

2018 SchweserNotes™

Part I

**FRM®**  
Exam Prep

Financial Markets  
and Products

eBook 3



## FRM® Exam Part I

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Sincerely,



Derek Burkett, CFA, FRM, CAIA

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# **FRM PART I BOOK 3:**

## **FINANCIAL MARKETS AND PRODUCTS**

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**FRM 2018 PART I BOOK 3: FINANCIAL MARKETS AND PRODUCTS**

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Published in 2018 by Kaplan, Inc.

Printed in the United States of America.

ISBN: 978-1-4754-7000-0

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# READING ASSIGNMENTS AND LEARNING OBJECTIVES

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*The following material is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by the Global Association of Risk Professionals.*

## READING ASSIGNMENTS

- John C. Hull, *Risk Management and Financial Institutions, 4th Edition* (Hoboken, NJ: John Wiley & Sons, 2015).
- 31. "Banks," Chapter 2 (page 1)
  - 32. "Insurance Companies and Pension Plans," Chapter 3 (page 10)
  - 33. "Mutual Funds and Hedge Funds," Chapter 4 (page 24)
- John C. Hull, *Options, Futures, and Other Derivatives, 10th Edition* (New York, NY: Pearson, 2017).
- 34. "Introduction," Chapter 1 (page 40)
  - 35. "Futures Markets and Central Counterparties," Chapter 2 (page 56)
  - 36. "Hedging Strategies Using Futures," Chapter 3 (page 70)
  - 37. "Interest Rates," Chapter 4 (page 82)
  - 38. "Determination of Forward and Futures Prices," Chapter 5 (page 98)
  - 39. "Interest Rate Futures," Chapter 6 (page 111)
  - 40. "Swaps," Chapter 7 (page 125)
  - 41. "Mechanics of Options Markets," Chapter 10 (page 142)
  - 42. "Properties of Stock Options," Chapter 11 (page 157)
  - 43. "Trading Strategies Involving Options," Chapter 12 (page 169)
  - 44. "Exotic Options," Chapter 26 (page 185)
- Robert McDonald, *Derivatives Markets, 3rd Edition* (Boston: Addison-Wesley, 2013).
- 45. "Commodity Forwards and Futures," Chapter 6 (page 196)

### Book 3

#### Reading Assignments and Learning Objectives

Jon Gregory, *Central Counterparties: Mandatory Clearing and Bilateral Margin Requirements for OTC Derivatives* (New York, NY: John Wiley & Sons, 2014).

46. "Exchanges, OTC Derivatives, DPCs and SPVs," Chapter 2 (page 219)

47. "Basic Principles of Central Clearing," Chapter 3 (page 229)

48. "Risks Caused by CCPs," Chapter 14 (section 14.4 only) (page 240)

Anthony Saunders and Marcia Millon Cornett, *Financial Institutions Management: A Risk Management Approach, 8th Edition* (New York, NY: McGraw-Hill, 2014).

49. "Foreign Exchange Risk," Chapter 13 (page 247)

Frank Fabozzi (editor), *The Handbook of Fixed Income Securities, 8th Edition* (New York, NY: McGraw-Hill, 2012).

50. "Corporate Bonds," Chapter 12 (page 262)

Bruce Tuckman and Angel Serrat, *Fixed Income Securities: Tools for Today's Markets, 3rd Edition* (Hoboken, NJ: John Wiley & Sons, 2011).

51. "Mortgages and Mortgage-Backed Securities," Chapter 20 (page 275)

## LEARNING OBJECTIVES

### 31. Banks

1. Identify the major risks faced by a bank. (page 1)
2. Distinguish between economic capital and regulatory capital. (page 2)
3. Explain how deposit insurance gives rise to a moral hazard problem. (page 2)
4. Describe investment banking financing arrangements including private placement, public offering, best efforts, firm commitment, and Dutch auction approaches. (page 3)
5. Describe the potential conflicts of interest among commercial banking, securities services, and investment banking divisions of a bank and recommend solutions to the conflict of interest problems. (page 4)
6. Describe the distinctions between the “banking book” and the “trading book” of a bank. (page 4)
7. Explain the originate-to-distribute model of a bank and discuss its benefits and drawbacks. (page 5)

### 32. Insurance Companies and Pension Plans

1. Describe the key features of the various categories of insurance companies and identify the risks facing insurance companies. (page 10)
2. Describe the use of mortality tables and calculate the premium payment for a policy holder. (page 12)
3. Calculate and interpret loss ratio, expense ratio, combined ratio, and operating ratio for a property-casualty insurance company. (page 15)
4. Describe moral hazard and adverse selection risks facing insurance companies, provide examples of each, and describe how to overcome the problems. (page 15)
5. Distinguish between mortality risk and longevity risk and describe how to hedge these risks. (page 16)
6. Evaluate the capital requirements for life insurance and property-casualty insurance companies. (page 16)
7. Compare the guaranty system and the regulatory requirements for insurance companies with those for banks. (page 17)
8. Describe a defined benefit plan and a defined contribution plan for a pension fund and explain the differences between them. (page 18)

### 33. Mutual Funds and Hedge Funds

1. Differentiate among open-end mutual funds, closed-end mutual funds, and exchange-traded funds (ETFs). (page 24)
2. Calculate the net asset value (NAV) of an open-end mutual fund. (page 28)
3. Explain the key differences between hedge funds and mutual funds. (page 28)
4. Calculate the return on a hedge fund investment and explain the incentive fee structure of a hedge fund including the terms hurdle rate, high-water mark, and clawback. (page 29)
5. Describe various hedge fund strategies, including long/short equity, dedicated short, distressed securities, merger arbitrage, convertible arbitrage, fixed income arbitrage, emerging markets, global macro, and managed futures, and identify the risks faced by hedge funds. (page 31)
6. Describe hedge fund performance and explain the effect of measurement biases on performance measurement. (page 34)

**34. Introduction (Options, Futures, and Other Derivatives)**

1. Describe the over-the-counter market, distinguish it from trading on an exchange, and evaluate its advantages and disadvantages. (page 41)
2. Differentiate between options, forwards, and futures contracts. (page 42)
3. Identify and calculate option and forward contract payoffs. (page 42)
4. Calculate and compare the payoffs from hedging strategies involving forward contracts and options. (page 47)
5. Calculate and compare the payoffs from speculative strategies involving futures and options. (page 48)
6. Calculate an arbitrage payoff and describe how arbitrage opportunities are temporary. (page 51)
7. Describe some of the risks that can arise from the use of derivatives. (page 51)
8. Differentiate among the broad categories of traders: hedgers, speculators, and arbitrageurs. (page 46)

**35. Futures Markets and Central Counterparties**

1. Define and describe the key features of a futures contract, including the asset, the contract price and size, delivery, and limits. (page 56)
2. Explain the convergence of futures and spot prices. (page 58)
3. Describe the rationale for margin requirements and explain how they work. (page 58)
4. Describe the role of a clearinghouse in futures and over-the-counter market transactions. (page 59)
5. Describe the role of central counterparties (CCPs) and distinguish between bilateral and centralized clearing. (page 60)
6. Describe the role of collateralization in the over-the-counter market and compare it to the margining system. (page 60)
7. Identify the differences between a normal and inverted futures market. (page 62)
8. Explain the different market quotes. (page 62)
9. Describe the mechanics of the delivery process and contrast it with cash settlement. (page 63)
10. Evaluate the impact of different trading order types. (page 64)
11. Compare and contrast forward and futures contracts. (page 64)

**36. Hedging Strategies Using Futures**

1. Define and differentiate between short and long hedges and identify their appropriate uses. (page 70)
2. Describe the arguments for and against hedging and the potential impact of hedging on firm profitability. (page 70)
3. Define the basis and explain the various sources of basis risk, and explain how basis risks arise when hedging with futures. (page 71)
4. Define cross hedging, and compute and interpret the minimum variance hedge ratio and hedge effectiveness. (page 71)
5. Compute the optimal number of futures contracts needed to hedge an exposure, and explain and calculate the “tailing the hedge” adjustment. (page 74)
6. Explain how to use stock index futures contracts to change a stock portfolio’s beta. (page 75)
7. Explain the term “rolling the hedge forward” and describe some of the risks that arise from this strategy. (page 76)

### **37. Interest Rates**

1. Describe Treasury rates, LIBOR, and repo rates, and explain what is meant by the “risk-free” rate. (page 82)
2. Calculate the value of an investment using different compounding frequencies. (page 83)
3. Convert interest rates based on different compounding frequencies. (page 83)
4. Calculate the theoretical price of a bond using spot rates. (page 84)
5. Derive forward interest rates from a set of spot rates. (page 88)
6. Derive the value of the cash flows from a forward rate agreement (FRA). (page 89)
7. Calculate the duration, modified duration, and dollar duration of a bond. (page 90)
8. Evaluate the limitations of duration, and explain how convexity addresses some of them. (page 91)
9. Calculate the change in a bond’s price given its duration, its convexity, and a change in interest rates. (page 92)
10. Compare and contrast the major theories of the term structure of interest rates. (page 93)

### **38. Determination of Forward and Futures Prices**

1. Differentiate between investment and consumption assets. (page 98)
2. Define short-selling and calculate the net profit of a short sale of a dividend-paying stock. (page 98)
3. Describe the differences between forward and futures contracts and explain the relationship between forward and spot prices. (page 99)
4. Calculate the forward price given the underlying asset’s spot price, and describe an arbitrage argument between spot and forward prices. (page 99)
5. Explain the relationship between forward and futures prices. (page 103)
6. Calculate a forward foreign exchange rate using the interest rate parity relationship. (page 102)
7. Define income, storage costs, and convenience yield. (page 104)
8. Calculate the futures price on commodities incorporating income/storage costs and/or convenience yields. (page 104)
9. Calculate, using the cost-of-carry model, forward prices where the underlying asset either does or does not have interim cash flows. (page 99)
10. Describe the various delivery options available in the futures markets and how they can influence futures prices. (page 105)
11. Explain the relationship between current futures prices and expected future spot prices, including the impact of systematic and nonsystematic risk. (page 105)
12. Define and interpret contango and backwardation, and explain how they relate to the cost-of-carry model. (page 106)

### **39. Interest Rate Futures**

1. Identify the most commonly used day count conventions, describe the markets that each one is typically used in, and apply each to an interest calculation. (page 111)
2. Calculate the conversion of a discount rate to a price for a US Treasury bill. (page 113)
3. Differentiate between the clean and dirty price for a US Treasury bond; calculate the accrued interest and dirty price on a US Treasury bond. (page 112)
4. Explain and calculate a US Treasury bond futures contract conversion factor. (page 114)

5. Calculate the cost of delivering a bond into a Treasury bond futures contract. (page 114)
6. Describe the impact of the level and shape of the yield curve on the cheapest-to-deliver Treasury bond decision. (page 114)
7. Calculate the theoretical futures price for a Treasury bond futures contract. (page 115)
8. Calculate the final contract price on a Eurodollar futures contract. (page 117)
9. Describe and compute the Eurodollar futures contract convexity adjustment. (page 117)
10. Explain how Eurodollar futures can be used to extend the LIBOR zero curve. (page 118)
11. Calculate the duration-based hedge ratio and create a duration-based hedging strategy using interest rate futures. (page 118)
12. Explain the limitations of using a duration-based hedging strategy. (page 119)

**40. Swaps**

1. Explain the mechanics of a plain vanilla interest rate swap and compute its cash flows. (page 125)
2. Explain how a plain vanilla interest rate swap can be used to transform an asset or a liability and calculate the resulting cash flows. (page 126)
3. Explain the role of financial intermediaries in the swaps market. (page 126)
4. Describe the role of the confirmation in a swap transaction. (page 126)
5. Describe the comparative advantage argument for the existence of interest rate swaps and evaluate some of the criticisms of this argument. (page 127)
6. Explain how the discount rates in a plain vanilla interest rate swap are computed. (page 128)
7. Calculate the value of a plain vanilla interest rate swap based on two simultaneous bond positions. (page 128)
8. Calculate the value of a plain vanilla interest rate swap from a sequence of forward rate agreements (FRAs). (page 130)
9. Explain the mechanics of a currency swap and compute its cash flows. (page 132)
10. Explain how a currency swap can be used to transform an asset or liability and calculate the resulting cash flows. (page 134)
11. Calculate the value of a currency swap based on two simultaneous bond positions. (page 132)
12. Calculate the value of a currency swap based on a sequence of FRAs. (page 133)
13. Describe the credit risk exposure in a swap position. (page 135)
14. Identify and describe other types of swaps, including commodity, volatility and exotic swaps. (page 135)

**41. Mechanics of Options Markets**

1. Describe the types, position variations, and typical underlying assets of options. (page 142)
2. Explain the specification of exchange-traded stock option contracts, including that of nonstandard products. (page 148)
3. Describe how trading, commissions, margin requirements, and exercise typically work for exchange-traded options. (page 150)

#### **42. Properties of Stock Options**

1. Identify the six factors that affect an option's price and describe how these six factors affect the price for both European and American options. (page 157)
2. Identify and compute upper and lower bounds for option prices on non-dividend and dividend paying stocks. (page 159)
3. Explain put-call parity and apply it to the valuation of European and American stock options. (page 160)
4. Explain the early exercise features of American call and put options. (page 162)

#### **43. Trading Strategies Involving Options**

1. Explain the motivation to initiate a covered call or a protective put strategy. (page 169)
2. Describe the use and calculate the payoffs of various spread strategies. (page 170)
3. Describe the use and explain the payoff functions of combination strategies. (page 175)

#### **44. Exotic Options**

1. Define and contrast exotic derivatives and plain vanilla derivatives. (page 185)
2. Describe some of the factors that drive the development of exotic products. (page 185)
3. Explain how any derivative can be converted into a zero-cost product. (page 186)
4. Describe how standard American options can be transformed into nonstandard American options. (page 186)
5. Identify and describe the characteristics and pay-off structure of the following exotic options: gap, forward start, compound, chooser, barrier, binary, lookback, shout, Asian, exchange, rainbow, and basket options. (page 187)
6. Describe and contrast volatility and variance swaps. (page 190)
7. Explain the basic premise of static option replication and how it can be applied to hedging exotic options. (page 191)

#### **45. Commodity Forwards and Futures**

1. Apply commodity concepts such as storage costs, carry markets, lease rate, and convenience yield. (page 196)
2. Explain the basic equilibrium formula for pricing commodity forwards. (page 197)
3. Describe an arbitrage transaction in commodity forwards, and compute the potential arbitrage profit. (page 198)
4. Define the lease rate and explain how it determines the no-arbitrage values for commodity forwards and futures. (page 201)
5. Define carry markets, and illustrate the impact of storage costs and convenience yields on commodity forward prices and no-arbitrage bounds. (page 204)
6. Compute the forward price of a commodity with storage costs. (page 204)
7. Compare the lease rate with the convenience yield. (page 206)
8. Identify factors that impact gold, corn, electricity, natural gas, and oil forward prices. (page 207)
9. Compute a commodity spread. (page 209)
10. Explain how basis risk can occur when hedging commodity price exposure. (page 210)
11. Evaluate the differences between a strip hedge and a stack hedge and explain how these differences impact risk management. (page 211)

12. Provide examples of cross-hedging, specifically the process of hedging jet fuel with crude oil and using weather derivatives. (page 212)
13. Explain how to create a synthetic commodity position, and use it to explain the relationship between the forward price and the expected future spot price. (page 197)

**46. Exchanges, OTC Derivatives, DPCs and SPVs**

1. Describe how exchanges can be used to alleviate counterparty risk. (page 219)
2. Explain the developments in clearing that reduce risk. (page 219)
3. Compare exchange-traded and OTC markets and describe their uses. (page 220)
4. Identify the classes of derivative securities and explain the risk associated with them. (page 221)
5. Identify risks associated with OTC markets and explain how these risks can be mitigated. (page 222)

**47. Basic Principles of Central Clearing**

1. Provide examples of the mechanics of a central counterparty (CCP). (page 229)
2. Describe advantages and disadvantages of central clearing of OTC derivatives. (page 231)
3. Compare margin requirements in centrally cleared and bilateral markets, and explain how margin can mitigate risk. (page 233)
4. Compare and contrast bilateral markets to the use of novation and netting. (page 233)
5. Assess the impact of central clearing on the broader financial markets. (page 234)

**48. Risks Caused by CCPs**

1. Identify and explain the types of risks faced by CCPs. (page 240)
2. Identify and distinguish between the risks to clearing members as well as non-members. (page 242)
3. Identify and evaluate lessons learned from prior CCP failures. (page 243)

**49. Foreign Exchange Risk**

1. Calculate a financial institution's overall foreign exchange exposure. (page 247)
2. Explain how a financial institution could alter its net position exposure to reduce foreign exchange risk. (page 248)
3. Calculate a financial institution's potential dollar gain or loss exposure to a particular currency. (page 248)
4. Identify and describe the different types of foreign exchange trading activities. (page 249)
5. Identify the sources of foreign exchange trading gains and losses. (page 249)
6. Calculate the potential gain or loss from a foreign currency denominated investment. (page 250)
7. Explain balance-sheet hedging with forwards. (page 252)
8. Describe how a non-arbitrage assumption in the foreign exchange markets leads to the interest rate parity theorem, and use this theorem to calculate forward foreign exchange rates. (page 255)
9. Explain why diversification in multicurrency asset-liability positions could reduce portfolio risk. (page 256)
10. Describe the relationship between nominal and real interest rates. (page 256)

## 50. Corporate Bonds

1. Describe a bond indenture and explain the role of the corporate trustee in a bond indenture. (page 262)
2. Explain a bond's maturity date and how it impacts bond retirements. (page 262)
3. Describe the main types of interest payment classifications. (page 263)
4. Describe zero-coupon bonds and explain the relationship between original-issue discount and reinvestment risk. (page 263)
5. Distinguish among the following security types relevant for corporate bonds: mortgage bonds, collateral trust bonds, equipment trust certificates, subordinated and convertible debenture bonds, and guaranteed bonds. (page 264)
6. Describe the mechanisms by which corporate bonds can be retired before maturity. (page 266)
7. Differentiate between credit default risk and credit spread risk. (page 267)
8. Describe event risk and explain what may cause it in corporate bonds. (page 268)
9. Define high-yield bonds, and describe types of high-yield bond issuers and some of the payment features unique to high yield bonds. (page 268)
10. Define and differentiate between an issuer default rate and a dollar default rate. (page 269)
11. Define recovery rates and describe the relationship between recovery rates and seniority. (page 270)

## 51. Mortgages and Mortgage-Backed Securities

1. Describe the various types of residential mortgage products. (page 275)
2. Calculate a fixed rate mortgage payment, and its principal and interest components. (page 278)
3. Describe the mortgage prepayment option and the factors that influence prepayments. (page 281)
4. Summarize the securitization process of mortgage backed securities (MBS), particularly formation of mortgage pools including specific pools and TBAs. (page 282)
5. Calculate weighted average coupon, weighted average maturity, and conditional prepayment rate (CPR) for a mortgage pool. (page 282)
6. Describe a dollar roll transaction and how to value a dollar roll. (page 287)
7. Explain prepayment modeling and its four components: refinancing, turnover, defaults, and curtailments. (page 290)
8. Describe the steps in valuing an MBS using Monte Carlo simulation. (page 292)
9. Define Option Adjusted Spread (OAS), and explain its challenges and its uses. (page 295)



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The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

# BANKS

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## Topic 31

### EXAM FOCUS

This topic introduces a number of concepts about banks that are developed more fully elsewhere in the FRM curriculum. For the exam, focus on understanding the major types of risk a bank faces and how they are addressed, both by banks themselves and by bank regulators. Be prepared to explain the differences between commercial banking and investment banking as well as the conflicts that exist in an organization that performs both of these services. Also, understand the distinctions between the lending and trading operations of a bank. Finally, be able to describe the implications of banks originating loans and distributing them to other parties.

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### TYPES OF BANKS

When we speak of “banks,” we include financial institutions that provide a variety of services. Banks can be categorized by the functions they perform and the customers they serve.

**Commercial banks** are those that take deposits and make loans. Commercial banks include **retail banks**, which primarily serve individuals and small businesses, and **wholesale banks**, which primarily serve corporate and institutional customers.

**Investment banks** are those that assist in raising capital for their customers (e.g., by managing the issuance of debt and equity securities) and advising them on corporate finance matters such as mergers and restructurings.

Whether a bank or bank holding company engages in both commercial banking and investment banking or must only do one or the other depends on the regulations where it does business.

### MAJOR RISKS FACED BY BANKS

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#### LO 31.1: Identify the major risks faced by a bank.

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The main risks faced by a bank include credit risk, market risk, and operational risk.

- **Credit risk** refers to the risk that borrowers may default on loans or that counterparties to contracts such as derivatives may default on their obligations. One measure of credit risk is a bank’s loan losses as a percentage of its assets.

- Market risk refers to the risk of losses from a bank's trading activities, such as declines in the value of securities the bank owns. Later in this topic, we will distinguish between the "trading book" and the "banking book" of a bank.
- Operational risk refers to the possibility of losses arising from external events or failures of a bank's internal controls. We will describe this risk in greater detail in Book 4, Topic 66.

Regulators in most jurisdictions require banks to hold adequate capital against these risks. Typically, they consider credit risk and operational risk with a time horizon of one year and market risk with a shorter time horizon.

## ECONOMIC CAPITAL VS. REGULATORY CAPITAL

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### LO 31.2: Distinguish between economic capital and regulatory capital.

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To mitigate the risk of bank failures caused by losses on loans or trading assets, banks must be funded by adequate sources of capital. Equity capital as a percentage of assets is a key measure of capital adequacy. Banks may also issue long-term debt to bolster their capital. This debt is subordinated to the claims of depositors if a bank faces financial distress.

Banks and their regulators may have different views about how much capital is sufficient in light of the risks a bank faces. **Regulatory capital** refers to the amount determined by bank regulators. In terms of bank regulation, equity is referred to as "Tier 1 capital" and subordinated long-term debt is referred to as "Tier 2 capital."



*Professor's Note: Regulations concerning bank capital, such as Basel I, Basel II, and Solvency II, are described in the FRM Part II curriculum.*

**Economic capital** refers to the amount of capital that a bank believes is adequate based on its own risk models. Even if economic capital is less than regulatory capital, as is often the case, a bank must maintain its capital at the regulatory minimum or greater.

## DEPOSIT INSURANCE AND MORAL HAZARD

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### LO 31.3: Explain how deposit insurance gives rise to a moral hazard problem.

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To increase public confidence in the banking system and prevent runs on banks, most countries have established systems of **deposit insurance**. Typically, a depositor's funds are guaranteed up to some maximum amount if a bank fails. These systems are funded by insurance premiums paid by banks.

Like other forms of insurance (as we will cover in the next topic on "Insurance Companies and Pension Plans"), deposit insurance brings an element of **moral hazard**. Moral hazard is the observed phenomenon that insured parties take greater risks than they would normally take if they were not insured. In the banking context, with deposit insurance in place, the moral hazard arises when depositors pay less attention to banks' financial health than they otherwise would. This allows banks to offer higher interest rates on deposits and make

higher-risk loans with the funds they attract. Losses on such loans contributed to increased bank failures in the United States in the 1980s and 2000s.

One way of mitigating moral hazard is by making insurance premiums risk-based. For example, in recent years, poorly-capitalized banks have been required to pay higher deposit insurance premiums than well-capitalized banks.

## INVESTMENT BANKING FINANCING ARRANGEMENTS

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### LO 31.4: Describe investment banking financing arrangements including private placement, public offering, best efforts, firm commitment, and Dutch auction approaches.

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When an investment bank arranges a securities issuance for a customer, it may try to place the entire issue with a particular buyer or group of buyers or sell the issue in the public market.

In a **private placement**, securities are sold directly to qualified investors with substantial wealth and investment knowledge. The investment bank earns fee income for arranging a private placement.

If the securities are sold to the investing public at large, the issuance is referred to as a **public offering**. Investment banks have two methods of assisting with a public offering. With a **firm commitment**, the investment bank agrees to purchase the entire issue at a price that is negotiated between the issuer and bank. The investment bank earns income by selling the issue to the public at a spread above the price it paid the issuer. An investment bank can also agree to distribute an issue on a **best efforts** basis rather than agreeing to purchase the whole issue. If only part of the issue can be sold, the bank is not obligated to buy the unsold portion. As with a private placement, the investment bank earns fee income for its services.

First-time issues of stock by firms whose shares are not currently publicly traded are called **initial public offerings** (IPOs). An investment bank can assist in determining an IPO price by analyzing the value of the issuer. An IPO price may also be discovered through a **Dutch auction** process. A Dutch auction begins with a price greater than what any bidder will pay, and this price is reduced until a bidder agrees to pay it. Each bidder may specify how many units they will purchase when accepting a price. The price continues to be reduced until bidders have accepted all the shares. The price at which the last of the shares can be sold becomes the price paid by all successful bidders.

## POTENTIAL CONFLICTS OF INTEREST

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**LO 31.5:** Describe the potential conflicts of interest among commercial banking, securities services, and investment banking divisions of a bank and recommend solutions to the conflict of interest problems.

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If a bank or a bank holding company provides commercial banking, investment banking, and securities services, several conflicts of interest may arise. For example, an investment banking division that is trying to sell newly issued stocks or bonds might want the securities division to sell these to their clients. The investment bankers may press the securities division's financial analysts to maintain "Buy" recommendations, or press its financial advisors to allocate these stocks and bonds to customer accounts. Such pressure may interfere with analysts' independence and objectivity or conflict with advisors' duties to clients.

Another clear conflict of interest among banking departments involves material nonpublic information. A commercial banking or investment banking division may acquire nonpublic information about a company when negotiating a loan or arranging a securities issuance. Other parts of the banking company, such as its trading desk, may benefit unfairly if they gain access to this information.

Because of these inherent conflicts, most bank regulators require some degree of separation among commercial banking, securities services, and investment banking. In some cases, they have prohibited firms from engaging in more than one of these activities, as was true in the United States when the Glass-Steagall Act was in force. Where banking firms are permitted to have commercial banking, securities, and investment banking units, the firms must implement Chinese walls, which are internal controls to prevent information from being shared among these units.

## BANKING BOOK VS. TRADING BOOK

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**LO 31.6:** Describe the distinctions between the "banking book" and the "trading book" of a bank.

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A bank's financial statements reflect accounting rules that apply to different aspects of its business. Revenue and income from its fee-based activities are recorded using the normal rules of accrual accounting, but other rules apply to its lending and trading activities.

The **banking book** refers to loans made, which are the primary assets of a commercial bank. Normally, the balance sheet value of a loan includes the principal amount to be repaid and accrued interest on the loan. However, for a **nonperforming loan** the value does not include accrued interest. A loan is typically classified as nonperforming if payments are more than 90 days overdue.

A bank will recognize a loss on a loan if it becomes likely that the borrower will not fully repay the principal. Bank financial statements reflect a reserve for loan losses that is determined by management, against which actual loan losses are charged. Increases or

decreases in the loan loss reserve are a potential tool for earnings manipulation, such as smoothing across business cycles, by a bank's management.

The **trading book** refers to assets and liabilities related to a bank's trading activities. Unlike other assets and liabilities, trading book items are marked to market daily. This is straightforward for items that trade in liquid markets and have readily available prices. For items that lack a liquid market, do not trade frequently, or are complex or custom instruments, marking to market involves estimating a price. Such items are sometimes said to be "marked to model."

## THE ORIGINATE-TO-DISTRIBUTE MODEL

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### LO 31.7: Explain the originate-to-distribute model of a bank and discuss its benefits and drawbacks.

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In contrast to a bank making loans and keeping them as assets, the **originate-to-distribute model** involves making loans and selling them to other parties. Many mortgage lenders in the United States operate on the originate-to-distribute model. Government agencies such as Ginnie Mae (GNMA), Fannie Mae (FNMA), and Freddie Mac (FHLMC) purchase mortgage loans from banks and issue securities backed by the cash flows from these mortgages.

The benefit of the originate-to-distribute model is that it increases liquidity in the sectors of the lending market where it is used. In addition to the residential mortgage market, this model has been applied in other areas such as student loans, credit card balances, and commercial loans and mortgages. For the banks that originate the loans, selling them to other parties is a way of freeing up capital with which they can meet regulatory requirements or make new loans.

A drawback of this model is that, in some cases, it has led banks to loosen lending standards. This was one of the factors that led to the credit crisis in the United States from 2007–2009.

## KEY CONCEPTS

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### LO 31.1

The major risks faced by a bank include the following:

- Credit risk from defaults on loans or by counterparties.
  - Market risk from declines in the value of trading book assets.
  - Operational risk from external events or failure of internal controls.
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### LO 31.2

Regulatory capital is the amount of capital that regulators require a bank to hold. This may include equity, or Tier 1 capital, and long-term subordinated debt, or Tier 2 capital.

Economic capital is the amount of capital a bank believes it needs to hold based on its own models. Regulatory capital is typically greater than economic capital.

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### LO 31.3

Deposit insurance exists to increase public trust in the banking system. However, it gives rise to moral hazard by decreasing the attention depositors pay to a bank's financial health and increasing the level of risk a bank is willing to take when its depositors are insured.

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### LO 31.4

In a private placement, securities are sold directly to qualified investors. In a public offering, securities are sold to the investing public.

When assisting a securities issuer on a best efforts basis, an investment bank sells as much of the issue to the public as it can. In a firm commitment, an investment bank buys an entire issue of securities from the issuer for one price and resells the securities to the public for a higher price. A Dutch auction process may be used to determine a price for an initial public offering.

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### LO 31.5

Within a firm that provides commercial banking, investment banking, and securities services, inherent conflicts of interest exist. Information may be acquired in a commercial banking or investment banking transaction that would give the other units an unfair advantage. An investment bank's task of selling newly issued stocks and bonds may conflict with a securities unit's duties to act in the best interests of its clients and recommend trading actions independently.

Bank regulators generally require commercial banking, investment banking, and securities activities to be kept separate, either by preventing firms from engaging in more than one of these activities or by requiring Chinese walls between these units of a bank.

**LO 31.6**

The banking book refers to loans made by a bank. The balance sheet value of a loan includes the principal amount to be repaid and accrued interest, unless the loan becomes nonperforming, in which case the value does not include accrued interest.

The trading book refers to assets and liabilities related to a bank's trading activities. Trading book items are marked to market daily based on actual market prices when they exist or on estimated prices when necessary.

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**LO 31.7**

The originate-to-distribute model involves banks making loans and selling them to other parties, many of which pool the loans and issue securities backed by their cash flows. This model frees up capital for the originating banks and may increase liquidity in sectors of the loan market. However, it has also led to decreased lending standards and lower credit quality of the loans sold.

## CONCEPT CHECKERS

1. The minimum level of capital a bank needs to maintain, according to its own estimates, models, and risk assessments, is best described as its:
  - A. equity capital.
  - B. financial capital.
  - C. economic capital.
  - D. regulatory capital.
2. Which of the following actions in the banking system is most likely intended to address the problem of moral hazard?
  - A. Deposit insurers charge risk-based premiums.
  - B. Banks increase loans to higher-risk borrowers.
  - C. Governments implement deposit insurance programs.
  - D. Banks increase the interest rates they offer to depositors.
3. An investment bank is most likely to earn a trading profit from buying and selling securities if it arranges a:
  - A. Dutch auction.
  - B. private placement.
  - C. best efforts offering.
  - D. firm commitment offering.
4. The purpose of a “Chinese wall” in banking is to:
  - A. prevent a bank failure from endangering other banks.
  - B. prevent a bank’s departments from sharing information.
  - C. restrict companies from offering both banking and securities services.
  - D. restrict companies from engaging in both commercial and investment banking.
5. A drawback of the originate-to-distribute banking model is that it has led to:
  - A. too little liquidity in certain sectors.
  - B. too much liquidity in certain sectors.
  - C. looser credit standards in certain sectors.
  - D. tighter credit standards in certain sectors.

## CONCEPT CHECKER ANSWERS

1. C Economic capital refers to a bank's own assessment of the minimum level of capital it needs to maintain. Economic capital is often less than regulatory capital, which is the minimum level a bank must maintain to comply with capital adequacy regulations.
2. A Charging risk-based premiums is a measure intended to address the problem of moral hazard, which exists when insured parties take greater risks than they would take in the absence of insurance.
3. D With a firm commitment offering, an investment bank buys an entire issue of securities from the issuer and attempts to sell them to the public at a higher price. In a private placement or a best efforts offering, an investment bank earns fee income rather than trading income. A Dutch auction is a method of price discovery for an initial public offering that does not involve buying and reselling shares.
4. B Chinese walls are internal controls to prevent a banking company's commercial banking, securities, and investment banking operations from sharing information.
5. C One drawback to the originate-to-distribute model is that it has led to looser credit standards in certain sectors, such as residential mortgages. A benefit of the model is that it has increased liquidity in certain sectors.

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The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

# INSURANCE COMPANIES AND PENSION PLANS

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Topic 32

## EXAM FOCUS

The focus of this topic is primarily on concepts related to life insurance and nonlife (property and casualty) insurance, such as moral hazard, adverse selection, mortality risk, and longevity risk. For the exam, be able to apply mortality tables to perform life expectancy computations and breakeven premium computations for life insurance companies. Also, be able to compute ratios relevant to property and casualty insurance companies. In addition, understand the risks facing insurance companies and be able to discuss specific ways to mitigate them.

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## CATEGORIES OF INSURANCE COMPANIES

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**LO 32.1: Describe the key features of the various categories of insurance companies and identify the risks facing insurance companies.**

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Insurance companies protect policyholders from specific loss events in exchange for the payment of periodic premiums. Three categories of insurance companies include life insurance, property and casualty (nonlife) insurance, and health insurance.

### Life Insurance

Life insurance companies usually provide long-term coverage and make a specified payment to the policyholder's beneficiaries upon the natural death (i.e., certain event) of the policyholder during the policy term. Coverage is also available for accidental death (i.e., uncertain event). **Term (temporary) life insurance** provides a specified amount of insurance coverage for a fixed period of time. No payments are made to the policyholder's beneficiaries if the policyholder survives the term of the policy; therefore, payment is not certain. **Whole (permanent) life insurance** provides a specified amount of insurance coverage for the life of the policyholder so payment will occur upon death, but there is uncertainty as to the timing. For both term and whole life insurance, it is most common for premiums and the amount of coverage to be fixed for the entire period in question.

In analyzing the relationship between the cost of one year of life insurance and whole life insurance premiums, assume a 30-year-old male purchases a \$2 million whole life policy with an annual premium of \$12,000. Based on mortality tables (as shown in LO 32.2), the probability of death within the year of a 30-year-old male is 0.001467, so the premium for one year of insurance should be \$2,934. The excess of \$9,066 is a surplus premium that is not required to cover the risk of a payout and is therefore invested by the insurance company for the policyholder. The process continues year after year while the cost of a one-

year policy increases as the policyholder ages. Later in the policyholder's life, the one-year policy cost will exceed the annual premium (\$12,000). From an overall perspective, the surplus in the earlier years is offset by the deficit in the later years.

## Property and Casualty (P&C) Insurance

P&C insurance companies usually provide annual and renewable coverage against loss events. The premiums may increase or decrease based on any changes in estimates of expected payout. **Property insurance** covers property losses such as fire and theft. Property insurers may be subject to catastrophic risks arising from many large claims due to natural disasters. Such risks could be managed using geographical, seismographical, and meteorological information to determine the probability and severity of catastrophic events. **Casualty (liability) insurance** covers third-party liability for injuries sustained while on a policyholder's premises or caused by the policyholder's use of a vehicle, for example. Liability insurance is subject to long-tail risk, which is the risk of legitimate claims being submitted years after the insurance coverage has ended. An example could include exposure to cancer-causing substances during the period of coverage but with the symptoms not occurring until years later.

Many property and casualty insurance companies insure a wide variety of risks, which in and of itself is a form of risk diversification. In addition, the expected payouts on claims can be estimated with a high degree of confidence if many policies are written on thousands of independent events. However, property damage claims from natural disasters and liability insurance claims are subject to fluctuating payouts and are very challenging to predict.

## Health Insurance

Health insurance companies provide coverage to policyholders for medical services that are not covered under a publicly funded health care system. Policyholders pay ongoing premiums and the insurance company will make payments for events such as necessary hospital treatment or prescription medication. Premiums may increase due to general increases in health care costs (similar to automobile insurance), but they typically will not increase due to the worsening of the policyholder's health (similar to life insurance). In some cases, insurance coverage may not be denied to individuals with pre-existing medical conditions. Some companies provide group health insurance plans through employers that cover both the employee and the employee's dependents.

