3D Computer Vision

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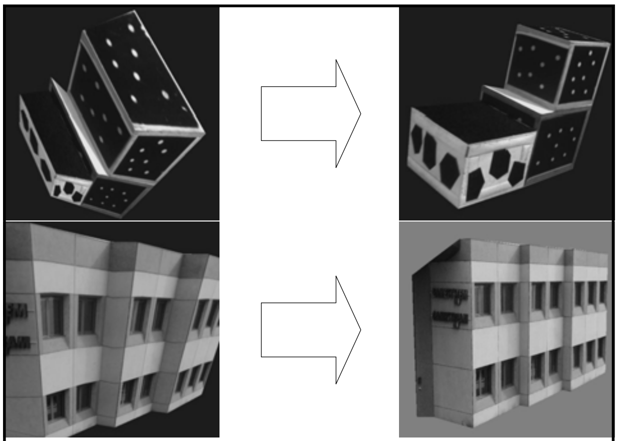
Lecture 24 – Camera Self-Calibration
From a Projective Reconstruction
(Chapters 8 and 19)



Introduction

Camera calibration can be computed from knowledge:

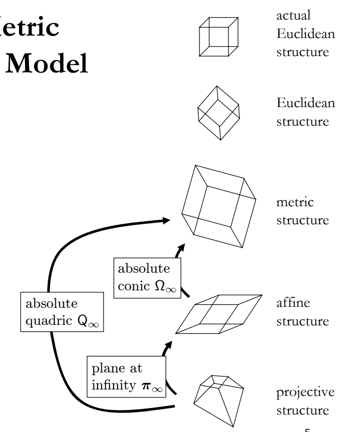
- On the scene, *e.g.* right angles, equal lengths
- On the camera motion, *e.g.* pure translation, pure rotation, stereo rig
- On the camera intrinsic parameters
  - Classical setup: unknown and constant
  - Other setup: all known besides the (varying) focal length

Introduction

- Self-calibration: determining internal camera parameters from multiple uncalibrated images
- Key idea: a camera moves rigidly, keeping fixed the absolute conic under the motion
- If a fixed conic can be determined by some means, the metric geometry can be computed

Modeling the Metric Properties of a 3D Model

- The metric properties of a projective reconstruction are contained in the absolute quadric
- The absolute quadric is equivalent to the plane at infinity + the absolute conic
- The affine properties of a projective reconstruction are contained in the plane at infinity
- The metric properties of an affine reconstruction at contained in the absolute conic



Possible Approaches

- Given a projective reconstruction
  - The *direct approach* determines the absolute quadric and upgrades the reconstruction to metric
  - The *stratified approach* computes the plane at infinity, upgrades the reconstruction to affine, compute the absolute conic and upgrades the reconstruction to metric
- Without an explicit projection reconstruction
  - *e.g.* the Kruppa equations

Scenarios

- Constant intrinsics

$$\mathsf{K}_i = \mathsf{K}(f, u_0, v_0) = \begin{pmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Varying focal length, know principal point

$$\mathsf{K}_i = \mathsf{K}(f_i) = \begin{pmatrix} f_i & 0 & u_0 \\ 0 & f_i & v_0 \\ 0 & 0 & 1 \end{pmatrix}$$

Problem Statement

★ The Projective 3D model is defined up to a projective basis

$$\mathsf{P}_i \mathbf{Q}_j \sim \mathsf{P}_i \mathsf{H}\mathsf{H}^{-1} \mathbf{Q}_j \sim \mathsf{P}'_i \mathbf{Q}'_j$$

where H is a (4×4) homography (with $\det(\mathsf{H}) \neq 0$) with 15 parameters

★ We are looking for a Euclidean 3D model defined up to a similarity $\mathsf{S} \in SE(3)$ with 7 parameters

★ The number of unknowns is thus $15 - 7 = 8$

★ We are looking for a projective transformation Z such that

$$\mathsf{P}_i \mathsf{Z} \quad \text{and} \quad \mathsf{Z}^{-1} \mathbf{Q}_j$$

are projection matrices and 3D points in a Euclidean coordinates frame

Unknown and Constant Intrinsic Parameters

$$\exists \mathsf{K} \quad \text{s.t.} \quad \forall i, \; \mathsf{P}_i \mathsf{Z} \sim (\; \mathsf{K}\mathsf{R}_i \quad -\mathsf{K}\mathsf{R}_i \mathbf{t}_i)$$

