MF Benchmark Functions

1、borehole:

$$f(\mathbf{x}) = \frac{2\pi T_u(H_u - H_l)}{\ln(r/r_w)\left(1 + \frac{2LT_u}{\ln(r/r_w)r_w^2K_w} + \frac{T_u}{T_l}\right)}$$

$$f_L(\mathbf{x}) = \frac{5T_u(H_u - H_l)}{\ln(r/r_w)\left(1.5 + \frac{2LT_u}{\ln(r/r_w)r_w^2K_w} + \frac{T_u}{T_l}\right)}$$

2、branin:

$$high: f(\mathbf{x}) = (x_2 - bx_1^2 + cx_1 - r)^2 + s(1-t)\cos(x_1) + s$$

$$mid: f_m(\mathbf{x}) = \sqrt{f_{high}(\mathbf{x}-2)} + \frac{2(x_1 - 0.5) - 3(3x_2 - 1) - 1}{100}$$

$$Low: f_m(1.2(\mathbf{x}+2)) - \frac{3x_2 + 1}{100}$$

3、currin:

$$high: f_{high}(\mathbf{x}) = \left[1 - \exp\left(-\frac{1}{2x_2}\right)\right]\frac{2300x_1^3 + 1900x_1^2 + 2092x_1 + 60}{100x_1^3 + 500x_1^2 + 4x_1 + 20}$$

$$Low: f_{low}(\mathbf{x}) = \frac{1}{4}[f_{high}(x_1 + 0.05, x_2 + 0.05) + f_{high}(x_1 + 0.05, \max(0, x_2 - 0.05)) + f_{high}(x_1 - 0.05, x_2 + 0.05) + f_{high}\left(x_1 - 0.05, \max\left(0,\right.\right.$$

4、hartmann:

$$f(x,\alpha) = -\sum_{i=1}^{4}\alpha_i \exp\left(-\sum_{j=1}^{3}A_{i,j}(x_j - P_{i,j})^2\right)$$

$$\mathbf{A} = \begin{bmatrix} 3.0 & 10 & 30 \\ 0.1 & 10 & 35 \\ 3.0 & 10 & 30 \\ 0.1 & 10 & 35 \end{bmatrix}$$

$$\mathbf{P} = 10^{-4}\begin{bmatrix} 3689 & 1170 & 2673 \\ 4699 & 4387 & 7470 \\ 1091 & 8732 & 5547 \\ 381 & 5743 & 8828 \end{bmatrix}$$

$$high: \alpha = (1.0, 1.2, 3.0, 3.2)^T$$

$$middle: \alpha = (1.01, 1.19, 2.9, 3.3)^T$$

$$low: \alpha = (1.02, 1.18, 2.8, 3.4)^T$$

5、park:

$$high: f_{high}(\mathbf{x}) = \frac{x_1}{2}\left[\sqrt{1 + (x_2 + x_3^2)\frac{x_4}{x_1^2}} - 1\right] + (x_1 + 3x_4)\exp[1 + \sin x_3]$$

$$Low: f_{low}(\mathbf{x}) = \left[1 + \frac{\sin x_1}{10}\right]f_{high}(\mathbf{x}) - 2x_1 + x_2^2 + x_3^2 + 0.5$$

6、forrester_my

from highest fidelity to 4 lowest fidelity level:

$$f_1(x) = (6x-2)^2 \sin(12x-4)$$
$$f_2(x) = (5.5x-2.5)^2 \sin(12x-4)$$
$$f_3(x) = 0.75 f_1(x) + 5(x-0.5) - 2$$
$$f_4(x) = 0.5 f_1(x) + 10(x-0.5) - 5$$

x: $[0,1]^{\mathrm{D}}$

from:

Forrester, A.I.; Sóbester, A.; Keane, A.J. Multi-fidelity optimization via surrogate modelling. Proc. R. Soc. A Math. Phys. Eng. Sci. 2007, 463, 3251–3269. [CrossRef]

两个版本？：

$y^L(x) = \begin{cases} 0.5(6x-2)^2 \sin(12x-4) \\ \quad + 10(x-0.5) - 5, \quad 0.5 < x \leq 1, \\ 0.5(6x-2)^2 \sin(12x-4) \\ \quad + 10(x-0.5) - 2, \quad 0.5 < x \leq 1; \end{cases}$	$y^H(x) = \begin{cases} 2y^L(x) - 20x + 20, \\ \qquad 0 \leq x \leq 0.5, \\ 4 + 2y^L(x) - 20x + 20, \\ \qquad 0.5 \leq x \leq 1. \end{cases}$	$x \in [0,1]$