## Risks Facing Insurance Companies

Major risks facing insurance companies include the following:

- *Insufficient funds to satisfy policyholders' claims.* The liability computations often provide a significant cushion, but it is always possible to have a sudden surge of payouts in a short period of time.
- *Poor return on investments.* Insurance companies often invest in fixed-income securities and if defaults suddenly increase, insurance companies will incur losses. Diversification of investments by industry sector and geography can help mitigate such losses.

- *Liquidity risk of investments.* Purchasing privately placed fixed-income securities, or publicly traded securities with a thinner market, may result in the inability to easily convert them to cash when most needed to satisfy a surge of claims.
- *Credit risk.* By transacting with banks and reinsurance companies, insurance companies face credit risk if the counterparty defaults on its obligations.
- *Operational risk.* Similar to banks, an insurance company faces losses due to failure of its systems and procedures or from external events outside the company's control (e.g., computer failure, human error).

## MORTALITY TABLES

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### LO 32.2: Describe the use of mortality tables and calculate the premium payment for a policy holder.

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An excerpt from mortality tables estimated by the U.S. Social Security Administration for 2013 is provided in Figure 1.

As an example, examine the row for a male aged 40. The second column indicates that the probability of a 40-year-old male dying within the next year is 0.002092 (or 0.2092%). The third column indicates that the probability of a male surviving to age 40 is 0.95908 (or 95.908%). The fourth column indicates that a 40-year-old male has a remaining life expectancy of 38.53 years so that, on average, he will live to age 78.53. The remaining three columns show the same estimates for a female and they appear slightly better than for a male.

Figure 1: Partial Mortality Table

Age (Years)	Male			Female		
	Probability of Death Within 1 Year	Survival Probability	Life Expectancy	Probability of Death Within 1 Year	Survival Probability	Life Expectancy
0	0.006519	1	76.28	0.005377	1	81.05
1	0.000462	0.99301	75.78	0.000379	0.99462	80.49
2	0.000291	0.99302	74.82	0.000221	0.99425	79.52
3	0.000209	0.99273	73.84	0.000162	0.99403	78.54
30	0.001467	0.97519	47.82	0.000664	0.98635	52.01
40	0.002092	0.95908	38.53	0.001287	0.97753	42.43
41	0.00224	0.95708	37.61	0.001393	0.97627	41.48
42	0.002418	0.95493	36.7	0.001517	0.97491	40.54
43	0.002629	0.95262	35.78	0.001662	0.97343	39.6
50	0.005038	0.9294	29.58	0.003182	0.95829	33.16
51	0.00552	0.92472	28.73	0.003473	0.95524	32.27
52	0.006036	0.91961	27.89	0.003767	0.95193	31.38
53	0.006587	0.91406	27.05	0.004058	0.94834	30.49
60	0.011197	0.86112	21.48	0.006545	0.91526	24.46
61	0.012009	0.85147	20.72	0.007034	0.90927	23.62
62	0.012867	0.84125	19.97	0.007607	0.90287	22.78
63	0.013772	0.83042	19.22	0.008281	0.896	21.95
70	0.023528	0.73461	14.24	0.015728	0.82864	16.43
71	0.025693	0.71732	13.57	0.017338	0.81561	15.68
72	0.028041	0.69889	12.92	0.019108	0.80147	14.95
73	0.030567	0.6793	12.27	0.021041	0.78616	14.23
80	0.059403	0.50629	8.2	0.043289	0.6388	9.64
90	0.167291	0.17735	4.03	0.132206	0.29104	4.8

Source: Social Security Administration, [www.ssa.gov/OACT/STATS/table4c6.html](http://www.ssa.gov/OACT/STATS/table4c6.html)

When examining the full table, the probability of death during the following year is a decreasing function of age until age 10 and then it increases. For an 80-year-old male, the probability of death within the next year is about 5.9% and increases to about 16.7% at age 90.

Some probabilities can be computed indirectly using other numbers in the table. For example, in the third column, the probability of a male surviving to age 70 is 0.73461 and the probability of the male surviving to age 71 is 0.71732. Therefore, the probability of death of a male between age 70 and 71 is  $0.73461 - 0.71732 = 0.01729$  (or about 1.73%). Given that a male reaches age 70, the probability of death within the following year is  $0.01729 / 0.73461 = 0.023536$  (or about 2.35%), which is consistent with the number in the second column.

Going further, the probability of the death of a 70-year-old male in the second year (between ages 71 and 72) is the probability that he does not die in the first year times the probability that he does die in the second year. Using the numbers in the second column, the probability is:  $(1 - 0.023528) \times 0.025693 = 0.025088$  (or about 2.51%).

With the information in the mortality tables, we can calculate the breakeven premium payment by equating the present value of the expected payout to the present value of the expected premium payments.

#### **Example: Breakeven premium payments**

The relevant interest rate for insurance contracts is 3% per annum (semiannual compounding applies), and all premiums are paid annually at the beginning of the year. A \$500,000 term insurance contract is being proposed for a 60-year-old male in average health. Assuming that payouts occur halfway throughout the year, calculate the insurance company's breakeven premium for a one-year term and a two-year term.

**Answer:**

*One-year term:*

The expected payout for a one-year term is  $0.011197 \times \$500,000 = \$5,598.50$ . Assuming the payout occurs in six months, the breakeven premium is:  $\$5,598.50 / 1.015 = \$5,515.76$ .

*Two-year term:*

The expected payout for a two-year term is the sum of the expected payouts in both the first year and the second year. The probability of death in the second year is  $(1 - 0.011197) \times 0.012009 = 0.011874$ , so the expected payout in the second year is  $0.011874 \times \$500,000 = \$5,937.27$ . If the payout occurs in 18 months, then the present value is  $\$5,937.27 / (1.015)^3 = \$5,677.91$ . The total present value of the payouts is then  $\$5,515.76 + \$5,677.91 = \$11,193.67$ .

The first premium payment occurs immediately (i.e., beginning of the first year) so it is certain to be received. However, the probability of the second premium payment being made at the beginning of the second year is the probability of not dying in the first year, which is  $1 - 0.011197 = 0.988803$ . The present value of the premium payments (using Y as the breakeven premium) =  $Y + (0.988803Y / 1.015^2) = 1.959793Y$ .

Computing the breakeven annual premium equates the present value of the payouts and the premium payments as follows:  $11,193.67 = 1.959793Y$ . Solving for Y, the breakeven annual premium is \$5,711.66.

## P&C INSURANCE RATIOS

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### LO 32.3: Calculate and interpret loss ratio, expense ratio, combined ratio, and operating ratio for a property-casualty insurance company.

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Property and casualty insurance companies compute the following ratios:

- The **loss ratio** for a given year is the percentage of payouts versus premiums generated, usually between 60–80% and increasing over time.
- The **expense ratio** for a given year is the percentage of expenses versus premiums generated, usually between 25–30% and decreasing over time. The largest expenses are usually loss adjustments (e.g., claims investigation and assessing payout amounts) and selling (e.g., broker commissions).
- The **combined ratio** for a given year is equal to the sum of the loss ratio and the expense ratio.
- The **combined ratio after dividends** for a given year is equal to the combined ratio plus the payment of dividends to policyholders as a percentage of premiums (if applicable).
- The **operating ratio** for a given year is the combined ratio (after dividends) less investment income as a percentage of premiums. The mismatch of the cash inflows (generally earlier) and outflows (generally later) for many insurance companies allows them to earn interest income. For example, policyholders tend to pay their premiums upfront at the beginning of the year, but insurance companies tend to pay out claims throughout the year or after year-end.

## MORAL HAZARD AND ADVERSE SELECTION

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### LO 32.4: Describe moral hazard and adverse selection risks facing insurance companies, provide examples of each, and describe how to overcome the problems.

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**Moral hazard** describes the risk to the insurance company that having insurance will lead the policyholder to act more recklessly than if the policyholder did not have insurance.

An example of moral hazard would be the existence of collision and liability coverage with automobile insurance. As a result of such coverage, some drivers would be willing to drive over the speed limits knowing that if an accident occurs, they would be covered for damage to the car and any resulting injury to a third party. Another example would be the existence of health insurance. As a result, some policyholders may request more health services than necessary.

Methods to mitigate against moral hazard include: deductibles (e.g., policyholder is responsible for a fixed amount of the loss), coinsurance provisions (e.g., insurance company will pay a fixed percentage of losses, less than 100%, over the deductible amount), and policy limits (e.g., fixed maximum payout).

**Adverse selection** describes the situation where an insurer is unable to differentiate between a good risk and a bad risk. By charging the same premiums to all policyholders, the insurer may end up insuring more bad risks (e.g., careless drivers, sick individuals).

Methods to mitigate against adverse selection include: (1) greater initial due diligence (e.g., mandatory physical examinations for life insurance, researching driving records for automobile insurance) and (2) ongoing due diligence (e.g., updating driving records and adjusting premiums to reflect changing risk).

## MORTALITY RISK VS. LONGEVITY RISK

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### LO 32.5: Distinguish between mortality risk and longevity risk and describe how to hedge these risks.

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**Mortality risk** refers to the risk of policyholders dying earlier than expected due to illness or disease, for example. From the perspective of the insurance company, the risk of losses increases due to the earlier-than-expected life insurance payout.

**Longevity risk** refers to the risk of policyholders living longer than expected due to better healthcare and healthier lifestyle choices, for example. From the perspective of the insurance company, the risk of losses increases due to the longer-than-expected annuity payout period.

## Hedging Mortality and Longevity Risks

There is a natural hedge (or offset) for insurance companies that deal with both life insurance products and annuity products. For example, longevity risk is bad for the annuity business but is good for the life insurance business due to the delayed payout (or no payout if the policyholder has term insurance and dies after the policy expires). Mortality risk is bad for the life insurance business but is good for the annuity business because of the earlier-than-expected termination of payouts.

To the extent that there is excessive net exposure to mortality risk, longevity risk, or both, an insurance company may consider **reinsurance contracts**. With this type of contract, the insurance company pays a fee to another insurance company to assume some or all of the risks that were originally insured.

Longevity derivatives are used to hedge longevity risk inherent in annuity contracts and defined benefit pensions. A good example would be a longevity bond (or a survivor bond) whereby the bond coupon is set to an amount that is linked to the number of people in a defined population group that are still alive.

## CAPITAL REQUIREMENTS FOR INSURANCE COMPANIES

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### LO 32.6: Evaluate the capital requirements for life insurance and property-casualty insurance companies.

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A life insurance company might have the following summarized balance sheet composition:

- *Assets*: investments (80%), other assets (20%)
- *Liabilities and Equity*: policy reserves (85%), subordinated long-term debt (5%), equity capital (10%)

Under an asset-liability management approach, the life insurance company attempts to equate asset duration with liability duration. There is risk associated with both sides of the balance sheet. On the asset side, corporate bonds comprise the bulk of the investments, so there is credit risk assumed. On the liability side, the policy reserves represent the present value of the future payouts as determined by actuaries. The risk is that the policy reserves are set too low if life insurance policyholders die too soon or annuity holders live too long. Equity capital represents contributed capital plus retained earnings and serves as a protection barrier if payouts are larger than loss reserves.

A P&C insurance company might have the following summarized balance sheet composition:

- *Assets*: investments (80%), other assets (20%)
- *Liabilities and Equity*: policy reserves (50%), unearned premiums (10%), subordinated long-term debt (5%), equity capital (35%)

On the asset side, the investments typically comprise of highly liquid bonds with shorter maturities than those used by life insurance companies. On the liability side, the unearned premiums represent prepaid insurance contracts whereby amounts are received but the coverage applies to future time periods; unearned premiums do not generally exist for life insurance companies. Finally, there is substantially more equity capital for a P&C insurance company than for a life insurance company. This is due to the highly unpredictable nature of claims (both timing and amount) for P&C insurance contracts.

## GUARANTY SYSTEM FOR INSURANCE COMPANIES

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### LO 32.7: Compare the guaranty system and the regulatory requirements for insurance companies with those for banks.

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In the United States, a **guaranty system** exists for both insurance companies and banks. Insurance companies are regulated at the state level while banks are regulated at the federal level.

For insurance companies, every insurer must be a member of the guaranty association in the state(s) in which it operates. If an insurance company becomes insolvent in a state, each of the other insurance companies must contribute an amount to the state guaranty fund based on the amount of premium income it earns in that state. The guaranty fund proceeds are distributed to the small policyholders of the insolvent company. In some cases, an annual limit may apply with regard to the contribution, which may contribute to a delay in accumulating sufficient funds to pay all of the policyholders. Most frequently, the policyholders of insolvent life insurance companies are transferred to other life insurance companies.

In contrast, the guaranty system for banks is a permanent fund to protect depositors and consists of amounts remitted by banks to the Federal Deposit Insurance Corporation (FDIC). No such permanent fund generally exists for insurance companies; therefore, insurance companies must make contributions whenever a default occurs.

## PENSION FUNDS

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**LO 32.8: Describe a defined benefit plan and a defined contribution plan for a pension fund and explain the differences between them.**

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Many companies establish pension plans on behalf of their employees with contributions being made by both parties. Upon retirement, the employee will receive periodic pension payments for the remainder of her life.

**Defined benefit plans** (i.e., employee benefit known, employer contribution unknown) explicitly state the amount of the pension that the employee will receive upon retirement. It is usually calculated as a fixed percentage times the number of years of employment times the annual salary for a specific period of time. There is significant risk borne by the employer because it is obligated to fund the benefit to the employee; therefore, when the present value of the pension obligation exceeds the market value of the pension assets, the employer must cover the deficiency. As a result, there is no risk borne by the employee (in theory). Additionally, some defined benefit plans may include one or more of the following features: (1) indexation of pension amounts to account for inflation, (2) continued pension payments (likely on a reduced basis) to the surviving spouse upon the death of a retired employee, or (3) a lump sum payment to an employee's dependents upon the death of a currently active employee.

**Defined contribution plans** (i.e., employer contribution known, employee benefit unknown) involve both employer and employee contributions being invested in one or more investment options selected by the employee. Upon retirement, the employee could opt to receive a lifetime pension (based on the ending value of the contributions) in the form of an annuity or, in some cases, simply to receive a lump sum. There is virtually no risk borne by the employer because it is obligated simply to make a set contribution and no more. The risk of underperformance of the plan's investments is borne solely by the employee.

A defined contribution plan involves one individual account associated with one employee. The individual pension is computed based only on the funds in that account. In contrast, a defined benefit plan involves one pooled account for all employees; all contributions go into and all payments come out of the one account.

## KEY CONCEPTS

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### LO 32.1

Three categories of insurance companies include life insurance, nonlife [property and casualty (P&C)] insurance, and health insurance. Life insurance companies usually provide long-term coverage and will make a specified payment to the policyholder's beneficiaries upon the death of the policyholder during the policy term. Term (temporary) life insurance provides a specified amount of insurance coverage for a fixed period of time. Whole (permanent) life insurance provides a specified amount of insurance coverage for the life of the policyholder.

Risks facing insurance companies include: (1) insufficient funds to satisfy policyholders' claims, (2) poor return on investments, (3) liquidity risk of investments, (4) credit risk, and (5) operational risk.

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### LO 32.2

Mortality tables can be used to compute life insurance premiums. Mortality tables include information related to the probability of an individual dying within the next year, the probability of an individual surviving to a specific age, and the remaining life expectancy of an individual of a specific age.

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### LO 32.3

P&C insurance companies compute the following ratios:

$$\text{loss ratio} + \text{expense ratio} = \text{combined ratio}$$

$$\text{combined ratio} + \text{dividends} = \text{combined ratio after dividends}$$

$$\text{combined ratio after dividends} - \text{investment income} = \text{operating ratio}$$

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### LO 32.4

Moral hazard describes the risk to the insurance company that having insurance will lead the policyholder to act more recklessly than if the policyholder did not have insurance. Methods to mitigate moral hazard include deductibles, coinsurance, and policy limits.

Adverse selection describes the situation where an insurer is unable to differentiate between a good risk and a bad risk. Methods to mitigate adverse selection include greater initial due diligence and ongoing due diligence.

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### LO 32.5

Mortality risk refers to the risk of policyholders dying earlier than expected. For the insurance company, the risk of losses increases due to the earlier-than-expected life

insurance payouts. Longevity risk refers to the risk of policyholders living longer than expected. For the insurance company, the risk of losses increases due to the longer-than-expected annuity payout period.

There is a natural hedge (or offset) for insurance companies that deal with both life insurance products and annuity products because longevity risk is bad for the annuity business but good for the life insurance business, and mortality risk is bad for the life insurance business but good for the annuity business. Other forms of hedging include reinsurance contracts with other insurance companies and longevity derivatives.

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#### LO 32.6

Under an asset-liability management approach, the life insurance company attempts to equate asset duration with liability duration. There is risk associated with both sides of the balance sheet. Equity capital represents contributed capital plus retained earnings and serves as a protection barrier if payouts are larger than loss reserves.

For P&C insurance companies, assets typically comprise of highly liquid bonds with shorter maturities than those used by life insurance companies. On the liability side, there are unearned premiums (non-existent with life insurance companies) that represent prepaid insurance contracts whereby amounts are received but the coverage applies to future time periods. Finally, there is substantially more equity capital than for a life insurance company because of the highly unpredictable nature of claims for P&C insurance contracts.

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#### LO 32.7

For insurance companies in the United States, every insurer must be a member of the guaranty association in the state(s) in which it operates. If an insurance company becomes insolvent in a state, then each of the other insurance companies must contribute an amount to the state guaranty fund based on the amount of premium income it earns in that state.

The guaranty system for banks in the United States is a permanent fund to protect depositors that consists of amounts remitted by banks to the Federal Deposit Insurance Corporation (FDIC). No such permanent fund exists for insurance companies.

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#### LO 32.8

Defined benefit plans explicitly state the amount of the pension that the employee will receive upon retirement. It is usually calculated as a fixed percentage times the number of years of employment times the annual salary for a specific period of time. There is significant risk borne by the employer because it is obligated to fund the benefit to the employee.

Defined contribution plans involve both employer and employee contributions being invested in one or more investment options selected by the employee. There is virtually no risk borne by the employer because it is obligated simply to make a set contribution and no more. The risk of underperformance of the plan's investments is borne solely by the employee.

## CONCEPT CHECKERS

1. Which of the following forms of insurance is most likely subject to long-tail risk?
  - A. Health insurance.
  - B. Life insurance.
  - C. Liability insurance.
  - D. Property insurance.
2. The relevant interest rate for insurance contracts is 2% per annum (semiannual compounding applies) and all premiums are paid annually at the beginning of the year. A \$2,000,000 term insurance contract is being proposed for a 40-year-old male in average health. Assume that payouts occur halfway throughout the year. Using the mortality rates estimated by the U.S. Social Security Administration (in Figure 1 on page 13), which of the following amounts is closest to the insurance company's breakeven premium for a two-year term?
  - A. \$4,246.
  - B. \$4,287.
  - C. \$4,332.
  - D. \$8,482.
3. The following information pertains to a property and casualty (P&C) insurance company:

Investment income	5%
Dividends	2%
Loss ratio	74%
Expense ratio	23%

Based on the information provided, what is this company's operating ratio?

- A. 90%.
  - B. 94%.
  - C. 97%.
  - D. 99%.
4. Which of the following problems would most likely be a concern for life insurance companies that are worried about differentiating between good risks and bad risks?
    - A. Adverse selection.
    - B. Catastrophic risk.
    - C. Longevity risk.
    - D. Moral hazard.
  5. Which of the following statements regarding the capital requirements and regulation of insurance companies is correct?
    - A. Insurance companies are regulated at both the state and federal level.
    - B. The guaranty system for insurance companies consists of a permanent fund created from premiums paid by insurers.
    - C. Unearned premiums can be found on the balance sheets of both life insurance and property and casualty insurance companies.
    - D. The amount of equity on the balance sheet of a life insurance company is typically lower than that of a property and casualty insurance company.

## CONCEPT CHECKER ANSWERS

1. C Liability insurance is subject to long-tail risk, which is the risk of legitimate claims being submitted years after the insurance coverage has ended. An example could include exposure to cancer-causing substances during the period of coverage but with the symptoms not occurring until years later.
2. B One-year term:

The expected payout for a one-year term is  $0.002092 \times \$2,000,000 = \$4,184$ . Assuming the payout occurs in six months, the breakeven premium is  $\$4,184 / 1.01 = \$4,142.57$ .

Two-year term:

The expected payout for a two-year term is the sum of the expected payouts in both the first year and the second year. The probability of death in the second year is  $(1 - 0.002092) \times 0.002224 = 0.0022353$ , so the expected payout in the second year is  $0.0022353 \times \$2,000,000 = \$4,470.63$ . If the payout occurs in 18 months, then the present value is  $\$4,470.63 / (1.01)^3 = \$4,339.15$ . The total present value of the payouts is then  $\$4,142.57 + \$4,339.15 = \$8,481.72$ .

The first premium payment occurs immediately (i.e., beginning of the first year) so it is certain to be received. However, the probability of the second premium payment being made at the beginning of the second year is the probability of not dying in the first year, which is  $1 - 0.002092 = 0.997908$ . The present value of the premium payments is as follows (using Y as the breakeven premium):  $Y + (0.997908Y / 1.01^2) = 1.978245Y$ .

Computing the breakeven annual premium equates the present value of the payouts and the premium payments as follows:  $8,481.72 = 1.978245Y$ . Solving for Y, the breakeven annual premium is \$4,287.50.

Response A (\$4,246) is not correct because it performs the computation on the assumption that all payouts occur at the end of the year instead of halfway throughout the year. Response C (\$4,332) is not correct because it did not apply any discounting (at the 1% semiannual rate). Response D (\$8,482) is not correct because it is simply the total present value of the payouts.

3. B The operating ratio is computed as follows:

$$\begin{aligned} &\text{loss ratio (74\%)} + \text{expense ratio (23\%)} + \text{dividends (2\%)} - \text{investment income (5\%)} \\ &= 94\% \end{aligned}$$

The combined ratio is computed as follows:

$$\text{loss ratio (74\%)} + \text{expense ratio (23\%)} = 97\%$$

The combined ratio after dividends is computed as follows:

$$\text{loss ratio (74\%)} + \text{expense ratio (23\%)} + \text{dividends (2\%)} = 99\%$$

4. A Adverse selection describes the situation where an insurer is unable to differentiate between a good risk and a bad risk. In the context of life insurance, by charging the same premiums to all policyholders (healthy and unhealthy individuals), the insurer may end up insuring more bad risks (e.g., unhealthy individuals). To mitigate adverse selection, a life insurance company might require physical examinations prior to providing coverage.
5. D Property and casualty insurance companies typically have a greater amount of equity than a life insurance company because of the highly unpredictable nature of P&C claims (both timing and amount).

Insurance companies are regulated at the state level only (and banks are regulated at the federal level only). The guaranty system for insurance companies is not a permanent fund; in contrast, banks have a permanent fund created from premiums paid by banks to the FDIC. On the liability side of a property and casualty insurance company's balance sheet, there are unearned premiums that represent prepaid insurance contracts whereby amounts are received but the coverage applies to future time periods. Unearned premiums do not exist with life insurance companies.

# MUTUAL FUNDS AND HEDGE FUNDS

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Topic 33

## EXAM FOCUS

Not every investor has the time or the skill to manage his own financial assets. For this reason, investors will sometimes hire a professional manager in the form of a mutual fund or perhaps a hedge fund. These pooled investment vehicles offer instant diversification and professional management to their investors. Mutual funds are often used by smaller investors while hedge funds are tools for wealthy individuals. Because hedge funds are limited only to those who can afford to lose their investment, they are subject to much less regulation. For the exam, be able to describe the various types of mutual funds and hedge funds along with their regulatory environments and typical fee structures.

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## TYPES OF MUTUAL FUNDS

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**LO 33.1: Differentiate among open-end mutual funds, closed-end mutual funds, and exchange-traded funds (ETFs).**

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Mutual funds are pooled investment vehicles that offer instant diversification for their investors. This diversification is very important because it spreads out risk to different sectors and asset classes. Most investors either do not have the time or the skill to properly diversify on their own. For this reason, investment vehicles like open-end mutual funds, closed-end mutual funds, and exchange-traded funds (ETFs) were created.

### Open-End Mutual Funds

Open-end mutual funds, which are often simply called *mutual funds*, are the most common pooled investment vehicle. Figure 1 shows the growth in open-end mutual fund assets since World War II. Essentially, investors are commingling their funds to be better diversified, to save on transaction fees, and to hire a professional management team. The professional management team will conduct research and ultimately invest commingled assets on behalf of their investors. These investors begin their investment by purchasing a set dollar amount of an open-end mutual fund and then they receive a proportional ownership interest (in the form of shares) in the mutual fund. This means that the number of shares goes up as new investors arrive and goes down as investors withdraw assets. When investors decide that they want to exit their investment in an open-end mutual fund, they can redeem their shares directly from the fund company, who will promptly send them either a check or a digital transfer of the value of their investment.

**Figure 1: Growth of Mutual Fund Assets**

<i>Year</i>	<i>Invested Assets (\$ billions)</i>
1940	0.5
1960	17.0
1980	134.8
2000	6,964.6
2015	15,652.0

Source: Investment Company Institute

At a high level, open-end mutual funds are broken down into four main categories: money market funds, equity funds, bond funds, and hybrid funds. Money market funds invest in short-term interest-bearing instruments, such as Treasury bills, commercial paper, and banker's acceptances. Money market investors are typically risk averse. This category is an alternative to interest-bearing bank accounts and is often the "cash" portion of an investor's asset allocation mix. Equity funds invest solely in stocks. Within this category you can find index funds that track a broad market index, such as the S&P 500 Index, funds that follow a certain style, such as medium company value funds, or sector funds, such as a health care sector fund. Bond funds invest only in fixed-income instruments, such as sovereign debt, corporate bonds, and asset-backed securities. Hybrid funds will blend stock and bond ownership into the same fund.

Open-end funds trade at the fund's **net asset value (NAV)**, which is essentially the sum of all assets owned minus any liabilities of the fund then divided by the shares outstanding. When investors decide they want to buy shares of an open-end mutual fund, they will transact at the next available NAV, which is not actually calculated until after the market closes at 4:00 pm in New York City. An investor who decides at 10:00 am that they want to buy shares will enter a buy order for a set dollar amount, but they will not know the price at which they will transact until after the market closes. For this reason, we say that open-end fund investors have poor price visibility. Since shares are transacted at an unknown price, investors cannot use stop orders or limit orders. They must place a market order to transact in shares of an open-end mutual fund.

Taxes are levied against open-end mutual fund investors as if they owned the diversified fund's holdings outright. If the underlying investment pays a dividend, then the investors must pay taxes on their proportional ownership interest in that dividend. The open-end fund may also buy and sell underlying investments and generate taxable short-term or long-term capital gains. These taxable events are also passed on to investors. Dividends and capital gains are distributed to investors typically toward the end of the calendar year, but they can be automatically reinvested in the fund to purchase more shares. Investors often choose reinvestment if they do not need the cash flow for current consumption.

The cost of investing is also a major consideration for any investment category. Open-end mutual funds have a management fee, an advertising surcharge (called a 12b-1 fee), and potentially a sales charge. The management fee covers the operational costs of the open-end mutual fund company, including the salaries of the management team. Management fees are typically around 1.0%, but they can be as high as 2.5–3.0% for international funds because they have increased complexity. The advertising surcharge is a stipend paid to the advisor who recommends the investment, and these fees can range from 0.0–1.0% with

the most common fee being 0.25%. Sales charges are commonly called *loads*. A **front-end load** is a set percentage that is charged to the investor when the asset is originally sold. Alternatively, some funds choose to charge a sales charge if an investor leaves a fund within a certain window of time. This is called a **back-end load**. Figure 2 shows the average cost of ownership for an open-end mutual fund per year over a five-year holding period.

**Figure 2: Average Total Cost of Ownership (% of Assets)**

Country	Bond Funds	Equity Funds
Australia	0.75	1.41
Austria	1.55	2.37
Belgium	1.60	2.27
Canada	1.84	3.00
Denmark	1.91	2.62
Finland	1.76	2.77
France	1.57	2.31
Germany	1.48	2.29
Italy	1.56	2.58
Luxembourg	1.62	2.43
Netherlands	1.73	2.46
Norway	1.77	2.67
Spain	1.58	2.70
Sweden	1.67	2.47
Switzerland	1.61	2.40
United Kingdom	1.73	2.48
United States	1.05	1.53
<i>Average</i>	<b>1.39</b>	<b>2.09</b>

Source: Khorana, Servaes, and Tufano, "Mutual Fund Fees Around the World," *Review of Financial Studies* 22 (March 2009): 1279–1310.

## Closed-End Mutual Funds

Closed-end mutual funds are a similar concept to open-end funds with a few notable differences. The first difference is that closed-end funds tend to invest in niche areas like specific emerging markets, while open-end mutual funds tend to invest in broader areas like a diversified emerging markets fund. Consider the difference between the China Fund (CHN), which is a closed-end fund that only invests in China, and the Vanguard Emerging Markets Index Fund (VEMAX), which is an open-end mutual fund with only 28.7% invested in China as of October 1, 2016.

The second difference is that a purchase of shares in an open-end mutual fund will increase the number of shares outstanding because new shares are created, but a closed-end fund's number of shares remains static. Investors who desire to purchase shares of a closed-end fund do not transact directly with the fund company but rather with other investors. Recall that investors who want to close their investment position in an open-end fund can simply redeem their shares from the fund company. This is where the fund gets the name "open-end."

The third difference is that closed-end fund investors cannot simply redeem their shares from the fund company. They must find another investor to buy their shares. This process is streamlined using a broker like Charles Schwab or Merrill Lynch.

The fourth difference is that, while open-end funds always transact at the next available NAV, a closed-end fund can transact at a price other than NAV. It is very common for a closed-end fund to trade at either a discount or a premium to its actual NAV. It then becomes important to know the historical norms for a closed-end fund's discount or premium before buying. For example, if a certain closed-end fund normally trades at a discount of 10% to its NAV but is now trading at a discount of only 3%, then it may still be overvalued and the investor would be wise to wait to make an investment.

*Professor's Note: In terms of trading, a closed-end fund behaves much like an individual stock. Investors can trade closed-end funds throughout the trading day, which means they have better price visibility and can utilize stop orders and limit orders if they so choose.*

## Exchange-Traded Funds

Exchange-traded funds (ETFs) represent an innovative twist on the open-end mutual fund. They enable instant diversification like an open-end fund, but they are exchange-traded, which means they trade throughout the day on the open market just like a closed-end fund does. Because they trade throughout the day, investors can utilize stop orders, limit orders, and even short selling in some cases.

A few ETFs also have call options and put options available. Unlike a closed-end fund, ETFs typically trade at their NAV. The vast majority of ETFs are passively managed index funds, although some new actively managed ETFs are beginning to come to market. One of the most widely known ETFs is the SPDR S&P 500 Index Fund (SPY).

Exchange-traded funds must disclose their holdings twice each day, which enables investors to have tremendous visibility into their underlying investments. Open-end mutual funds, on the other hand, disclose their holdings very infrequently, perhaps as delayed as once per quarter.

Another big difference is the management fees. Exchange-traded funds often have a considerably lower internal expense ratio, which means less of a hurdle for the investment to rise above. Lower fees equal higher potential after-fee returns for investors.

Because open-end funds, closed-end funds, and exchange-traded funds all solicit investment from small retail customers, they are subject to significant regulatory oversight. They are all regulated by the Securities and Exchange Commission (SEC) and must register with the SEC and provide a very detailed disclosure document, called a prospectus, to all investors prior to investing. The SEC also enforces the prevention of conflicts of interest, fraud, and excessive fees. Regulatory oversight theoretically helps protect investors and causes increased costs for the funds as they hire compliance specialists to ensure that all regulations are being followed.

## NET ASSET VALUE

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### LO 33.2: Calculate the net asset value (NAV) of an open-end mutual fund.

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In order to calculate the net asset value (NAV), the fund needs to know the current value of all investment holdings (including cash positions), any liabilities like management fees payable, and the total number of shares outstanding. Calculation of the NAV is shown as follows:

$$\text{NAV} = \frac{\text{fund assets} - \text{fund liabilities}}{\text{total shares outstanding}}$$

#### Example: Computing NAV

Consider an open-end mutual fund that owns \$1.1 billion in equities, \$350 million in bonds, and \$35 million in cash. They owe \$1.85 million in management fees payable at this point in the quarter and they have 39.635 million shares outstanding. Calculate this fund's NAV.

**Answer:**

$$\$37.42 = \frac{(\$1,100 + \$350 + \$35) - 1.85}{39.635}$$

Investors who wish to buy or sell this fund will transact at exactly \$37.42 per share, which is not calculated until after the market closes on the trading day in question. If they wanted to invest \$25,000, then they would buy exactly 668.092 (= \$25,000 / \$37.42) shares after the market closes on the relevant trading day.

Recall that the NAV for an open-end mutual fund is only calculated after the close of trading on any given day, while the NAV for closed-end funds and exchange-traded funds is calculated continuously throughout the day.

## HEDGE FUNDS VS. MUTUAL FUNDS

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### LO 33.3: Explain the key differences between hedge funds and mutual funds.

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Hedge funds and mutual funds share some common characteristics, but several nuances between them are very different. Both hedge funds and mutual funds offer professional management, instant diversification, and the ability to commingle funds with other investors. However, mutual funds are marketed to any and all investors, while hedge funds are restricted to only wealthy and sophisticated investors. Because of this, hedge funds

escape certain regulations that apply to mutual funds. Specifically, they do not need to provide the redemption of shares at any time the investor chooses, a daily calculated NAV, or the full disclosure of their investment policies and strategies. Hedge funds are also permitted to use leverage while mutual funds are not. Because hedge funds can use leverage and are also permitted to use both long and short investment strategies, they are considered to be an alternative investment class.



*Professor's Note: The term "hedge fund" implies that the fund is hedging some form of risk. This may be the case if the fund is using both long and short positions, but not all hedge funds focus on risk reduction. Some, like distressed debt funds, actually focus on risk enhancement.*

Hedge funds have become much more mainstream for institutional investors as of late. One attraction is that many hedge funds have registered in tax-favorable jurisdictions. For example, a little over 30% of all hedge funds are domiciled in the Cayman Islands. Institutional investors have been using hedge funds to invest in short-selling, convertible debt instruments, credit default swaps, distressed debt, non-investment grade bonds, and sometimes illiquid assets.

Since hedge funds are not required to redeem shares any time an investor requests, they have implemented advance notification requirements and lock-up periods for any withdrawal requests. The advance notification could mean that the investor must wait 90 days after requesting a withdrawal before they can expect to have access to their money. The **lock-up period** is a certain amount of time in which the investor is not able to withdraw his funds. This could be one year, two years, or some other customized time period. The advance notification time period and the lock-up period will be disclosed to investors before they invest. They exist for one key reason—many hedge fund investments are not very easy to unwind on short notice. Some hedge fund investments are illiquid, which means managers cannot sell them quickly and retain a proper value. In addition, some hedge fund investments are bets on certain asset mispricing, and those trades can take time to unwind.

## HEDGE FUND EXPECTED RETURNS AND FEE STRUCTURES

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**LO 33.4: Calculate the return on a hedge fund investment and explain the incentive fee structure of a hedge fund including the terms hurdle rate, high-water mark, and clawback.**

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While mutual funds charge fees as a set percentage of assets under management (AUM), hedge funds deploy a more complex compensation structure centered around incentive fees. These **incentive fees** are engineered to give hedge fund managers significant payouts based on their performance. The typical hedge fund fee structure is known as “**2 plus 20%**,” which means that they charge a flat 2% of all assets that they manage plus an additional 20% of all profits above a specified benchmark. This compares to a typical American open-end equity mutual fund that charges roughly 1.5% and an exchange-traded fund that typically charges less than 0.5%. This fee differential can prove very lucrative for hedge fund managers. In 2015, the top 25 hedge funds earned in excess of \$12 billion dollars from incentive fees.

Hedge funds do soften the incentive fee structure with a few safeguards for investors. The first safeguard is the **hurdle rate**, which is the benchmark that must be beaten before incentive fees can be charged. The hurdle rate could be zero (used for absolute return strategies), Treasuries plus a premium, LIBOR plus a premium, or some other custom benchmark. It is usually not the S&P 500 Index.

The second safeguard is a **high-water mark clause**, which essentially states that previous losses must first be recouped and hurdle rates surpassed before incentive fees once again apply. Consider a hedge fund that just began with \$100 million in assets from investors. Their hurdle rate is the 10-year Treasury, which is currently yielding 1.5%. In the first year of operation, this hedge fund made some bad decisions and ended up losing \$10 million (ending balance of \$90 million). This means that the managers get to charge the 2% flat fee, but no incentive fees apply. Incentive fees would only have applied to any profits earned above a 1.5% return, meaning that only an ending balance higher than \$101.5 million would have triggered the 20% incentive fee. In year two, this hedge fund would need to get its fund up above \$103 million (two years of beating Treasuries) in order for incentive fees to apply. In this case, the high-water mark for year one is \$101.5 million, and for year two it is \$103 million.

