge against a decline in the value of the euro against the pound over the next 60 days. The U.K. risk-free rate is 3% and the EU risk-free rate is 4%. No change is expected in these rates over the next 60 days. The current spot rate is 0.9230 £ per €. To hedge the currency risk, the U.K. company should take a short position in a Euro contract at a forward price of:
A. 0.9205.
B. 0.9215.
C. 0.9244.
D. 0.9141.

15. A level-payment, fixed-rate mortgage has the following characteristics:
- Term 30 years.
- Mortgage rate 9.0%.
- Servicing fee 0.5%.
- Original mortgage loan balance $150,000.

The monthly mortgage payment is:
A. $416.67.
B. $1,125.00.
C. $1,206.93.
D. $1,216.70.

Self-Test Answers: Financial Markets and Products

1. **D** The short position loses when the price rises.

   ($329.22 – $318.60) × 100 = 1,062 loss

   Margin account will change as follows: $5,000 – $1,062 = $3,938

   Variation margin of $1,062 is required because the balance has fallen below the maintenance margin level. This variation margin payment is required in order to restore the account back to the initial level.

   (See Topic 35)

2. **C** The portfolio manager wants to reduce exposure to systematic risk so she will want to sell S&P index futures. This will reduce the current beta to her target beta of 0.90.

   number of contracts = (target beta – current beta) × (portfolio value / futures value)

   number of contracts = (0.9 – 1.2) × [$56 million / (1,050 × 250)]

   number of contracts = –64 (i.e., sell 64 contracts)

   (See Topic 36)

3. **C** Here are the cash flows associated with the three bonds:

	0	1	2	3
1-year	–$95.2381	+$100		
2-year	–$100.9232	+$6	+$106	
3-year	–$102.6730	+$7	+$7	+$107

   To find Z_2, the 2-year spot rate:

$$\$100.9232 = \frac{\$6}{1.05^1} + \frac{\$106}{(1+Z_2)^2} \Rightarrow Z_2 = 5.51\%$$

   To find Z_3, the 3-year spot rate:

$$\$102.6730 = \frac{\$7}{1.05^1} + \frac{\$7}{1.0551^2} + \frac{\$107}{(1+Z_3)^3} \Rightarrow Z_3 = 6.05\%$$

   (See Topic 37)

4. **B** If the expectations theory of the term structure is correct, altering the maturity of the government's borrowing will not affect the government's borrowing cost (i.e., borrowing once for 30 years is the same as borrowing 30 times for one year at a time). If the liquidity theory is correct, the government's borrowing cost will go down, as it no longer has to compensate lenders with the liquidity premium for borrowing long term.

   (See Topic 37)

5. **A** The dividend in 125 days is irrelevant because it occurs after the forward contract matures.

   $$PVD = \$0.50e^{-0.04\times(35/365)} = \$0.4981$$
   $$FP = (\$35 - \$0.4981)\times e^{(0.04)(120/365)} = \$34.96$$
   $$V_{45}(\text{short position}) = -(\$27.50 - \$34.96e^{-0.04\times(75/365)}) = \$7.17$$

   (See Topic 38)

6. **A** $F = S \times e^{(\text{risk-free rate} - \text{dividend yield}) \times t}$

   $F = 1{,}250 \times e^{(0.05 - 0.017) \times 0.5}$

   $F = 1{,}270.80$

   The actual futures price is 1,276, so selling the futures and buying the underlying index nets a profit of 1,276 – 1,270.80 = 5.20.

   (See Topic 38)

7. **A** Floating = \$100 million × 0.005 × 0.5 = \$250,000

   Fixed = \$100 million × 0.011 × 0.5 = \$550,000

   (See Topic 40)

8. **A** According to put/call parity, the put's value is:

   $$p_0 = c_0 - S_0 + \left(X \times e^{-R_c^f \times T}\right) = \$4.09 - \$60.00 + \left|\$60.00 \times e^{-(0.05 \times 1.0)}\right| = \$1.16$$

   (See Topic 42)

9. **A** ABC and Chefron stock are identical in all respects except Chefron pays a dividend. Therefore, the call option on Chefron stock must be worth less than the call on ABC (i.e., less than \$4.09). \$3.51 is the only possible answer.