(和tl1重复了 ---

7、Non linear Sin

$$high: f_{high}(x) = (x - \sqrt{2}) f_{low}(x)^2$$
$$low: f_{low}(x) = \sin(8\pi x)$$

8~17: tl2~tl10

2	HF	$y_h = \sin(2\pi(x-0.1)) + x^2$	1	$[0,1]^{\mathrm{D}}$
	LF	$y_l = \sin(2\pi(x-0.1))$		
3	HF	$y_h = x\sin(x)/10$	1	$[0,10]^{\mathrm{D}}$
	LF	$y_l = x\sin(x)/10 + x/10$		
4	HF	$y_h = \cos(3.5\pi x)\exp(-1.4x)$	1	$[0,1]^{\mathrm{D}}$
	LF	$y_l = \cos(3.5\pi x)\exp(-1.4x) + 0.75x^2$		
5	HF	$y_h = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$	2	$[-2,2]^{\mathrm{D}}$
	LF	$y_l = 2x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + 0.5x_1x_2 - 4x_2^2 + 2x_2^4$		
6	HF	$y_h = \frac{1}{6}[(30 + 5x_1 \sin(5x_1))(4 + \exp(-5x_2)) - 100]$	2	$[0,1]^{\mathrm{D}}$
	LF	$y_l = \frac{1}{6}\left[(30 + 5x_1 \sin(5x_1))\left(4 + \frac{2}{5}\exp(-5x_2)\right) - 100\right]$		
7	HF	$y_h = \sum_{i=1}^{2} x_i^4 - 16x_i^2 + 5x_i$	2	$[-3,4]^{\mathrm{D}}$
	LF	$y_l = \sum_{i=1}^{2} x_i^4 - 16x_i^2$		
8	HF	$y_h = [1 - 2x_1 + 0.05\sin(4\pi x_2 - x_1)]^2 + [x_2 - 0.5\sin(2\pi x_1)]^2$	2	$[0,1]^{\mathrm{D}}$
	LF	$y_l = [1 - 2x_1 + 0.05\sin(4\pi x_2 - x_1)]^2 + 4[x_2 - 0.5\sin(2\pi x_1)]^2$		
9	HF	$y_h = (x_1 - 1)^2 + (x_1 - x_2)^2 + x_2x_3 + 0.5$	3	$[0,1]^{\mathrm{D}}$
	LF	$y_l = 0.2y_h - 0.5x_1 - 0.2x_1x_2 - 0.1$		
10	HF	$y_h = \sum_{i=1}^{8} x_i^4 - 16x_i^2 + 5x_i$	8	$[-3,3]^{\mathrm{D}}$
	LF	$y_l = \sum_{i=1}^{8} 0.3x_i^4 - 16x_i^2 + 5x_i$		

Note: D stands for the dimension of the functions, and S is the design space.