The third safeguard for investors is a **clawback clause**, which enables investors to retain a portion of previously paid incentive fees in an escrow account that is used to offset investment losses should they occur.

The incentive fee structure of a hedge fund certainly encourages hedge fund managers to reach for profits, but this comes at the expense of also encouraging them to take risks. A hedge fund manager essentially owns a call option against the assets of the hedge fund and payoff for options are higher if volatility is higher. Consider an example where a hedge fund manager is presented with an opportunity that offers a 40% probability of returning 50% and a 60% probability of losing 50%. The expected return of the fund can be calculated as follows:

$$(0.4 \times 50\%) + (0.6 \times -50\%) = -10\%$$

In this example, the hedge fund manager might be willing to take a big risk (60% probability) of losing money, which would end in him only collecting his 2% flat fee. The alternative is that if he were to end up making a huge return with the lower probability event, then he would potentially earn a substantial incentive fee. If this hedge fund generates a 50% profit, then he could potentially earn fees of 11.6% [= 2% (flat fee) + 0.20 × 48% (incentive fee on return above the 2% flat fee)]. The expected payoff for fees then becomes 5.84%:

$$(0.4 \times 11.6\%) + (0.6 \times 2\%) = 5.84\%$$

From the investor's perspective, the expected payoff is -15.84%:

$$[0.4 \times (50\% - 11.6\%)] + [0.6 \times (-50\% - 2\%)] = 0.1536 - 0.312 = -15.84\%$$

The expected return for the hedge fund is 5.84% and the expected return for the hedge fund investor is -15.84%. When these two numbers are added together, we arrive back at the original return of -10%. This shows the disproportionate payoff for the hedge fund manager. Why would investors be willing to make this investment? Clearly they are hoping that the incentive fees will motivate the hedge fund manager to do everything within their power to produce significant returns for both the investor and the hedge fund manager.

## HEDGE FUND STRATEGIES

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**LO 33.5: Describe various hedge fund strategies, including long/short equity, dedicated short, distressed securities, merger arbitrage, convertible arbitrage, fixed income arbitrage, emerging markets, global macro, and managed futures, and identify the risks faced by hedge funds.**

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Hedge funds deploy numerous different strategies in their attempt to capture incentive fees. Not all hedge funds fall easily into a specific category, but the discussion in this section follows the classification system used by the Dow Jones, which provides indices to track various hedge fund strategies. Throughout this section, you will see the term **arbitrage**, which (in the hedge fund context) involves short selling an asset that is believed to be overvalued and buying an asset that is believed to be undervalued in an attempt to exploit a pricing differential.

### Long/Short Equity

Long/short equity hedge funds endeavor to find mispriced securities. Managers of a long/short equity fund spend a great deal of time conducting fundamental analysis on stocks, that are largely ignored by most analysts, in an attempt to find mispricings. They will buy (go long) a stock that they believe to be undervalued, and they will short sell (go short) a stock that they believe to be overvalued. Sometimes funds can have a net long bias or a net short bias depending on what opportunities they see in the markets. Funds can also be sector neutral, where they net long and short positions that cancel out sector exposure. *Market neutral funds* are where long and short positions make the fund ambivalent to market direction, and *factor neutral funds* are where positions are isolated from a specific factor like oil or interest rate policy.

### Dedicated Short

Dedicated short hedge funds are focused exclusively on finding a company that they think is overvalued and then short selling the stock. Traditionally, short sellers are looking for companies with weak financials, those that switch auditors frequently, those that delay SEC filings, those in industries with overcapacity, or those engaged in lawsuits that could go horribly wrong.

### Distressed Securities

Bonds with a credit rating of BB or less are considered to be “junk” bonds, while those with a CCC rating are considered to be “distressed.” Distressed bonds usually trade at deep discounts to par value and often offer yields upwards of 10% greater than a comparable

Treasury. Of course, an investment in a distressed bond could prove worthless if the wrong events happen. Distressed securities hedge funds are searching for distressed bonds with the potential to turn things around. Many of these distressed companies are in or close to being in bankruptcy proceedings. Some distressed bond investors passively wait for the investment to turn around, while others take an active approach to influencing the target company's reorganization. Distressed bond investors do their homework to figure out if they can gain an advantage by buying specific debt tranches. If they own more than one-third of any class of a bond, then they can block any reorganization plan that is not in their best interest. There is tremendous profit to be made in this area for investors who know what they are doing.

### Merger Arbitrage

Merger arbitrage hedge funds try to find arbitrage opportunities after mergers are announced. These are primarily positive deals where the managers are planning on the deal going through. There are two different types of mergers: *cash deals* and *stock deals*.

Consider an all-cash deal where company A announces that it will buy company B at \$50 per share. Pre-announcement, company B was trading at \$37.50 and post-announcement company B will typically be trading somewhere near \$48. Why not at \$50, which is where the deal was announced? It could be because some market participants are slow to adjust to the new information, but it could also be that there is some cushion left in company B's price on the chance that the deal falls through. Either way, a merger arbitrage fund would buy the shares of company B and wait for the full \$50 (or better) price to be achieved.

Now consider an all-stock deal where company A offers one share of its stock for every four shares of company B's stock. This could be a realistic ratio if company B had a considerably lower market capitalization than company A. In this case, a merger arbitrage fund would buy a certain amount of company B's shares and, at the same time, they would short sell one-quarter of this number of shares in company A's stock. This is because the acquirer usually pays too much and their stock usually goes down after a merger. The merger arbitrage strategy is a very lucrative strategy, but there is also great potential for insider trading issues. The SEC will actively pursue insider trading violations, so these managers must make certain that they are only factoring public information into their decision-making process.

### Convertible Arbitrage

Some hedge funds invest using convertible bonds, which are fixed-income instruments that can be converted into shares of stock if the stock price rises above a pre-specified value. If convertible bonds are not converted into shares of stock, then they simply retain their bond status and continue to offer interest payments and a certain principal repayment at maturity. This debt instrument conceptually merges a bond with a call option on the stock. Sometimes, if the convertible bond is also callable, the issuer will announce its intention to call the bond in order to force convertible bondholders to convert to stock. A conversion into stock will shift the investor from being a debtholder to an equity holder and will therefore reduce the debt burden of the issue without them actually repaying any debt. A convertible arbitrage hedge fund develops a sophisticated model to value convertible bonds that factors everything from default risk to interest rate risk. Sometimes they offset

investment risk by shorting the issuer's stock or by using more sophisticated assets like credit default swaps and interest rate swaps.

### Fixed Income Arbitrage

Fixed income arbitrage hedge funds attempt to exploit perceived mispricings in the realm of fixed-income securities. Some hedge funds try to find arbitrage opportunities in estimating shifts in the Treasury yield curve. Others look for overvalued and undervalued positions with U.S. Treasuries, with other country's sovereign debt obligations, or with credit default swap rates. However, this strategy is risky business. Recall that Long-Term Capital Management (LTCM) realized 40% plus returns using this strategy in the 1990s only to have their trades move the wrong way and cause such panic in the broader markets that several Wall Street banks had to bail them out. The LTCM scenario turned from a profitable fixed income arbitrage opportunity to a significant systemic risk overnight. Some have compared fixed income arbitrage to picking up nickels in front of a steamroller.

### Emerging Market

Emerging market hedge funds focus on investments in developing countries. These managers often expend great effort to research their investments by visiting potential investment targets, attending conferences, meeting with analysts, talking directly with management, and possibly hiring consultants with local knowledge. Some hedge funds choose to invest in developing country securities in their local market while others invest using *American depository receipts* (ADRs), which are certificates issued in America that provide ownership in foreign countries coupled with currency exposure. There are occasionally pricing discrepancies between the ADR and the underlying asset that an adept hedge fund manager can exploit as well. If managers decide to invest using emerging market debt, then they need to consider default risk because several countries have defaulted multiple times, including Russia, Argentina, Brazil, and Venezuela.

### Global Macro

Several of the most financially successful hedge fund managers have made their fortunes with a global macro hedge fund strategy. In this strategy, hedge fund managers attempt to profit from a global macroeconomic trend that they feel is not in equilibrium (priced correctly and rationally). They will place very large dollar bets on the equilibrium being reestablished. Typically, the investment focus of global macro funds is either on foreign exchange rates or on interest rates. The biggest challenge for these funds is that there is no way to know for certain when a perceived deviation from equilibrium will be corrected. There is a saying that the markets can stay irrational (out of equilibrium) longer than most investors can stay solvent. In other words, a deviation from equilibrium could take a long time to correct itself and some hedge funds will not be able to wait out the trend.

### Managed Futures

Managed futures hedge funds attempt to predict future movements in commodity prices based on either technical analysis or fundamental analysis. Technical analysis attempts to infer patterns from past price movements and use those patterns as a basis for predictions.

When technical analysis is used, fund managers will backtest their trading rules using historical data. Fundamental analysis studies economic, political, and other relevant measurable factors to determine a valuation for the given commodity and then buy or short sell based on the outcome of this fundamental research.

## HEDGE FUND PERFORMANCE AND MEASUREMENT BIAS

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### LO 33.6: Describe hedge fund performance and explain the effect of measurement biases on performance measurement.

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Hedge fund performance is not as easy to assess as mutual fund performance, which is readily available and accurately reported by numerous independent parties. Participation in hedge fund indices is voluntary. If the fund had good performance, then they will report their results to the index vendor. If they did not have good results, then they simply do not report their results to the index. In the Barclay's Hedge Fund Index, the data for August 2016 had 2,914 funds reporting information, while September 2016 only had 617. This is known as the **measurement bias** of hedge fund index reporting. When returns are reported by a hedge fund, the database is then backfilled with the fund's previous returns. This is known as **backfill bias** and it creates an issue with reliability for hedge fund benchmarks. It is very common for a hedge fund to have a string of several good years and then have a meltdown. LTCM, for example, reported returns (before fees) of 28%, 59%, 57%, and 17% in 1994, 1995, 1996, and 1997, respectively. However, in 1998, the fund lost virtually everything.

Prior to 2008, most hedge funds performed very well. However, after the financial crisis of 2007–2009, hedge funds have underperformed relative to the S&P 500. Figure 3 shows historical performance for the Barclay's Hedge Fund Index from 2006–2015 relative to the S&P 500 Index.

**Figure 3: Hedge Fund Historical Performance**

	<i>Barclay's Hedge Fund Index</i>	<i>S&amp;P 500 Index</i>
2006	12.39%	15.61%
2007	10.22%	5.48%
2008	-21.63%	-36.55%
2009	23.74%	25.94%
2010	10.88%	14.82%
2011	-5.48%	2.10%
2012	8.25%	15.89%
2013	11.12%	32.15%
2014	2.88%	13.52%
2015	0.04%	1.36%

Source: Barclay<sup>1</sup> and the Stern School of Business at NYU<sup>2</sup>

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1. [http://www.barclayhedge.com/research/indices/ghs/Hedge\\_Fund\\_Index.html](http://www.barclayhedge.com/research/indices/ghs/Hedge_Fund_Index.html).
  2. [http://pages.stern.nyu.edu/~adamodar/New\\_Home\\_Page/datafile/histretSP.html](http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/histretSP.html).

In Figure 3, we see that hedge funds outperformed the S&P 500 in two out of ten years from 2006–2015. One of the years of outperformance was during a significant negative year for the S&P 500 Index. This is one reason why hedge funds remain attractive diversifiers for pension funds and other interested investors. Protection during periods of stock market volatility is one hallmark of hedge funds as they actively pursue short selling when traditional mutual funds are not allowed to do so.

## KEY CONCEPTS

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### LO 33.1

There are three primary types of commingled pools of investments that are available to investors. They are open-end mutual funds, closed-end mutual funds, and exchange-traded funds (ETFs). Open-end funds transact at the next available net asset value (NAV), which occurs after the market has closed for the day. Shares may be redeemed directly from the fund company with an open-end fund. Closed-end funds transact throughout the trading day, but shares cannot be redeemed at the fund company and their price may differ substantially from their NAV—the shares must be bought or sold by other investors. Exchange-traded funds also trade throughout the day, but their shares do trade at the NAV. ETFs usually have the lowest internal fees, which is a big component of investment returns.

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### LO 33.2

The NAV is easily calculated as the total invested assets of the fund minus any liabilities (typically management fees payable) all divided by the total shares outstanding. The NAV for an open-end fund is set after the trading day is over, while the NAV for a closed-end fund and an exchange-traded fund is calculated continuously throughout the trading day. The NAV is used to determine the number of shares purchased or sold in a fund.

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### LO 33.3

Both mutual funds and hedge funds offer professional management, instant diversification, and the ability to commingle funds with other investors. However, there are some notable differences between mutual funds and hedge funds. Hedge funds are only marketed to wealthy and sophisticated investors. Because of this, hedge funds escape certain regulatory oversight, which enables them to avoid allowing investors to redeem shares at any time they want, calculating the NAV daily, and disclosing investment policies and strategies. They are also permitted to use leverage and short selling, which are commonly not permitted for mutual funds. In addition, hedge funds use lock-up periods to prevent investor withdrawals at the wrong time for the fund.

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### LO 33.4

Hedge funds commonly deploy a 2% and 20% incentive fee structure, where they earn management fees for investment results relative to a given hurdle rate. Investors are partially protected with the use of high-water marks and clawback clauses.

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### LO 33.5

There are many different types of hedge fund strategies. They all search for perceived mispricings in different corners of the markets and then try to exploit them for profit.

Long/short equity funds take both long and short positions in the equity markets, diversifying or hedging across sectors, regions, or market capitalizations, and have directional exposure to the overall market.

Dedicated short funds tend to take net short positions in equities, and their returns are negatively correlated with equities.

Distressed hedge funds invest across the capital structure of firms that are under financial or operational distress or are in the middle of bankruptcy. These hedge fund managers try to profit from an issuer's ability to improve its operation or come out of a bankruptcy successfully.

Merger arbitrage funds bet on spreads related to proposed merger and acquisition transactions.

Convertible arbitrage funds attempt to profit from the purchase of convertible securities and the shorting of corresponding stock.

Fixed income arbitrage funds try to obtain profits by exploiting inefficiencies and price anomalies between related fixed income securities.

Emerging market funds invest in currencies, debt, equities, and other instruments in countries with emerging or developing markets.

Global macro managers make large bets on directional movements in interest rates, exchange rates, commodities, and stock indices and do better during extreme moves in the currency markets.

Managed futures funds attempt to predict future movements in commodity prices based on either technical analysis or fundamental analysis.

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### LO 33.6

Hedge fund benchmarks are problematic due to measurement bias and backfill bias. Over the last ten years, reported hedge fund performance suggests that they have only beaten the S&P 500 Index in two of those years.

## CONCEPT CHECKERS

1. Which of the following statements is not correct regarding investment funds available to all investors?
  - A. Open-end mutual funds always transact at the next available net asset value.
  - B. Stop orders can be used on closed-end funds.
  - C. Open-end mutual funds can be purchased with a limit order.
  - D. Short selling is available for some exchange-traded funds.
2. Which of the follow characteristics is a key differentiator between mutual funds and hedge funds?
  - A. Professional asset management.
  - B. Immediate access to withdrawals from the fund.
  - C. Charging a fee for providing investment services.
  - D. Easy diversification for an investor.
3. What is the expected return to a hedge fund if the fund uses a standard 2 and 20 incentive fee structure with an investment that has a 35% probability of making 55% and a 65% probability of losing 45%?
  - A. 5.71%.
  - B. 6.12%.
  - C. 3.78%.
  - D. 5.28%.
4. Which type of hedge fund focuses on isolating mispricings in foreign exchange markets?
  - A. Fixed income arbitrage hedge funds.
  - B. Global macro hedge funds.
  - C. Managed futures hedge funds.
  - D. Convertible arbitrage hedge funds.
5. Which of the following statements is/are most accurate regarding hedge fund performance reporting?
  - I. When a hedge fund's performance is recorded in an index, all of its prior results are also included.
  - II. Hedge funds are permitted to self-select if their performance is reported in index averages.
  - A. I only.
  - B. II only.
  - C. Both I and II.
  - D. Neither I nor II.

## CONCEPT CHECKER ANSWERS

1. C Open-end mutual funds have very low price transparency because they trade at the next available NAV, which is not calculated until after the market closes. As such, they can only be bought or sold using a market order. Closed-end funds can be bought or sold using stop orders and limit orders. In some cases, ETFs can be sold short.
2. B Mutual funds must offer immediate access to withdrawals from their fund. This is an SEC requirement. Hedge funds have advance notification and lock-up periods, which prevent immediate access to withdrawals from the fund.
3. A The hedge fund could potentially earn fees of 12.6% [2% (flat fee) +  $0.20 \times 53\%$  (incentive fee on return above the 2% flat fee)]. The expected payoff for fees then becomes 5.71% computed as follows:

$$(0.35 \times 12.6\%) + (0.65 \times 2\%) = 5.71\%$$

4. B Global macro funds focus on finding mispricings at the level of the global macro economy. They materialize in foreign exchange pricing and interest rates. Fixed income arbitrage funds focus on various mispricings with fixed-income securities. Managed futures funds focus on forecasting commodity prices. Convertible arbitrage funds focus on valuing convertible bonds.
5. C Statement I describes backfill bias and Statement II describes measurement bias. Backfill bias arises when the database is backfilled with the fund's previous returns. Measurement bias indicates that not all hedge funds report their performance to index providers.

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The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

# INTRODUCTION (OPTIONS, FUTURES, AND OTHER DERIVATIVES)

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Topic 34

## EXAM FOCUS

In this topic, we present the basic concepts of derivative securities and derivative markets. For the exam, know the basic derivative terms as well as the terms related to derivative markets. Also, be able to compute payoffs for the different derivative securities. Finally, be able to create a hedge and know how to take advantage of an arbitrage situation.

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## COMMON TERMS RELATED TO DERIVATIVES

The following section discusses common terms associated with derivatives. Understanding these concepts will be helpful going forward as you progress through the derivatives material.

A **derivative security** is a financial security (e.g., options) whose value is derived in part from another security's characteristics or value. This other security is referred to as the **underlying asset**. A derivative effectively "derives" its price from some other variable.

A **market maker** is the individual that "makes a market" in a security. The market maker maintains bid and offer prices in a given security and stands ready to buy or sell lots of said security at publicly quoted prices.

A **spot contract** is an agreement to buy or sell an asset *today*. A **forward contract** specifies the price and quantity of an asset to be delivered on or before a future pre-specified date. A **futures contract** is a legally binding agreement to buy or sell a commodity or financial instrument in a designated future month at a previously agreed upon price by the buyer and seller.

A **call option** gives its holder the right, but not the obligation, to buy a specified number of shares of the underlying security at the given strike price, on or before the option contract's expiration date. A **put option** gives the investor the right, but not the obligation, to sell a fixed number of shares at a fixed price within a given pre-specified time period. An investor may wish to have the option to sell shares of a stock at a certain price and time in order to hedge an existing investment.

An American-styled option contract can be exercised any time between issue date and expiration date. In contrast, a European-styled option contract may be exercised only on the actual expiration date. **American options** will be worth more than **European options** when the right to early exercise is valuable, and they will have equal value when it is not.

A **long position** refers to actually owning the security, while a **short position** is when a person sells a security he does not own. An investor taking a short position anticipates a drop in the price of the security.

The exercise, or **strike price**, is the price at which the security underlying an options contract may be bought or sold.

**Expiration date** is the last date on which an option may be exercised.

The **bid price** is the “quoted bid,” or the highest price a dealer is willing to pay to purchase a security. This is essentially the available price at which an investor can sell shares of stock. The **offer price** is the price at which the security is offered for sale, also known as the “asking price.” The **bid-ask spread** is the difference between the ask (i.e., offer) price and the bid price.

## DERIVATIVE MARKETS

An **open outcry system** and **electronic trading system** are different forms of trading securities (matching buyers with sellers). The open outcry system (e.g., CBOT) is the more traditional system, which involves traders actually indicating their trades through hand signals and shouting. Electronic trading does not involve an actual “physical” exchange location, but rather involves matching buyers and sellers electronically via computers (e.g., NASDAQ).

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### LO 34.1: Describe the over-the-counter market, distinguish it from trading on an exchange, and evaluate its advantages and disadvantages.

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An **over-the-counter (OTC) market** differs from a traditional exchange. It is a customized trading market which utilizes telephone and computers to make trades. This market typically involves much larger trades than traditional exchanges. The most typical OTC trade is conducted over the phone. Since terms are not specified by an “exchange,” participants have more flexibility to negotiate the most mutually agreeable or attractive trade.

The OTC market is several times the size of the traditional exchange market. For example, in 2007, the OTC market was over \$500 trillion, while the exchange-traded market was under \$100 trillion.

Advantages of over-the-counter trading:

- Terms are not set by any exchange.
- Participants have flexibility to negotiate.
- In the event of a misunderstanding, calls are recorded.

Disadvantages of over-the-counter trading:

- OTC trading has more credit risk than exchange trading. Exchanges are organized in such a way that credit risk is eliminated.

## BASICS OF DERIVATIVE SECURITIES

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### LO 34.2: Differentiate between options, forwards, and futures contracts.

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An **option contract** is a contract that, in exchange for the option price, gives the option buyer the right, but not the obligation, to buy (sell) an asset at the exercise price from (to) the option seller within a specified time period, or depending on the type of option, a precise date (i.e., expiration date). A call option gives the option holder the right to purchase the underlying asset by a certain specified date for a specified (in advance) price. A put option gives the option holder the right to sell the underlying asset by a selected date for a pre-selected price.

A **forward contract** is a contract that specifies the price and quantity of an asset to be delivered sometime in the future. There is no standardization for forward contracts, and these contracts are traded in the over-the-counter market. One party takes the long position, agreeing to purchase the underlying asset at a future date for a specified price, while the other party is the short, agreeing to sell the asset on that same date for that same price. Forward contracts are often used in foreign exchange situations as these contracts can be used to hedge foreign currency risk.

A **futures contract** is a more formalized, legally binding agreement to buy/sell a commodity/financial instrument in a pre-designated month in the future, at a price agreed upon today by the buyer/seller. Futures contracts are highly standardized regarding quality, quantity, delivery time, and location for each specific commodity. These contracts are typically traded on an exchange.



*Professor's Note: Remember that a futures contract is an obligation/promise to actually complete a transaction, while an option is simply the right to buy/sell.*

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### LO 34.3: Identify and calculate option and forward contract payoffs.

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#### Call Option Payoff

The payoff on a **call option** to the option buyer is calculated as follows:

$$C_T = \max (0, S_T - X)$$

where:

$C_T$  = payoff on call option

$S_T$  = stock price at maturity

$X$  = strike price of option

The payoff to the option seller is  $-C_T$  [i.e.,  $-\max (0, S_T - X)$ ]. We should note that  $\max (0, S_t - X)$ , where time,  $t$ , is between 0 and  $T$ , is also the payoff if the owner decides to exercise the call option early (in the case of an American option as we will discuss later).

The price paid for the call option,  $C_0$ , is referred to as the **call premium**. Thus, the profit to the option buyer is calculated as follows:

$$\text{profit} = C_T - C_0$$

where:

$C_T$  = payoff on call option

$C_0$  = call premium

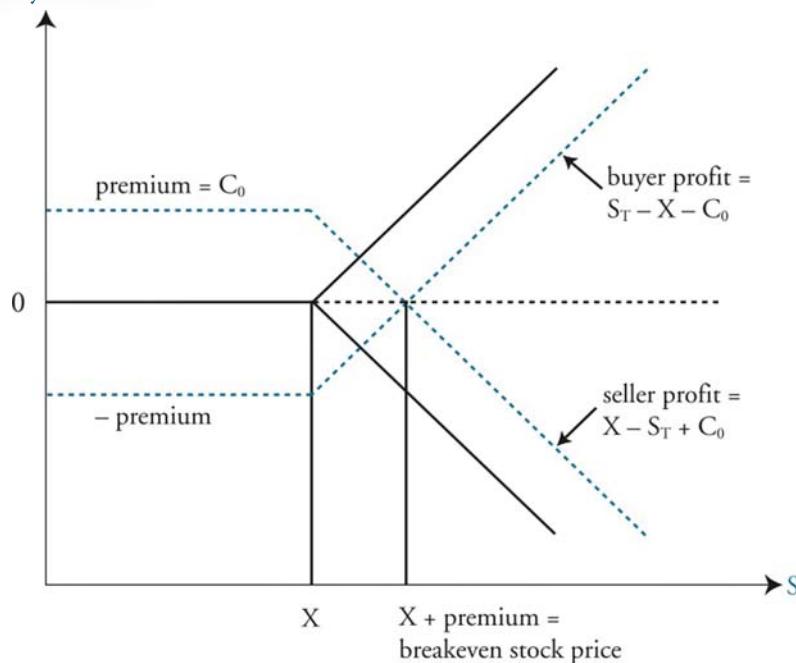
Conversely, the profit to the option seller is:

$$\text{profit} = C_0 - C_T$$

Figure 1 depicts the payoff and profit for the buyer and seller of a call option.

**Figure 1: Profit Diagram for a Call at Expiration**

#### Call Payoff/Profit



## Put Option Payoff

The payoff on a **put option** is calculated as follows:

$$P_T = \max(0, X - S_T)$$

where:

$P_T$  = payoff on put option

$S_T$  = stock price at maturity

$X$  = strike price of option

The payoff to the option seller is  $-P_T$  [i.e.,  $-\max(0, X - S_T)$ ]. We should note that  $\max(0, X - S_t)$ , where  $0 < t < T$ , is also the payoff if the owner decides to exercise the put option early.

The price paid for the put option,  $P_0$ , is referred to as the **put premium**. Thus, the profit to the option buyer is calculated as follows:

$$\text{profit} = P_T - P_0$$

where:

$P_T$  = payoff on put option

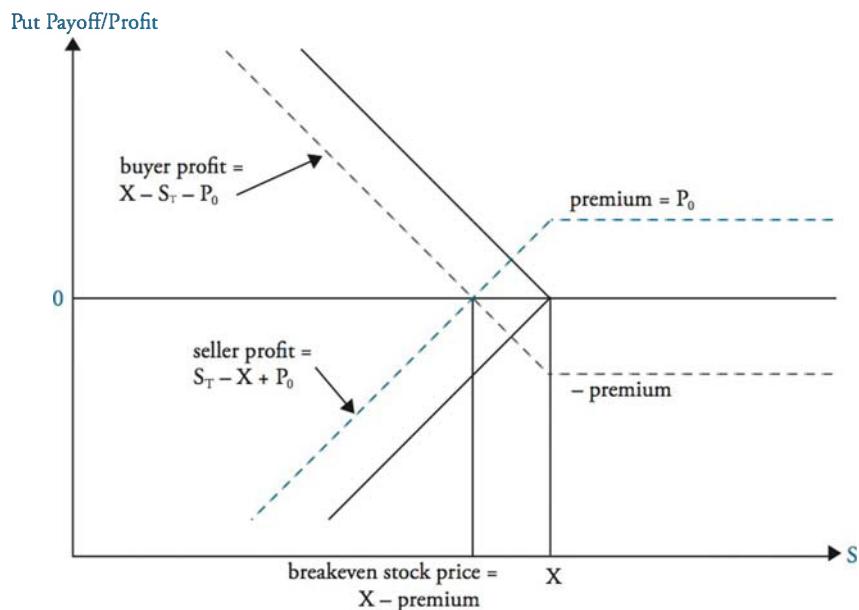
$P_0$  = put premium

The profit to the option seller is:

$$\text{profit} = P_0 - P_T$$

Figure 2 depicts the payoff and profit for the buyer and writer of a put option.

**Figure 2: Profit Diagram for a Put at Expiration**



**Example: Calculating profit and payoffs from options**

Compute the payoff and profit to a call buyer, a call writer, put buyer, and put writer if the strike price for both the put and the call is \$45, the stock price is \$50, the call premium is \$3.50, and the put premium is \$2.50.

**Answer:**

Call buyer:

$$\text{payoff} = C_T = \max(0, S_T - X) = \max(0, \$50 - \$45) = \$5$$

$$\text{profit} = C_T - C_0 = \$5 - \$3.50 = \$1.50$$

Call writer:

$$\text{payoff} = -C_T = -\max(0, S_T - X) = -\max(0, \$50 - \$45) = -\$5$$

$$\text{profit} = C_0 - C_T = \$3.50 - \$5 = -\$1.50$$

Put buyer:

$$\text{payoff} = P_T = \max(0, X - S_T) = \max(0, \$45 - \$50) = \$0$$

$$\text{profit} = P_T - P_0 = \$0 - \$2.50 = -\$2.50$$

Put writer:

$$\text{payoff} = -P_T = -\max(0, X - S_T) = -\max(0, \$45 - \$50) = \$0$$

$$\text{profit} = P_0 - P_T = \$2.50 - \$0 = \$2.50$$

**Forward Contract Payoff**

The payoff to a long position in a forward contract is calculated as follows:

$$\text{payoff} = S_T - K$$

where:

$S_T$  = spot price at maturity

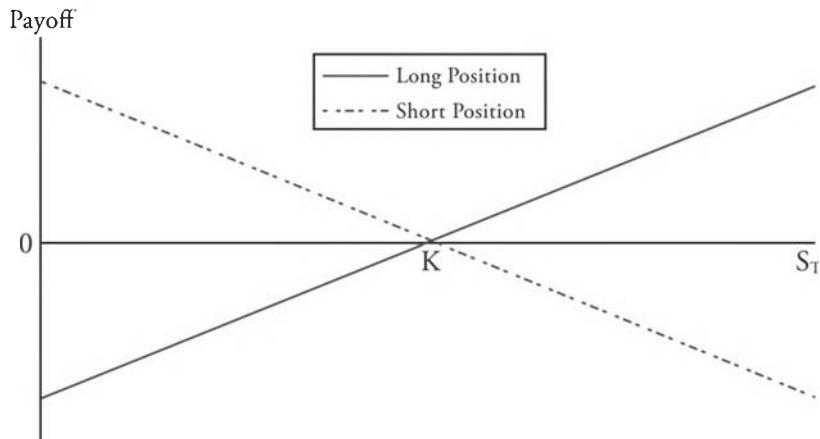
$K$  = delivery price

Conversely, the payoff to a short position in a forward contract is calculated as follows:

$$\text{payoff} = K - S_T$$

Figure 3 depicts the payoff for the long and short positions in a forward contract.

**Figure 3: Forward Contract Payoff**



**Example: Calculating forward contract payoffs**

Compute the payoff to the long and short positions in a forward contract given that the forward price is \$25 and the spot price at maturity is \$30.

**Answer:**

Payoff to long position:

$$\text{payoff} = S_T - K = \$30 - \$25 = \$5$$

Payoff to short position:

$$\text{payoff} = K - S_T = \$25 - \$30 = -\$5$$

## CATEGORIES OF TRADERS

### LO 34.8: Differentiate among the broad categories of traders: hedgers, speculators, and arbitrageurs.

**Hedgers** typically reduce their risks through the use of forward contracts or options. By using forward contracts, the trader is attempting to neutralize risk by fixing the price the hedger will pay or receive for the underlying asset. Option contracts, in contrast, are more of an insurance policy.

Speculators want to take a position in the market and profit from this position. Speculators are effectively betting on future price movement. When a speculator uses futures, there is a large possible gain or loss. Speculating using options is less risky since the maximum loss is the cost of the option itself.

Arbitrageurs take offsetting positions in financial instruments in order to lock in a riskless profit.

## HEDGING STRATEGIES

### LO 34.4: Calculate and compare the payoffs from hedging strategies involving forward contracts and options.

Hedgers use forward contracts and options to reduce or eliminate financial exposure. An investor or business with a long exposure to an asset can hedge exposure by either entering into a short futures contract or by buying a put option. An investor or business with a short exposure to an asset can hedge exposure by either entering into a long futures contract or by buying a call option.

Hedgers use forward contracts to lock in the price of the underlying security. Forward contracts do not require an initial investment, but hedgers give up any price movement that may have had positive results in the event that the position was left unhedged. Option contracts on the other hand function as insurance for the underlying by providing the downside protection that the hedger seeks and allowing for price movement in the direction that could yield positive results. This insurance does not come without a cost, as we described earlier, since hedgers are required to pay a premium to purchase options.

#### Example: Hedging with a forward contract

Suppose that a company based in the United States will receive a payment of €10M in three months. The company is worried that the euro will depreciate and is contemplating using a forward contract to hedge this risk. Compute the following:

1. The value of the €10M in U.S. dollars at maturity given that the company hedges the exchange rate risk with a forward contract at 1.25 \$/€.
2. The value of the €10M in U.S. dollars at maturity given that the company did not hedge the exchange rate risk and the spot rate at maturity is 1.2 \$/€.

#### Answer:

1. The value at maturity for the hedged position is:  
$$\text{€}10,000,000 \times 1.25 \text{ $/€} = \$12,500,000$$
2. The value at maturity for the unhedged position is:  
$$\text{€}10,000,000 \times 1.2 \text{ $/€} = \$12,000,000$$

**Example: Hedging with a put option**

Suppose that an investor owns one share of ABC stock currently priced at \$30. The investor is worried about the possibility of a drop in share price over the next three months and is contemplating purchasing put options to hedge this risk. Compute the following:

1. The profit on the unhedged position if the stock price in three months is \$25.
2. The profit on the unhedged position if the stock price in three months is \$35.
3. The profit for a hedged stock position if the stock price in three months is \$25, the strike price on the put is \$30, and the put premium is \$1.50.
4. The profit for a hedged stock position if the stock price in three months is \$35, the strike price on the put is \$30, and the put premium is \$1.50.

**Answer:**

1. Profit =  $S_T - S_0 = \$25 - \$30 = -\$5$
2. Profit =  $S_T - S_0 = \$35 - \$30 = \$5$
3. Profit =  $S_T - S_0 + \max(0, X - S_T) - P_0$   
 $= \$25 - \$30 + \max(0, \$30 - \$25) - \$1.50 = -\$1.50$
4. Profit =  $S_T - S_0 + \max(0, X - S_T) - P_0$   
 $= \$35 - \$30 + \max(0, \$30 - \$35) - \$1.50 = \$3.50$



*Professor's Note: Notice that the max term is \$5 in Case #3 and \$0 in Case #4.*

## SPECULATIVE STRATEGIES

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### LO 34.5: Calculate and compare the payoffs from speculative strategies involving futures and options.

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Speculators have a different motivation for using derivatives than hedgers. They use derivatives to make bets on the market, while hedgers try to eliminate exposures.

The motivation for using futures in speculation is that the limited amount of initial investment creates significant leverage. The amount of investment required for futures is the amount of the initial margin required by the exchange. This is generally a small percentage of the notional value of the underlying, and Treasury securities can typically be posted as margin. Futures contracts can result in large gains or large losses, and contract payoffs are symmetrical.

Options also create significant leverage as investors only need to pay the option premium to purchase an option instead of the face value of the underlying. Options differ from futures in that options have asymmetrical payoffs. Gains can be quite large going long options, but losses from long option positions are limited to the option premium.

### Example: Speculating with futures

An investor believes that the euro will strengthen against the dollar over the next three months and would like to take a position with a value of €250,000. He could purchase euros in the spot market at 0.80 \$/€ or purchase two futures contracts at 0.83 \$/€ with an initial margin of \$10,000. Compute the profit from the following:

1. Purchasing euros in the spot market if the spot rate in three months is 0.85 \$/€.
2. Purchasing euros in the spot market if the spot rate in three months is 0.75 \$/€.
3. Purchasing the futures contract if the spot rate in three months is 0.85 \$/€.
4. Purchasing the futures contract if the spot rate in three months is 0.75 \$/€.

### Answer:

1. Profit = €250,000 × (0.85 \$/€ – 0.80 \$/€) = \$12,500
2. Profit = €250,000 × (0.75 \$/€ – 0.80 \$/€) = -\$12,500
3. Profit = €250,000 × (0.85 \$/€ – 0.83 \$/€) = \$5,000
4. Profit = €250,000 × (0.75 \$/€ – 0.83 \$/€) = -\$20,000

A summary of these four transactions is as follows:

	<i>Purchase Euros in Spot Market</i>	<i>Purchase Long Forward Position</i>
Investment	\$200,000	\$10,000
Profit if spot at maturity = 0.85 \$/€	\$12,500	\$5,000
Profit if spot at maturity = 0.75 \$/€	-\$12,500	-\$20,000

**Example: Speculating with options**

An investor who has \$30,000 to invest believes that the price of stock XYZ will increase over the next three months. The current price of the stock is \$30. The investor could directly invest in the stock, or she could purchase 3-month call options with a strike price of \$35 for \$3. Compute the profit from the following:

1. Investing directly in the stock if the price of the stock is \$45 in three months.
2. Investing directly in the stock if the price of the stock is \$25 in three months.
3. Purchasing call options if the price of the stock is \$45 in three months.
4. Purchasing call options if the price of the stock is \$25 in three months.