(1) – Keep the three first columns only:

$$\mathsf{P}_i \mathsf{Z} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \sim \mathsf{K}\mathsf{R}_i$$

(2) – Multiply each side by its transpose:

$$\mathsf{P}_i \mathsf{Z} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \mathsf{Z}^\mathsf{T} \mathsf{P}_i^\mathsf{T} \sim \mathsf{K}\mathsf{K}^\mathsf{T}$$

Unknown and Constant Intrinsic Parameters

(3) – Estimate X instead of Z:

$$\mathsf{X} \sim \mathsf{Z} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \mathsf{Z}^\mathsf{T}$$

(4) — Solve the (quadratic) equations on X:

$$\forall i \;\; \forall l \;\; \mathsf{P}_i \mathsf{X} \mathsf{P}_i^\mathsf{T} \sim \mathsf{P}_l \mathsf{X} \mathsf{P}_l^\mathsf{T}$$

(5) – Extract Z from X using *e.g.* Cholesky decomposition

Note 1: X is (4×4) rank 3 and represents the dual absolute quadric
Note 2: $\mathsf{K}\mathsf{K}^\mathsf{T}$ is the dual image of the absolute conic

Varying Focal Length Calibration

$$\mathsf{K}_i \sim \begin{pmatrix} \tau f_i & s f_i & u_0 \\ 0 & f_i & v_0 \\ 0 & 0 & 1 \end{pmatrix} \sim \underbrace{\begin{pmatrix} \tau & s & u_0 \\ 0 & 1 & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_{\tilde{\mathsf{K}}} \begin{pmatrix} f_i & 0 & 0 \\ 0 & f_i & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

aspect ratio $\tau = 1$; skew $s = 0$; principal point $(u_0, v_0) = \frac{1}{2}(X, Y)$

(1) – From previous method:

$$\mathsf{P}_i \mathsf{X} \mathsf{P}_i^\mathsf{T} \sim \mathsf{K}\mathsf{K}^\mathsf{T}$$

(2) – Mutliply by the inverve of the known, shared part $\tilde{\mathsf{K}}$ of the K_i:

$$\tilde{\mathsf{K}}^{-1} \mathsf{P}_i \mathsf{X} \mathsf{P}_i^\mathsf{T} \tilde{\mathsf{K}}^{-\mathsf{T}} \sim \begin{pmatrix} f_i^2 & 0 & 0 \\ 0 & f_i^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Varying Focal Length Calibration

(3) – Solve the set of linear equations on X:

$$\forall i \; \begin{cases} \left(\tilde{\mathsf{K}}^{-1} \mathsf{P}_i \mathsf{X} \mathsf{P}_i^\mathsf{T} \tilde{\mathsf{K}}^{-\mathsf{T}}\right)_{(2,1)} & = \; 0 \\ \left(\tilde{\mathsf{K}}^{-1} \mathsf{P}_i \mathsf{X} \mathsf{P}_i^\mathsf{T} \tilde{\mathsf{K}}^{-\mathsf{T}}\right)_{(3,1)} & = \; 0 \\ \left(\tilde{\mathsf{K}}^{-1} \mathsf{P}_i \mathsf{X} \mathsf{P}_i^\mathsf{T} \tilde{\mathsf{K}}^{-\mathsf{T}}\right)_{(3,2)} & = \; 0 \\ \left(\tilde{\mathsf{K}}^{-1} \mathsf{P}_i \mathsf{X} \mathsf{P}_i^\mathsf{T} \tilde{\mathsf{K}}^{-\mathsf{T}}\right)_{(1,1)} & = \; \left(\tilde{\mathsf{K}}^{-1} \mathsf{P}_i \mathsf{X} \mathsf{P}_i^\mathsf{T} \tilde{\mathsf{K}}^{-\mathsf{T}}\right)_{(2,2)} \end{cases}$$

(4) – Extract Z from X using Cholesky decomposition

Singularities

- Multiple solutions to the self-calibration problem
- Generic, *i.e.* algorithm independent
- Cause numerical instability
- Critical Motion Sequences (CMS) – [Sturm, 1997 ; Kahl *et al.*, 1999]
- For the variable focal length scenario, there exists three types of CMS

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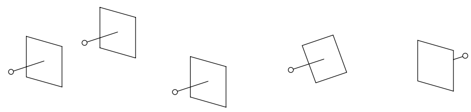
Critical Motion Sequences – Type I

- Position: Arbitrary
- Orientation: Parallel optical axes
- Description: Purely translational motion, combined with rotations around the optical axis or a change of viewing direction