18~25: test3~test9

Test-3 (*cf.* [32])	$y^L(x) = \mathrm{e}^{1.4x}\cos(3.5\pi x)$	$y^H(x) = \mathrm{e}^{x}\cos(x) + \frac{1}{x^2}$	$x \in [0, 1]$
Test-4 (*cf.* [33])	$y^L(x) = \sin\left(\frac{2\pi x}{10}\right) + 0.2\sin\left(\frac{2\pi x}{2.5}\right)$	$y^H(x) = \sin\left(\frac{2\pi x}{2.5}\right) + \cos\left(\frac{2\pi x}{2.5}\right)$	$x \in [0, 10]$
Test-5 (*cf.* [34])	$y^L(x_1, x_2) = y^H(0.7x_1, 0.7x_2) + x_1x_2 - 65$	$y^H(x_1, x_2) = 4x_1^2 - 2.1x_1^4 + \frac{x_1^6}{3} - 4x_2^2 + 4x_2^4 + x_1x_2$	$(x_1, x_2) \in [-2, 2]^2$
Test-6 (*cf.* [35])	$y^L(\mathbf{x}) = 100\mathrm{e}^{\sin(x_1)} + 5x_2x_3 + x_4 + \mathrm{e}^{x_5x_6}$	$y^H(\mathbf{x}) = \mathrm{e}^{\sin[(0.9(x_1+0.48))^{10}]} + x_2x_3 + x_4$	$\mathbf{x} \in [0, 1]^6$
Test-7 (*cf.* [36])	$y^L(\mathbf{x}) = \sum_{i=5}^8 x_i\cos\left(\sum_{j=1}^4 x_j\right) + \sum_{i=5}^8 x_i\sin\left(\sum_{j=1}^4 x_j\right)$	$y^H(\mathbf{x}) = \left[\left(\sum_{i=5}^8 x_i\cdot\cos\left(\sum_{j=1}^4 x_j\right)\right)^2 + \left(\sum_{i=5}^8 x_i\cdot\sin\left(\sum_{j=1}^4 x_j\right)\right)^2\right]^{\frac{1}{2}}$	$(x_1, \cdots, x_8) \in [0, 2\pi]^4 \times [0, 1]^4$
Test-8 (*cf.* [20])	$y^H(\mathbf{x}) = (x_1)^2 + \sum_{i=2}^{20}(2x_i^2 - x_{i-1})^2$	$y^L(\mathbf{x}) = 0.8y^H(\mathbf{x}) - \sum_{i=1}^{19} 0.4x_ix_{i+1} - 50$	$\mathbf{x} \in [-3, 3]^{20}$
Test-9 (*cf.* [37])	$y^L(\mathbf{x}) = (y^H(\mathbf{x}))^3 + (y^H(\mathbf{x}))^2 + y^H(\mathbf{x})$	$y^H(\mathbf{x}) = (x_1 - 1)^2 + (x_{30} - 1)^2 + 30\sum_{i=1}^{29}(30 - i)(x_i^2 - x_{i+1})^2$	$\mathbf{x} \in [-3, 2]^{30}$

26~30:

Test	Formulation	Ref.	Domain	D
P_1	$f_1(x) = \sin(30(x-0.9)^4)\cos(2(x-0.9)) + (x-0.9)/2 + \eta_1\lambda_1(x)$	[19]	$x \in [0, 1]$	1
	$f_2(x) = (f_1(x) - 1 + x)/(1 + 0.25x) + \eta_2\lambda_1(x)$	[19]		
	$f_3(x) = \sin(20(x-0.87)^4)\cos(2(x-0.87)) + (x-0.87)/2 - (2.5 - (0.7x - 0.14)^2) + 2x + \eta_3\lambda_1(x)$	–		
P_2	$f_1(x) = \sin(30(x-0.9)^4)\cos(2(x-0.9)) + (x-0.9)/2 + \eta_1\lambda_2(x)$	[19]	$x \in [0, 1]$	1
	$f_2(x) = (f_1(x) - 1 + x)/(1 + 0.25x) + \eta_2\lambda_2(x)$	[19]		
	$f_3(x) = \sin(20(x-0.87)^4)\cos(2(x-0.87)) + (x-0.87)/2 - (2.5 - (0.7x - 0.14)^2) + 2x + \eta_3\lambda_2(x)$	–		
P_3	$f_1(\mathbf{x}) = \sum_{j=1}^{\mathbf{D}-1}[100(x_{j+1} - x_j^2)^2 + (1 - x_j)^2] + \eta_1\lambda_1(x_1)$	[9]	$\mathbf{x} \in [-2, 2]$	2
	$f_2(\mathbf{x}) = \sum_{j=1}^{\mathbf{D}-1}[50(x_{j+1} - x_j^2)^2 + (-2 - x_j)^2] - \sum_{j=1}^{\mathbf{D}} 0.5x_j + \eta_2\lambda_1(x_1)$	–		
	$f_3(\mathbf{x}) = (f_1(\mathbf{x}) - 4 - \sum_{j=1}^{\mathbf{D}} 0.5x_j)/(10 + \sum_{j=1}^{\mathbf{D}} 0.25x_j) + \eta_3\lambda_1(x_j)$	[9]		
P_4	$f_1(\mathbf{x}) = \sum_{j=1}^{\mathbf{D}} \frac{x_j^2}{25} - \prod_{j=1}^{\mathbf{D}}\cos\left(\frac{x_j}{\sqrt{j}}\right) + 1 + \eta_1\lambda_1(x_1)$	[21]	$\mathbf{x} \in [-6, 5]$	2
	$f_2(\mathbf{x}) = -\prod_{j=1}^{\mathbf{D}}\cos\left(\frac{x_j}{\sqrt{j}}\right) + 1 + \eta_2\lambda_1(x_1)$	–		
	$f_3(\mathbf{x}) = \sum_{j=1}^{\mathbf{D}} \frac{x_j^2}{20} - \prod_{j=1}^{\mathbf{D}}\cos\left(\frac{x_j}{\sqrt{j+1}}\right) - 1 + \eta_3\lambda_1(x_1)$	–		
P_5	$f_H(\mathbf{z}) = \sum_{j=1}^{\mathbf{D}}(z_j^2 + 1 - \cos(10\pi z_j))$	[20]	$\mathbf{x} \in [-0.1, 0.2]$	2
	$f_i(\mathbf{z}) = f_H(\mathbf{z}) + e_r(\mathbf{z}, \phi_i) + \eta_i\lambda_3(x_1), \quad i = 1, \ldots, N$	[20]		
	$e_r(\mathbf{z}, \phi_i) = \sum_{j=1}^{\mathbf{D}} a(\phi_i)\cos^2\omega(\phi_i)z_j + b(\phi_i) + \pi, \quad i = 1, \ldots, N$	[20]		
	with $a(\phi_i) = \Theta(\phi_i)$, $\omega(\phi_i) = 10\pi\Theta(\phi_i)$, $b(\phi_i) = 0.5\pi\Theta(\phi_i)$	[20]		
	and $\Theta(\phi_i) = 1 - 0.0001\phi_i$	[20]		
	with $\phi = \{10000, 5000, 2500\}$	-		