**Answer:**

1. Number of stocks to purchase =  $\$30,000 / \$30 = 1,000$   
Profit =  $1,000 \times (\$45 - \$30) = \$15,000$
2. Profit =  $1,000 \times (\$25 - \$30) = -\$5,000$
3. Number of call options to purchase =  $\$30,000 / \$3 = 10,000$   
Profit =  $10,000 \times [\max(0, \$45 - \$35) - \$3] = \$70,000$
4. Profit =  $10,000 \times [\max(0, \$25 - \$35) - \$3] = -\$30,000$



*Professor's Note: Since option contracts are traded in amounts of 100 options, the transactions in #3 and #4 above would entail the purchase of 100 call option contracts (i.e.,  $10,000 / 100 = 100$ ).*

A summary of these four transactions is as follows:

	Purchase Stock	Purchase Call Option
# Shares/Call option	1,000	10,000
Profit if stock at maturity = \$45	\$15,000	\$70,000
Profit if spot at maturity = \$25	-\$5,000	-\$30,000

## ARBITRAGE OPPORTUNITIES

### LO 34.6: Calculate an arbitrage payoff and describe how arbitrage opportunities are temporary.

Arbitrageurs are also frequent users of derivatives. Arbitrageurs seek to earn a risk-free profit in excess of the risk-free rate through the discovery and manipulation of mispriced securities. They earn a riskless profit by entering into equivalent offsetting positions in one or more markets. Arbitrage opportunities typically do not last long as supply and demand forces will adjust prices to quickly eliminate the arbitrage situation.

#### Example: Arbitrage of stock trading on two exchanges

Assume stock DEF trades on the New York Stock Exchange (NYSE) and the Tokyo Stock Exchange (TSE). The stock currently trades on the NYSE for \$32 and on the TSE for ¥2,880. Given the current exchange rate is 0.0105 \$/¥, determine if an arbitrage profit is possible.

#### Answer:

Value in dollars of DEF on TSE =  $\text{¥}2,880 \times 0.0105 \text{ $/¥} = \$30.24$

Arbitrageur could purchase DEF on TSE for \$30.24 and sell on NYSE for \$32.

Profit per share =  $\$32 - \$30.24 = \$1.76$

## RISK FROM DERIVATIVES

### LO 34.7: Describe some of the risks that can arise from the use of derivatives.

Derivatives are versatile and can be used for hedging, arbitrage, and pure speculation. If, however, the “bet” one makes starts going in the wrong direction, the results can be catastrophic. Additionally, the risk exists that a trader with instructions to hedge a position may actually use derivatives to speculate. This risk is known as operational risk. Controls need to be carefully established and monitored within both financial and nonfinancial corporations to prevent misuse of derivatives. Risk limits should be set, and adherence to risk limits should be monitored.

## KEY CONCEPTS

### LO 34.1

The over-the-counter (OTC) market is used for large trades, and a typical OTC trade is conducted over the phone. Terms are not set by an “exchange,” giving traders more flexibility to negotiate mutually agreeable terms. The OTC market has more credit risk. Exchanges are organized to eliminate credit risk.

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### LO 34.2

A call option gives its holder the right to buy a specified number of shares of the underlying security at the given strike price, on or before the option contract's expiration date, while a put option is the right to sell a fixed number of shares at a fixed price within a given pre-specified time period.

A forward contract is an agreement to buy or sell an asset at a pre-selected future time for a certain price.

A futures contract is a more formalized, legally binding agreement to buy or sell a commodity or financial asset in a pre-designated month in the future, at a price agreed upon today by the buyer/seller.

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### LO 34.3

The payoff on a call option to the option buyer is calculated as follows:

$$\text{Call}_T = \max (0, S_T - X)$$

where:

$S_T$  = stock price at maturity

X = strike price of option

The payoff on a put option is calculated as follows:

$$\text{Put}_T = \max (0, X - S_T)$$

where:

$S_T$  = stock price at maturity

X = strike price of option

The payoff to a long position in a forward contract is calculated as follows:

$$\text{payoff} = S_T - K$$

where:

$S_T$  = spot price at maturity

K = delivery price

**LO 34.4**

Hedgers use derivatives to control or eliminate a financial exposure. Futures lock in the price of the underlying security and do not allow for any upside potential. Options hedge negative price movements and allow for upside potential since they have asymmetric payouts.

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**LO 34.5**

Speculators use derivatives to make bets on the market. Futures require a small initial investment, which is the initial margin requirement. Futures contracts can result in large gains or large losses as futures have a symmetrical payout function.

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**LO 34.6**

Arbitrageurs seek to earn a riskless profit through the discovery and manipulation of mispriced securities. Riskless profit is earned by entering into equivalent offsetting positions in one or more markets. Arbitrage opportunities do not last long as the act of arbitrage brings prices back into equilibrium quickly.

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**LO 34.7**

Derivatives are versatile instruments and can be used for hedging, arbitrage, and pure speculation. Controls need to be carefully established to prevent misuse of derivatives. Risk limits must be carefully established and scrupulously enforced.

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**LO 34.8**

There are three broad types of traders: hedgers, speculators, and arbitrageurs.

Hedging is used for risk management. The hedger has a risk associated with the underlying commodity or financial instrument. The use of futures helps mitigate those risks.

Speculating does not mitigate risk but is risk-taking. Profit is the motive of the speculator since he has no risk before entering into the futures transactions.

Arbitrage ensures that futures and cash markets stay in balance. Buying in the cheaper market and selling in the overpriced market will bring markets back into alignment and provide a riskless profit for an arbitrageur.

## CONCEPT CHECKERS

1. Which of the following statements is an advantage of an exchange trading system?  
On an exchange system:
  - A. terms are not specified.
  - B. trades are made in such a way as to reduce credit risk.
  - C. participants have flexibility to negotiate.
  - D. in the event of a misunderstanding, calls are recorded between parties.
  
2. Which of the following statements regarding futures contracts is most likely correct?  
A business with a long exposure to an asset would hedge this exposure by either entering into a:
  - A. long futures contract or by buying a call option.
  - B. long futures contract or by buying a put option.
  - C. short futures contract or by buying a call option.
  - D. short futures contract or by buying a put option.
  
3. Which of the following statements is least likely correct regarding the use of derivatives?
  - A. Misuse of derivatives is not a very significant risk.
  - B. Risk limits for derivatives should be set, and adherence to these limits should be monitored.
  - C. Due to leverage inherent in derivatives, if a bet goes wrong, results can be catastrophic.
  - D. There is a risk that traders may use derivatives for unintended purposes.
  
4. An individual that maintains bid and offer prices in a given security and stands ready to buy or sell lots of said security is a(n):
  - A. hedger.
  - B. arbitrageur.
  - C. speculator.
  - D. market maker.
  
5. An agreement sold over an exchange to buy/sell a commodity or financial instrument at a designated future date is known as a(n):
  - A. spot contract.
  - B. option contract.
  - C. futures contract.
  - D. forward contract.

## CONCEPT CHECKER ANSWERS

1. B Exchanges are organized to reduce credit risk. The other answer choices are advantages of over-the-counter trading.
2. D A business with a long exposure to an asset would hedge the exposure by either entering into a short futures contract or by buying a put option.
3. A Misuse of derivatives can be a significant risk for firms that engage in derivatives trading.
4. D A market maker maintains bid and offer prices in a security and stands ready to buy or sell lots of the given security.
5. C A futures contract is an agreement sold on an exchange to buy/sell a commodity or financial instrument in a designated future month.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

# FUTURES MARKETS AND CENTRAL COUNTERPARTIES

Topic 35

## EXAM FOCUS

In this topic, candidates should focus on the terminology of futures markets, how futures differ from forwards, the mechanics of margin deposits, and the process of marking to market. Limit price moves, delivery options, and convergence of spot prices to futures prices are also likely exam concepts. It is also important to understand the ways a futures position can be terminated prior to contract expiration and how cash settlement is accomplished by the final mark-to-market at contract expiration.

## FEATURES OF FUTURES CONTRACTS

**LO 35.1: Define and describe the key features of a futures contract, including the asset, the contract price and size, delivery, and limits.**

Futures contracts are exchange-traded obligations to buy or sell a certain amount of an underlying good at a specified price and date. The underlying asset varies from agricultural products to stock indices. Most futures positions are not held to take delivery of the underlying good. Instead, they are closed out or reversed prior to the settlement date.

The purchaser of a futures contract is said to have gone long or taken a **long position**, while the seller of a futures contract is said to have gone short or taken a **short position**. For each contract traded, there is a buyer and a seller. The long has contracted to buy the asset at the contract price at contract expiration, and the short has an obligation to sell at that price. Futures contracts are used by **speculators** to gain exposure to changes in the price of the asset underlying a futures contract. A **hedger**, in contrast, will use futures contracts to reduce exposure to price changes in the asset (i.e., hedge their asset price risk). An example is a wheat farmer who sells wheat futures to reduce the uncertainty about the price of wheat at harvest time.

**Open interest** is the total number of long positions in a given futures contract. It also equals the total number of short positions in a futures contract. An open interest of 200 would imply that there are 200 short positions in existence and 200 long positions in existence. It is possible, on any given day, for the trading volume on a contract to be higher than its open interest.

## Trading Futures Contracts

To illustrate how a futures contract is created, let's use a contract on gold as an example. Each contract represents 100 ounces and is quoted on a U.S. dollar per-ounce basis. Suppose an investor instructs a broker to sell one futures contract on gold with an April

delivery date. At about the same time another investor instructs a broker to buy an identical futures contract. The seller of the futures contract has a short-futures position and is obligated to sell 100 ounces of gold at the futures price at contract expiration. The buyer of the futures contract has a long futures position and is obligated to buy 100 ounces of gold at the futures price at maturity. They agree on a price of \$993.60 per ounce. The two parties in this example have no idea of one another's existence because the clearinghouse (discussed in LO 35.4) takes the opposite side of every transaction. In the futures market there is always the same number of long and short positions. This means that if a long position wins, the corresponding short position loses.

### Characteristics Specified in Futures Contracts

Futures contracts are similar to forward contracts in that both allow for a transaction to take place at a future date at a price agreed upon today. The difference between the two is that forward contracts are private, customized contracts, while futures trade on an organized exchange and have terms that are highly standardized. When a new futures contract is introduced to the marketplace, the futures exchange must specify the exact terms of the contract. Futures contract characteristics specified by the exchange include the following:

- *Quality of the underlying asset.* When the underlying asset for the contract is a financial asset, such as Japanese yen, the definition of the asset is straightforward. However, when the underlying asset is a commodity, there may be different levels of quality for that good available in the marketplace (e.g., different types of wheat). The futures exchange stipulates the quality of a good that will be acceptable for settling the contract.
- *Contract size.* The contract size specifies the quantity of the asset that must be delivered to settle a futures contract (e.g., one grain contract = 5,000 bushels).
- *Delivery location.* The exchange specifies the place where delivery will take place.
- *Delivery time.* Futures contracts are referred to by the month in which delivery is to take place (e.g., a December corn contract). Some contracts are not settled by delivery but by payment in cash, based on the difference between the futures price and the market price at settlement.
- *Price quotations and tick size.* The exchange determines how the price of a contract will be quoted as well as the minimum price fluctuation for the contract, which is referred to as the *tick size*. For example, grain is quoted in dollars per bushel, and the minimum tick size is  $\frac{1}{4}$  cent per bushel. Since a grain contract consists of 5,000 bushels, the minimum tick size is \$12.50 ( $= 5,000 \times \$0.0025$ ) per contract.
- *Daily price limits.* The exchange sets the maximum price movement for a contract during a day. For example, wheat cannot move more than \$0.20 from its close the preceding day, for a daily price limit of \$1,000. When a contract moves down by its daily price limit, it is said to be *limit down*. When the contract moves up by its price limit, it is said to be *limit up*.
- *Position limits.* The exchange sets a maximum number of contracts that a speculator may hold in order to prevent speculators from having an undue influence on the market. Such limits do not apply to hedgers.

## FUTURES/SPOT CONVERGENCE

### LO 35.2: Explain the convergence of futures and spot prices.

The spot (cash) price of a commodity or financial asset is the price for immediate delivery. The futures price is the price today for delivery at some future point in time (i.e., the maturity date). The **basis** is the difference between the spot price and the futures price.

$$\text{basis} = \text{spot price} - \text{futures price}$$

As the maturity date nears, the basis converges toward zero. At expiration, the spot price must equal the futures price because the futures price has become the price today for delivery today, which is the same as the spot. Arbitrage will force the prices to be the same at contract expiration.

#### Example: Why the futures price must equal the spot price at expiration

Suppose the current spot price of silver is \$4.65. Demonstrate by arbitrage that the futures price of a futures silver contract that expires in one minute must equal the spot price.

#### Answer:

Suppose the futures price was \$4.70. We could buy the silver at the spot price of \$4.65, sell the futures contract, and deliver the silver under the contract at \$4.70. Our profit would be  $\$4.70 - \$4.65 = \$0.05$ . Because the contract matures in one minute, there is virtually no risk to this arbitrage trade.

Suppose instead the futures price was \$4.61. Now we would buy the silver contract, take delivery of the silver by paying \$4.61, and then sell the silver at the spot price of \$4.65. Our profit is  $\$4.65 - \$4.61 = \$0.04$ . Once again, this is a riskless arbitrage trade.

Therefore, in order to prevent arbitrage, the futures price at the maturity of the contract must be equal to the spot price of \$4.65.

## MARGIN REQUIREMENTS

### LO 35.3: Describe the rationale for margin requirements and explain how they work.

Margin is cash or highly liquid collateral placed in an account to ensure that any trading losses will be met. Marking to market is the daily procedure of adjusting the margin account balance for daily movements in the futures price. The amount required to open a futures position is called the **initial margin**. The **maintenance margin** is the minimum margin

account balance required to retain the futures position. When the margin account balance falls below the maintenance margin, the investor gets a margin call, and he must bring the margin account back to the initial margin amount. The amount necessary to do this is called the **variation margin**.

#### Example: Margin trading

Let's return to our investor with the long gold contract. The investor entered the position at \$993.60. Each contract controls 100 troy ounces for a current market value of \$99,360. Assume that the initial margin is \$2,500, the maintenance margin is \$2,000, and the futures price drops to \$991.00 at the end of the first day and \$985.00 on the end of the second day. Compute the amount in the margin account at the end of each day for the long position and any variation margin needed.

#### Answer:

At the end of the first day, the loss is computed as  $(\$991 - \$993.6)100 = -\$260$ , so when the account is marked to market, \$260 is withdrawn from the buyer's margin account and \$260 deposited in the seller's margin account. The buyer's (long) margin account balance is now \$2,240 ( $= \$2,500 - \$260$ ). The margin account balance for the short position is now \$2,760 ( $= \$2,500 + \$260$ ).

At the end of the second day, the daily loss is  $(\$985 - \$991)100 = -\$600$ , and the buyer's margin account balance is reduced to \$1,640 ( $= \$2,240 - \$600$ ). At \$1,640 the investor will get a margin call since the margin account balance is less than the maintenance margin. The variation margin is the amount necessary to bring the margin account back up to the initial margin. In this case, it is \$860 ( $= \$2,500 - \$1,640$ ).

Depending on the client, brokers may require the posting of a balance in the margin account more than the maintenance margin requirements established by exchanges. For example, hedgers are usually required to post smaller margins than speculators. To ensure that the daily cash flows are withdrawn or contributed appropriately, the exchange has a clearinghouse.

## CLEARINGHOUSES IN FUTURES TRANSACTIONS

### LO 35.4: Describe the role of a clearinghouse in futures and over-the-counter market transactions.

Each exchange has a **clearinghouse**. The clearinghouse guarantees that traders in the futures market will honor their obligations. The clearinghouse does this by splitting each trade once it is made and acting as the opposite side of each position. The clearinghouse acts as the buyer to every seller and the seller to every buyer. By doing this, the clearinghouse allows either side of the trade to reverse positions at a future date without having to contact the other side of the initial trade. This allows traders to enter the market knowing that they will be able to reverse their position. Traders are also freed from having to worry about the

counterparty defaulting since the counterparty is now the clearinghouse. In the history of U.S. futures trading, the clearinghouse has never defaulted on a trade.

The clearinghouse has members that collateralize it, ensuring that no defaults take place. All trades eventually go through the clearinghouse members, who must have a **clearing margin** posted at the clearinghouse in the same way an investor has a margin account with a broker. This ensures that the clearinghouse is liquid enough at all times to honor all obligations under futures contracts.

## CENTRAL COUNTERPARTIES IN OVER-THE-COUNTER TRANSACTIONS

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**LO 35.5: Describe the role of central counterparties (CCPs) and distinguish between bilateral and centralized clearing.**

**LO 35.6: Describe the role of collateralization in the over-the-counter market and compare it to the margining system.**

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The **over-the-counter (OTC) market** includes the trading in all securities not listed on one of the registered exchanges. This market is subject to a good deal of credit risk since the party on the other side of an OTC contract could default on its payments. In an effort to reduce credit risk, the OTC market has adopted some practices from futures exchanges.

Clearinghouses for standard OTC transactions are referred to as **central counterparties (CCPs)**. These CCPs operate in a similar fashion to clearinghouses on futures exchanges. After two parties (X and Y) negotiate an OTC agreement, it is submitted to the CCP for acceptance. Assuming the transaction is accepted, the CCP will become the counterparty to both parties X and Y. Thus, it assumes the credit risk of both parties in an OTC transaction. This risk is managed by requiring the parties to post initial margin and any variation margins on a daily basis.

Arguments for the use of CCPs in OTC markets include (1) collateralized positions with reserves and margins, (2) reduction of financial system credit risk, and (3) increased transparency of OTC trades. Regulators have pushed for the use of CCPs in OTC markets in an attempt to reduce systemic risk, which is the risk that a failure by a significant financial institution will impact other institutions and potentially lead to a collapse of the overall financial system. An example of systemic risk occurred during the 2007–2009 credit crisis, when the OTC transactions for insurance corporation AIG led to large losses and an eventual bailout of the company by the U.S. government.

Historically, OTC markets functioned as a series of bilateral agreements between parties through a process known as **bilateral clearing**. If a CCP was instead used for every OTC transaction, each market participant would only deal with a central clearing party. However, because only some OTC transactions are currently required to use CCPs, in practice, the current OTC market is a mix of both bilateral agreements and transactions that use centralized clearing.

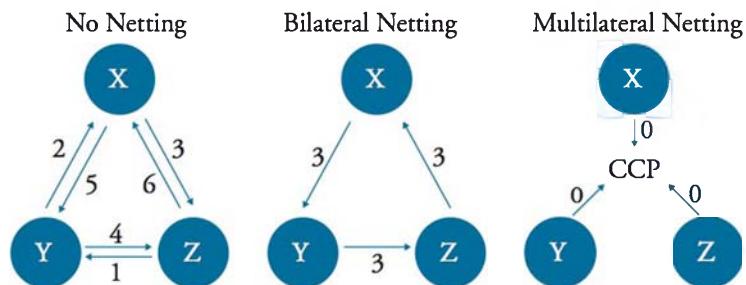
Bilateral clearing usually includes a master agreement with a credit support annex (CSA), which outlines the use of collateral between parties. Providing collateral is a means of reducing credit risk in OTC markets. This **collateralization** is basically a marked-to-market

feature for the OTC market where any loss is settled in cash at the end of the trading day. A cash payment is made to the counterparty with a positive account balance. This is a similar system to trading on margin, where the futures trader needs to restore funds if the value of the contract drops below the maintenance margin.

## Counterparty Risk Exposures

Figure 1 illustrates how counterparty risk exposures are reduced through the use of a CCP. Suppose X, Y, and Z represent three different counterparties. The arrows in the figure represent the amount of money owed (i.e., counterparty risk exposure) between the entities. For example, under a *no netting* framework, entity X has an exposure of 2 to entity Y, and entity Y has an exposure of 5 to entity X.

**Figure 1: Reduction of Risk Exposure Through Multilateral Netting**



Suppose entity X defaults under the *no netting* framework. This will result in a loss of 5 for entity Y and a loss of 3 to entity Z. Under the no netting framework, entity X still claims a total of 8 from both entities Y and Z (i.e., 2 + 6). With *bilateral netting*, trades between two entities are cleared, reducing the total exposure in the market. If entity X defaults under the bilateral framework, then entity Y will have a reduced loss of 3 and entity Z will have no loss because there is no risk exposure remaining from entity X for entity Z.

If entity X defaults under the *multilateral netting* framework, entities Y and Z do not have losses because there is no outstanding counterparty risk exposure from other CCP members. All trades are cleared through the CCP in the multilateral netting framework.

Clearly both the bilateral and multilateral netting frameworks significantly reduce risk exposures compared to the no netting framework. However, the biggest advantage of a CCP is the ability to mitigate systemic risk through multilateral netting. Figure 1 implies that systemic risk exposures are reduced more under multilateral netting than bilateral netting. However, the reduction in risk exposures for the multilateral netting framework as opposed to the bilateral framework are only possible if a relatively small number of CCPs clear a relatively large number of transactions.

## FUTURES MARKET QUOTES

**LO 35.7:** Identify the differences between a normal and inverted futures market.

**LO 35.8:** Explain the different market quotes.

Futures quotes can be found from exchanges as well as various online websites. Figure 2 contains a subset of gold futures quotes posted on the CME Group website (<http://www.cmegroup.com>). The first column indicates the maturity month for a given futures contract. Recall that each gold futures contract represents 100 ounces and is priced in U.S. dollars per ounce.

**Figure 2: Gold Futures Quotes**

Month	Last Trade	Change	Prior Settlement	Open	High	Low	Volume
Dec-17	1280.6	+2.9	1277.7	1278.0	1281.8	1274.9	245,277
Feb-18	1285.1	+3.0	1282.1	1282.8	1286.1	1279.4	13,715
Apr-18	1289.1	+2.8	1286.3	1287.4	1289.7	1284.0	1,983
Jun-18	1292.4	+1.8	1290.6	1290.4	1294.1	1288.2	2,075
Dec-18	1306.5	+3.1	1303.4	1304.1	1306.5	1304.1	526

The current trading price of a given futures contract is shown in the second column. The change between the previous day's settlement price and the last trade is reflected in the third column. The **settlement price** is typically computed as the price right before the end of the previous trading day. This price is used for computing daily gains and losses as well as determining margin requirements.

Figure 2 also indicates the opening prices for a given day and the highest and lowest prices during the trading day. As you can see, the December 2017 gold futures contract for this particular day opened at \$1,278/oz, increased to a high of \$1,281.8/oz and decreased to a low of \$1,274.9/oz. This contract last traded at \$1,280.6/oz, which indicates a change of \$2.9/oz over the previous day's settlement price.

The last column in Figure 2 reflects the trading volume in each futures contract, which is the number of contracts that have been traded on a given day. This amount differs from **open interest**, discussed earlier, which is the total number of long (or short) positions in a given futures contract. Trading volume could potentially be higher than open interest on any particular day if a large amount of day trading took place, where traders open and close a futures position on the same day.

Figure 2 also shows the pattern of futures prices as a function of contract maturity. Depending on the direction of futures settlement prices, the market may be normal or inverted. In this case, gold futures contract prices are moving higher with increasing time horizons. Increasing settlement prices over time would indicate a **normal futures market**. Conversely, decreasing settlement prices over time would indicate an **inverted futures market**.

## THE DELIVERY PROCESS

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### LO 35.9: Describe the mechanics of the delivery process and contrast it with cash settlement.

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There are four ways to terminate a futures contract:

1. A short can terminate the contract by delivering the goods. When the long accepts this delivery, he pays the contract price to the short. This is called **delivery**. The location for delivery (for physical assets), terms of delivery, and details of exactly what is to be delivered are all specified in the **notice of intention to deliver** file. Each exchange has specific rules as to the conditions for making an intent to deliver. However, the price paid or received will be dictated by the settlement period on the exchange-determined last trading day of the contract.
2. In a **cash-settlement contract**, delivery is not an option. The futures account is marked to market based on the settlement price on the last day of trading.
3. You may make a **reverse**, or **offsetting**, trade in the futures market. With futures, the other side of your position is held by the clearinghouse—if you make an exact opposite trade (maturity, quantity, and good) to your current position, the clearinghouse will net your positions out, leaving you with a zero balance. This is how most futures positions are settled. The contract price can differ between the two contracts. If you initially are long one contract at \$970 per ounce of gold and subsequently sell (i.e., take the short position in) an identical gold contract when the price is \$950 per ounce, \$20 multiplied by the number of ounces of gold specified in the contract will be deducted from the margin deposit(s) in your account. The sale of the futures contract ends the exposure to future price fluctuations on the first contract. Your position has been *reversed*, or **closed out**, by a *closing* trade.
4. A position may also be settled through an **exchange for physicals**. Here you find a trader with an opposite position to your own and deliver the goods and settle up between yourselves, off the floor of the exchange (i.e., an ex-pit transaction). This is the sole exception to the federal law that requires that all trades take place on the floor of the exchange. You must then contact the clearinghouse and tell them what happened. An exchange for physicals differs from a delivery in that the traders actually exchange the goods, the contract is not closed on the floor of the exchange, and the two traders privately negotiate the terms of the transaction. Regular delivery involves only one trader and the clearinghouse.

## TYPES OF ORDERS

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### LO 35.10: Evaluate the impact of different trading order types.

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There are several different types of orders in the marketplace:

**Market orders** are orders to buy or sell at the best price available. A **discretionary order** is a market order where the broker has the option to delay transaction in search of a better price.

**Limit orders** are orders to buy or sell away from the current market price. A *limit buy order* is placed below the current price. A *limit sell order* is placed above the current price. Limit orders have a time limit, such as instantaneous, one day, one week, one month, or good till canceled. Limit orders are turned over to the specialist by the commission broker.

**Stop-loss orders** are used to prevent losses or to protect profits. Suppose you own a stock currently selling for \$40. You are afraid that it may drop in price, and if it does, you want your broker to sell it, thereby limiting your losses. You would place a *stop loss sell* order at a specific price (e.g., \$35); if the stock price drops to this level, your broker will place a sell market order. A *stop loss buy* order is usually combined with a short sale to limit losses. If the stock price rises to the “stop” price, the broker enters a market order to buy the stock.

Variations on these order types also exist. **Stop-limit orders** are a combination of a stop and limit order. The stop price and limit price must be specified, so that once the stop level is reached, or bettered, the order would turn into a limit order and hopefully transact at the limit price. **Market-if-touched orders**, or MIT orders, are orders that would become market orders once a specified price is reached in the marketplace.

For those orders that remain outstanding until the designated price range is reached, the trader making the order needs to indicate the time period for the order (**time-of-day order**). **Good-till-canceled (GTC) orders** (a.k.a. **open orders**) are orders that remain open until they either transact or are canceled. A popular method of submitting a limit order is to have it automatically canceled at the end of the trading day in which it was submitted. **Fill-or-kill orders** must be executed immediately or the trade will not take place.

## FORWARDS AND FUTURES CONTRACTS

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### LO 35.11: Compare and contrast forward and futures contracts.

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A **forward contract** is a contract between two parties in which they agree to perform a future action. For example, in a currency forward contract, one party agrees to deliver a given amount of one currency on a future date and the other party (the *counterparty*) agrees to deliver a given amount of another currency on a future date.

**Futures contracts**, like forward contracts, also obligate one party to sell an asset at a specific price in the future, but they are standardized and traded on an exchange. Futures contracts have currencies, bonds, stock indices, and physical commodities as their underlying assets. Because futures trade on exchanges, they are more regulated and have more liquidity compared to forward contracts, which trade in the OTC market.

Forward and futures contracts differ in the following ways:

- Forwards are private transactions between two parties; futures are traded on organized exchanges.
- Forwards are customizable to satisfy both parties; futures are standardized for underlying asset, size, and maturity.
- Forwards are bilateral agreements with counterparty risk; futures trade with clearinghouses and have no counterparty risk.
- Forwards are usually not regulated; futures are regulated by securities regulators.
- Forwards transaction costs are embedded in the contract prices set by dealers; futures transaction costs are commissions paid to brokers and are relatively low.
- It is difficult to offset or cancel a forward contract because trading and liquidity are low; it is easy to offset or cancel futures because the market is active and provides good liquidity.
- Forwards settle at expiration; futures are marked to market and settle daily.

## KEY CONCEPTS

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### LO 35.1

A long (short) futures position obligates the owner to buy (sell) the underlying asset at a specified price and date. Most futures positions are reversed (or closed out) as opposed to satisfying the contract by making (or taking) delivery.

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### LO 35.2

The spot price of a commodity or financial asset is the price for immediate delivery. The futures price is the price today for delivery at some future point in time (i.e., the maturity date). The basis is the difference between the spot price and the futures price. As the maturity date nears, the basis converges toward zero. Arbitrage will force the spot and futures prices to be the same at contract expiration.

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### LO 35.3

Futures are traded on margin (leveraged):

- Initial margin is the necessary collateral to trade the futures.
- Maintenance margin is the minimum collateral amount required to retain trading privileges.
- Variation margin is the collateral amount that must be deposited to replenish the margin account back to the initial margin.

The futures market is a zero-sum game in that the short's losses are the long's gains and vice versa. Gains and losses due to changes in futures prices are computed at the end of each trading day in a process known as marking to market.

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### LO 35.4

The clearinghouse maintains an orderly and liquid market by acting as the counterparty to each long or short futures position. In the over-the-counter (OTC) markets, the central counterparty (i.e., clearinghouse) becomes the counterparty to both parties in an OTC transaction.

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### LO 35.5

Historically, OTC markets functioned as a series of bilateral agreements between parties through a process known as bilateral clearing. Regulators have pushed for the use of centralized clearing in OTC markets through the use of central counterparties (CCPs) in an attempt to reduce systemic risk. CCPs operate in a similar fashion to clearinghouses on futures exchanges.

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### LO 35.6

Collateralization is a means of reducing credit risk in bilateral OTC contracts. It is a marked-to-market feature where any loss is settled in cash at the end of the trading day. A cash payment is made to the counterparty with a positive account balance.

**LO 35.7**

Increasing settlement prices over time indicate a normal market, while decreasing settlement prices over time indicate an inverted market.

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**LO 35.8**

Futures quotes can be found from exchanges as well as various online sources. Quotes typically contain the last trading price, the previous day's settlement price, and the open, high, and low prices for a particular trading day. The settlement price is typically computed as the price right before the end of the previous trading day. The quote would also include the trading volume of each futures contract, which indicates the number of contracts that have been traded on a given day.

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**LO 35.9**

A short can terminate the futures contract by delivering the goods. When the long accepts this delivery, he pays the contract price to the short. This is known as the delivery process. In a cash-settlement contract, delivery is not an option.

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**LO 35.10**

Several different types of orders exist in the marketplace including: market, limit, stop-loss, stop-limit, and market-if-touched orders. Market orders are orders to buy or sell at the best price available. Limit orders are orders to buy or sell away from the current market price. Stop-loss orders are used to prevent losses or to protect profits. Stop-limit orders are a combination of a stop and limit order. Market-if-touched orders are orders that would become market orders once a specified price is reached.

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**LO 35.11**

Futures contracts are similar to forward contracts in that both allow for a transaction to take place at a future date at a price agreed upon today. The difference between the two is that forward contracts are private, customized contracts, while futures trade on an organized exchange and have terms that are highly standardized.

## CONCEPT CHECKERS

1. When an investor is obligated to buy the underlying asset in a futures position, it is a:
  - A. basis trade.
  - B. long-futures position.
  - C. short-futures position.
  - D. hedged-futures position.
2. Which of the following are characteristics specified by a futures contract?
  - I. Asset quality and asset quantity.
  - II. Delivery arrangements and delivery time.
  - A. I only.
  - B. II only.
  - C. Both I and II.
  - D. Neither I nor II.
3. An investor enters into a short position in a gold futures contract with the following characteristics:
  - The initial margin is \$3,000.
  - The maintenance margin is \$2,250.
  - The contract price is \$1,300.
  - Each contract controls 100 troy ounces.If the price drops to \$1,295 at the end of the first day and \$1,290 at the end of the second day, which of the following is closest to the variation margin required at the end of the second day?
  - A. \$0.
  - B. \$250.
  - C. \$500.
  - D. \$1,000.
4. Which of the following items are functions of the clearinghouse?
  - I. Determine which contracts trade.
  - II. Receive margin deposits from brokers.
  - A. I only.
  - B. II only.
  - C. Both I and II.
  - D. Neither I nor II.
5. It is possible that which of the following types of orders may never be executed?
  - A. Limit orders.
  - B. Market-if-touched (MIT) orders.
  - C. Stop-limit orders.
  - D. All of the above.

## CONCEPT CHECKER ANSWERS

1. B When an investor is obligated to buy the underlying asset in a futures position, it is a long futures position.
2. C Delivery time, asset quality, asset quantity, and delivery arrangements are all characteristics specified by the futures contract.
3. A Note that the investor in this question has a short position that profits from price declines. The short position margin account has increased by \$1,000 over the two days, so there is no variation margin required.
4. B The clearinghouse acts as buyer to every seller and seller to every buyer, thus virtually eliminating default risk. It also collects margin payments from clearing members (brokers). Determining which contracts will trade is a function of the exchange, not the clearinghouse.
5. D All of these orders require that the price reach a certain range before being activated. If the price never reaches that range, the order will never be activated.

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The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

# HEDGING STRATEGIES USING FUTURES

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Topic 36

## EXAM FOCUS

Futures contracts are used extensively for implementing hedging strategies. This topic presents the calculations for determining the optimal hedge ratio and shows how to use it to determine the number of futures contracts necessary to hedge a spot market exposure. This topic also addresses basis risk, the change in the relationship between spot prices and futures prices over a hedge horizon. Basis risk arises because an asset being hedged may not be exactly the same as the asset underlying the futures contract.

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## HEDGING WITH FUTURES

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**LO 36.1: Define and differentiate between short and long hedges and identify their appropriate uses.**

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A **short hedge** occurs when the hedger shorts (sells) a futures contract to hedge against a price decrease in the existing long position. When the price of the hedged asset decreases, the short futures position realizes a positive return, offsetting the decline in asset value. Therefore, a short hedge is appropriate when you have a long position and expect prices to decline.

A **long hedge** occurs when the hedger buys a futures contract to hedge against an increase in the value of the asset that underlies a short position. In this case, an increase in the value of the shorted asset will result in a loss to the short seller. The objective of the long hedge is to offset the loss in the short position with a gain from the long futures position. A long hedge is therefore appropriate when you have a short position and expect prices to rise.

## Advantages and Disadvantages of Hedging

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**LO 36.2: Describe the arguments for and against hedging and the potential impact of hedging on firm profitability.**

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The objective of hedging with futures contracts is to reduce or eliminate the price risk of an asset or a portfolio. For example, a farmer with a large corn crop that will be harvested in a few months could wait until the end of the growing season and sell his corn at the prevailing spot price, *or* he could sell corn futures and “lock in” the price of his corn at a predetermined rate. By taking a short position in a corn futures contract, the farmer eliminates—or at least reduces—exposure to fluctuating corn prices. This is an example of a *short hedge*, where the user locks in a future selling price.

Alternatively, a cereal company will need to purchase corn in the future. The company could wait to buy corn in the spot market and face the volatility of future corn spot prices or lock in its purchase price by buying corn futures in advance. This demonstrates an *anticipatory hedge*. The cereal company has an anticipated need for corn and buys corn futures to lock in the price of those future corn purchases. This is an example of a *long hedge*, where the user locks in a future purchasing price.

It is easy to see that the benefit from hedging leads to less uncertainty regarding future profitability. However, there are some arguments against hedging. The main issue is that hedging can lead to less profitability if the asset being hedged ends up increasing in value. The increase in value will be offset by a corresponding loss in the futures contract used for the hedge.

Another argument against hedging is the questionable benefit that accrues to shareholders. Clearly, hedging reduces risk for a company and its shareholders, but there is reason to believe that shareholders can more easily hedge risk on their own. A third argument deals with the nature of the hedging company's industry. For example, assume that prices in an industry frequently adjust for changes in input prices and exchange rates. If competitors do not hedge, then there is an incentive to keep the status quo. In this way, the company ensures that profitability will remain more stable than if it were to hedge frequent changes.

