

# Pierre de Fermat

**Pierre de Fermat**, (born August 17, 1601, Beaumont-de-Lomagne, France—died January 12, 1665, Castres), French mathematician who is often called the founder of the modern theory of numbers. Together with René Descartes, Fermat was one of the two leading mathematicians of the first half of the 17th century. Independently of Descartes, Fermat discovered the fundamental principle of analytic geometry. His methods for finding tangents to curves and their maximum and minimum points led him to be regarded as the inventor of the differential calculus. Through his correspondence with Blaise Pascal he was a co-founder of the theory of probability.

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Fermat, portrait by Roland Lefèvre;  
in the Narbonne City Museums,  
France  
  
*Courtesy of the Musée de la Ville de  
Narbonne, France*

## Life and early work

Little is known of Fermat's early life and education. He was of Basque origin and received his primary education in a local Franciscan school. He studied law, probably at Toulouse and perhaps also at Bordeaux. Having developed tastes for foreign languages, classical literature, and ancient science and mathematics, Fermat followed the custom of his day in composing conjectural "restorations" of lost works of antiquity. By 1629 he had begun a reconstruction of the long-lost *Plane Loci* of Apollonius, the Greek geometer of the 3rd century BCE. He soon found that the study of loci, or sets of points with certain characteristics, could be facilitated by the application of algebra to geometry through a coordinate system. Meanwhile, Descartes had observed the same basic principle of analytic geometry, that equations in two

variable quantities define plane curves. Because Fermat's *Introduction to Loci* was published posthumously in 1679, the exploitation of their discovery, initiated in Descartes's *Géométrie* of 1637, has since been known as Cartesian geometry.

In 1631 Fermat received the baccalaureate in law from the University of Orléans. He served in the local parliament at Toulouse, becoming councillor in 1634. Sometime before 1638 he

became known as Pierre de Fermat, though the authority for this designation is uncertain. In 1638 he was named to the Criminal Court.

## Analyses of curves

Fermat's study of curves and equations prompted him to generalize the equation for the ordinary parabola  $ay = x^2$ , and that for the rectangular hyperbola  $xy = a^2$ , to the form  $a^{n-1}y = x^n$ . The curves determined by this equation are known as the parabolas or hyperbolas of Fermat according as  $n$  is positive or negative. He similarly generalized the Archimedean spiral  $r = a\theta$ . These curves in turn directed him in the middle 1630s to an algorithm, or rule of mathematical procedure, that was equivalent to differentiation. This procedure enabled him to find equations of tangents to curves and to locate maximum, minimum, and inflection points of polynomial curves, which are graphs of linear combinations of powers of the independent variable. During the same years, he found formulas for areas bounded by these curves through a summation process that is equivalent to the formula now used for the same purpose in the integral calculus. Such a formula is:

$$A = \int_0^a x^n dx = a^{n+1}/(n+1).$$

It is not known whether or not Fermat noticed that differentiation of  $x^n$ , leading to  $na^{n-1}$ , is the inverse of integrating  $x^n$ . Through ingenious transformations he handled problems involving more general algebraic curves, and he applied his analysis of infinitesimal quantities to a variety of other problems, including the calculation of centres of gravity and finding the lengths of curves. Descartes in the *Géométrie* had reiterated the widely held view, stemming from Aristotle, that the precise rectification or determination of the length of algebraic curves was impossible; but Fermat was one of several mathematicians who, in the years 1657–59, disproved the dogma. In a paper entitled “De Linearum Curvarum cum Lineis Rectis Comparatione” (“Concerning the Comparison of Curved Lines with Straight Lines”), he showed that the semicubical parabola and certain other algebraic curves were strictly rectifiable. He also solved the related problem of finding the surface area of a segment of a paraboloid of revolution. This paper appeared in a supplement to the *Veterum Geometria Promota*, issued by the mathematician Antoine de La Loubère in 1660. It was Fermat's only mathematical work published in his lifetime.