from: R. Pellegrini et al. Assessing the Performance of an Adaptive Multi-Fidelity Gaussian Process with Noisy Training Data: A Statistical Analysis

28、P3?:

highest to lowest:

$$f_1(\mathbf{x}) = \sum_{i=1}^{D-1} 100\left(x_{i+1} - x_i^2\right)^2 + (1 - x_i)^2$$

$$f_2(\mathbf{x}) = \sum_{i=1}^{D-1} 50\left(x_{i+1} - x_i^2\right)^2 + (-2 - x_i)^2 - \sum_{i=1}^{D} 0.5x_i$$

$$f_3(\mathbf{x}) = \frac{f_1(\mathbf{x}) - 4 - \sum_{i=1}^{D} 0.5x_i}{10 + \sum_{i=1}^{D} 0.25x_i}$$

x: $[-2, 2]^D$

D can be 2\5\10

from: **Multi-Fidelity Sparse Polynomial Chaos and Kriging SurrogateModels Applied to Analytical Benchmark Problems**

31、?not sure:

$$f_{\mathrm{LF}}(x) = \sin(8\pi x),$$
$$f_{\mathrm{HF}}(x) = (x - \sqrt{2}) f_{\mathrm{LF}}^2(x).$$

$$\Omega = [0, 1].$$

from: Multi-fifidelity regression using artifificial neural networks: efficient approximation of parameter-dependent output quantities

32、Colville function (和P2很像)

$$y_h = 100\left(x_1^2 - x_2\right)^2 + (x_1-1)^2 + (x_3-1)^2 + 90\left(x_3^2 - x_4\right) + 10.1\left((x_2-1)^2 + (x_4-1)^2\right) + 19.8(x_2-1)(x_4-1)$$

$$y_l = y_h\left(A^2(x_1, x_2, x_3, x_4)\right) - (A + 0.5)\left(5x_1^2 + 4x_2^2 + 3x_3^2 + x_4^2\right)$$

$$x_i \in [-1, 1]$$

$$A \leq 0.68$$

$$A \geq 0.68,$$

from: A radial basis function-based multi-fidelity surrogate model: exploring correlation between high-fidelity and low-fidelity modelse

33~44 、maolin1~maolin20:

No	LF/HF	Test functions	D	S	r^2
1	HF	$y_H = \frac{sin(10\pi x)}{2x} + (x-1)^4$	1	$[0, 1]^D$	0.733
	LF	$y_L = \frac{sin(10\pi x)}{x} + 2(x-1)^4$			
5	HF	$y_H = \left(x_2 - \frac{5.1x_1^2}{4\pi^2} + \frac{5.1x_1}{\pi} - 6\right)^2 + 10(1-0.125\pi)\cos(x_1) + 10$	2	$[0, 5]^D$	0.771
	LF	$y_L = (1-0.125\pi)\cos(x_1)$			
6	HF	$y_H = 101x_1^2 + 101\left(x_1^2 + x_1^2\right)^2$	2	$[-1, 1]^D$	0.771
	LF	$y_L = x_1^2 + 100\left(x_1^2 + x_1^2\right)^4$			
7	HF	$y_H = [1 - 0.2x_2 + 0.05\sin(4\pi x_2 - x_1)]^2 + [x_2 - 0.5\sin(2\pi x_1)]^2$	2	$[-5, 10]^D$	0.706
	LF	$y_L = [1 - 0.2x_2 + 0.05\sin(4\pi x_2 - x_1)]^2 + 4[x_2 - 0.5\sin(2\pi x_1)]^2$			
8	HF	$y_H = (1.5 - x_1 + x_1x_2)^2 + \left(2.25 - x_1 + x_1x_2^2\right)^2 + \left(2.625 - x_1 + x_1x_2^3\right)^2$	2	$[0, 1]^D$	0.445
	LF	$y_L = (1.5 - x_1 + x_1x_2)^2 + x_1 + x_2$			
10	HF	$y_H = \left[1 - \exp\left(-\frac{1}{2x_2}\right)\right]\frac{2300x_1^3 + 1900x_1^2 + 2092x_2 + 60}{100x_1^3 + 500x_1^2 + 4x_2 + 20}$	2	$[0, 0.5]^D$	0.752
	LF	$y_L = -\frac{2}{5}[y_H(x_1 + 0.05, x_2 + 0.05)] + \frac{1}{4}\left[\begin{matrix} y_H(x_1 + 0.05, \max(0, x_2 - 0.05)) \\ + y_H(x_1 - 0.05, x_2 + 0.05) + y_H(x_1 - 0.05, \max(0, x_2 - 0.05)) \end{matrix}\right]$			
12	HF	$y_H = x_1\exp\left(-x_1^2 - x_2^2\right)$	2	$[-2, 2]^D$	0.828
	LF	$y_L = x_1\exp\left(-x_1^2 - x_2^2\right) + \frac{x_1}{10}$			
13	HF	$y_H = \exp(x_1 + x_2)\cos(x_1x_2)$	2	$[-1, 1]^D$	0.927
	LF	$y_L = \exp(x_1 + x_2)\cos(x_1x_2) + \cos\left(x_1^2 + x_2^2\right)$			
14	HF	$y_H = \sum_{i=1}^3 \alpha_i \exp\left(-\sum_{j=1}^3 A_{ij}\left(x_j - p_{ij}\right)^2\right)$	2	$[-1, 1]^D$	0.927
	LF	$y_L = \sum_{i=1}^3 \exp\left(-\sum_{j=1}^3 A_{ij}\left(x_j - p_{ij}\right)^2\right)$			
15	HF	$y_H = 100\left(\exp\left(-\frac{2}{x_1^{1.75}}\right) + \exp\left(-\frac{2}{x_2^{1.75}}\right) + \exp\left(-\frac{2}{x_3^{1.75}}\right)\right)$	3	$[0, 1]^D$	0.865
	LF	$y_L = 100\left(\exp\left(-\frac{2}{x_1^{1.75}}\right) + \exp\left(-\frac{2}{x_2^{1.75}}\right) + 0.2{*}\exp\left(-\frac{2}{x_3^{1.75}}\right)\right)$			
19	HF	$y_H = \sum_{i=1}^{d-1}\left[100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2\right]$	6	$[-5, 10]^D$	0.761
	LF	$y_L = \sum_{i=1}^{d-1}\left[100(x_{i+1} - x_i)^2 + 4(x_i - 1)^4\right]$			
20	HF	$y_H = 4\left(x_1 - 2 + 8x_2 - 8x_2^2\right)^2 + (3 - 4x_2)^2 + 16\sqrt{x_3 + 1}(2x_3 - 1)^2\sum_{i=4}^8 i\ln\left(1 + \sum_{j=3}^i x_j\right)$	8	$[0, 1]^D$	0.731
	LF	$y_H = 4\left(x_1 - 2 + 8x_2 - 8x_2^2\right)^2 + (3 - 4x_2)^2 + 16\sqrt{x_3 + 1}(2x_3 - 1)^2\sum_{i=4}^8 \ln\left(1 + \sum_{j=3}^i x_j\right)$			

from: Maolin Shi1 & Liye Lv1 & Wei Sun1 & Xueguan Song1. A multi-fidelity surrogate model based on support vector regression