## BASIS RISK

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**LO 36.3: Define the basis and explain the various sources of basis risk, and explain how basis risks arise when hedging with futures.**

**LO 36.4: Define cross hedging, and compute and interpret the minimum variance hedge ratio and hedge effectiveness.**

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When all of the existing position characteristics match perfectly with those of the futures contract specifications, we have a perfect hedge. With a perfect hedge, the loss on a hedged position will be perfectly offset by the gain on the futures position. Perfect hedges are not very common. There are two major reasons why this is so: (1) the asset in the existing position is often not the same as that underlying the futures (e.g., we may be hedging a corporate bond portfolio with a futures contract on a U.S. Treasury bond), and (2) the hedging horizon may not match perfectly with the maturity of the futures contract. The existence of either one of these conditions leads to what is called **basis risk**.

The basis in a hedge is defined as the difference between the spot price on a hedged asset and the futures price of the hedging instrument (e.g., futures contract). Basis is calculated as:

$$\text{basis} = \text{spot price of asset being hedged} - \text{futures price of contract used in hedge}$$

When the hedged asset and the asset underlying the hedging instrument are the same, the basis will be zero at maturity.



*Professor's Note: This is the typical definition for basis (where basis equals spot price minus futures price). However, basis is also sometimes defined as: futures price minus spot price, mostly when dealing with financial asset futures.*

When the spot price increases faster than the futures price over the hedging horizon, basis increases and a strengthening of the basis is said to occur. When the futures price increases faster than the spot price and the basis decreases, a weakening of the basis occurs. When hedging, a change in basis is unavoidable. The change in basis over the hedge horizon is termed *basis risk*, and it can work either for or against a hedger.

To minimize basis risk, hedgers should select the contract on an asset that is most highly correlated with the spot position and a contract maturity that is closest to the hedging horizon. Contract liquidity must also be considered when selecting a futures contract for hedging.

Three sources of basis risk are: (1) interruption in the convergence of the futures and spot prices, (2) changes in the cost of carry, and (3) imperfect matching between the cash asset and the hedge asset. Let's discuss each of these sources in more detail.

1. *Interruption in the convergence of the futures and spot prices.* Normally, spot prices and futures prices will converge as the time to maturity decreases, and basis reduces to zero at maturity. However, if the position is unwound prior to maturity, the return to the futures position could be different from the return to the cash position. A more rapid convergence results in a more rapid transfer of margin payments, while a less rapid convergence would delay payments. An interruption in the convergence could result in payments from the seller to the buyer. All of these effects are types of basis risk.
2. *Changes in the cost of carry.* Significant basis risk can arise due to changes in the components of the cost of carry. The cost of carry includes storage and safekeeping, interest, insurance, and related costs. Perhaps the most volatile of these costs is interest costs. An increase in the interest rates increases the opportunity cost of holding the asset, so the cost of carry and, hence, the difference in the basis of the contract rises.
3. *Imperfect matching between the cash asset and the hedge asset.* Sometimes it may be more efficient to cross hedge or hedge a cash position with a hedge asset that is closely related but different from the cash asset. For example, Eurodollar deposits are closely related to T-bill rates and may be considered a good hedge. However, if there is a structural shock that changes the close relationship of these two assets, the position may not be hedged as effectively as originally believed. This is the most common form of basis risk. Other forms of mismatch include maturity or duration mismatches, liquidity mismatches, and credit risk mismatches:
  - *Maturity or duration mismatch.* Hedging a portfolio of mortgages with 10-year Treasury notes (T-notes) may seem reasonable if the effective duration of the mortgages matches the duration of the T-notes. However, if rates fall and the mortgages prepay faster (resulting in a shorter duration), the position will not be matched.
  - *Liquidity mismatch.* Hedging an illiquid asset with a more liquid one will result in greater basis risk. Although over the long term the prices may be comparable, the difference in liquidity may result in large gaps between the pricing of the two assets. Hence, basis risk is inversely proportional to the liquidity of the hedged asset.

- *Credit risk mismatch.* The widening or narrowing of credit spreads constitutes another form of basis risk when the credit risk of the hedged asset is different (or becomes different) from the credit risk of the hedge instrument.

All of these represent basis risk. The size and type of basis risk can vary during the term of the contract, even if the position is perfectly hedged at maturity.

## The Optimal Hedge Ratio

We can account for an imperfect relationship between the spot and futures positions by calculating an **optimal hedge ratio** that incorporates the degree of correlation between the rates.

A hedge ratio is the ratio of the size of the futures position relative to the spot position. The *optimal hedge ratio*, which minimizes the variance of the combined hedge position, is defined as follows:

$$HR = \rho_{S,F} \frac{\sigma_S}{\sigma_F}$$

This is also the beta of spot prices with respect to futures contract prices since:

$$\rho = \frac{\text{Cov}_{S,F}}{\sigma_S \sigma_F} \text{ and } \frac{\text{Cov}_{S,F}}{\sigma_S \sigma_F} \times \frac{\sigma_S}{\sigma_F} = \frac{\text{Cov}_{S,F}}{\sigma_F^2} = \beta_{S,F}$$

where:

$\rho_{S,F}$  = the correlation between the spot prices and the futures prices

$\sigma_S$  = the standard deviation of the spot price

$\sigma_F$  = the standard deviation of the futures price

### Example: Minimum variance hedge ratio

Suppose a currency trader computed the correlation between the spot and futures to be 0.925, the annual standard deviation of the spot price to be \$0.10, and the annual standard deviation of the futures price to be \$0.125. Compute the hedge ratio.

**Answer:**

$$HR = 0.925 \times \frac{0.100}{0.125} = 0.74$$

The ratio of the size of the futures to the spot should be 0.74.

The **effectiveness of the hedge** measures the variance that is reduced by implementing the optimal hedge. This effectiveness can be evaluated with a coefficient of determination ( $R^2$ ) term where the independent variable is the change in futures prices and the dependent variable is the change in spot prices. Recall that  $R^2$  measures the goodness-of-fit of a regression. As shown previously, the beta of spot prices with respect to futures prices is equal

to the hedge ratio (HR), which is also the slope of this regression. The  $R^2$  measure for this simple linear regression is the square of the correlation coefficient ( $\rho^2$ ) between spot and futures prices.

## HEDGING WITH STOCK INDEX FUTURES

### LO 36.5: Compute the optimal number of futures contracts needed to hedge an exposure, and explain and calculate the “tailing the hedge” adjustment.

A common hedging application is the hedging of equity portfolios using futures contracts on stock indices (index futures). In this application, it is important to remember that the hedged portfolio's beta serves as a hedge ratio when determining the correct number of contracts to purchase or sell. The number of futures contracts required to completely hedge an equity position is determined with the following formula:

$$\begin{aligned}\text{number of contracts} &= \beta_{\text{portfolio}} \times \left( \frac{\text{portfolio value}}{\text{value of futures contract}} \right) \\ &= \beta_{\text{portfolio}} \times \left( \frac{\text{portfolio value}}{\text{futures price} \times \text{contract multiplier}} \right)\end{aligned}$$

#### Example: Hedging with stock index futures

You are a portfolio manager with a \$20 million growth portfolio that has a beta of 1.4, relative to the S&P 500. The S&P 500 futures are trading at 1,150, and the multiplier is 250. You would like to hedge your exposure to market risk over the next few months. Identify whether a long or short hedge is appropriate, and determine the number of S&P 500 contracts you need to implement the hedge.

#### Answer:

You are long the S&P 500, so you should construct a short hedge and sell the futures contract. The number of contracts to sell is equal to:

$$1.4 \times \frac{\$20,000,000}{1,150 \times 250} \approx 97 \text{ contracts}$$

## Tailing the Hedge

A hedger may actually over-hedge the underlying exposure if daily settlement is not properly accounted for. To correct for the possibility of over-hedging, a hedger can implement a **tailing the hedge** strategy. The extra step needed to carry out this strategy is to multiply the hedge ratio by the daily spot price to futures price ratio. In practice, it is not efficient to adjust the hedge for every daily change in the spot-to-futures ratio.

**Example: Tailing the hedge**

Suppose that you would like to make a tailing the hedge adjustment to the number of contracts needed in the previous example. Assume that when evaluating the next daily settlement period you find that the S&P 500 spot price is 1,095 and the futures price is now 1,160. Determine the number of S&P 500 contracts needed after making a tailing the hedge adjustment.

**Answer:**

The number of contracts to sell is equal to:

$$1.4 \times [(\$20,000,000) / (1,150 \times 250)] \times (1,095 / 1,160) = 92 \text{ contracts}$$

**Adjusting the Portfolio Beta****LO 36.6: Explain how to use stock index futures contracts to change a stock portfolio's beta.**

Hedging an existing equity portfolio with index futures is an attempt to reduce the *systematic risk* of the portfolio. If the beta of the capital asset pricing model is used as the systematic risk measure, then hedging boils down to a reduction of the portfolio beta. Let  $\beta$  be our portfolio beta,  $\beta^*$  be our target beta after we implement the strategy with index futures,  $P$  be our portfolio value, and  $A$  be the value of the underlying asset (i.e., the stock index futures contract). To compute the appropriate number of futures, we use the following equation:

$$\text{number of contracts} = (\beta^* - \beta) \frac{P}{A}$$

This equation can result in either positive or negative values. Negative values indicate selling futures (decreasing systematic risk), and positive values indicate buying futures contracts (increasing systematic risk).

**Example: Adjusting portfolio beta**

Suppose we have a well-diversified \$100 million equity portfolio. The portfolio beta relative to the S&P 500 is 1.2. The current value of the 3-month S&P 500 Index is 1,080. The portfolio manager wants to completely hedge the systematic risk of the portfolio over the next three months using S&P 500 Index futures. Demonstrate how to adjust the portfolio's beta.

**Answer:**

In this instance, our target beta,  $\beta^*$ , is 0, since a complete hedge is desired.

$$\text{number of contracts} = (0 - 1.2) \frac{100,000,000}{1,080 \times 250} = -444.44$$

The negative sign tells us we need to sell 444 contracts.

## ROLLING A HEDGE FORWARD

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**LO 36.7: Explain the term “rolling the hedge forward” and describe some of the risks that arise from this strategy.**

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When the hedging horizon is long relative to the maturity of the futures used in the hedging strategy, hedges have to be rolled forward as the futures contracts in the hedge come to maturity or expiration. Typically, as a maturity date approaches, the hedger must close out the existing position and replace it with another contract with a later maturity. This is called **rolling the hedge forward**.

When rolling a hedge forward, hedgers are not only exposed to the basis risk of the original hedge, they are also exposed to the basis risk of a new position each time the hedge is rolled forward. This is referred to as rollover basis risk, or simply **rollover risk**.

## KEY CONCEPTS

### LO 36.1

Hedging may be achieved by shorting futures to protect an underlying position against price deterioration or by buying futures to hedge against unanticipated price increases in an underlying asset.

### LO 36.2

Investors hedge with futures contracts to reduce or eliminate the price risk of an asset or a portfolio. The advantage of hedging is that it leads to less uncertainty regarding future profitability. The disadvantage of hedging is that it can lead to less profitability if the asset being hedged ends up increasing in value.

### LO 36.3

Basis risk is the risk that a difference may occur between the spot price of a hedged asset and the futures price of the contract used to implement the hedge. Basis risk is zero only when there is a perfect match between the hedged asset and the contract's underlying instrument in terms of maturity and asset type.

Three sources of basis risk are: (1) interruption in the convergence of the futures and spot prices, (2) changes in the cost of carry, and (3) imperfect matching between the cash asset and the hedge asset.

### LO 36.4

Sometimes it may be more efficient to cross hedge or hedge a cash position with a hedge asset that is closely related but different from the cash asset.

A hedge ratio is the ratio of the size of the futures position relative to the spot position necessary to provide a desired level of protection.

$$HR = \rho_{\text{spot,futures}} \times \frac{\sigma_{\text{spot}}}{\sigma_{\text{futures}}}$$

The effectiveness of the hedge measures the variance that is reduced by implementing the optimal hedge.

### LO 36.5

A common hedging application is the hedging of equity portfolios using futures contracts on stock indices (index futures). The number of futures contracts required to completely hedge an equity position is determined as follows:

$$\# \text{ of contracts} = \beta_{\text{portfolio}} \times \left( \frac{\text{portfolio value}}{\text{futures price} \times \text{contract multiplier}} \right)$$

**LO 36.6**

When hedging an equity portfolio with a short position in stock index futures, the beta of the portfolio is reduced. To change a stock portfolio's beta, use the following formula:

$$\text{number of contracts} = (\beta^* - \beta) \times \frac{\text{portfolio value}}{\text{value of futures contract}}$$

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**LO 36.7**

When the hedging horizon is longer than the maturity of the futures, the hedge must be rolled forward to retain the hedge. This exposes the hedger to rollover risk, the basis risk when the hedge is re-established.

## CONCEPT CHECKERS

Use the following data to answer Questions 1 and 2.

An equity portfolio is worth \$100 million with the benchmark of the Dow Jones Industrial Average. The Dow is currently at 10,000, and the corresponding portfolio beta is 1.2. The futures multiplier for the Dow is 10.

1. Which of the following is the closest to the number of contracts needed to double the portfolio beta?
  - A. 1,100.
  - B. 1,168.
  - C. 1,188.
  - D. 1,200.
2. To cut the beta in half, the correct trade is:
  - A. long 600 contracts.
  - B. short 600 contracts.
  - C. long 1,200 contracts.
  - D. short 1,200 contracts.
3. Which of the following situations describe a hedger with exposure to basis risk?
  - I. A portfolio manager for a large-cap growth fund knows he will be receiving a significant cash investment from a client within the next month and wants to pre-invest the cash using stock index futures.
  - II. A farmer has a large crop of corn he is looking to sell before June 30. The farmer uses a June futures contract to lock in his sales price.
  - A. I only.
  - B. II only.
  - C. Both I and II.
  - D. Neither I nor II.
4. The standard deviation of price changes in a wheat futures contract is 0.6, while the standard deviation of changes in the price of wheat is 0.75. The covariance between the spot price changes and the futures price changes is 0.3825. Which of the following is closest to the optimal hedge ratio?
  - A. 0.478.
  - B. 0.850.
  - C. 1.063.
  - D. 1.250.

5. A large-cap value equity manager has a \$6,500,000 equity portfolio with a beta of 0.92. An S&P 500 futures contract is available with a current value of 1,175 and a multiplier of 250. What position should the manager take to completely hedge the portfolio's market risk?
- A. Short 20 contracts.
  - B. Short 22 contracts.
  - C. Short 24 contracts.
  - D. Long 22 contracts.

## CONCEPT CHECKER ANSWERS

1. D  $(2.4 - 1.2) \frac{100,000,000}{10,000 \times 10} = 1.2 \times 1,000 = 1,200$

where beta = 1.2, target beta = 2.4, A =  $10 \times 10,000$ , P = \$100 million

2. B  $(0.6 - 1.2) \frac{100,000,000}{10,000 \times 10} = -0.6 \times 1,000 = -600$

where beta = 1.2, target beta = 0.6, A =  $10 \times 10,000$ , P = \$100 million

3. C Both of these situations describe exposure to basis risk—the risk that the difference between the spot price and futures delivery price will change. The portfolio manager using futures to pre-invest the cash does not know the exact date he will receive the cash and may need to sell or hold the futures contract for a longer time period than intended. The farmer may need to sell his June futures contract early if he sells his corn earlier than the June futures expiration date.

4. C Notice in this problem, we were given the covariance but not the correlation. We can calculate the correlation using the formula learned back in the Quantitative Analysis material, as follows:

$$\rho = \frac{\text{COV}_{S,F}}{(\sigma_S)(\sigma_F)} = \frac{0.3825}{(0.75)(0.60)} = 0.85$$

Now that we have our correlation value, we can calculate the minimum hedge ratio as:

$$0.85 \left( \frac{0.75}{0.60} \right) = 1.0625, \text{ or, directly, } \frac{\text{Cov}_{S,F}}{\sigma_F^2} = \frac{0.3825}{0.6^2} = 1.0625$$

5. A  $0.92 \times \frac{6,500,000}{1,175 \times 250} \approx 20 \text{ contracts}$

Because the manager has a long position in the market, she will want to take a short position in the futures.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

# INTEREST RATES

Topic 37

## EXAM FOCUS

Spot, or zero, rates are computed from coupon bonds using a method known as bootstrapping. Forward rates can then be computed from the spot or zero curve. For the exam, understand how to use the bootstrapping method and how to compute forward rates from spot rates. Also, be familiar with the discrete and continuous compounding methods. Note that the fixed income readings in Book 4 will provide more information on the calculation of spot and forward rates as well as constructing the spot and forward rate curves. Duration and convexity are also mentioned in this topic but will be discussed in much more detail in Book 4.

## TYPES OF RATES

**LO 37.1: Describe Treasury rates, LIBOR, and repo rates, and explain what is meant by the “risk-free” rate.**

Three interest rates play a key role in interest rate derivatives: Treasury rates, LIBOR, and repo rates. Keep in mind that interest rates increase as the credit risk of the underlying instrument increases.

- **Treasury rates.** Treasury rates are the rates that correspond to government borrowing in its own currency. They are considered risk-free rates.
- **LIBOR.** The London Interbank Offered Rate (LIBOR) is the rate at which large international banks fund their activities. Some credit risk exists with LIBOR.
- **Repo rates.** The “repo” or repurchase agreement rate is the implied rate on a repurchase agreement. In a repo agreement, one party agrees to sell a security to another with the understanding that the selling party will buy it back later at a specified higher price. The interest rate implied by the price differential is the repo rate. The most common repo is the overnight repurchase agreement. Longer-term agreements are called term repos. Depending on the parties and structure involved, there is some credit risk with repurchase agreements.



*Professor's Note: You may see reference to an inverse floater (a.k.a. reverse floater) on the exam. Just know that an inverse floater is a debt instrument whose coupon payments fluctuate inversely with the reference rate (e.g., LIBOR). For example, the inverse floater's coupon rate will increase when LIBOR decreases and vice versa.*

As mentioned, Treasury rates (such as T-bill and T-bond rates) are often considered the benchmark for nominal risk-free rates. However, derivative traders view these rates as being too low to be considered risk free (since part of the demand for these bonds comes from fulfilling regulatory requirements, which drives prices up and rates down). As a result, traders instead use LIBOR rates for short-term risk-free rates, because LIBOR better reflects a trader's opportunity cost of capital.

## COMPOUNDING

**LO 37.2: Calculate the value of an investment using different compounding frequencies.**

**LO 37.3: Convert interest rates based on different compounding frequencies.**

Derivative pricing often uses a framework called continuous time mathematics. In this framework, it is assumed that returns are continuously compounded. This is a theoretical construct only, as returns cannot literally be compounded continuously. Fortunately, converting discrete compounding to continuous compounding is straightforward.

If we have an initial investment of  $A$  that earns an annual rate  $R$ , compounded  $m$  times a year for  $n$  years, then it has a future value of:

$$FV_1 = A \left(1 + \frac{R}{m}\right)^{m \times n}$$

If our same investment is continuously compounded over that period, it has a future value of:

$$FV_2 = Ae^{R \times n}$$

For any rate,  $R$ ,  $FV_2$  will always be greater than  $FV_1$ . The difference will decrease as  $m$  increases. In fact, as  $m$  becomes infinitely large, the difference goes to zero.

In most circumstances rates are discretely compounded, so we need to find the continuously compounded rate that gives the same future value. Using the previous two equations, the goal is to solve the following:

$$A \left(1 + \frac{R}{m}\right)^{m \times n} = Ae^{R_c n}$$

where:

$R_c$  = the continuous rate

We can solve for  $R_c$  as:

$$R_c = m \times \ln\left(1 + \frac{R}{m}\right)$$

We can also solve for  $R$  as:

$$R = m \left( e^{\frac{R_c}{m}} - 1 \right)$$

*Professor's Note: In order to algebraically solve for  $R$  or  $R_c$ , given one of the equations above, it is helpful to understand that  $e$  is the base of the natural log ( $\ln$ ). In other words, the natural log is the inverse function of the exponential function:  $e^{\ln(x)} = \ln(e^x) = x$ . So if you are given an equation such that  $R = e^x$ ;  $x$  will be equal to:  $\ln(R)$ .*



**Example: Computing continuous rates**

Suppose we have a 5% rate that is compounded semiannually. Compute the corresponding continuous rate. Repeat this for quarterly, monthly, weekly, and daily compounding.

**Answer:**

$$R_c = 2 \ln\left(1 + \frac{0.05}{2}\right) = 0.049385$$

The results for other compounding frequencies are shown in Figure 1.

**Figure 1: Compounding Frequencies and Returns**

<i>m</i>	<i>R<sub>c</sub></i>
4	0.049690
12	0.049896
52	0.049976
250	0.049995

Notice that as *m* increases, the difference between the rates decreases.

**Example: Discrete compounding rate**

A loan is quoted at 12% annually with continuous compounding. Interest is paid monthly. Calculate the equivalent rate with monthly compounding.

**Answer:**

$$R = 12(e^{0.12/12} - 1) = 12.06\%$$

## SPOT (ZERO) RATES AND BOND PRICING

### LO 37.4: Calculate the theoretical price of a bond using spot rates.

**Spot rates** are the rates that correspond to zero-coupon bond yields. They are the appropriate discount rates for a single cash flow at a particular future time or maturity. Spot rates are also often called zero rates. Most interest rates that are observed in the market, such as coupon bond yields, are not spot rates.

## Bond Pricing

A coupon bond makes a series of cash flows. Each cash flow considered in isolation is equivalent to a zero-coupon bond. Using this interpretation, a coupon bond is a series of zero-coupon bonds, and its value, assuming continuous compounding and semiannual coupons, is:

$$B = \left( \frac{c}{2} \times \sum_{j=1}^N e^{-\frac{z_j \times j}{2}} \right) + \left( FV \times e^{-\frac{z_N \times N}{2}} \right)$$

where:

$c$  = the annual coupon

$N$  = the number of semiannual payment periods

$z_j$  = the bond equivalent spot rate that corresponds to  $j$  periods ( $j/2$  years) on a continuously compounded basis

$FV$  = the face value of the bond

Don't let this formula intimidate you. It simply says that the value of a bond is the present value of its cash flows, where each cash flow is discounted at the appropriate spot rate for its maturity. Notice that the negative sign on the rate just means that the coupon and principal payments are being discounted back to the present in a continuous fashion. The following example is a good illustration of the process.

### Example: Calculating bond price

Compute the price of a \$100 face value, 2-year, 4% semiannual coupon bond using the annualized spot rates in Figure 2.

Figure 2: Spot Rates

Maturity (Years)	Spot Rate (%)
0.5	2.5
1.0	2.6
1.5	2.7
2.0	2.9

Answer:

$$B = \left( \$2 \times e^{-\frac{0.025 \times 1}{2}} \right) + \left( \$2 \times e^{-\frac{0.026 \times 2}{2}} \right) + \left( \$2 \times e^{-\frac{0.027 \times 3}{2}} \right) + \left( \$102 \times e^{-\frac{0.029 \times 4}{2}} \right) = \$102.10$$

## Bond Yield

The yield of a bond is the single discount rate that equates the present value of a bond to its market price. You can use a financial calculator to compute bond yield, as in the following example.

**Example: Calculating bond yield**

Compute the yield for the bond in the previous example.

**Answer:**

$$\text{PMT} = 2; N = 4; PV = -102.10; FV = 100; \text{CPT} \rightarrow I/Y = 1.456; \\ Y = 1.456\% \times 2 \approx 2.91\%$$

The bond's **par yield** is the rate which makes the price of a bond equal to its par value. When the bond is trading at par, the coupon will be equal to the bond's yield.

### BOOTSTRAPPING SPOT RATES

The theoretical spot curve is derived by interpreting each Treasury bond (T-bond) as a package of zero-coupon bonds. Using the prices for each bond, the spot curve is computed using the bootstrapping methodology.

For example, suppose there is a T-bond maturing on a coupon date in exactly six months. Further assume that the bond is priced at 102.2969% of par and has a semiannual coupon of 6.125%. How is the corresponding spot rate computed? In this case, this is truly a zero-coupon bond, since there is only one cash flow, which occurs in six months. Simply solve for  $z_1$  in the bond valuation equation, given the price, as follows:

$$102.2969 = \left( \$100 + \frac{\$6.125}{2} \right) \times e^{-\frac{z_1}{2}}$$

Solving this for  $z_1$ :

$$z_1 = -2 \times \ln \left[ \frac{\$102.2969}{\left( \$100 + \frac{\$6.125}{2} \right)} \right] = 1.491\%$$

The 6-month spot rate on a bond equivalent basis is 1.491%. Also note that the yield to maturity did not need to be computed in this case because the yield to maturity (YTM) and the spot rate are the same.

How is the spot rate that corresponds to one year found? Suppose a T-bond that matures in one year is priced at 104.0469% of par and has a semiannual coupon of 6.25%. From the previous computation, the 6-month spot rate is known, so the bond valuation equation can be written as:

$$104.0469 = \left( \frac{\$6.25}{2} \times e^{-\frac{0.01491}{2}} \right) + \left( \$100 + \frac{\$6.25}{2} \right) \times e^{-\frac{z_2 \times 2}{2}}$$

$$\Rightarrow z_2 = 0.02136 = 2.136\%$$

The 1-year spot rate with continuous compounding is 2.136%.

**Example: Bootstrapping spot rates**

Compute the corresponding spot rate curve using the information in Figure 3. Note that we've already computed the first two spot rates.

**Figure 3: Input Information to Bootstrap Spot Rates**

<i>Price as a Percentage of Par</i>	<i>Coupon</i>	<i>Semiannual Period</i>	<i>Maturity (Years)</i>
102.2969	6.125	1	0.5
104.0469	6.250	2	1.0
104.0000	5.250	3	1.5
103.5469	4.750	4	2.0

**Answer:**

The spot rates derived by bootstrapping are shown in Figure 4.

**Figure 4: Bootstrapped Spot Rate Curve**

<i>Price as a Percentage of Par</i>	<i>Coupon</i>	<i>Semiannual Period</i>	<i>Maturity (Years)</i>	<i>Spot Rates</i>
102.2969	6.125	1	0.5	1.491%
104.0469	6.250	2	1.0	2.136%
104.0000	5.250	3	1.5	2.515%
103.5469	4.750	4	2.0	2.915%

An alternative verification is to use the spot rates to check if they result in the same prices using the bond valuation equation. For example, using the spot rates will ensure computation of the same price for the 2-year bond:

$$B = \left( \frac{\$4.75}{2} \times e^{-\frac{0.01491}{2} \times 1} \right) + \left( \frac{\$4.75}{2} \times e^{-\frac{0.02136}{2} \times 2} \right) + \left( \frac{\$4.75}{2} \times e^{-\frac{0.02515}{2} \times 3} \right) + \\ \left( \$100 + \frac{\$4.75}{2} \right) \times e^{-\frac{0.02915}{2} \times 4} = \$103.5469$$

This results in a bond price of \$103.5469. Notice that this is exactly the price of the 2-year bond.

## FORWARD RATES

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### LO 37.5: Derive forward interest rates from a set of spot rates.

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**Forward rates** are interest rates implied by the spot curve for a specified future period. The spot rates in Figure 4 are the appropriate rates that an investor should expect to realize for various maturities. Suppose an investor is faced with the following two investments, which are based on the spot curve in Figure 4.

1. Invest for two years at 2.915%.
2. Invest for a year at 2.136% and then roll over that investment for another year at the forward rate.

It does not matter which investment is chosen if they both offer the same return at the end of two years. This is the same as stating that both strategies give the same future value at the end of two years. Equating the two future values:

$$e^{\frac{0.02915}{2} \times 4} = e^{\frac{0.02136}{2} \times 2} \times e^{\frac{R_{\text{Forward}}}{2} \times 2}$$

where:

$R_{\text{Forward}}$  = the 1-year forward rate one year from now

As we will show, for the two strategies to be equal,  $R_{\text{Forward}}$  must be 3.693%.

We can simplify this calculation by using the following equation:

$$R_{\text{Forward}} = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1} = R_2 + (R_2 - R_1) \times \left( \frac{T_1}{T_2 - T_1} \right)$$

where:

$R_i$  = the spot rate corresponding with  $T_i$  periods

$R_{\text{Forward}}$  = the forward rate between  $T_1$  and  $T_2$

For example, if the 1-year rate is 2.136% and the 2-year rate is 2.915%, the 1-year forward rate one year from now is:

$$R_{\text{Forward}} = 0.02915 + (0.02915 - 0.02136) \times \left( \frac{1}{2-1} \right) = 0.03694 = 3.694\%$$

This is the same forward rate (with slight rounding error) that was calculated before.

As a further example, consider the problem of finding the 1-year forward rate three years from now, given a 3-year spot rate of 7.424% and a 4-year spot rate of 8.216% (both continuously compounded annual rates). Based on the previous formula, the continuously compounded 1-year rate three years from now is:

$$0.08216 + (0.08216 - 0.07424) \times \frac{3}{4-3} = 0.10592$$

With this equation, generalizations can be made between the shape of the spot curve and the forward curve. The second term is always positive for an upward-sloping spot curve. Therefore, when there is an upward-sloping spot curve, the corresponding forward rate curve is upward-sloping and above the spot curve. Similarly, when there is a downward-sloping spot curve, the corresponding forward-rate curve is downward-sloping and below the spot curve.

## FORWARD RATE AGREEMENTS

### LO 37.6: Derive the value of the cash flows from a forward rate agreement (FRA).

A **forward rate agreement** (FRA) is a forward contract obligating two parties to agree that a certain interest rate will apply to a principal amount during a specified future time. Obviously, forward rates play a crucial role in the valuation of FRAs. The  $T_2$  cash flow of an FRA that promises the receipt or payment of  $R_K$  is:

$$\text{cash flow (if receiving } R_K) = L \times (R_K - R) \times (T_2 - T_1)$$

$$\text{cash flow (if paying } R_K) = L \times (R - R_K) \times (T_2 - T_1)$$

where:

$L$  = principal

$R_K$  = annualized rate on  $L$ , expressed with compounding period  $T_1 - T_2$

$R$  = annualized actual rate, expressed with compounding period  $T_1 - T_2$

$T_i$  = time  $i$ , expressed in years

The value of an FRA if we're receiving or paying is:

$$\text{value (if receiving } R_K) = L \times (R_K - R_{\text{Forward}}) \times (T_2 - T_1) \times e^{-R_2 \times T_2}$$

$$\text{value (if paying } R_K) = L \times (R_{\text{Forward}} - R_K) \times (T_2 - T_1) \times e^{-R_2 \times T_2}$$

where:

$R_{\text{Forward}}$  = forward rate between  $T_1$  and  $T_2$

Note that  $R_2$  is expressed as a continuously compounded rate.

#### Example: Computing the payoff from an FRA

Suppose an investor has entered into an FRA where he has contracted to pay a fixed rate of 3% on \$1 million based on the quarterly rate in three months. Assume that rates are compounded quarterly. Compute the payoff of the FRA if the quarterly rate is 1% in three months.

**Answer:**

For this FRA, the payoff will take place in six months. The net payoff will be the difference between the fixed-rate payment and the floating rate receipt. If the floating rate is 1% in three months, the payoff at the end of the sixth month will be:

$$\$1,000,000 (0.01 - 0.03)(0.25) = -\$5,000$$

**Example: Computing the value of an FRA**

Suppose the 3-month and 6-month LIBOR spot rates are 4% and 5%, respectively (continuously compounded rates). An investor enters into an FRA in which she will receive 8% (assuming quarterly compounding) on a principal of \$5,000,000 between months 3 and 6. Calculate the value of the FRA.

**Answer:**

$$R_{\text{Forward}} = 0.05 + (0.05 - 0.04) \times \left( \frac{1}{2-1} \right) = 0.06 = 6\%$$

$$R_{\text{Forward}} (\text{with quarterly compounding}) = 4 \times \left( e^{\frac{0.06}{4}} - 1 \right) = 0.060452 = 6.05\%$$

$$\text{value} = \$5,000,000 \times (0.0800 - 0.0605) \times (0.50 - 0.25) \times e^{-(0.05)(0.5)} = \$23,773$$

**DURATION****LO 37.7: Calculate the duration, modified duration, and dollar duration of a bond.**

The duration of a bond is the average time until the cash flows on the bond are received. For a zero-coupon bond, this is simply the time to maturity. For a coupon bond, its duration will be necessarily shorter than its maturity. The weights on the time in years until each cash flow is to be received are the proportion of the bond's value represented by each of the coupon payments and the maturity payment. The formula for duration using continuously compounded discounting of the cash flows is:

$$\text{duration} = \sum_{i=1}^n t_i \left[ \frac{c_i e^{-yt_i}}{B} \right]$$

where:

$t_i$  = the time (in years) until cash flow  $c_i$  is to be received

$y$  = the continuously compounded yield (discount rate) based on a bond price of  $B$

The usefulness of the duration measure lies in the fact that the approximate change in a bond's price,  $B$ , for a parallel shift in the yield curve of  $\Delta y$  is:

$$\frac{\Delta B}{B} = -\text{duration} \times \Delta y$$

The change in yield is often expressed as a **basis point** change. One basis point is equivalent to 0.01%. So a 100 basis point change is a change of 1% in the yield.

**Modified duration** is used when the yield given is something other than a continuously compounded rate. When the yield is expressed as a semiannually compounded rate, for example, modified duration = duration/(1 +  $y/2$ ). In general we can express this relation as: modified duration =  $\frac{\text{duration}}{1+\frac{y}{m}}$ , where  $m$  is the number of compounding periods per year.

Note that as  $m$  goes to infinity (continuous compounding), the two measures are equal and there is no difference between the two.

On the exam, you may also see a reference to **dollar duration**. Dollar duration is simply modified duration multiplied by the price of the bond.

## CONVEXITY

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### LO 37.8: Evaluate the limitations of duration and explain how convexity addresses some of them.

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Duration is a good approximation of price changes for an option-free bond, but it's only good for relatively small changes in interest rates. As rate changes grow larger, the curvature of the bond price/yield relationship becomes more important, meaning that a linear estimate of price changes, such as duration, will contain errors.

In fact, the relationship between bond price and yield is not linear (as assumed by duration) but convex. This convexity shows that the difference between actual and estimated prices widens as the yield swings grow. That is, the widening error in the estimated price is due to the curvature of the actual price path. This is known as the **degree of convexity**.

Fortunately, the amount of convexity in a bond can be measured and used to supplement duration in order to achieve a more accurate estimate of the change in price. It's important to note that all convexity does is account for the amount of error in the estimated price change based on duration. In other words, it picks up where duration leaves off and converts the straight (estimated price) line into a curved line that more closely resembles the convex (actual price) line.

## Using Convexity to Improve Price Change Estimates

In order to obtain an estimate of the percentage change in price due to convexity, or the amount of price change that is not explained by duration, the following calculation will need to be made:

$$\text{convexity effect} = 1/2 \times \text{convexity} \times \Delta y^2$$

The convexity effect is typically quite small. However, remember that convexity is simply correcting for the error embedded in the duration, so you would expect convexity to have a much smaller effect than duration. Also note that for an option-free bond, the convexity effect is always positive, no matter which direction interest rates move. Thus, for option-free bonds, convexity is always added to duration to modify the price volatility errors embedded in duration. This decreases the drop in price (due to an increase in yields) and adds to the rise in price (due to a fall in yields).

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**LO 37.9: Calculate the change in a bond's price given its duration, its convexity, and a change in interest rates.**

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By combining duration and convexity, we can obtain a far more accurate estimate of the percentage change in the price of a bond, especially for large swings in yield. That is, you can account for the amount of convexity embedded in a bond by adding the convexity effect to the duration effect.

**Example: Estimating price changes with the duration/convexity approach**

Estimate the effect of a 100 basis point increase and decrease on a 10-year, 5%, option-free bond currently trading at par, using the duration/convexity approach. The bond has a duration of 7 and a convexity of 90.

**Answer:**

Using the duration/convexity approach:

$$\text{percentage bond price change} \approx \text{duration effect} + \text{convexity effect}$$

$$\Delta B_{+\Delta y} \approx [-7 \times 0.01] + [(1/2) \times 90 \times (0.01^2)]$$

$$\approx -0.07 + 0.0045 = -0.0655 = -6.55\%$$

$$\Delta B_{-\Delta y} \approx [-7 \times -0.01] + [(1/2) \times 90 \times (-0.01^2)]$$

$$\approx 0.07 + 0.0045 = 0.0745 = 7.45\%$$

## THEORIES OF THE TERM STRUCTURE

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**LO 37.10: Compare and contrast the major theories of the term structure of interest rates.**

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The expectations theory suggests that forward rates correspond to expected future spot rates. That is, forward rates are good predictors of expected future spot rates. In reality, the expectations theory fails to explain all future spot rate expectations. The **market segmentation theory** states that the bond market is segmented into different maturity sectors and that supply and demand for bonds in each maturity range dictate rates in that maturity range. The **liquidity preference theory** suggests that most depositors prefer short-term liquid deposits. In order to coax them to lend longer term, the intermediary will raise longer-term rates by adding a liquidity premium.