## Disagreement with other Cartesian views

Fermat differed also with Cartesian views concerning the law of refraction (the sines of the angles of incidence and refraction of light passing through media of different densities are in a constant ratio), published by Descartes in 1637 in *La Dioptrique*; like *La Géométrie*, it was an appendix to his celebrated *Discours de la méthode*. Descartes had sought to justify the sine law through a premise that light travels more rapidly in the

denser of the two media involved in the refraction. Twenty years later Fermat noted that this appeared to be in conflict with the view espoused by Aristotelians that nature always chooses the shortest path. Applying his method of maxima and minima and making the assumption that light travels less rapidly in the denser medium, Fermat showed that the law of refraction is consonant with his "principle of least time." His argument concerning the speed of light was found later to be in agreement with the wave theory of the 17th-century Dutch scientist Christiaan Huygens, and in 1849 it was verified experimentally by A.-H.-L. Fizeau.

Through the mathematician and theologian Marin Mersenne, who, as a friend of Descartes, often acted as an intermediary with other scholars, Fermat in 1638 maintained a controversy with Descartes on the validity of their respective methods for tangents to curves. Fermat's views were fully justified some 30 years later in the calculus of Sir Isaac Newton. Recognition of the significance of Fermat's work in analysis was tardy, in part because he adhered to the system of mathematical symbols devised by François Viète, notations that Descartes's *Géométrie* had rendered largely obsolete. The handicap imposed by the awkward notations operated less severely in Fermat's favourite field of study, the theory of numbers; but here, unfortunately, he found no correspondent to share his enthusiasm. In 1654 he had enjoyed an exchange of letters with his fellow mathematician Blaise Pascal on problems in probability concerning games of chance, the results of which were extended and published by Huygens in his *De Ratiociniis in Ludo Aleae* (1657).

## Work on theory of numbers

Fermat vainly sought to persuade Pascal to join him in research in number theory. Inspired by an edition in 1621 of the *Arithmetic* of Diophantus, the Greek mathematician of the 3rd century AD, Fermat had discovered new results in the so-called higher arithmetic, many of which concerned properties of prime numbers (those positive integers that have no factors other than 1 and themselves). One of the most elegant of these had been the theorem that every prime of the form  $4n + 1$  is uniquely expressible as the sum of two squares. A more important result, now known as Fermat's lesser theorem, asserts that if  $p$  is a prime number and if  $a$  is any positive integer, then  $a^p - a$  is divisible by  $p$ . Fermat seldom gave demonstrations of his results, and in this case proofs were provided by Gottfried Leibniz, the 17th-century German mathematician and philosopher, and Leonhard Euler, the 18th-century Swiss mathematician. For occasional demonstrations of his theorems Fermat used a device that he called his method of "infinite descent," an inverted form of reasoning by recurrence or mathematical induction. One unproved conjecture by Fermat turned out to be false. In 1640, in letters to mathematicians and to other knowledgeable thinkers of the day, including Blaise Pascal, he announced his belief that numbers of the form  $2^{2^n} + 1$ , known since as "numbers of Fermat," are necessarily

prime; but a century later Euler showed that  $2^{2^5} + 1$  has 641 as a factor. It is not known if there are any primes among the Fermat numbers for  $n > 5$ . Carl Friedrich Gauss in 1796 in Germany found an unexpected application for Fermat numbers when he showed that a regular polygon of  $N$  sides is constructible in a Euclidean sense if  $N$  is a prime Fermat number or a product of distinct Fermat primes. By far the best known of Fermat's many theorems is a problem known as his "great," or "last," theorem. This appeared in the margin of his copy of Diophantus' *Arithmetica* and states that the equation  $x^n + y^n = z^n$ , where  $x, y, z$ , and  $n$  are positive integers, has no solution if  $n$  is greater than 2. This theorem remained unsolved until the late 20th century.

Fermat was the most productive mathematician of his day. But his influence was circumscribed by his reluctance to publish.

Carl B. Boyer

#### CITATION INFORMATION

ARTICLE TITLE: Pierre de Fermat

WEBSITE NAME: Encyclopaedia Britannica

PUBLISHER: Encyclopaedia Britannica, Inc.

DATE PUBLISHED: 13 August 2019

URL: <https://www.britannica.com/biography/Pierre-de-Fermat>

ACCESS DATE: January 06, 2020