45、Toal:

HF function:

$$y_h = \sum_{i=1}^{10} (x_i - 1)^2 - \sum_{i=2}^{10} x_i x_{i-1}$$

LF function:

$$y_l = \sum_{i=1}^{10} (x_i - A)^2 - (A - 0.65) \sum_{i=2}^{10} i x_i x_{i-1}$$

where $x_i \in [-100,100]$, $i = 1,2,\ldots,10$. The parameter A varies from 0 to 1.

from: Toal DJ (2015) Some considerations regarding the use of multi-fdelity Kriging in the construction of surrogate models. Struct Multidisc Optim 51(6):1223–1245

46~49：

No.	HF/LF	Test functions	D	S
6	HF	$y_h = \left[x_2 - 1.275\left(\frac{x_1}{\pi}\right)^2 + 5\frac{x_1}{\pi} - 6\right]^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos(x_1)$	2	$x_1 \in [-5,10]$ $x_2 \in [0,15]$
	LF	$y_l = \frac{1}{2}[x_2 - 1.275\left(\frac{x_1}{\pi}\right)^2 + 5\frac{x_1}{\pi} - 6]^2 + 10(1 - \frac{1}{8\pi})\cos(x_1)$		
11	HF	$y_h = \sum_{i=1}^{3} 0.3\sin\left(\frac{16}{15}x_i - 1\right) + \left[\sin\left(\frac{16}{15}x_i - 1\right)\right]^2$	3	$[-1,1]^D$
	LF	$y_l = \sum_{i=1}^{3} 0.3\sin\left(\frac{16}{15}x_i - 1\right) + 0.2\left[\sin\left(\frac{16}{15}x_i - 1\right)\right]^2$		
15	HF	$y_h = \sum_{i=1}^{2}\left[\left(x_{4i-3} + 10x_{4i-2}\right)^2 + 5\left(x_{4i-1} - x_{4i}\right)^2 + \left(x_{4i-2} - 2x_{4i-1}\right)^4 + 10\left(x_{4i-3} - x_{4i}\right)^4\right]$	8	$[0,1]^D$
	LF	$y_l = \sum_{i=1}^{2}\left[\left(x_{4i-3} + 10x_{4i-2}\right)^2 + 125\left(x_{4i-1} - x_{4i}\right)^2 + \left(x_{4i-2} - 2x_{4i-1}\right)^4 + 10\left(x_{4i-3} - x_{4i}\right)^4\right]$		
16	HF	$y_h = \sum_{i=1}^{10}\exp(x_i)\left[A(i) + x_i - \ln\left(\sum_{k=1}^{10}\exp(x_k)\right)\right]$ $A = [-6.089, -17.164, -34.054, -5.914, -24.721, -14.986, -24.100, -10.708, -26.662, -22.662, -22.179]$	10	$[-2,3]^D$
	LF	$y_l = \sum_{i=1}^{10}\exp(x_i)\left[B(i) + x_i - \ln\left(\sum_{k=1}^{10}\exp(x_k)\right)\right]$ $B = [-10, -10, -20, -10, -20, -20, -20, -10, -20, -20]$		

和maolin13撞车:

10	HF	$y_h = \cos(x_1 + x_2)\exp(x_1 x_2)$	2	$[0,1]^D$
	LF	$y_l = \cos[0.6(x_1 + x_2)]\exp(0.6x_1 x_2)$		

和maolin19撞车:

13	HF	$y_h = \sum_{i=1}^{5}\left[100\left(x_i^2 - x_{i+1}\right)^2 + \left(x_i - 1\right)^2\right]$	6	$[0,1]^D$
	LF	$y_l = \sum_{i=1}^{5}\left[100\left(x_i^2 - 4x_{i+1}\right)^2 + \left(x_i - 1\right)^2\right]$		

All from: Shuo Wang, et al. A multi-fdelity surrogate model based on moving least squares: fusing diferent fdelity data for engineering design.