## KEY CONCEPTS

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### LO 37.1

Three types of interest rates are particularly relevant in the interest rate derivative markets: Treasury rates, London Interbank Offered Rate (LIBOR), and repo rates. Treasury rates (such as T-bill and T-bond rates) are often considered the benchmark for nominal risk-free rates.

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### LO 37.2

If we have an initial investment of  $A$  that earns an annual rate  $R$ , compounded  $m$  times a year for  $n$  years, then it has a future value of:

$$FV = A \left(1 + \frac{R}{m}\right)^{m \times n}$$

---

### LO 37.3

In most circumstances, rates are discretely compounded so we need to find the continuously compounded rate that gives the same future value. The continuous rate can be solved as follows:

$$R_c = m \times \ln\left(1 + \frac{R}{m}\right)$$

---

### LO 37.4

Zero (spot) rates correspond to the interest earned on a single cash flow at a single point in time. Bond prices are computed using the spot curve by discounting each cash flow at the appropriate spot rate.

The yield of a bond is the single discount rate that equates the present value of a bond to its market price.

Zero rates are computed using the bootstrapping methodology.

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### LO 37.5

Forward rates are computed from spot rates. When the spot curve is upward-sloping, the corresponding forward rate curve is upward-sloping and above the spot curve. When the spot curve is downward-sloping, the corresponding forward rate curve is downward-sloping and below the spot curve.

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**LO 37.6**

A forward-rate agreement is a contract between two parties that an interest rate will apply to a specific principal during some future time period.

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**LO 37.7**

Duration and modified duration are the same when continuously compounded yields are used, and they both estimate the percentage price change of a bond from an absolute change in yield. Dollar duration is modified duration multiplied by the price of the bond.

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**LO 37.8**

Duration is only good for relatively small changes in interest rates. As rate changes grow larger, the curvature of the bond price/yield relationship becomes more important, meaning that a linear estimate of price changes, such as duration, will contain errors. The amount of convexity in a bond can be measured and used to supplement duration in order to achieve a more accurate estimate of the change in price.

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**LO 37.9**

The approximate change in a bond's price,  $B$ , for a parallel shift in the yield curve of  $\Delta y$  is:

$$\frac{\Delta B}{B} = -\text{duration} \times \Delta y$$

In order to obtain an estimate of the percentage change in price due to convexity, the following calculation will need to be made:

$$\text{convexity effect} = \frac{1}{2} \times \text{convexity} \times \Delta y^2$$

Combining duration and convexity creates a more accurate estimate of the percentage change in the price of a bond:

$$\text{percentage bond price change} \approx \text{duration effect} + \text{convexity effect}$$

---

**LO 37.10**

The expectations theory suggests that forward rates correspond to expected future spot rates. The market segmentation theory states that bonds are segmented into different maturity sectors and that supply and demand dictate rates in the segmented maturity sectors. The liquidity preference theory suggests that longer-term rates incorporate a liquidity premium.

## CONCEPT CHECKERS

1. What is the continuously compounded rate of return for an investment that has a value today of \$86.50 and will have a future value of \$100 in one year?
  - A. 13.62%.
  - B. 14.50%.
  - C. 15.61%.
  - D. 16.38%.
2. Assume that the continuously compounded 10-year spot rate is 5% and the 9-year spot rate is 4.9%. Which of the following is closest to the 1-year forward rate nine years from now?
  - A. 4.1%.
  - B. 5.1%.
  - C. 5.9%.
  - D. 6.0%.
3. An investor enters into a 1-year forward rate agreement (FRA) where she will receive the contracted rate on a principal of \$1 million. The contracted rate is a 1-year rate at 5%. Which of the following is closest to the cash flow if the actual rate is 6% at maturity of the underlying asset (loan)?
  - A. -\$10,000.
  - B. -\$1,000.
  - C. +\$1,000.
  - D. +\$10,000.
4. What is the bond price of a \$100 face value, 2.5-year, 3% semiannual coupon bond using the following annual continuously compounded spot rates:  $z_1 = 3\%$ ,  $z_2 = 3.1\%$ ,  $z_3 = 3.2\%$ ,  $z_4 = 3.3\%$ , and  $z_5 = 3.4\%$ ?
  - A. \$97.27.
  - B. \$97.83.
  - C. \$98.15.
  - D. \$98.99.
5. A \$100 face value, 1-year, 4% semiannual bond is priced at 99.806128. If the annualized 6-month spot rate ( $z_1$ ) is 4.1%, what is the 1-year spot rate ( $z_2$ )? (Both spots are continuously compounded rates.)
  - A. 4.07%.
  - B. 4.16%.
  - C. 4.20%.
  - D. 4.26%.

## CONCEPT CHECKER ANSWERS

1. B The formula to solve this problem is:

$$R_c = m \times \ln\left(1 + \frac{R}{m}\right)$$

First, we need to compute  $R$  as the rate earned on the \$86.50 investment:

$$R = \frac{\$100 - \$86.50}{\$86.50} = 0.15607$$

This is essentially the effective rate earned over one year with annual compounding.

So,  $m = 1$ , and  $R_c = 1 \times \ln(1.15607) = 0.1450$ . Alternatively, since  $m = 1$ ,

$$\ln\left(\frac{100}{86.50}\right) = 0.1450 = 14.50\%$$

2. C  $R_{Forward} = R_2 + (R_2 - R_1) \times [T_1 / (T_2 - T_1)] = 0.05 + (0.05 - 0.049) \times [9 / (10 - 9)] = 5.9\%$

3. A  $\$1,000,000 (0.05 - 0.06)(1) = -\$10,000$

4. D  $B = 1.5 \times e^{[-0.03/2] \times 1} + 1.5 \times e^{[-0.031/2] \times 2} + 1.5 \times e^{[-0.032/2] \times 3} + 1.5 \times e^{[-0.033/2] \times 4} + 101.5 \times e^{[-0.034/2] \times 5} = 1.48 + 1.45 + 1.43 + 1.40 + 93.23 = \$98.99$

5. B  $B = 2 \times e^{[-z_1/2] \times 1} + 102 \times e^{[-z_2/2] \times 2}; \$99.806128 = 2 \times e^{[-0.041/2] \times 1} + 102 \times e^{[-z_2/2] \times 2}; \$97.846711 = 102 \times e^{[-z_2/2] \times 2}; z_2 = 0.0415707 = 4.16\%$

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The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

# DETERMINATION OF FORWARD AND FUTURES PRICES

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Topic 38

## EXAM FOCUS

Both forward and futures contracts are obligations regarding a future transaction. Because the difference in pricing between these contract types is small, forward contract pricing and futures contract pricing are often presented interchangeably. The basic model for forward prices is the cost-of-carry model, which essentially connects the forward price to the cost incurred from purchasing and storing the underlying asset until the contract maturity date. Cash flows over the life of the contract are easily incorporated into the pricing model. Futures contracts contain delivery options that benefit the short seller of the contract. These delivery options must be incorporated into the futures pricing model.

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## INVESTMENT AND CONSUMPTION ASSETS

### LO 38.1: Differentiate between investment and consumption assets.

An **investment asset** is an asset that is held for the purpose of investing. This type of asset is held by many different investors for the sake of investment. Examples of investment assets include stocks and bonds. A **consumption asset** is an asset that is held for the purpose of consumption. Examples of consumption assets include commodities such as oil and natural gas.

## SHORT-SELLING AND SHORT SQUEEZE

### LO 38.2: Define short-selling and calculate the net profit of a short sale of a dividend-paying stock.

**Short sales** are orders to sell securities that the seller does not own. Short selling is also known as “shorting” and is possible with some investment assets. For a short sale, the short seller (1) simultaneously borrows and sells securities through a broker, (2) must return the securities at the request of the lender or when the short sale is closed out, and (3) must keep a portion of the proceeds of the short sale on deposit with the broker.

The short seller may be forced to close his position if the broker runs out of securities to borrow. This is known as a **short squeeze**, and the seller will need to close his short position immediately.

Why would anyone ever want to sell securities short? The seller thinks the current price is too high and that it will fall in the future, so the short seller hopes to sell high and then buy low. If a short sale is made at \$30 per share and the price falls to \$20 per share, the short seller can buy shares at \$20 to replace the shares borrowed and keep \$10 per share as profit.

Two rules currently apply to short selling:

1. The short seller must pay all dividends due to the lender of the security.
2. The short seller must deposit collateral to guarantee the eventual repurchase of the security.

**Example: Net profit of a short sale of a dividend-paying stock**

Assume that trader Alex Rodgers sold short XYZ stock in March by borrowing 200 shares and selling them for \$50/share. In April, XYZ stock paid a dividend of \$2/share. Calculate the net profit from the short sale assuming Rodgers bought back the shares in June for \$40/share in order to replace the borrowed shares and close out his short position.

**Answer:**

The cash flows from the short sale on XYZ stock are as follows:

March: borrow 200 shares and sell them for \$50/share	+\$10,000
April: short seller dividend payment to lender of \$2/share	-\$400
June: buyback shares for \$40/share to close short position	<u>-\$8,000</u>
Total net profit =	+\$1,600

## FORWARD AND FUTURES CONTRACTS

**LO 38.3: Describe the differences between forward and futures contracts and explain the relationship between forward and spot prices.**

**LO 38.4: Calculate the forward price given the underlying asset's spot price, and describe an arbitrage argument between spot and forward prices.**

**LO 38.9: Calculate, using the cost-of-carry model, forward prices where the underlying asset either does or does not have interim cash flows.**

Futures contracts and forward contracts are *similar* in that both:

- Can be either deliverable or cash settlement contracts.
- Are priced to have zero value at the time an investor enters into the contract.

Futures contracts *differ* from forward contracts in the following ways:

- Futures contracts trade on organized exchanges. Forwards are private contracts and do not trade on an exchange.
- Futures contracts are highly standardized. Forwards are customized contracts satisfying the needs of the parties involved.

- A single clearinghouse is the counterparty to all futures contracts. Forwards are contracts with the originating counterparty.
- The government regulates futures markets. Forward contracts are usually not regulated.

## FORWARD PRICES

The pricing model used to compute forward prices makes the following assumptions:

- No transaction costs or short-sale restrictions.
- Same tax rates on all net profits.
- Borrowing and lending at the risk-free rate.
- Arbitrage opportunities are exploited as they arise.

For the development of a forward pricing model, we will use the following notation:

- $T$  = time to maturity (in years) of the forward contract.
- $S_0$  = underlying asset price today ( $t = 0$ ).
- $F_0$  = forward price today.
- $r$  = continuously compounded risk-free annual rate.

The forward price may be written as:

### Equation 1

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$$F_0 = S_0 e^{rT}$$

The right-hand side of Equation 1 is the cost of borrowing funds to buy the underlying asset and carrying it forward to time  $T$ . Equation 1 states that this cost must equal the forward price. If  $F_0 > S_0 e^{rT}$ , then arbitrageurs will profit by selling the forward and buying the asset with borrowed funds. If  $F_0 < S_0 e^{rT}$ , arbitrageurs will profit by selling the asset, lending out the proceeds, and buying the forward. Hence, the equality in Equation 1 must hold. Note that this model assumes perfect markets.

As it turns out, actual short sales are not necessary for Equation 1 to hold. All that is necessary is a sufficient number of investors who are not only holding the investment asset but also are willing to sell the asset if the forward price becomes too low. In the event that the forward price is too low, the investor will sell the asset and take a long position in the forward contract. This is important since the arbitrage relationship in Equation 1 must hold for all investment assets even though short selling is not available for every asset.

#### Example: Computing a forward price with no interim cash flows

Suppose we have an asset currently worth \$1,000. The current continuously compounded rate is 4% for all maturities. Compute the price of a 6-month forward contract on this asset.

**Answer:**

$$F_0 = \$1,000 e^{0.04(0.5)} = \$1,020.20$$

## Forward Price With Carrying Costs

If the underlying pays a known amount of cash over the life of the forward contract, a simple adjustment is made to Equation 1. Since the owner of the forward contract does not receive any of the cash flows from the underlying asset between contract origination and delivery, the present value of these cash flows must be deducted from the spot price when calculating the forward price. This is most easily seen when the underlying asset makes a periodic payment. With this in mind, we let  $I$  represent the *present value* of the cash flows over  $T$  years. Equation 1 then becomes:

### Equation 2

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$$F_0 = (S_0 - I) e^{rT}$$

The same arbitrage arguments used for Equation 1 are used here. The only modification is that the arbitrageur must account for the known cash flows.

#### Example: Forward price when underlying asset has a cash flow

Compute the price of a 6-month forward on a coupon bond worth \$1,000 that pays a 5% coupon semiannually. A coupon is to be paid in three months. Assume the risk-free rate is 4%.

**Answer:**

The cost of carry (income) in this case is computed as:

$$I = 25e^{-0.04(0.25)} = \$24.75125$$

Using Equation 2:

$$F_0 = (\$1,000 - \$24.75125)e^{0.04(0.5)} = \$994.95$$

## The Effect of a Known Dividend

When the underlying asset for a forward contract pays a dividend, we assume that the dividend is paid continuously. Letting  $q$  represent the continuously compounded dividend yield paid by the underlying asset expressed on a per annum basis, Equation 1 becomes:

### Equation 3

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$$F_0 = S_0 e^{(r-q)T}$$

Once again, the same arbitrage arguments are used to prove that Equation 3 must be true.

**Example: Forward price when the underlying asset pays a dividend**

Compute the price of a 6-month forward contract for which the underlying asset is a stock index with a value of 1,000 and a continuous dividend yield of 1%. Assume the risk-free rate is 4%.

**Answer:**

Using Equation 3:

$$F_0 = 1,000e^{(0.04 - 0.01)0.5} = 1,015.11$$

**VALUE OF A FORWARD CONTRACT**

The initial value of a forward contract is zero. After its inception, the contract can have a positive value to one counterparty (and a negative value to the other). Since the forward price at every moment in time is computed to prevent arbitrage, the value at inception of the contract must be zero. The forward contract can take on a non-zero value only after the contract is entered into and the obligation to buy or sell has been made. If we denote the obligated delivery price after inception as  $K$ , then the value of the long contract on an asset with no cash flows is computed as  $S_0 - Ke^{-rT}$ ; with cash flows (with present value  $I$ ) it is  $S_0 - I - Ke^{-rT}$ ; and with a continuous dividend yield of  $q$ , it is  $S_0e^{-qT} - Ke^{-rT}$ .

**Example: Value of a stock index forward contract**

Using the stock index forward in the previous example, compute the value of a long position if the index increases to 1,050 immediately after the contract is purchased.

**Answer:**

In this case,  $K = 1,015.11$  and  $S_0 = 1,050$ , so the value is:

$$1,050e^{-0.01(0.5)} - 1,015.11e^{-0.04(0.5)} = 49.75$$

**CURRENCY FUTURES****LO 38.6: Calculate a forward foreign exchange rate using the interest rate parity relationship.**

Interest rate parity (IRP) states that the forward exchange rate,  $F$  [measured in domestic currency (DC) per unit of foreign currency (FC)], must be related to the spot exchange rate,  $S$ , and to the interest rate differential between the domestic and the foreign country,  $r_{DC} - r_{FC}$ .

The general form of the interest rate parity condition is expressed as:

$$F = S e^{(r_{DC} - r_{FC})T}$$

This equation is a no-arbitrage relationship. Using our notation from earlier, we can state the interest rate parity relationship as:

#### Equation 4

$$F_0 = S_0 e^{(r_{DC} - r_{FC})T}$$

Note that this is equivalent to Equation 3 with  $r_{FC}$  replacing  $q$ . Just as the continuous dividend yield  $q$  was used to adjust the cost of carry, we use the continuous yield on a foreign currency deposit here.

#### Example: Currency futures pricing

Suppose we wish to compute the futures price of a 10-month futures contract on the Mexican peso. Each contract controls 500,000 pesos and is quoted in terms of dollar/peso. Assume that the continuously compounded risk-free rate in Mexico ( $r_{FC}$ ) is 14%, the continuously compounded risk-free rate in the United States is 2%, and the current exchange rate is 0.12.

**Answer:**

Applying Equation 4:

$$F_0 = \$0.12 e^{(0.02 - 0.14) \frac{10}{12}} = \$0.10858 / \text{peso}$$



*Professor's Note: The concept of interest rate parity will show up again in the foreign exchange risk topic (Topic 49).*

## FORWARD PRICES VS. FUTURES PRICES

### LO 38.5: Explain the relationship between forward and futures prices.

The most significant difference between forward contracts and futures contracts is the daily marking to market requirement on futures contracts. When interest rates are known over the life of a contract,  $T$ , forward and futures prices can be shown to be the same. Various relationships can be derived, depending on the assumptions made between the value of the underlying and the level of change in interest rates. In general, when  $T$  is small, the price differences are usually very small and can be ignored. Empirical research comparisons of forwards and futures prices are mixed. Some studies conclude a significant difference and others do not. The important concept to understand here is that assuming the two are the same is an approximation, and under certain circumstances the approximation can be inaccurate.

## COMMODITY FUTURES

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**LO 38.7: Define income, storage costs, and convenience yield.**

**LO 38.8: Calculate the futures price on commodities incorporating income/storage costs and/or convenience yields.**

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*Professor's Note: Topic 45 later in this book is devoted to commodity forwards and futures. In that topic, you will learn more about storage costs and convenience yield as well as the arbitrage relationships that must hold with commodity futures.*

### Income and Storage Costs

When the underlying is considered a consumption asset, the pricing relationships developed above do not adequately capture all the necessary characteristics of the asset. *Consumption assets have actual storage costs associated with them.* These costs increase the carrying costs. The costs can be expressed either as a known cash flow or as a yield. Let  $U$  denote the present value of known storage cost over the life of the forward contract. Equation 1 then becomes:

#### Equation 5

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$$F_0 = (S_0 + U)e^{rT}$$

If we express the storage costs in terms of a continuous yield,  $u$ :

#### Equation 6

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$$F_0 = S_0 e^{(r+u)T}$$

The arbitrage relationships are the same except we need to account for the additional carrying costs over  $T$  years. However, when the owner of these assets is reluctant to sell the asset, Equations 5 and 6 are replaced by:

#### Equation 7

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$$F_0 \leq (S_0 + U)e^{rT}$$

And:

#### Equation 8

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$$F_0 \leq S_0 e^{(r+u)T}$$

## CONVENIENCE YIELD

Equations 7 and 8 suggest there is a *benefit to owning the underlying consumable asset compared to owning the futures contract*. If we introduce a **convenience yield**,  $y$ , to balance Equations 7 and 8, we have:

$$F_0 e^{yT} = (S_0 + U) e^{rT} = S_0 e^{(r+u)T}$$

This formula can be reduced to:

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### Equation 9

$$F_0 = S_0 e^{(r+u-y)T}$$

In other words, the convenience yield is simply the yield required to produce an equality and is thus a measure of the benefit of owning spot, or physical, consumption commodities.

## DELIVERY OPTIONS IN THE FUTURES MARKET

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### LO 38.10: Describe the various delivery options available in the futures markets and how they can influence futures prices.

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Some futures contracts grant **delivery options** to the short—options on what, where, and when to deliver. Some Treasury bond contracts give the short a choice of several bonds that are acceptable to deliver and options as to when to deliver during the expiration month. Physical assets, such as gold or corn, may offer a choice of delivery locations to the short. These options can be of significant value to the holder of the short position in a futures contract.

As shown in the previous discussion on commodity futures, if the cost of carrying the asset is greater than the convenience yield (benefit from holding the physical asset), it is ideal for the short position to deliver the contract early. This scenario suggests that the futures price will increase over time; hence, the short has an incentive to deliver early. The opposite relationship holds true when the cost of carry is less than the convenience yield. In this case, the short position will delay delivery since the futures price is expected to fall over time.

## FUTURES AND EXPECTED FUTURE SPOT PRICES

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### LO 38.11: Explain the relationship between current futures prices and expected future spot prices, including the impact of systematic and nonsystematic risk.

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The cost of carry model is a widely used method for estimating the appropriate price of a futures contract, but other theories exist for explaining the futures price. One intuitively appealing model expresses the futures price as a function of the expected spot price ( $S_T$ ).

$$F_0 = E(S_T)$$

For obvious reasons, this is called the **expectations model** and states that the current futures price for delivery at time  $T$  is equal to the expected spot price at time  $T$ . Similar to the no-arbitrage rule, this model acts to keep the current futures price in line with the expected spot rate at that time. If the futures price is less than the expected price, aggressive buying of the futures would push up the futures price. If the futures price is greater than the expected spot rate, aggressive selling of the futures would lead to lower the futures price. Although intuitively appealing, other factors probably play a role in the pricing mechanism. Indeed, if the expectations model limited traders to a risk-free rate of return, there would be no incentive to buy or sell contracts.

### Cost of Carry vs. Expectations

Economist John Maynard Keynes found the expectations model to be flawed precisely because it provided no justification for speculators to enter the market. Futures contracts provide a mechanism to transfer risk from those who need to hedge their positions (e.g., farmers who are long the commodities) to speculators. In order to entice speculators to bear the risk of these contracts, there has to exist an expectation of profit greater than the risk-free rate. For this to occur, the futures contract price must be less than the expected spot rate at maturity [ $F_0 < E(S_T)$ ] and must continually increase during the term of the contract. Keynes referred to this as **normal backwardation**. This relationship suggests that the asset underlying the futures contract exhibits positive systematic risk, since this is the risk that remains after diversifying away all nonsystematic risk.

On the other side of the contracts are those who are users of the commodity who want to shift some of the risk of rising market prices to speculators. They wish to purchase futures contracts from speculators. The speculators have to be enticed into assuming this risk by the expectation of profits that would exceed the risk-free rate. From this perspective, the futures price must be higher than the expected spot price at maturity [ $F_0 > E(S_T)$ ] and must continually decrease during the term of the contract. Keynes referred to this expectation as **contango** (a.k.a. normal contango). This relationship suggests that the asset underlying the futures contract exhibits negative systematic risk.

### CONTANGO AND BACKWARDATION

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#### LO 38.12: Define and interpret contango and backwardation, and explain how they relate to the cost-of-carry model.

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**Backwardation** refers to a situation where the futures price is below the spot price. For this to occur, there must be a significant benefit to holding the asset. Backwardation might occur if there are benefits to holding the asset that offset the opportunity cost of holding the asset (the risk-free rate) and additional net holding costs.

**Contango** refers to a situation where the futures price is above the spot price. If there are no benefits to holding the asset (e.g., dividends, coupons, or convenience yield), contango will occur because the futures price will be greater than the spot price.



*Professor's Note: In this case, the reference to backwardation and contango refers to the relationship between the futures price and the current spot price, not the expected spot price.*

## KEY CONCEPTS

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### LO 38.1

An investment asset is an asset that is held for the purpose of investing. A consumption asset is an asset that is held for the purpose of consumption.

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### LO 38.2

Short sales are orders to sell securities that the seller does not own. A short squeeze results if the broker runs out of securities to borrow.

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### LO 38.3

Forward and futures contracts are similar because they are both future obligations to transact an asset on some future date. Forward contracts do not trade on an exchange, are not standardized, and do not normally close out prior to expiration.

The relationship between forward and spot prices is as follows:

$$F = S_0 e^{rT}$$

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### LO 38.4

The cost-of-carry model is used to price forward and futures contracts. It states that the total cost of carrying the underlying asset to expiration must be the futures price. Any other price results in arbitrage.

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### LO 38.5

When interest rates are known over the life of a contract, forward and futures prices can be shown to be the same. Various relationships can be derived, depending on the assumptions made between the value of the underlying and the level of change in interest rates.

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### LO 38.6

Interest rate parity states that the forward exchange rate,  $F$  [measured in domestic currency (DC) per unit of foreign currency (FC)], must be related to the spot exchange rate,  $S$ , and to the interest rate differential between the domestic and the foreign country:

$$F = S_0 e^{(r_{DC} - r_{FC})T}$$

**LO 38.7**

Consumption assets have actual storage costs (known as carrying costs) associated with them.

If there is a benefit to owning the underlying consumable asset compared to owning the futures, the futures price will incorporate a convenience yield.

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**LO 38.8**

Futures price with storage costs,  $u$ :  $F = S_0 e^{(r+u)T}$

Futures price with convenience yield,  $y$ :  $F = S_0 e^{(r+u-y)T}$

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**LO 38.9**

The futures price or cost-of-carry model is easily accommodated for interim cash flows from the underlying asset. If the underlying asset pays a known amount of cash,  $I$ , over the life of the forward contract, a simple adjustment is made to the cost-of-carry model:

$$F = (S_0 - I)e^{rT}$$

When the underlying asset pays a dividend,  $q$ , we assume that the dividend is paid continuously:

$$F = S_0 e^{(r-q)T}$$

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**LO 38.10**

Physical assets, such as gold or corn, may offer a choice of delivery locations to the short. These options can be of significant value to the holder of the short position in a futures contract. Futures contracts are typically “offset” by buying or selling a contract before the delivery date. Only a small percentage of contracts result in physical delivery.

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**LO 38.11**

The expectations model states that the current futures price for delivery at time  $T$  is equal to the expected spot price at time  $T$ . This model acts to keep the current futures price in line with the expected spot rate at that time.

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**LO 38.12**

Contango is the situation in which the futures price is above the current spot price. Backwardation is the opposite relationship.

## CONCEPT CHECKERS

Use the following data to answer Questions 1 and 2.

An investor has an asset that is currently worth \$500, and the continuously compounded risk-free rate at all maturities is 3%.

1. Which of the following is the closest to the no-arbitrage price of a 3-month forward contract?
  - A. \$496.26.
  - B. \$500.00.
  - C. \$502.00.
  - D. \$503.76.
2. If the asset pays a continuous dividend of 2%, which of the following is the closest to the no-arbitrage price of a 3-month forward contract?
  - A. \$494.24.
  - B. \$498.75.
  - C. \$501.25.
  - D. \$506.29.
3. A bond pays a semiannual coupon of \$40 and has a current value of \$1,109. The next payment on the bond is in four months and the interest rate is 6.50%. Using the continuous time model, the price of a 6-month forward contract on this bond is closest to:
  - A. \$995.62.
  - B. \$1,011.14.
  - C. \$1,035.65.
  - D. \$1,105.20.
4. The owner of 300,000 bushels of corn wishes to hedge his position for a sale in 150 days. The current price of corn is \$1.50/bushel and the contract size is 5,000 bushels. The interest rate is 7%, compounded daily. The storage cost for the corn is \$18/day. Assume the cost of storage as a percentage of the contract per year is 1.46%. The price for the appropriate futures contract used to hedge the position is closest to:
  - A. \$6,635.
  - B. \$7,248.
  - C. \$7,656.
  - D. \$7,765.
5. Backwardation refers to a situation where:
  - A. spot prices are above futures prices.
  - B. spot prices are below futures prices.
  - C. expected future spot prices are above futures prices.
  - D. expected future spot prices are below futures prices.

## CONCEPT CHECKER ANSWERS

1. D Using Equation 1:

$$500e^{(0.03)(0.25)} = \$503.76$$

where S = 500, T = 0.25, and r = 0.03

2. C Using Equation 3:

$$500e^{(0.03-0.02)0.25} = \$501.25$$

3. D Use the formula  $F_0 = (S_0 - I)e^{rT}$ , where I is the present value of \$40 to be received in 4 months, or 0.333 years. At a discount rate of 6.50%:

$$I = \$40 \times e^{-0.065 \times 0.333} = \$39.14$$

$$F_0 = (\$1,109 - 39.14) \times e^{(0.065 \times 0.5)} = \$1,105.20$$

4. D Since both the interest and the storage costs compound on a daily basis, a continuous time model is appropriate to approximate the price of the contract.

The cost of storage as a percentage of the contract per year is:

$$u = 365 \times \frac{18}{1.50 \times 300,000} = 0.0146$$

Using Equation 6, the futures price per bushel is:

$$F = \$1.50 \times e^{(0.07 + 0.0146)(150/365)} = \$1.553 \times 5,000 \text{ bushels per contract} = \$7,765.34$$

5. A Backwardation refers to a situation where spot prices are higher than futures prices. Significant monetary benefits of the asset or a relatively high convenience yield can lead to this result.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

# INTEREST RATE FUTURES

## Topic 39

### EXAM FOCUS

In this topic, we examine Treasury bonds (T-bonds) and Eurodollar futures contracts. These instruments are two of the most popular interest rate futures contracts that trade in the United States. Be able to define the cheapest-to-deliver bond for T-bonds and know how to use the convexity adjustment for Eurodollar futures. Duration-based hedging using interest rate futures is also discussed. Be familiar with the equation to calculate the number of contracts needed to conduct a duration-based hedge.

### DAY COUNT CONVENTIONS

**LO 39.1: Identify the most commonly used day count conventions, describe the markets that each one is typically used in, and apply each to an interest calculation.**

Day count conventions play a role when computing the interest that accrues on a fixed income security. When a bond is purchased, the buyer must pay any accrued interest earned through the settlement date.

$$\text{accrued interest} = \text{coupon} \times \frac{\# \text{ of days from last coupon to the settlement date}}{\# \text{ of days in coupon period}}$$

In the United States, there are three commonly used day count conventions.

1. U.S. Treasury bonds use **actual/actual**.
2. U.S. corporate and municipal bonds use **30/360**.
3. U.S. money market instruments (Treasury bills) use **actual/360**.

The following examples demonstrate the use of day count conventions when computing accrued interest.

#### Example: Day count conventions

Suppose there is a semiannual-pay bond with a \$100 par value. Further assume that coupons are paid on March 1 and September 1 of each year. The annual coupon is 6%, and it is currently July 13. Compute the accrued interest of this bond as a T-bond and a U.S. corporate bond.

**Answer:**

The T-bond uses actual/actual (in period), and the reference (March 1 to September 1) period has 184 days. There are 134 actual days from March 1 to July 13, so the accrued interest is:

$$\frac{134}{184} \times \$3 = \$2.1848$$

The corporate bond uses 30/360, so the reference period now has 180 days. Using this convention, there are 132 ( $= 30 \times 4 + 12$ ) days from March 1 to July 13, so the accrued interest is:

$$\frac{132}{180} \times \$3 = \$2.20$$

## QUOTATIONS FOR T-BONDS

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**LO 39.3: Differentiate between the clean and dirty price for a US Treasury bond; calculate the accrued interest and dirty price on a US Treasury bond.**

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T-bond prices are quoted relative to a \$100 par amount in dollars and 32nds. So a 95–05 is 95 5/32, or 95.15625. The quoted price of a T-bond is not the same as the cash price that is actually paid to the owner of the bond. In general:

$$\text{cash price} = \text{quoted price} + \text{accrued interest}$$

### Clean and Dirty Prices

The cash price (a.k.a. **invoice price** or **dirty price**) is the price that the seller of the bond must be paid to give up ownership. It includes the present value of the bond (a.k.a. **quoted price** or **clean price**) plus the accrued interest. This relationship is shown in the equation above. Conversely, the clean price is the cash price less accrued interest:

$$\text{quoted price} = \text{cash price} - \text{accrued interest}$$

This relationship can also be expressed as:

$$\text{clean price} = \text{dirty price} - \text{accrued interest}$$

**Example: Calculate the cash price of a bond**

Assume the bond in the previous example is a T-bond currently quoted at 102–11. Compute the cash price.

**Answer:**

$$\text{cash price} = \$102.34375 + \$2.1848 = \$104.52855$$

For a \$100,000 par amount, this is \$104,528.55.

**QUOTATIONS FOR T-BILLS****LO 39.2: Calculate the conversion of a discount rate to a price for a US Treasury bill.**

T-bills and other money-market instruments use a discount rate basis and an actual/360 day count. A T-bill with a \$100 face value with  $n$  days to maturity and a cash price of  $Y$  is quoted as:

$$\text{T-bill discount rate} = \frac{360}{n}(100 - Y)$$

This is referred to as the discount rate in annual terms. However, this discount rate is not the actual rate earned on the T-bill. The following example shows the calculation of the annualized yield on a T-bill, given its price.

**Example: Calculating the cash price on a T-bill**

Suppose you have a 180-day T-bill with a discount rate, or quoted price, of five (i.e., the annualized rate of interest earned is 5% of face value). If face value is \$100, what is the true rate of interest and the cash price?

**Answer:**

Interest is equal to  $\$2.5$  ( $= \$100 \times 0.05 \times 180 / 360$ ) for a 180-day period. The true rate of interest for the period is therefore 2.564% [ $= 2.5 / (100 - 2.5)$ ].

Cash price:  $5 = (360 / 180) \times (100 - Y)$ ;  $Y = \$97.5$ .

## TREASURY BOND FUTURES

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**LO 39.4: Explain and calculate a US Treasury bond futures contract conversion factor.**

**LO 39.5: Calculate the cost of delivering a bond into a Treasury bond futures contract.**

**LO 39.6: Describe the impact of the level and shape of the yield curve on the cheapest-to-deliver Treasury bond decision.**

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In a T-bond futures contract, any government bond with more than 15 years to maturity on the first of the delivery month (and not callable within 15 years) is deliverable on the contract. This produces a large supply of potential bonds that are deliverable on the contract and reduces the likelihood of market manipulation. Since the deliverable bonds have very different market values, the Chicago Board of Trade (CBOT) has created **conversion factors**. The conversion factor defines the price received by the short position of the contract (i.e., the short position is delivering the contract to the long). Specifically, the cash received by the short position is computed as follows:

$$\text{cash received} = (\text{QFP} \times \text{CF}) + \text{AI}$$

where:

QFP = quoted futures price (most recent settlement price)

CF = conversion factor for the bond delivered

AI = accrued interest since the last coupon date on the bond delivered

Conversion factors are supplied by the CBOT on a daily basis. Conversion factors are calculated as: (discounted price of a bond – accrued interest) / face value. For example, if the present value of a bond is \$142, accrued interest is \$2, and face value is \$100, the conversion factor would be:  $(142 - 2) / 100 = 1.4$ .

### Cheapest-to-Deliver Bond

The conversion factor system is not perfect and often results in one bond that is the cheapest (or most profitable) to deliver. The procedure to determine which bond is the cheapest-to-deliver (CTD) is as follows:

$$\text{cash received by the short} = (\text{QFP} \times \text{CF}) + \text{AI}$$

$$\text{cost to purchase bond} = (\text{quoted bond price} + \text{AI})$$

The CTD bond minimizes the following: quoted bond price –  $(\text{QFP} \times \text{CF})$ . This expression calculates the cost of delivering the bond.

**Example: The cheapest-to-deliver bond**

Assume an investor with a short position is about to deliver a bond and has four bonds to choose from which are listed in the following table. The last settlement price is \$95.75 (this is the quoted futures price). Determine which bond is the cheapest-to-deliver.

Bond	Quoted Bond Price	Conversion Factor
1	99	1.01
2	125	1.24
3	103	1.06
4	115	1.14

**Answer:**

Cost of delivery:

$$\text{Bond 1: } 99 - (95.75 \times 1.01) = \$2.29$$

$$\text{Bond 2: } 125 - (95.75 \times 1.24) = \$6.27$$

$$\text{Bond 3: } 103 - (95.75 \times 1.06) = \$1.51$$

$$\text{Bond 4: } 115 - (95.75 \times 1.14) = \$5.85$$

Bond 3 is the cheapest-to-deliver with a cost of delivery of \$1.51.

Finding the cheapest-to-deliver bond does not require any arcane procedures but could involve searching among a large number of bonds. The following guidelines give an indication of what type of bonds tend to be the cheapest-to-deliver under different circumstances:

- When yields > 6%, CTD bonds tend to be low-coupon, long-maturity bonds.
- When yields < 6%, CTD bonds tend to be high-coupon, short-maturity bonds.
- When the yield curve is upward sloping, CTD bonds tend to have longer maturities.
- When the yield curve is downward sloping, CTD bonds tend to have shorter maturities.

## TREASURY BOND FUTURES PRICE

### LO 39.7: Calculate the theoretical futures price for a Treasury bond futures contract.

Recall the cost-of-carry relationship, where the underlying asset pays a known cash flow, as was presented in the previous topic. The futures price is calculated in the following fashion:

$$F_0 = (S_0 - I)e^{rT}$$

where:

I = present value of cash flow

We can use this equation to calculate the theoretical futures price when accounting for the CTD bond's accrued interest and its conversion factor.