   (See Topic 42)

10. **C** A collar is the combination of a protective put and a covered call. Ignoring transaction costs, at levels below the put strike price or above the call strike price, the profit from a collar levels off. Between the put strike price and the call strike price, the profit level is gradually rising.

    (See Topic 43)

11. **B** Consider each option separately:

$50 long put:	$50 – $33 = +$17
$42 short put:	$42 – $33 = –$9 × 2 = –$18
$37 long put:	$37 – $33 = +$4
Net cost of options:	(–7 + 8 – 2) = –$1
Overall profit per share:	$2 per share

(See Topic 43)

12. **C** The shout option allows the buyer to choose the date when he "shouts" to the option seller that the intrinsic value should be determined. At expiration, the option buyer receives the maximum of the shout value or the intrinsic value at expiration.

(See Topic 44)

13. **C** Note that while the USD has the lower interest rate, it is also trading at a forward discount relative to the CHF. Since the USD will earn less interest *and* depreciate in value, we definitely want to invest in CHF (not in USD), and no calculation is necessary.

As an illustration of covered interest arbitrage, we have:

$$(1+R_A) < \frac{(1+R_B)(\text{forward rate})}{\text{spot rate}}$$

$$1.052 < \frac{(1.076)(0.72)}{0.7102} = 1.0908$$

Today:
(1) Borrow USD1 at 5.2% and purchase CHF at $0.7102 to get $1 / 0.7102 = 1.408 CHF at spot rate.
(2) Lend the purchased CHF at 7.6% and sell forward 1.5150 CHF at the forward rate of 0.7200 USD/CHF.

In one year:
(1) Use the proceeds of the savings account [(1.408)(1.076) = 1.5150 CHF] to purchase USD1.0908 at the forward rate (1.515 CHF × 0.72 USD/CHF).
(2) Pay off the loan of USD1 × 1.052 = USD1.052 and earn a riskless profit = USD1.0908 – USD1.052 = USD0.0388.

(See Topic 49)

14. **B** The U.K. company will be receiving euros in 60 days, so it should short the 60-day forward on the euro as a hedge. The no-arbitrage forward price is:

$$F_T = £0.923 \times \frac{1.03^{60/365}}{1.04^{60/365}} = 0.9215$$

(See Topic 49)

15. **C** N = 360; I = 9/12 = 0.75; PV = 150,000; CPT → PMT = $1,206.93

(See Topic 51)

Formulas

Topic 32

combined ratio: loss ratio + expense ratio

combined ratio after dividends: combined ratio + dividends

operating ratio: combined ratio after dividends – investment income

Topic 33

net asset value: $NAV = \frac{\text{fund assets} - \text{fund liabilities}}{\text{total shares outstanding}}$

Topic 34

call option payoff: $C_T = \max(0, S_T - X)$

put option payoff: $P_T = \max(0, X - S_T)$

forward contract payoff: payoff = $S_T - K$

where:
S_T = spot price at maturity
K = delivery price

Topic 36

basis = $S_t - F_0$

where:
S_t = cash (or spot) price of the underlying asset at time t
F_0 = current price of the futures contract

hedge ratio: $HR = \rho_{S,F} \frac{\sigma_S}{\sigma_F}$

beta: $\frac{Cov_{S,F}}{\sigma_F^2} = \beta_{S,F}$

correlation: $\rho = \frac{Cov_{S,F}}{\sigma_S \sigma_F}$

hedging with stock index futures:

$$\text{number of contracts} = \beta_{\text{portfolio}} \times \left(\frac{\text{portfolio value}}{\text{value of futures contract}} \right)$$

$$= \beta_{\text{portfolio}} \times \left(\frac{\text{portfolio value}}{\text{futures price} \times \text{contract multiplier}} \right)$$

adjusting the portfolio beta: $\text{number of contracts} = (\beta^* - \beta)\frac{P}{A}$