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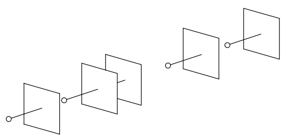
Critical Motion Sequences – Type II

- Position: Aligned optical centres
- Orientation: The optical axes coincide with the line, except for two positions where the orientation is arbitrary
- Description: Translation along the optical axis



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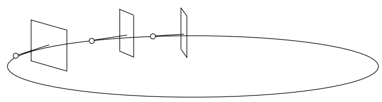
Critical Motion Sequences – Type III

- Position: Optical centres on two conics
- Orientation: Optical axes tangent to the conics
- Description: Motion on a conic and gaze in the motion direction



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Other Approaches to Camera Self-Calibration

- The Kruppa equations
  - Not based on a projective reconstruction
  - Use only the fundamental matrices between pairs of images
- Minimal solutions for two views
  - Focal lengths from the fundamental matrix
  - Two cases: Different or identical focal lengths

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Self-calibration For a Rotating Camera

Video of a track and field event

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Background Panorama

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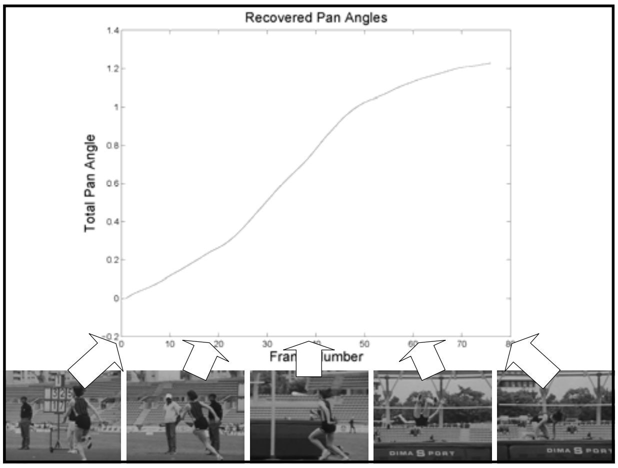

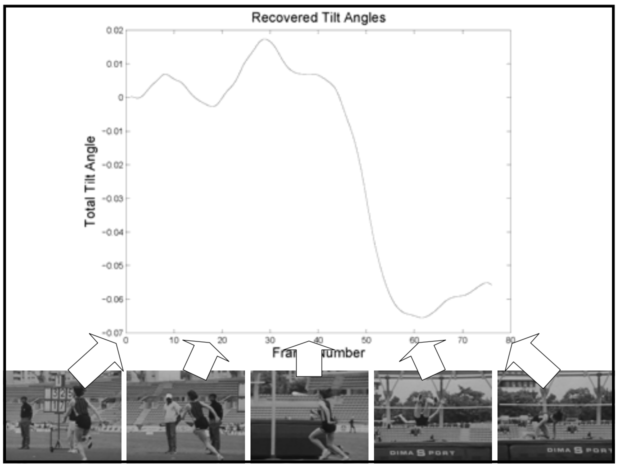

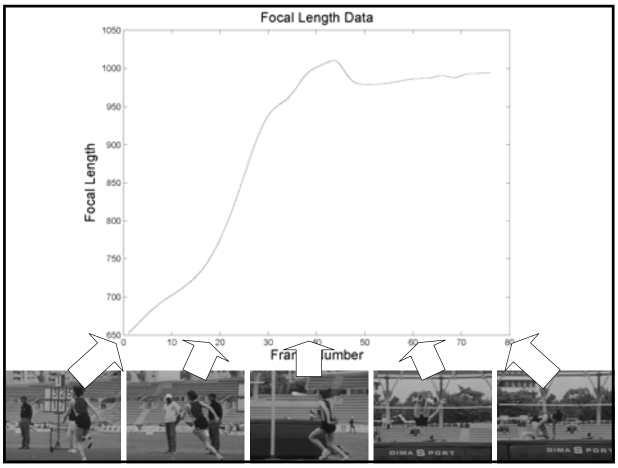

Closure

- Self-calibration makes simple assumptions on the camera
- It makes flexible metric Structure-from-Motion: metric reconstruction is possible given a sequence of images for which (almost) nothing is known
- Further reading:
  - The absolute quadric [Triggs, 1997]
  - Linear self-calibration with varying focal length [Pollefeys *et al.*, 1998]

Note: Some of the slides on camera self-calibration are strongly inspired from slides kindly provided by P. Sturm from INRIA, France.

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