**Example: Theoretical futures price**

Suppose that the CTD bond for a Treasury bond futures contract pays 10% semiannual coupons. This CTD bond has a conversion factor of 1.1 and a quoted bond price of 100. Assume that there are 180 days between coupons and the last coupon was paid 90 days ago. Also assume that Treasury bond futures contract is to be delivered 180 days from today, and the risk-free rate of interest is 3%. Calculate the theoretical price for this T-bond futures contract.

**Answer:**

The cash price of the CTD bond is equal to the quoted bond price plus accrued interest. Accrued interest is computed as follows:

$$AI = \text{coupon} \times \left( \frac{\text{number of days from last coupon to settlement date}}{\text{number of days in coupon period}} \right)$$

$$AI = 5 \times \frac{90}{180} = 2.5$$

$$\text{cash price} = 100 + 2.5 = 102.5$$

Since the next coupon will be received 90 days from today, that cash flow should be discounted back to the present using the familiar present value equation which discounts the cash flow using the risk-free rate:

$$5e^{-0.03 \times (90/365)} = \$4.96$$

Using the cost-of-carry model, the cash futures price (which expires 180 days from today) is then calculated as follows:

$$F_0 = (102.5 - 4.96)e^{(0.03)(180/365)} = 98.99$$

We are not done, however, since the futures contract expires 90 days after the last coupon payment. The quoted futures price at delivery is calculated after subtracting the amount of accrued interest (recall: QFP = cash futures price – AI).

$$98.99 - \left( 5 \times \frac{90}{180} \right) = \$96.49$$

Finally, the conversion factor is utilized, producing a theoretical price for this T-bond futures contract of:

$$QFP = \frac{96.49}{1.1} = \$87.72$$

## EURODOLLAR FUTURES

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**LO 39.8:** Calculate the final contract price on a Eurodollar futures contract.

**LO 39.9:** Describe and compute the Eurodollar futures contract convexity adjustment.

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The 3-month **eurodollar futures** contract trades on the Chicago Mercantile Exchange (CME) and is the most popular interest rate futures in the United States. This contract settles in cash and the minimum price change is one “tick,” which is a price change of one basis point, or \$25 per \$1 million contract. Eurodollar futures are based on a eurodollar deposit (a eurodollar is a U.S. dollar deposited outside the United States) with a face amount of \$1 million. The interest rate underlying this contract is essentially the 3-month (90-day) forward LIBOR. If  $Z$  is the quoted price for a eurodollar futures contract, the contract price is:

$$\text{eurodollar futures price} = \$10,000[100 - (0.25)(100 - Z)]$$

For example, if the quoted price,  $Z$ , is 97.8:

$$\text{contract price} = \$10,000[100 - (0.25)(100.0 - 97.8)] = \$994,500$$

### Convexity Adjustment

The corresponding 90-day forward LIBOR (on an annual basis) for each contract is  $100 - Z$ . For example, assume that the previous eurodollar contract was for a futures contract that matured in six months. Then the 90-day forward LIBOR six months from now is approximately 2.2% ( $100 - 97.8$ ). However, the daily marking to market aspect of the futures contract can result in differences between actual forward rates and those implied by futures contracts. This difference is reduced by using the convexity adjustment. In general, long-dated eurodollar futures contracts result in implied forward rates larger than actual forward rates. The two are related as follows:

$$\text{actual forward rate} = \text{forward rate implied by futures} - (\frac{1}{2} \times \sigma^2 \times T_1 \times T_2)$$

where:

$T_1$  = the maturity on the futures contract

$T_2$  = the time to the maturity of the rate underlying the contract ( $T_1 + 90$  days)

$\sigma$  = the annual standard deviation of the change in the rate underlying the futures contract, or 90-day LIBOR

Notice that as  $T_1$  increases, the convexity adjustment will need to increase. So as the maturity of the futures contract increases, the necessary convexity adjustment increases. Also, note that the  $\sigma$  and the  $T_2$  are largely dictated by the specifications of the futures contract.

---

**LO 39.10: Explain how Eurodollar futures can be used to extend the LIBOR zero curve.**


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Forward rates implied by convexity-adjusted eurodollar futures can be used to produce a LIBOR spot curve (also called a LIBOR zero curve since spot rates are sometimes referred to as zero rates). Recall the equation presented previously in Topic 37, which was used to generate the shape of the *futures* rate curve:

$$R_{\text{Forward}} = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

where:

$R_i$  = spot rate corresponding with  $T_i$  periods  
 $R_{\text{Forward}}$  = the forward rate between  $T_1$  and  $T_2$

This forward rate equation can be rearranged to solve for the *spot* rate for the next time period ( $T_2$ ):

$$R_2 = \frac{R_{\text{Forward}}(T_2 - T_1) + R_1 T_1}{T_2}$$

Given the first LIBOR spot rate ( $R_1$ ) and the length of each forward contract period, we can calculate the next spot rate ( $R_2$ ). The rate at  $T_2$  can then be used to find the rate at  $T_3$  and so on. The end result is a generated LIBOR spot (zero) curve.

## DURATION-BASED HEDGING

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**LO 39.11: Calculate the duration-based hedge ratio and create a duration-based hedging strategy using interest rate futures.**


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The objective of a **duration-based hedge** is to create a combined position that does not change in value when yields change by a small amount. In other words, a position that has a duration of zero needs to be produced. The combined position consists of our portfolio with a hedge horizon value of  $P$  and a futures position with a contract value of  $F$ . Denote the duration of the portfolio at the hedging horizon as  $D_P$  and the corresponding duration of the futures contract as  $D_F$ . Using this notation, the duration-based hedge ratio can be expressed as follows:

$$N = -\frac{P \times D_P}{F \times D_F}$$

where:

$N$  = number of contracts to hedge

The minus sign suggests that the futures position is the opposite of the original position. In other words, if the investor is long the portfolio, he must short  $N$  contracts to produce a position with a zero duration.

**Example: Duration-based hedge**

Assume there is a 6-month hedging horizon and a portfolio value of \$100 million. Further assume that the 6-month T-bond contract is quoted at 105–09, with a contract size of \$100,000. The duration of the portfolio is 10, and the duration of the futures contract is 12. Outline the appropriate hedge for small changes in yield.

**Answer:**

$$N = -\frac{100,000,000 \times 10}{105,281.25 \times 12} = -791.53$$

Rounding up to the nearest whole number means the manager should short 792 contracts.

**LIMITATIONS OF DURATION****LO 39.12: Explain the limitations of using a duration-based hedging strategy.**

The price/yield relationship of a bond is convex, meaning it is nonlinear in shape. Duration measures are linear approximations of this relationship. Therefore, as the change in yield increases, the duration measures become progressively less accurate. Moreover, duration implies that all yields are perfectly correlated. Both of these assumptions place limitations on the use of duration as a single risk measurement tool. When changes in interest rates are both large and nonparallel (i.e., not perfectly correlated), duration-based hedge strategies will perform poorly.

## KEY CONCEPTS

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### LO 39.1

Day count conventions play a role when computing the interest that accrues on a fixed income security. When a bond is purchased, the buyer must pay any accrued interest earned through the settlement date. The most common day count conventions are Actual/Actual, 30/360, and Actual/360.

---

### LO 39.2

T-bills are quoted on a discount rate basis. A T-bill with a \$100 face value with  $n$  days to maturity and a cash price of  $Y$  is quoted as:

$$\text{T-bill discount rate} = \frac{360}{n}(100 - Y)$$

---

### LO 39.3

For a U.S. Treasury bond, the dirty price is the price that the seller of the bond must be paid to give up ownership. It includes the present value of the bond plus the accrued interest. Conversely, the clean price is the dirty price less accrued interest.

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### LO 39.4

Since deliverable bonds have very different market values, the Chicago Board of Trade (CBOT) has created conversion factors. Conversion factors are supplied by the CBOT on a daily basis. They are calculated as:

$$(\text{bond discounted price} - \text{accrued interest}) / \text{face value}$$

---

### LO 39.5

The conversion factor system is not perfect and often results in one bond that is the cheapest (or most profitable) to deliver. The cheapest-to-deliver (CTD) bond is the bond that minimizes the following:

$$\text{quoted bond price} - (\text{quoted futures price} \times \text{conversion factor})$$

---

### LO 39.6

When the yield curve is not flat, there is a single bond that is the cheapest-to-deliver (CTD). When the yield curve is upward sloping, CTD bonds tend to have longer maturities. When the yield curve is downward sloping, CTD bonds tend to have shorter maturities.

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**LO 39.7**

The theoretical price for a T-bond futures contract is calculated as:

$$(\text{cash futures price} - \text{accrued interest}) / \text{conversion factor}$$


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**LO 39.8**

Eurodollar contracts are based on LIBOR and are quoted on a discount rate basis. If  $Z$  is the quoted price for a eurodollar futures contract, the contract price is:

$$\text{eurodollar futures price} = \$10,000 \times [100 - (0.25) \times (100 - Z)]$$


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**LO 39.9**

Long-dated eurodollar contracts must be adjusted for convexity before being used to estimate the corresponding forward rates. As the maturity of the futures contract increases, the necessary convexity adjustment increases.

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**LO 39.10**

Forward rates implied by convexity-adjusted eurodollar futures can be used to produce a LIBOR spot curve. The following equation is used to generate the shape of the futures rate curve:

$$R_{\text{Forward}} = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

where :

$R_i$  = spot rate corresponding with  $T_i$  periods

$R_{\text{Forward}}$  = the forward rate between  $T_1$  and  $T_2$

---

**LO 39.11**

Duration can be used to compute the number of futures contracts needed to implement a duration-based hedging strategy. The duration-based hedge ratio can be expressed as follows:

$$\text{number of contracts} = -\frac{\text{portfolio value} \times \text{duration}_{\text{portfolio}}}{\text{futures value} \times \text{duration}_{\text{futures}}}$$


---

**LO 39.12**

The effectiveness of duration-based hedging strategies is limited when there are large changes in yield or nonparallel shifts in the yield curve.

## CONCEPT CHECKERS

1. Assume a 6-month hedging horizon and a portfolio value of \$30 million. Further assume that the 6-month Treasury bond (T-bond) contract is quoted at 100–13, with a contract size of \$100,000. The duration of the portfolio is 8, and the duration of the futures contract is 12. Which of the following is closest to the appropriate hedge for small changes in yield?
  - A. Long 298 contracts.
  - B. Short 298 contracts.
  - C. Long 199 contracts.
  - D. Short 199 contracts.
  
2. Which of the following items limits the use of duration as a risk metric?
  - I. It assumes the price/yield relationship is linear.
  - II. It assumes interest rate volatility is constant.
  - A. I only.
  - B. II only.
  - C. Both I and II.
  - D. Neither I nor II.
  
3. Consider day count convention and, specifically, the following example: A semiannual bond with \$100 face value has a 4% coupon. Today is August 3. Assume coupon dates of March 1 and September 1. Which of the following statements is true?
  - A. Corporate bonds accrue more interest in July than T-bonds.
  - B. Corporate bonds accrue more interest from March 1 to September 1 than September 1 to March 1.
  - C. Corporate bonds accrue more interest than T-bonds for this period (March 1 to August 3).
  - D. The T-bond accrued interest is \$1.76 for this period (March 1 to August 3).
  
4. Assume an investor is about to deliver a short bond position and has four options to choose from which are listed in the following table. The settlement price is \$92.50 (i.e., the quoted futures price). Determine which bond is the cheapest-to-deliver.

<i>Bond</i>	<i>Quoted Bond Price</i>	<i>Conversion Factor</i>
1	98	1.02
2	122	1.27
3	105	1.08
4	112	1.15

- A. Bond 1.
- B. Bond 2.
- C. Bond 3.
- D. Bond 4.

5. Assume the cash price on a 90-T-bill is quoted as 98.75. The discount rate is closest to:
- A. 4%.
  - B. 7%.
  - C. 6%.
  - D. 5%.

## CONCEPT CHECKER ANSWERS

1. D  $N = -\frac{(\$30,000,000 \times 8)}{(\$100,406.25 \times 12)} = -199$

The appropriate hedge is to short 199 contracts.

2. A The limitations of duration include: (1) that it is valid for only *small changes in yield*, (2) that it assumes the price/yield relationship is linear, and (3) it assumes that changes in yield are the same across all maturities and risk levels (i.e., they're perfectly correlated).

3. C July accrued T-bond interest is  $31/184 = 0.1685$ ; July accrued corporate bond interest is  $30/180 = 0.1667$ . T-bonds accrue  $155/184 = 0.8424 \times \$2 = \$1.6848$ ; C-bonds accrue  $152/180 = 0.8444 \times \$2 = \$1.6889$ .

4. A Cost of delivery:

$$\text{Bond 1: } 98 - (92.50 \times 1.02) = \$3.65$$

$$\text{Bond 2: } 122 - (92.50 \times 1.27) = \$4.53$$

$$\text{Bond 3: } 105 - (92.50 \times 1.08) = \$5.10$$

$$\text{Bond 4: } 112 - (92.50 \times 1.15) = \$5.63$$

Bond 1 is the cheapest-to-deliver with a cost of delivery of \$3.65.

5. D The discount rate on a U.S. T-bill is calculated using the following equation:

$$\text{discount rate} = \frac{360}{n} \times (100 - \text{cash price})$$

$$\text{discount rate} = \frac{360}{90} \times (100 - 98.75) = 5\%$$

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The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

# SWAPS

## Topic 40

### EXAM FOCUS

An interest rate swap is an agreement between two parties to exchange interest payments based on a specified principal over a period of time. In a plain vanilla interest rate swap, one of the interest rates is floating, and the other is fixed. Swaps can be used to efficiently alter the interest rate risk of existing assets and liabilities. A currency swap exchanges interest rate payments in two different currencies. For valuation purposes, swaps can be thought of as a long and short position in two different bonds or as a package of forward rate agreements. Credit risk in swaps cannot be ignored.

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### MECHANICS OF INTEREST RATE SWAPS

#### LO 40.1: Explain the mechanics of a plain vanilla interest rate swap and compute its cash flows.

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The most common interest rate swap is the **plain vanilla interest rate swap**. In this swap arrangement, Company X agrees to pay Company Y a periodic fixed rate on a notional principal over the tenor of the swap. In return, Company Y agrees to pay Company X a periodic floating rate on the same notional principal. Both payments are in the same currency. Therefore, only the net payment is exchanged. Most interest rate swaps use the London Interbank Offered Rate (LIBOR) as the reference rate for the floating leg of the swap. Finally, since the payments are based in the same currency, there is no need for the exchange of principal at the inception of the swap. This is why it is called notional principal.

For example, companies X and Y enter into a 2-year plain vanilla interest rate swap. The swap cash flows are exchanged semiannually, and the reference rate is 6-month LIBOR. The LIBOR rates are shown in Figure 1. The fixed rate of the swap is 3.784%, and the notional principal is \$100 million. We will compute the cash flows for Company X, the fixed payer of this swap.

Figure 1: 6-Month LIBOR

<i>Beginning of Period</i>	<i>LIBOR</i>
1	3.00%
2	3.50%
3	4.00%
4	4.50%
5	5.00%

The first cash flow takes place at the end of period one and uses the LIBOR at the beginning of that same period. In other words, at the beginning of each period, both payments for the end of the period are known. The gross cash flows for the end of the first period for both parties are calculated in the following manner:

$$\text{floating} = \$100 \text{ million} \times 0.03 \times 0.5 = \$1.5 \text{ million}$$

$$\text{fixed} = \$100 \text{ million} \times 0.03784 \times 0.5 = \$1.892 \text{ million}$$

Note that 0.5 is the semiannual day count. The net payment for Company X is an outflow of \$0.392 million. Note that we are ignoring the many day-count and business-day conventions associated with swaps. Figure 2 shows the other cash flows.

**Figure 2: Swap Cash Flows**

<i>End of Period</i>	<i>LIBOR at Beginning of Period</i>	<i>Floating Cash Flow</i>	<i>Fixed Cash Flow</i>	<i>Net X Cash Flow</i>
1	3.00%	\$1,500,000	\$1,892,000	-\$392,000
2	3.50%	\$1,750,000	\$1,892,000	-\$142,000
3	4.00%	\$2,000,000	\$1,892,000	\$108,000
4	4.50%	\$2,250,000	\$1,892,000	\$358,000

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**LO 40.2: Explain how a plain vanilla interest rate swap can be used to transform an asset or a liability and calculate the resulting cash flows.**

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Let's continue with companies X and Y. Suppose that X has a 2-year floating-rate liability, and Y has a 2-year fixed-rate liability. After they enter into the swap, interest rate risk exposure from their liabilities has completely changed for each party. X has transformed the floating-rate liability into a fixed-rate liability, and Y has transformed the fixed-rate liability to a floating-rate liability. Note that X pays fixed and receives floating, so X's liability becomes fixed.

Similarly, assume that X has a fixed-rate asset and Y has a floating-rate asset tied to LIBOR. After entering into the swap, X has transformed the fixed-rate asset into a floating-rate asset, and Y has transformed the floating-rate asset into a fixed-rate asset.

## FINANCIAL INTERMEDIARIES

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**LO 40.3: Explain the role of financial intermediaries in the swaps market.**

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**LO 40.4: Describe the role of the confirmation in a swap transaction.**

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In many respects, swaps are similar to forwards:

- Swaps typically require no payment by either party at initiation.
- Swaps are custom instruments.

- Swaps are not traded in any organized secondary market.
- Swaps are largely unregulated.
- Default risk is an important aspect of the contracts.
- Most participants in the swaps market are large institutions.
- Individuals are rarely swap market participants.

There are swap intermediaries who bring together parties with needs for the opposite side of a swap. Dealers, large banks, and brokerage firms, act as principals in trades just as they do in forward contracts. In many cases, a swap party will not be aware of the other party on the offsetting side of the swap since both parties will likely only transact with the intermediary. Financial intermediaries, such as banks, will typically earn a spread of about 3 to 4 basis points for bringing two nonfinancial companies together in a swap agreement. This fee is charged to compensate the intermediary for the risk involved. If one of the parties defaults on its swap payments, the intermediary is responsible for making the other party whole.

Confirmations, as drafted by the International Swaps and Derivatives Association (ISDA), outline the details of each swap agreement. A representative of each party signs the confirmation, ensuring that they agree with all swap details (such as tenor and fixed/floating rates) and the steps taken in the event of default.

## COMPARATIVE ADVANTAGE

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### LO 40.5: Describe the comparative advantage argument for the existence of interest rate swaps and evaluate some of the criticisms of this argument.

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Let's return to companies X and Y and assume that they have access to borrowing for two years as specified in Figure 3.

**Figure 3: Borrowing Rates for X and Y**

Company	Fixed Borrowing	Floating Borrowing
Y	5.0%	LIBOR + 10 bps
X	6.5%	LIBOR + 100 bps

Company Y has an **absolute advantage** in both markets but a comparative advantage in the fixed market. Notice that the differential between X and Y in the fixed market is 1.5%, or 150 basis points (bps), and the corresponding differential in the floating market is only 90 basis points. When this is the case, Y has a comparative advantage in the fixed market, and X has a comparative advantage in the floating market. When a **comparative advantage** exists, a swap arrangement will reduce the costs of both parties. In this example, the net potential borrowing savings by entering into a swap is the difference between the differences, or 60 bps. In other words, by entering into a swap, the total savings shared between X and Y is 60 bps.

To better understand where the 60 bps comes from, suppose Y borrows fixed at 5% for two years, X borrows floating for two years at LIBOR + 1%, and then X and Y enter into a swap to transform their liabilities. Specifically, X pays Y fixed and Y pays X floating based on LIBOR. If we assume the net savings is split evenly, the net borrowing costs for X are then 6.2% and LIBOR – 20 bps for Y. Each has saved 30 bps for a total of 60 bps. If an intermediary were used, part of the 60 bps would be used to pay the bid-ask spread.

## PROBLEMS WITH COMPARATIVE ADVANTAGE

A problem with the comparative advantage argument is that it assumes X can borrow at LIBOR + 1% over the life of the swap. It also ignores the credit risk taken on by Y by entering into the swap. If X were to raise funds by borrowing directly in the capital markets, no credit risk is taken, so perhaps the savings is compensation for that risk. The same criticisms exist when an intermediary is involved.

## VALUING INTEREST RATE SWAPS

### The Discount Rate

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**LO 40.6: Explain how the discount rates in a plain vanilla interest rate swap are computed.**

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Since a swap is nothing more than a sequence of cash flows, its value is determined by discounting each cash flow back to the valuation date. The question is, what is the appropriate *discount rate* to use? It turns out that the forward rates implied by either forward rate agreements (FRAs) or the convexity-adjusted Eurodollar futures are used to produce a LIBOR spot curve. The swap cash flows are then discounted using the corresponding spot rate from this curve. The following connection between forward rates and spot rates exists when continuous compounding is used:

$$R_{\text{forward}} = R_2 + (R_2 - R_1) \frac{T_1}{T_2 - T_1}$$

where:

$R_i$  = spot rate corresponding with  $T_i$  years  
 $R_{\text{forward}}$  = forward rate between  $T_1$  and  $T_2$

We will utilize this equation later when we value an interest rate swap using a sequence of forward rate agreements.

### Valuing an Interest Rate Swap With Bonds

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**LO 40.7: Calculate the value of a plain vanilla interest rate swap based on two simultaneous bond positions.**

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Let's return to our two companies, X and Y, in our 2-year swap arrangement. From X's perspective, there are two series of cash flows—one fixed going out and one floating coming in. Essentially, X has a long position in a floating-rate note (since it is an inflow) and a short position in a fixed-rate note (since it is an outflow). From Y's perspective, it is exactly the opposite—Y has a short position in a floating-rate note (since it is an outflow) and a long position in a fixed-rate note (since it is an inflow).

If we denote the present value of the fixed-leg payments as  $B_{\text{fix}}$  and the present value of the floating-leg payments as  $B_{\text{flt}}$ , the value of the swap can be written for both X and Y as:

$$V_{\text{swap}}(X) = B_{\text{flt}} - B_{\text{fix}}$$

$$V_{\text{swap}}(Y) = B_{\text{fix}} - B_{\text{flt}}$$

Note that  $V_{\text{swap}}(X) + V_{\text{swap}}(Y) = 0$ . This is by design since the two positions are mirror images of one another. At inception of the swap, it is convention to select the fixed payment so that  $V_{\text{swap}}(X) = V_{\text{swap}}(Y) = 0$ . As expected floating rates in the future change, the swap value for each party is no longer zero.

Valuing an interest rate swap in terms of bond positions involves understanding that the value of a floating rate bond will be equal to the notional amount at any of its periodic settlement dates when the next payment is set to the market (floating) rate. Since  $V_{\text{swap}} = \text{Bond}_{\text{fixed}} - \text{Bond}_{\text{floating}}$ , we can value the fixed-rate bond using the spot rate curve and then discount the next (known) floating-rate payment plus the notional amount at the current discount rate. The following example illustrates this method.

#### Example: Valuing an interest rate swap

Consider a \$1 million notional swap that pays a floating rate based on 6-month LIBOR and receives a 6% fixed rate semiannually. The swap has a remaining life of 15 months with pay dates at 3, 9, and 15 months. Spot LIBOR rates are as follows: 3 months at 5.4%; 9 months at 5.6%; and 15 months at 5.8%. The LIBOR at the last payment date was 5.0%. Calculate the value of the swap to the fixed-rate receiver using the bond methodology.

**Answer:**

$$B_{\text{fixed}} = \left( \text{PMT}_{\text{fixed}, 3 \text{ months}} \times e^{-(r \times t)} \right) + \left( \text{PMT}_{\text{fixed}, 9 \text{ months}} \times e^{-(r \times t)} \right) + \\ \left[ (\text{notional} + \text{PMT}_{\text{fixed}, 15 \text{ months}}) \times e^{-(r \times t)} \right]$$

$$B_{\text{fixed}} = \left( \$30,000 \times e^{-(0.054 \times 0.25)} \right) + \left( \$30,000 \times e^{-(0.056 \times 0.75)} \right) + \\ \left[ (\$1,000,000 + \$30,000) \times e^{-(0.058 \times 1.25)} \right] \\ = \$29,598 + \$28,766 + \$957,968 = \$1,016,332$$

$$B_{\text{floating}} = \left[ \text{notional} + \left( \text{notional} \times \frac{r_{\text{floating}}}{2} \right) \right] \times e^{-(r \times t)} \\ = \left[ \$1,000,000 + \left( \$1,000,000 \times \frac{0.05}{2} \right) \right] \times e^{-(0.054 \times 0.25)} = \$1,011,255$$

$$V_{\text{swap}} = (B_{\text{fixed}} - B_{\text{floating}}) = \$1,016,332 - \$1,011,255 = \$5,077$$

Figure 4 sums up the payments and present value factors.

**Figure 4: Valuing an Interest Rate Swap With Two Bond Positions**

Time	Fixed Cash Flow	Floating Cash Flow	Present Value Factor	PV Fixed CF	PV Floating CF
0.25 (3 months)	30,000	1,025,000	0.9866*	29,598	1,011,255
0.75 (9 months)	30,000		0.9589*	28,766	
1.25 (15 months)	1,030,000		0.9301*	957,968	
Total				1,016,332	1,011,255

\* Note that some rounding has occurred.

Again we see that the value of the swap = 1,016,332 – 1,011,255 = \$5,077.

### Valuing an Interest Rate Swap With FRAs

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**LO 40.8: Calculate the value of a plain vanilla interest rate swap from a sequence of forward rate agreements (FRAs).**

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At settlement, the payment made on a forward rate agreement is the notional amount multiplied by the difference between a market (floating) rate such as LIBOR and the contract (fixed) rate specified in the FRA. This is identical to a periodic payment on an interest rate swap when the reference floating rates and notional principal amounts are the same and the swap fixed rate is equal to the contract rate specified in the FRA. Viewed this way, we can see that an interest rate swap is equivalent to a series of FRAs. One way to value a swap would be to use expected forward rates to forecast the expected net cash flows and then discount these expected cash flows at the corresponding spot rates, consistent with forward rate expectations.

#### Example: Valuing an interest rate swap with FRAs

Consider the previous example on valuing an interest rate swap with two bond positions. An investor has a \$1 million notional swap that pays a floating rate based on 6-month LIBOR and receives a 6% fixed rate semiannually. The swap has a remaining life of 15 months with pay dates at 3, 9, and 15 months. Spot LIBOR rates are as follows: 3 months at 5.4%; 9 months at 5.6%; and 15 months at 5.8%. The LIBOR at the last payment date was 5.0%. Calculate the value of the swap to the fixed-rate receiver using the FRA methodology.

**Answer:**

To calculate the value of the swap, we'll need to find the floating rate cash flows by calculating the expected forward rates via the LIBOR based spot curve.

The first floating rate cash flow is calculated in a similar fashion to the previous example.

LIBOR rate (last payment date): 5%.

Floating rate cash flow in 3 months:  $1,000,000 \times 0.05 / 2 = \$25,000$ .

The second floating rate cash flow is calculated by finding the forward rate that corresponds to the period between 3 months and 9 months. To calculate forward rate for the period between 3 and 9 months, use the previously mentioned forward rate formula:

$$R_{\text{forward}} = R_2 + (R_2 - R_1) \frac{T_1}{T_2 - T_1}$$

$$R_{\text{forward}} = 0.056 + (0.056 - 0.054) \frac{0.25}{0.75 - 0.25} = 0.057 = 5.7\%$$

This rate is a continuously compounded rate, so we need to find the equivalent forward rate with semiannual compounding:

$$R_{\text{forward (SC)}} = 2 \times [e^{(0.057/2)} - 1] = 0.05782 = 5.782\%$$

Floating rate cash flow in 9 months:  $1,000,000 \times 0.05782 / 2 = \$28,910$

The third floating rate cash flow is calculated by finding the forward rate that corresponds to the period between 9 months and 15 months.

$$R_{\text{forward}} = 0.058 + (0.058 - 0.056) \frac{0.75}{1.25 - 0.75} = 0.061 = 6.1\%$$

$$R_{\text{forward (SC)}} = 2 \times [e^{(0.061/2)} - 1] = 0.06194 = 6.1939\%$$

Floating rate cash flow in 15 months:  $1,000,000 \times 0.061939 / 2 = \$30,969$

**Figure 5: Valuing an Interest Rate Swap Based on a Sequence of FRAs**

Time	Fixed Cash Flow	Floating Cash Flow	Present Value Factor	PV Fixed CF	PV Floating CF
0.25 (3 months)	30,000	25,000	0.9866*	29,598	24,665
0.75 (9 months)	30,000	28,910	0.9589*	28,766	27,721
1.25 (15 months)	30,000	30,969	0.9301*	27,902	28,803
Total				86,266	81,189

\* Note that some rounding has occurred.

The value of the swap based on a sequence of FRAs =  $86,266 - 81,189 = \$5,077$ .

As you can see from the previous two examples, valuing a swap based on a sequence of forward rate agreements produces the same result as valuing a swap based on two simultaneous bond positions.

## CURRENCY SWAPS

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**LO 40.9: Explain the mechanics of a currency swap and compute its cash flows.**

**LO 40.11: Calculate the value of a currency swap based on two simultaneous bond positions.**

---

A currency swap exchanges both principal and interest rate payments with payments in different currencies. The exchange rate used in currency swaps is the spot exchange rate. The valuation and application of currency swaps is similar to the interest rate swap. However, since the principals in a currency swap are not the same currency, they are exchanged at the inception of the currency swap so that they have equal value using the spot exchange rate. Also, the periodic cash flows throughout the swap are not netted as they are in the interest rate swap.

Suppose we have two companies, A and B, that enter into a fixed-for-fixed currency swap with periodic payments annually. Company A pays 6% in Great Britain pounds (GBP) to Company B and receives 5% in U.S. dollars (USD) from Company B. Company A pays a principal amount to B of USD175 million, and B pays GBP100 million to A at the outset of the swap. Notice that A has effectively borrowed GBP from B and so it must pay interest on that loan. Similarly, B has borrowed USD from A. The cash flows in this swap are actually more easily computed than in an interest rate swap since both legs of the swap are fixed. Every period (12 months), A will pay GBP6 million to B, and B will pay USD8.75 million to A. At the end of the swap, the principal amounts are re-exchanged.

From Company A's perspective, there are two series of cash flows: one fixed GBP cash flow stream going out and one fixed USD cash flow stream coming in. Essentially, A has a long position in a USD-denominated note (since it's an inflow) and a short position in a GBP-denominated note (since it's an outflow).

If we denote the present value of the GBP-denominated payments as  $B_{\text{GBP}}$  and the present value of the USD payments as  $B_{\text{USD}}$ , the value of the swap in USD to Company A is:

$$V_{\text{swap}}(\text{USD}) = B_{\text{USD}} - (S_0 \times B_{\text{GBP}})$$

where:

$S_0$  = spot rate in USD per GBP

**Example: Calculate the value of a currency swap**

Suppose the yield curves in the United States and Great Britain are flat at 2% and 4%, respectively, and the current spot exchange rate is USD1.50 = GBP1. Value the currency swap just discussed assuming the swap will last for three more years.

**Answer:**

$$B_{USD} = 8.75e^{-0.02 \times 1} + 8.75e^{-0.02 \times 2} + 183.75e^{-0.02 \times 3} = \text{USD}190.03 \text{ million}$$

$$B_{GBP} = 6e^{-0.04 \times 1} + 6e^{-0.04 \times 2} + 106e^{-0.04 \times 3} = \text{GBP}105.32 \text{ million}$$

$$V_{\text{swap}} (\text{to A in USD}) = 190.03 - (1.5 \times 105.32) = \text{USD}32.05 \text{ million}$$

**LO 40.12: Calculate the value of a currency swap based on a sequence of FRAs.**

The value of a currency swap can also be calculated based on a sequence of FRAs.

**Example: Value of a currency swap with FRAs**

Suppose the yield curves in the United States and Great Britain are flat at 2% and 4%, respectively, and the current spot exchange rate is USD1.50 = GBP1.

Compute the value of the currency swap discussed previously using a sequence of FRAs to Company A. Assume the swap will last for three more years.

The corresponding forward rates are as follows:

**Figure 6: Forward Rates**

Year 1	\$1.47/£
Year 2	\$1.44/£
Year 3	\$1.41/£

*Professor's Note: The year 1 forward rate is calculated as follows:*

  $F_1 = 1.5e^{(0.02 - 0.04) \times 1} = \$1.47/\text{£}$ . Interest rate parity suggests that the dollar will appreciate relative to the pound, so the \$/£ forward rate will decline (i.e., it will take fewer USD to buy 1 GBP). We will discuss interest rate parity in the foreign exchange risk topic (Topic 49).

**Answer:**

Figure 7 denotes the cash flows and forward rates for this currency swap.

**Figure 7: Valuing a Currency Swap Based on a Sequence of FRAs**

Time	USD Cash Flow	GBP Cash Flow	Forward Rate	\$ Value of £	Net Cash Flows	PV of Net CF
1	8.75	6	1.47	8.82	-0.07	-0.069
2	8.75	6	1.44	8.64	0.11	0.106
3	8.75	6	1.41	8.46	0.29	0.273
	175	100	1.41	141	34	32.02
Total						32.33*

\* Note some rounding has occurred.

Ignoring the rounding differences, we see that the value of the currency swap to Company A is 32 million using both the two simultaneous bond positions and the forward rate agreements.

### Using a Currency Swap to Transform Existing Positions

**LO 40.10: Explain how a currency swap can be used to transform an asset or liability and calculate the resulting cash flows.**

Currency swaps can be combined with existing positions to completely alter the risk of a liability or an asset. For example, suppose that Company A has a dollar-based liability. By entering into a currency swap, the liability has become a pound-based liability at the GBP fixed (or floating) rate.

### Comparative Advantage

Comparative advantage is also used to explain the success of currency swaps. Typically, a domestic borrower will have an easier time borrowing in his own currency. This often results in comparative advantages that can be exploited by using a currency swap. The argument is directly analogous to that used for interest rate swaps. Suppose A and B have the 5-year borrowing rates in the United States and Germany (EUR) shown in Figure 8.

**Figure 8: Borrowing Rates**

Borrowing Rates for A and B		
Company	USD Borrowing	EUR Borrowing
A	5.0%	7.0%
B	6.0%	7.5%

Company A needs EUR, and Company B needs USD. Company A has an absolute advantage in both markets but a comparative advantage in the USD market. Notice that the differential between A and B in the USD market is 1%, or 100 basis points (bps), and the corresponding differential in the EUR market is only 50 basis points. When this is the case, A has a comparative advantage in the USD market, and B has a comparative advantage in the EUR market. The net potential borrowing savings by entering into a swap is the difference between the differences, or 50 bps. In other words, by entering into a currency swap, the savings for both A and B totals 50 bps.

## SWAP CREDIT RISK

### LO 40.13: Describe the credit risk exposure in a swap position.

Because  $V_{\text{swap}}(A) + V_{\text{swap}}(B) = 0$ , whenever one side of a swap has a positive value, the other side must be negative. For example, if  $V_{\text{swap}}(A) > 0$ ,  $V_{\text{swap}}(B) < 0$ . As  $V_{\text{swap}}(A)$  increases in value,  $V_{\text{swap}}(B)$  must become more negative. This results in increased credit risk to A since the likelihood of default increases as B has larger and larger payments to make to A. However, the potential losses in swaps are generally much smaller than the potential losses from defaults on debt with the same principal. This is because the value of swaps is generally much smaller than the value of the debt.

## OTHER TYPES OF SWAPS

### LO 40.14: Identify and describe other types of swaps, including commodity, volatility and exotic swaps.

In an **equity swap**, the return on a stock, a portfolio, or a stock index is paid each period by one party in return for a fixed-rate or floating-rate payment. The return can be the capital appreciation or the total return including dividends on the stock, portfolio, or index.

In order to reduce equity risk, a portfolio manager might enter into a 1-year quarterly pay S&P 500 index swap and agree to receive a fixed rate. The percentage increase in the index each quarter is netted against the fixed rate to determine the payment to be made. If the index return is negative, the fixed-rate payer must also pay the percentage decline in the index to the portfolio manager. Uniquely among swaps, equity swap payments can be floating on both sides and the payments are not known until the end of the quarter. With interest rate swaps, both the fixed and floating payments are known at the beginning of the period for which they will be paid.

A swap on a single stock can be motivated by a desire to protect the value of a position over the period of the swap. To protect a large capital gain in a single stock, and to avoid a sale for tax or control reasons, an investor could enter into an equity swap as the equity-returns payer and receive a fixed rate in return. Any decline in the stock price would be paid to the investor at the settlement dates, plus the fixed-rate payment. If the stock appreciates, the investor must pay the appreciation less the fixed payment.