Topic 37

discrete compounding: $FV = A\left(1 + \frac{R}{m}\right)^{m \times n}$

continuous compounding: $FV = Ae^{R \times n}$

forward rate agreement: $\text{cash flow (if receiving } R_K) = L \times (R_K - R) \times (T_2 - T_1)$

$\text{cash flow (if paying } R_K) = L \times (R - R_K) \times (T_2 - T_1)$

Topic 38

forward price: $F_0 = S_0 e^{rT}$

forward price with carrying costs: $F_0 = (S_0 - I)\, e^{rT}$

forward price when the underlying asset pays a dividend: $F_0 = S_0 e^{(r-q)T}$

Topic 39

$$\text{accrued interest} = \text{coupon} \times \frac{\text{\# of days from last coupon to the settlement date}}{\text{\# of days in coupon period}}$$

cash price of a bond: cash price = quoted price + accrued interest

annual rate on a T-Bill: T-bill discount rate = $\frac{360}{n}(100 - Y)$

cheapest-to-deliver bond: quoted bond price – (QFP × CF)

Eurodollar futures price = $\$10{,}000[100 - (0.25)(100 - Z)]$

convexity adjustment:

actual forward rate = forward rate implied by futures – $(0.5 \times \sigma^2 \times t_1 \times t_2)$

duration-based hedge ratio: $N = -\frac{P \times D_P}{F \times D_F}$

Topic 40

forward rate between T_1 and T_2: $R_{forward} = R_2 + (R_2 - R_1)\frac{T_1}{T_2 - T_1}$

Topic 42

put-call parity:

$S = c - p + Xe^{-rT}$
$p = c - S + Xe^{-rT}$
$c = S + p - Xe^{-rT}$
$Xe^{-rT} = S + p - c$

lower and upper bounds for options:

Option	*Minimum Value*	*Maximum Value*
European call	$c \geq \max(0, S_0 - Xe^{-rT})$	S_0
American call	$C \geq \max(0, S_0 - Xe^{-rT})$	S_0
European put	$p \geq \max(0, Xe^{-rT} - S_0)$	Xe^{-rT}
American put	$P \geq \max(0, X - S_0)$	X

Topic 43

bull call spread: profit = $\max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0}$

bear put spread: profit = $\max(0, X_H - S_T) - \max(0, X_L - S_T) - P_{H0} + P_{L0}$

butterfly spread with calls:

profit = $\max(0, S_T - X_L) - 2\max(0, S_T - X_M) + \max(0, S_T - X_H) - C_{L0} + 2C_{M0} - C_{H0}$

straddle: profit = $\max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0$

strangle: profit = $\max(0, S_T - X_H) + \max(0, X_L - S_T) - C_0 - P_0$

Topic 45

pricing a commodity forward with a lease payment: $F_{0,T} = S_0 e^{(r - \delta)T}$

commodity forward pricing with storage costs: $F_{0,T} = S_0 e^{(r + \lambda)T}$

commodity forward pricing with convenience yield: $F_{0,T} = S_0 e^{(r - c)T}$

Topic 49

interest rate parity: $\text{forward} = \text{spot}\left[\frac{(1+r_{DC})}{(1+r_{FC})}\right]^{T}$

$$\text{forward} = \text{spot} \times e^{(r_{DC}-r_{FC})T}$$

nominal interest rate:

$$\text{exact methodology: } (1+r) = (1+\text{real } r)[1+E(i)]$$

$$\text{linear approximation: } r \approx \text{real} + E(i)$$

Topic 50

original-issue discount (OID) = face value – offering price

dollar default rate:

$$\frac{\text{cumulative dollar value of all defaulted bonds}}{(\text{cumulative dollar value of all issuance}) \times (\text{weighted average \# of years outstanding})}$$

Topic 51

single monthly mortality rate: $SMM = 1 - (1 - CPR)^{1/12}$

option cost = zero-volatility spread – OAS

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