A **swaption** is an option which gives the holder the right to enter into an interest rate swap. Swaptions can be American- or European-style options. Like any option, a swaption is purchased for a premium that depends on the strike rate (the fixed rate) specified in the swaption.

Firms may enter into **commodity swap** agreements where they agree to pay a fixed rate for the multi-period delivery of a commodity and receive a corresponding floating rate based on the average commodity spot rates at the time of delivery. Although many commodity swaps exist, the most common use is to manage the costs of purchasing energy resources such as oil and electricity.

A **volatility swap** involves the exchanging of volatility based on a notional principal. One side of the swap pays based on a pre-specified volatility while the other side pays based on historical volatility.

As you can see, many different types of swaps exist. Some additional examples include: accrual swaps, cancelable swaps, index amortizing rate swaps, and constant maturity swaps. Swaps are also sometimes created for exotic structures. An example of an **exotic swap** was between Procter and Gamble and Banker's Trust where P&G's payments were based on the commercial paper rate.

## KEY CONCEPTS

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### LO 40.1

A plain vanilla interest rate swap exchanges floating-rate payments (LIBOR) for fixed-rate payments over the life of the swap. The floating rate payments at time  $t$  in a plain vanilla interest rate swap are computed using the floating rate at time  $t - 1$ .

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### LO 40.2

Interest rate swaps can be combined with existing asset and liability positions to drastically change the interest rate risk.

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### LO 40.3

A swap dealer or financial intermediary facilitates the ability to enter into swaps.

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### LO 40.4

Confirmations outline the details of each swap agreement. A representative of each party signs the confirmation, ensuring that they agree with all swap details and the steps taken in the event of default.

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### LO 40.5

The comparative advantage argument suggests that when one of two borrowers has a comparative advantage in either the fixed- or floating-rate market, both borrowers will be better off by entering into a swap to exploit the advantage. The comparative advantage argument is flawed in that it assumes rates can be borrowed for the life of the swap. It also ignores the credit risk associated with the swap that does not exist if funds were raised directly in the capital markets.

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### LO 40.6

Since a swap is nothing more than a sequence of cash flows, its value is determined by discounting each cash flow back to the valuation date. The cash flows are discounted using the corresponding spot rate from the LIBOR spot curve.

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### LO 40.7

The value of a swap to the fixed-rate receiver at a point in time is the difference between the present value of the remaining fixed-rate payments and the present value of the remaining floating-rate payments.

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### LO 40.8

Valuing a swap based on a sequence of forward rate agreements (FRAs) produces the same result as valuing a swap based on two simultaneous bond positions.

**LO 40.9**

A currency swap exchanges interest rate payments in two different currencies. The exchange rate used in currency swaps is the spot exchange rate.

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**LO 40.10**

Currency swaps can be combined with existing positions to completely alter the risk of a liability or an asset.

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**LO 40.11**

Since the principals in a currency swap are not the same currency, they are exchanged at the inception of the currency swap so that they have equal value using the spot exchange rate. Also, the periodic cash flows throughout the swap are not netted as they are in an interest rate swap.

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**LO 40.12**

In addition to valuing a currency swap based on two simultaneous bond positions, the value of a currency swap can also be calculated based on a sequence of FRAs.

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**LO 40.13**

Credit risk is an important factor in existing swap positions, although potential losses are usually smaller than that with debt agreements.

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**LO 40.14**

Many different types of swaps exist. Examples of swaps, in addition to interest rate swaps and currency swaps, include: equity swaps, commodity swaps, and volatility swaps.

## CONCEPT CHECKERS

Use the following data to answer Question 1.

Two companies, C and D, have the borrowing rates shown in the following table.

Borrowing Rates for C and D		
Company	Fixed Borrowing	Floating Borrowing
C	10%	LIBOR + 50 bps
D	12%	LIBOR + 100 bps

- According to the comparative advantage argument, what is the total potential savings for C and D if they enter into an interest rate swap?
  - 0.5%.
  - 1.0%.
  - 1.5%.
  - 2.0%.
- Which of the following is most accurate regarding the credit risk of a currency swap?  
As the value of the:
  - domestic currency leg increases, so does the credit risk of the domestic currency payer.
  - foreign currency leg increases, so does the credit risk of the foreign currency payer.
  - I only.
  - II only.
  - Both I and II.
  - Neither I nor II.
- Which of the following would properly transform a floating-rate liability to a fixed-rate liability? Enter into a pay:
  - foreign currency swap.
  - fixed interest rate swap.
  - domestic currency swap.
  - floating interest rate swap.
- Use the following information to determine the value of the swap to the floating rate payer using the bond methodology. Assume we are at the floating rate reset date.
  - \$1 million notional value, semiannual, 18-month maturity.
  - Spot LIBOR rates: 6 months, 2.6%; 12 months, 2.65%; 18 months, 2.75%.
  - The fixed rate is 2.8%, with semiannual payments.
  - \$66.
  - \$476.
  - \$3,425.
  - \$5,077.

5. Suppose Company X pays 5% annually (in euros) to Company Y and receives 4% annually (in dollars). Company X pays a principal amount of \$150 million to Y, and Y pays a €100 million to X at the inception of the swap. Assume the yield curve is flat in the United States and in Germany (Europe). The U.S. rate is 3%, and the German rate is 5%. The current spot exchange rate is \$1.45/€. What is the value of the currency swap to Company X using the bond methodology if it is expected to last for two more years?
- A. \$3.53 million.
  - B. \$52.98 million.
  - C. \$8.09 million.
  - D. \$12.74 million.

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## CONCEPT CHECKER ANSWERS

1. C The difference of the differences is  $(12\% - 10\%) - [\text{LIBOR} + 1\% - (\text{LIBOR} + 0.5\%)] = 1.5\%$ .
2. D As one currency (A) appreciates relative to another currency (B), the value of a currency swap increases on behalf of the currency A payer. As a result, the credit risk of the currency B payer increases.
3. B The fixed interest rate swap will allow for the conversion of a floating-rate liability to a fixed-rate liability.
4. B  $B_{\text{fix}} = [\$14,000 \times e^{-(0.026 \times 0.5)}] + [\$14,000 \times e^{-(0.0265 \times 1.0)}] + [(\$1,000,000 + \$14,000) \times e^{-(0.0275 \times 1.5)}] = \$13,819 + \$13,634 + \$973,023 = \$1,000,476$

Note that we are at a (semiannual) reset date, so the floating rate portion has a value equal to the notional amount.

$$V_{\text{swap}} = (B_{\text{fix}} - B_{\text{floating}}) = \$1,000,476 - \$1,000,000 = \$476$$

5. C  $B_{\$} = 6e^{-0.03 \times 1} + 156e^{-0.03 \times 2} = \$5.82 + \$146.92 = \$152.74$   
 $B_{\text{\euro}} = 5e^{-0.05 \times 1} + 105e^{-0.05 \times 2} = \text{\euro}4.76 + \text{\euro}95.00 = \text{\euro}99.76$

$$V_{\text{swap}} (\text{to X}) = 152.74 - (1.45 \times 99.76) = \$8.09 \text{ million}$$

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The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

# MECHANICS OF OPTIONS MARKETS

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Topic 41

## EXAM FOCUS

Stock options give the owner the right, but not the obligation, to buy or sell a stock at a specific price on or before a specific date. Call options give the owner the right to buy the stock, and put options give the owner the right to sell the stock. An option is exercised when the owner executes the right to buy or sell the stock. This topic covers the basic mechanics of option trading. You should understand the different kinds of options and the system by which exchange-traded options are bought and sold.

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## OPTION TYPES

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### LO 41.1: Describe the types, position variations, and typical underlying assets of options.

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Option contracts have asymmetric payoffs. The buyer of an option has the right to exercise the option but is not obligated to exercise. Therefore, the maximum loss for the buyer of an option contract is the loss of the price (premium) paid to acquire the position, while the potential gains in some cases are theoretically infinite. Because option contracts are a zero-sum game, the seller of the option contract could incur substantial losses, but the maximum potential gain is the amount of the premium received for writing the option. *American options* may be exercised at any time up to and including the contract's expiration date, while *European options* can be exercised only on the contract's expiration date.

To understand the potential returns, we need to introduce the standard symbols used to represent the relevant factors:

- X = strike price or exercise price specified in the option contract (a fixed value)
- $S_t$  = price of the underlying asset at time  $t$
- $C_t$  = the market value of a call option at time  $t$
- $P_t$  = the market value of a put option at time  $t$
- $t$  = the time subscript, which can take any value between 0 and  $T$ , where  $T$  is the maturity or expiration date of the option

## Call Options

A *call option* gives the *owner* the right, but not the obligation, to buy the stock from the seller of the option. The owner is also called the *buyer* or the holder of the *long position*. The buyer benefits, at the expense of the option *seller*, if the underlying stock price is greater than the exercise price. The option *seller* is also called the *writer* or holder of the *short position*.

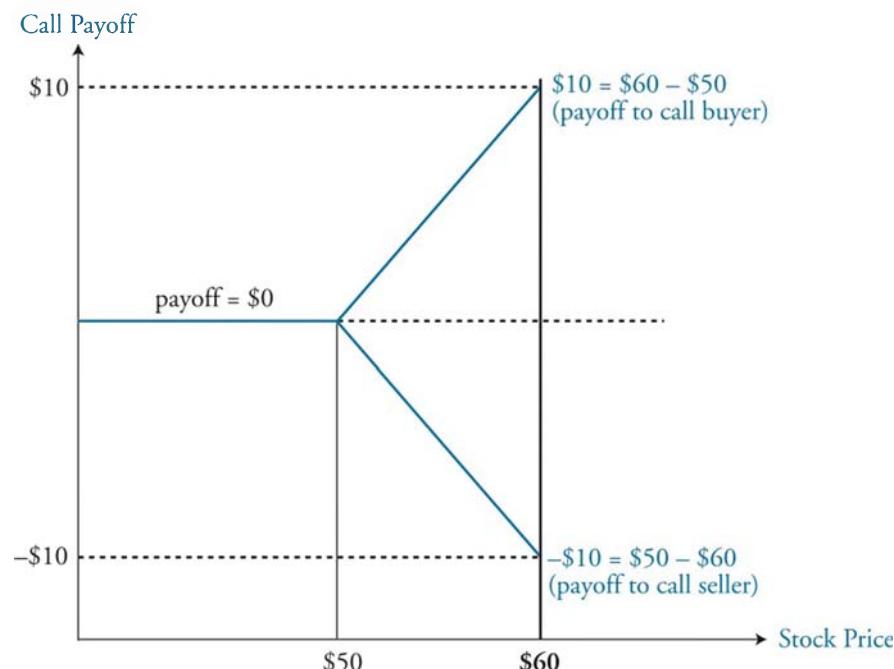
At maturity time  $T$ , if the price of the underlying stock is less than or equal to the strike price of a call option (i.e.,  $S_T \leq X$ ), the payoff is zero, so the option owner would not exercise the option. On the other hand, if the stock price is higher than the exercise price (i.e.,  $S_T > X$ ) at maturity, then the payoff of the call option is equal to the difference between the market price and the strike price ( $S_T - X$ ). The “payoff” (at the option’s maturity) to the call option seller, is the mirror image (opposite sign) of the payoff to the buyer.

Because of the linear relationships between the value of the option and the price of the underlying asset, simple graphs can clearly illustrate the possible value of option contracts at the expiration date. Figure 1 illustrates the payoff of a call with an exercise price equal to 50.



*Professor's Note: An option payoff graph ignores the initial cost of the option.*

Figure 1: Payoff of Call With Exercise Price Equal to \$50



**Example: Payoff of a call option**

An investor writes an at-the-money call option on a stock with an exercise price of 50 ( $X = 50$ ). If the stock price rises to \$60, what will be the *payoff* to the owner and seller of the call option?

**Answer:**

The call option may be exercised with the holder of the long position buying the stock from the writer at 50 for a \$10 gain. The payoff to the option buyer is \$10, and the payoff to the option writer is *negative* \$10. This is illustrated in Figure 1 and, as mentioned, does not include the premium paid for the option.

This example shows just how easy it is to determine option payoffs. At expiration time  $T$  (the option's maturity), the payoff to the option owner, represented by  $C_T$ , is:

$$C_T = S_T - X \quad \text{if} \quad S_T > X$$

$$C_T = 0 \quad \text{if} \quad S_T \leq X$$

Another popular way of writing this is with the “max (0, variable)” notation. If the variable in this expression is greater than zero, then  $\max (0, \text{variable}) = \text{variable}$ ; if the variable’s value is less than zero, then  $\max (0, \text{variable}) = 0$ . Thus, letting the variable be the quantity  $S_0 - X$ , we can write:

$$C_T = \max (0, S_T - X)$$

The payoff to the option seller is the negative value of these numbers. In what follows, we will always talk about payoff in terms of the option owner unless otherwise stated. We should note that  $\max (0, S_t - X)$ , where  $0 < t < T$ , is also the payoff if the owner decides to exercise the call option early. In this topic, we will only consider time  $T$  in our analysis.

Although our focus here is not to calculate  $C_t$ , we should clearly define it as the initial cost of the call when the investor purchases at time 0, which is  $T$  units of time before  $T$ .  $C_0$  is also called the premium. Thus, we can write that the profit to the owner at  $t = T$  is:

$$\text{profit} = C_T - C_0$$

This says that at time  $T$ , the owner’s profit is the option payoff minus the premium paid at time 0. Incorporating  $C_0$  into Figure 1 gives us the profit diagram for a call at expiration, and this is Figure 2.

Figure 2 illustrates an important point, which is that the profit to the owner is negative when the stock price is less than the exercise price plus the premium. At expiration, we can say that:

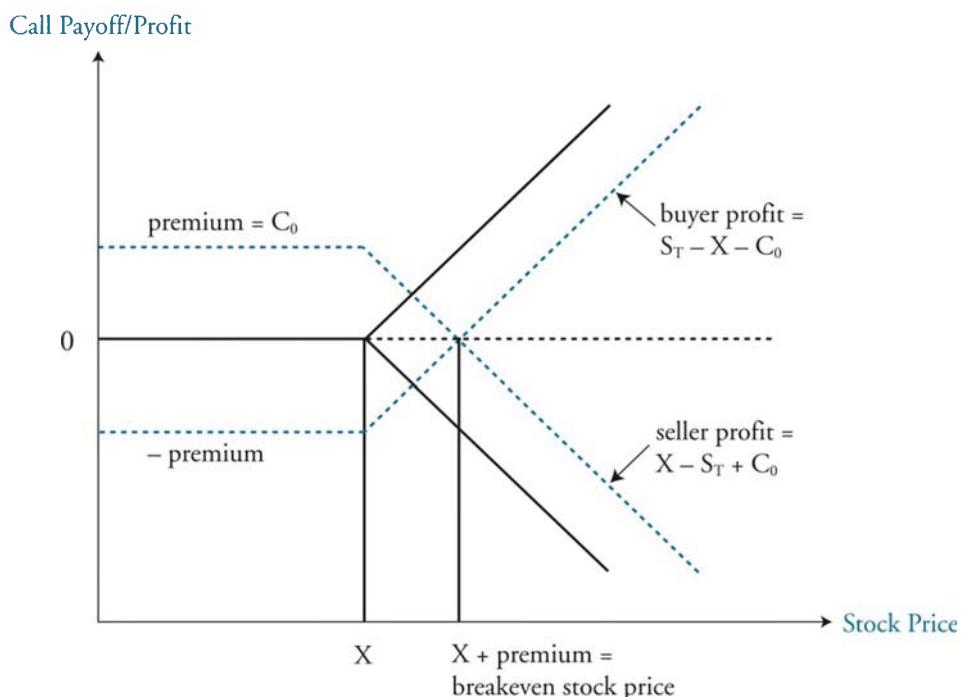
$$\text{if } S_T < X + C_0 \quad \text{then: call buyer profit} < 0 < \text{call seller profit}$$

if  $S_T = X + C_0$  then: call buyer profit = 0 = call seller profit

if  $S_T > X + C_0$  then: call buyer profit > 0 > call seller profit

The **breakeven price** is a very descriptive term that we use for  $X + C_0$ , or  $X + \text{premium}$ .

Figure 2: Profit Diagram for a Call at Expiration



## Put Options

If you understand the properties of a call, the properties of a put should come to you fairly easily. A put option gives the owner the right to sell a stock to the seller of the put at a specific price. At expiration, the buyer benefits if the price of the underlying is less than the exercise price  $X$ :

$$\begin{aligned} P_T &= X - S_T && \text{if } S_T < X \\ P_T &= 0 && \text{if } X \leq S_T \end{aligned}$$

or:

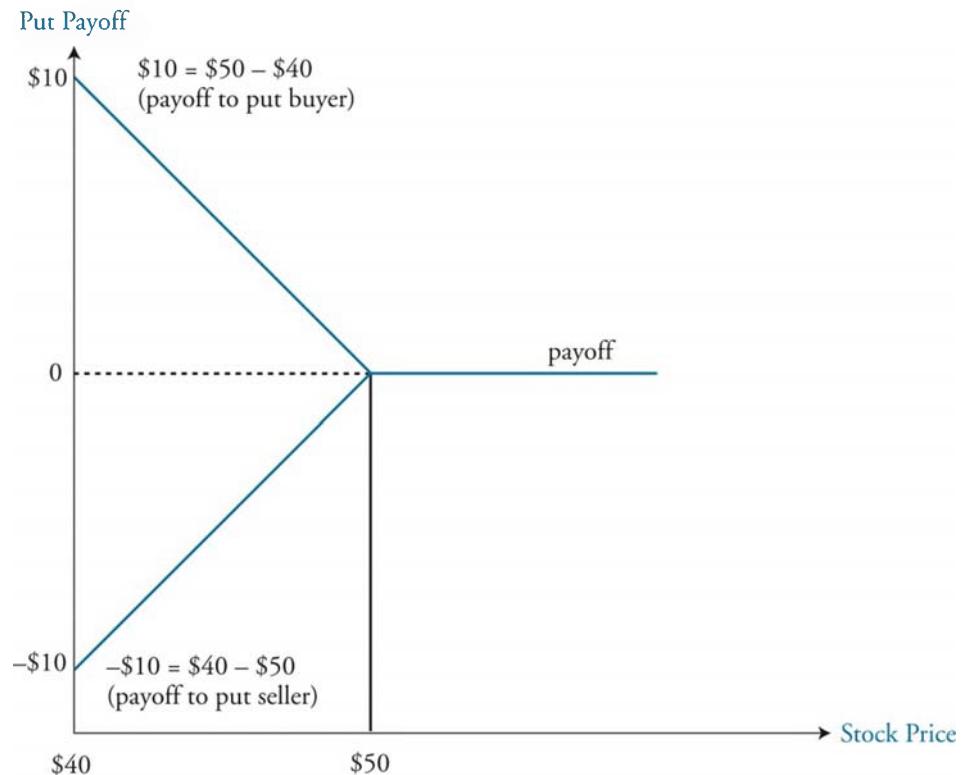
$$P_T = \max(0, X - S_T)$$

For example, an investor writes a put option on a stock with a strike price of  $X = 50$ . If the stock stays at \$50 or above, the payoff of the put option is zero (because the holder may receive the same or better price by selling the underlying asset on the market rather than exercising the option). But if the stock price falls below \$50, say to \$40, the put option may be exercised with the option holder buying the stock from the market at \$40 and selling it to the put writer at \$50 for a \$10 gain. The writer of the put option must pay the put price of \$50 when it can be sold in the market at only \$40, resulting in a \$10 loss. The gain to the option holder is the same magnitude as the loss to the option writer. Figure 3 illustrates this example, excluding the initial cost of the put and

transaction costs. Figure 4 includes the cost of the put (but not transaction costs) and illustrates the profit to the put owner.

**Figure 3: Put Payoff to Buyer and Seller**

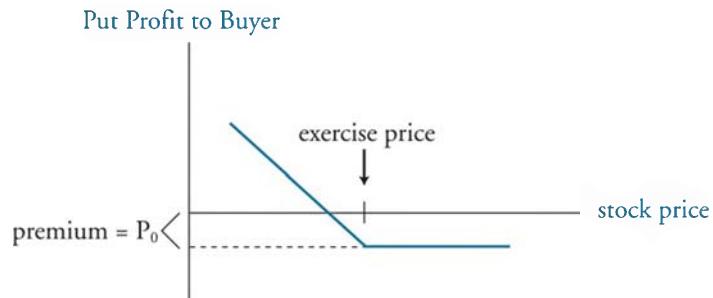
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Given the “mirror image quality” that results from the “zero-sum game” nature of options, we often just draw the profit to the buyer as shown in Figure 4. Then, we can simply remember that each positive (negative) value is a negative (positive) value for the seller.

**Figure 4: Put Profit to Buyer**

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The breakeven price for a put position upon expiration is the exercise price minus the premium paid,  $X - P_0$ .

## UNDERLYING ASSETS

Exchange-traded options trade on four primary assets: individual stocks, foreign currency, stock indices, and futures.

**Stock options.** Stock options are typically exchange-traded, American-style options. Each option contract is normally for 100 shares of stock. For example, if the last trade on a call option occurred at \$3.60, the option contract would cost \$360. After issuance, stock option contracts are adjusted for stock splits but not cash dividends. The primary U.S. exchanges for stock options are the Chicago Board Options Exchange (CBOE), Boston Options Exchange, NYSE Euronext, and the International Securities Exchange.

**Currency options.** Investors holding currency options receive the right to buy or sell an amount of foreign currency based on a domestic currency amount. For calls, a currency option is going to pay off only if the actual exchange rate is above a specified exercise rate. For puts, a currency option is going to pay off only if the actual exchange rate is below a specified exercise rate. The majority of currency options are traded on the over-the-counter market, while the remainder are exchange traded. The NASDAQ OMX trades European-style options for several currencies. Note that the unit size for currency options is considerably larger than stock options (i.e., 1 million units for yen and 10,000 units for other currencies).

**Index options.** Options on stock indices are typically European-style options and are cash settled. Index options can be found on both the over-the-counter markets and the exchange-traded markets. The payoff on an index call is the amount (if any) by which the index level at expiration exceeds the index level specified in the option (the strike price), multiplied by the contract multiplier (typically 100).

### Example: Index options

Assume you own a call option on an index with an exercise price equal to 950. The multiplier for this contract is 100. Compute the payoff on this option assuming that the index is 956 at expiration.

### Answer:

The payoff on an index call (long) is the amount (if any) by which the index level at expiration exceeds the index level specified in the option (the exercise price), multiplied by the contract multiplier. An equal amount will be deducted from the account of the index call option writer. In this example, the expiration date payoff is  $(956 - 950) \times \$100 = \$600$ .

**Futures options.** American-style, exchange-traded options are most often utilized for futures contracts. Typically, the futures option expiration date is set to a date shortly before the expiration date of the futures contract. The market value of the underlying asset for futures options is the value of the underlying futures contract. The payoff for call options is calculated as the futures price less the strike price, while the payoff for put options is calculated as the strike price less the futures price.

## STOCK OPTIONS SPECIFICATIONS

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**LO 41.2: Explain the specification of exchange-traded stock option contracts, including that of nonstandard products.**

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### Expiration

Options can be either American or European style. As mentioned previously, American options can be exercised throughout the life of the option, while European options can only be exercised on the expiration date of the option. For this reason, American options are always at least as valuable as corresponding European options. Exchange-traded stock options are typically American-style options. The expiration dates of these options dictate how the option is named. For example, a June put option on Intel means that the option expires in June. The actual day of expiration is the Saturday following the third Friday of the expiration month. Different expiration cycles dictate the actual expiration months of a stock option over a given year. **Long-term equity anticipation securities (LEAPS®)** are simply long-dated options with expirations greater than one year. All LEAPS have January expirations.

### Strike Prices

Strike prices are dictated by the value of the stock. Low-value stocks have smaller strike increments than higher-value stocks. Typically, stocks that are priced around \$20 have increments of \$2.50, stocks that are priced around \$50 have increments of \$5.00, and so on. The strike price is usually denoted as  $X$  and the underlying stock as  $S$ .

### Moneyness, Time Value, and Intrinsic Value

An *option class* refers to all options of the same type, whether calls or puts. An *option series* refers to an option class with the same expiration. For a call (put), when the underlying asset price is less (greater) than the strike price, the option is said to be out of the money. For both a call and put, when the underlying asset price is equal to the strike price, the option is said to be at the money. For a call (put), when the underlying asset is greater (less) than the strike price, the option is said to be in the money. An option price (or premium) prior to expiration has two components: the time value and the intrinsic value. The *intrinsic value* is the maximum of zero or the difference between the underlying asset and the strike price [i.e., intrinsic value of a call option =  $\max(0, S - X)$  and intrinsic value of a put option =  $\max(0, X - S)$ ]. The *time value* is the difference between the option premium and the intrinsic value.

## Nonstandard Products

Nonstandard option products include flexible exchange (FLEX) options, exchange-traded fund (ETF) options, weekly options, binary options, credit event binary options (CEBOs), and deep out-of-the-money (DOOM) options.

**FLEX options.** FLEX options are exchange-traded options on equity indices and equities that allow some alteration of the options contract specifications. The nonstandard terms include alteration of the strike price, different expiration dates, or European-style (rather than the standard American-style). FLEX options were developed in order for the exchanges to better compete with the nonstandard options that trade over the counter. The minimum size for FLEX trades is typically 100 contracts.

**ETF options.** While similar to index options, ETF options are typically American-style options and utilize delivery of shares rather than cash at settlement.

**Weekly options.** *Weeklys* are short-term options that are created on a Thursday and have an expiration date on the Friday of the next week.

**Binary options.** Binary options generate discontinuous payoff profiles because they pay only one price (\$100) at expiration if the asset value is above the strike price. The term binary means the option payoff has one of two states: the option pays \$100 at expiration if the option is above the strike price or the option pays nothing if the price is below the strike price. Hence, a payoff discontinuity results from the fact that the payoff is only one value—it does not increase continuously with the price of the underlying asset as in the case of a traditional option.

**CEBOs.** A CEBO is a specific form of credit default swap. The payoff in a CEBO is triggered if the reference entity suffers a qualifying credit event (e.g., bankruptcy, missed debt payment, or debt restructuring) prior to the option's expiration date (which always occurs in December). Option payoff, if any, occurs on the expiration date. CEBOs are European options that are cash settled.

**DOOM options.** These put options are structured to only be in the money in the event of a large downward price movement in the underlying asset. Due to their structure, the strike price of these options is quite low. In terms of protection, DOOM options are similar to credit default swaps. Note that this option type is always structured as a put option.

## The Effect of Dividends and Stock Splits

In general, options are not adjusted for cash dividends. This will have option pricing consequences that will need to be incorporated into a valuation model. Options are adjusted for *stock splits*. For example, if a stock has a 2-for-1 stock split, then the strike price will be reduced by one-half and the number of shares underlying the option will double. In general, if a stock experiences a  $b$ -for- $a$  stock split, the strike price becomes  $(a/b)$  of its previous value and the number of shares underlying the option is increased by multiples of  $(b/a)$ . Stock dividends are dealt with in the same manner. For example, if a stock pays a 25% stock dividend, this is treated in the same manner as a 5-for-4 stock split.

## Position and Exercise Limits

The number of options a trader can have on one stock is limited by the exchange. This is called a position limit. Additionally, short calls and long puts are considered to be part of the same position. The exercise limit equals the position limit and specifies the maximum number of option contracts that can be exercised by an individual over any five consecutive business days.

## OPTION TRADING

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### LO 41.3: Describe how trading, commissions, margin requirements, and exercise typically work for exchange-traded options.

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As mentioned, options are quoted relative to one underlying stock. To compute the actual option cost, the quote needs to be multiplied by 100. This is because an options contract represents an option on 100 shares of the underlying stock. The quotes will also include the strike, expiration month, volume, and the option class.

Market makers will quote bid and offer (or ask) prices whenever necessary. They profit on the bid-offer spread and add liquidity to the market. Floor brokers represent a particular firm and execute trades for the general public. The order book official enters limit orders relayed from the floor broker. An offsetting trade takes place when a long (short) option position is offset with a sale (purchase) of the same option. If a trade is not an offsetting trade, then open interest increases by one contract.

## Commissions

Option investors must consider the commission costs associated with their trading activity. Commission costs often vary based on trade size and broker type (discount vs. full service). Brokers typically structure commission rates as a fixed amount plus a percentage of the trade amount. The following example provides an illustration on how commission costs affect an option trade's profitability.

**Example: Commission costs**

An investor buys a call contract with a strike price of \$260. The current price of the underlying stock is \$245. Assume the option price is \$10 and the contract is settled with shares rather than cash. Using the commission schedule for a discount broker below, calculate (1) the commission costs incurred by the investor based on the initial trade and (2) the investor's net profit if the stock price increases to \$280 prior to expiration. Assume the cost to exercise the option is 1% of the trade amount and the cost to sell stock is also 1% of the trade amount.

**Figure 5: Commission Schedule**

<i>Trade Amount</i>	<i>Commission Rate</i>
$\leq \$3,000$	\$30 + 0.8% of trade amount
\$3,001 to \$14,999	\$30 + 0.6% of trade amount
$\geq \$15,000$	\$30 + 0.4% of trade amount

*Other details:*

Minimum charge per contract: \$4  
Maximum charge per contract: \$35

**Answer:**

$$1. \text{ Contract cost} = \$10 \times 100 = \$1,000$$

Initial commission costs =  $\$30 + (\$1,000 \times 0.8\%) = \$38$ . Because this exceeds the maximum contract charge, \$35 is charged (i.e., the maximum contract charge).

$$2. \text{ Gross profit: } \$280 - \$260 = \$20 \text{ per share. } \$20 \times 100 \text{ shares} = \$2,000$$

$$\text{Additional commission costs} = 1\% \times 2 \times \$280 \times 100 = \$560$$

$$\text{Total commission costs} = \$35 + \$560 = \$595$$

$$\text{Net profit} = \$2,000 - \$1,000 - \$595 = \$405$$

Due to the costs associated with exercising the option and then selling the stock, some retail investors may find it more efficient to simply sell the option to another investor.

One final note on option commission costs is that they fail to account for the cost embedded in the bid-offer spread. The cost associated with this spread for options can be calculated by multiplying the spread by 50%. For example, if the bid price is \$12 and the offer price is \$12.20, the associated cost for both the option buyer and option seller would be \$0.10 per contract [ $(\$12.20 - \$12.00) \times 50\%$ ]. This cost is also present in stock transactions.

## Margin Requirements

Options with maturities nine months or fewer cannot be purchased on margin. This is because the leverage would become too high. For options with longer maturities, investors can borrow a maximum of 25% of the option value.

Investors who engage in writing options must have a margin account due to the high potential losses and potential default. The required margin for option writers is dependent on the amount and position of option contracts written.

**Naked options** (or *uncovered options*) refers to options in which the writer does not also own a position in the underlying asset. The size of the initial and maintenance margin for naked option writing is equal to the option premium plus a percentage of the underlying share price. Writing *covered calls* (selling a call option on a stock that is owned by the seller of the option) is far less risky than naked call writing.

## The Options Clearing Corporation

Similar to a clearinghouse for futures, the **Options Clearing Corporation (OCC)** guarantees that buyers and sellers in the exchange-traded options market will honor their obligations and records all option positions. Exchange-traded options have no default risk because of the OCC, while over-the-counter options possess default risk. The OCC requires that all trades are cleared by one of its clearing members. OCC members must meet net capital requirements and help finance an emergency fund that is utilized in the event of a member default. Non-member brokers must contact a clearing member to clear their option trades. The OCC guarantees contract performance and therefore requires option writers to post margin as a means of supporting their obligation and option buyers to deposit required funds by the morning of the business day immediately following the day the option is purchased.

## Exercising an Option

When an investor decides to exercise an option prior to contract expiration, her broker contacts the assigned OCC member responsible for clearing that broker's trades. This OCC member then submits an exercise order to the OCC which matches it with a clearing member who identifies an investor who has written a stock option. This assigned investor then must sell (if a call option) or buy (if a put option) the underlying at the specified strike price on the third business day after the order to exercise is received. Exercising an option results in the open interest being reduced by one. At contract expiration, unexercised options that are in the money after accounting for transaction costs will be exercised by brokers.

## Other Option-Like Securities

Exchange-traded options are not issued by the company and delivery of shares associated with the exercise of exchange-traded options involves shares that are already outstanding. *Warrants* are often issued by a company to make a bond issue more attractive and will typically trade separately from the bond at some point. Warrants are like call options

except that, upon exercise, the company receives the strike price and may issue new shares to deliver. The same distinction applies to *employee stock options*, which are issued as an incentive to company employees and provide a benefit if the stock price rises above the exercise price. When an employee exercises incentive stock options, any shares issued by the company will increase the number of shares outstanding.

*Convertible bonds* contain a provision that gives the bondholder the option of exchanging the bond for a specified number of shares of the company's common stock. At exercise, the newly issued shares increase the number of shares outstanding and debt is retired based on the amount of bonds exchanged for the shares. There is a potential for dilution of the firm's common shares from newly issued shares with warrants, employee stock options, and convertible bonds that does not exist for exchange-traded options.

## KEY CONCEPTS

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### LO 41.1

A call (put) option gives the owner the right to purchase (sell) the underlying asset at a strike price. When the owner executes this right, the option is said to be exercised. Because long (buy, purchase) option positions give the owner the right to exercise, the seller (short, writer) of the option has the obligation to meet the terms of the option.

American options may be exercised at any time up to and including the contract's expiration date, while European options can be exercised only on the contract's expiration date. Exchange-traded options are typically American options.

Primary types of exchange-traded options include option on individual stocks, foreign currency, stock indices, and futures.

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### LO 41.2

For a call (put), when the underlying asset price is less (greater) than the strike price, the option is said to be out of the money. For both a call and put, when the underlying asset price is equal to the strike price, the option is said to be at the money. For a call (put), when the underlying asset price is greater (less) than the strike price, the option is said to be in the money. Options are not adjusted for cash dividends, but are adjusted for stock splits.

LEAPS are options with expiration dates greater than a year. Nonstandard option products include FLEX options, ETF options, weekly options, binary options, CEOBs, and DOOM options.

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### LO 41.3

Options with a maturity of nine months or fewer cannot be purchased on margin and must be paid in full due to the leverage effect of options. For options with longer maturities, investors can borrow up to 25% of the option value. Writers of options are required to have margin accounts with a broker.

Investors must account for commission costs when utilizing option. Commissions vary based on trade size and broker type. Commission rates typically are structured as a fixed dollar amount plus a percentage of the trade amount. In some instances, investors can earn higher profits by selling in-the-money options rather than exercising the options.

The Options Clearing Corporation (OCC) guarantees that buyers and sellers in the options market will honor their obligations and records all option positions. This minimizes default risk.

Warrants, employee stock options, and convertible bonds are option-like securities. Unlike options, these securities are issued by financial institutions or companies. The cost to the issuer of these securities is the possibility of increased dilution of the stock.

## CONCEPT CHECKERS

Use the following data to answer Questions 1 and 2.

An investor owns a stock option that currently has a strike price of \$100.

1. If the stock experiences a 4-to-1 split, the strike price becomes:
  - A. \$20.
  - B. \$25.
  - C. \$50.
  - D. \$100.
2. The number of shares now covered by each option contract is:
  - A. 100.
  - B. 200.
  - C. 300.
  - D. 400.
3. If an option is quoted at \$2.75, the cost of one contract to the potential buyer is closest to:
  - A. \$0.275.
  - B. \$2.75.
  - C. \$275.00.
  - D. \$2,750.00.
4. Which of the following statements regarding option value or expiration is correct?
  - I. American-style options are less valuable than European options.
  - II. All options expire on the third Wednesday of the expiration month.
  - A. I only.
  - B. II only.
  - C. Both I and II.
  - D. Neither I nor II.
5. Which of the following option characteristics is correct?
  - I. A put option is in the money when the asset price is less than the strike price.
  - II. LEAPS are long-term (over one-year) options that expire in December of each year.
  - A. I only.
  - B. II only.
  - C. Both I and II.
  - D. Neither I nor II.

**CONCEPT CHECKER ANSWERS**

1. B  $\frac{a}{b} = \frac{1}{4} \times \$100 = \$25$
2. D  $\frac{b}{a} = \frac{4}{1} \times \$100 = \$400$  (Each option contract is originally for 100 shares.)
3. C Multiply the quote by 100 because each option contract is for 100 shares.  $\$2.75 \times 100 = \$275.00$
4. D American-style options are at least as valuable as European-style options. Options expire on the Saturday after the third Friday.
5. A A put option is in the money when the asset price is less than the strike price. LEAPS expire in January.