QAP Reformulation

- **Without road capacity**
  - **Input**

In the reformulation of QAP, we introduce virtual node 100 as government except buildings and candidate locations as shown in Fig.1. We also add two residential developers and two commercial developers. Agent 5 and 6 represent residential developer of home building 101 and 102 respectively. Agent 7 and 8 represent commercial developer of office building 401 and 402 respectively. Residential developer can go along with building-to-location arc and planning arc. The origin of residential developer of home building is home building, the destination is government. The origin of commercial developer of office building is government, the destination is office building. Of course, the traveler's origin and destination are also home building and office building. Traveler can go on building-to-location arc and physical link. In this example, we assume the cost on building-to-location arc and planning arc is equal to zero. Therefore, we just only consider transportation cost of traveler. For agent of traveler type, an agent represents all travelers between one home building and one office building. For example, the origin of traveler 3 and 4 is both home building 102, their destination is both office building 401. So, we regard traveler 3 and 4 as one agent. According to the rule, we signed all travelers again. We mark traveler 1 as agent 1, traveler 2 as agent 2, traveler 3 and 4 as agent 3 and traveler 5 as agent 4 as shown in 1. The traffic flow matrix between any two buildings of different type is

$$\begin{array}{cc} & \text{Office building } j \\ & \begin{array}{cc} 401 & 402 \end{array} \\ \text{Home builidng } i \begin{array}{c} 101 \\ 102 \end{array} & \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = \left[f_{ij} \right] \end{array}$$

The value on physical link is transportation cost of unit flow. The transportation cost matrix of per unit traffic flow between any two locations of different type is

$$\begin{array}{cc} & \text{Locations for office buildings} \\ & \begin{array}{cc} 301 & 302 \end{array} \\ \text{Locations for home buildings} \begin{array}{c} 201 \\ 202 \end{array} & \begin{bmatrix} 6 & 7 \\ 7 & 5 \end{bmatrix} = \left[c_{kl} \right] \end{array}.$$

The possible transportation cost between locations for each traveler agent $c_{uv}(a)$ is equal to $f_{ij} \times [c_{kl}]$. For agent 1, 2, 3 and 4, the cost matrix is

$$\begin{array}{cc} & \begin{array}{cc} 301 & 302 \end{array} \\ \begin{array}{c} 201 \\ 202 \end{array} & \begin{bmatrix} 7 & 6 \\ 6 & 5 \end{bmatrix} \end{array}, \quad \begin{array}{cc} & \begin{array}{cc} 301 & 302 \end{array} \\ \begin{array}{c} 201 \\ 202 \end{array} & \begin{bmatrix} 7 & 6 \\ 6 & 5 \end{bmatrix} \end{array}, \quad \begin{array}{cc} & \begin{array}{cc} 301 & 302 \end{array} \\ \begin{array}{c} 201 \\ 202 \end{array} & \begin{bmatrix} 14 & 12 \\ 12 & 10 \end{bmatrix} \end{array} \text{ and } \begin{array}{cc} & \begin{array}{cc} 301 & 302 \end{array} \\ \begin{array}{c} 201 \\ 202 \end{array} & \begin{bmatrix} 7 & 6 \\ 6 & 5 \end{bmatrix} \end{array},$$

respectively.

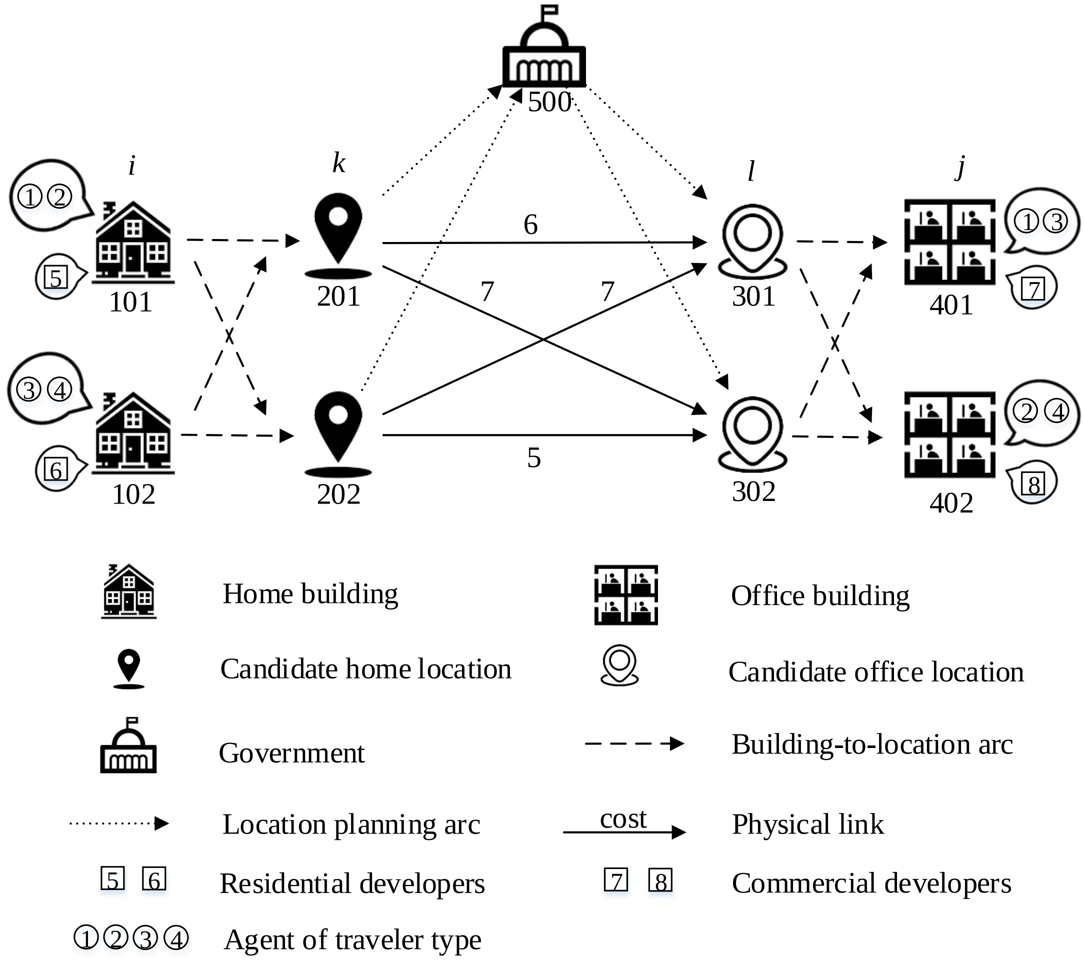

Fig.1. Illustrative example of reformulation QAP without road capacity

- **Output**

The optimal layout is shown in Fig.2. Table 1 shows the optimal path of each agent. Building 101, 102, 301, 302 are assigned location 202, 201, 402, 401 respectively. The total cost is 30.

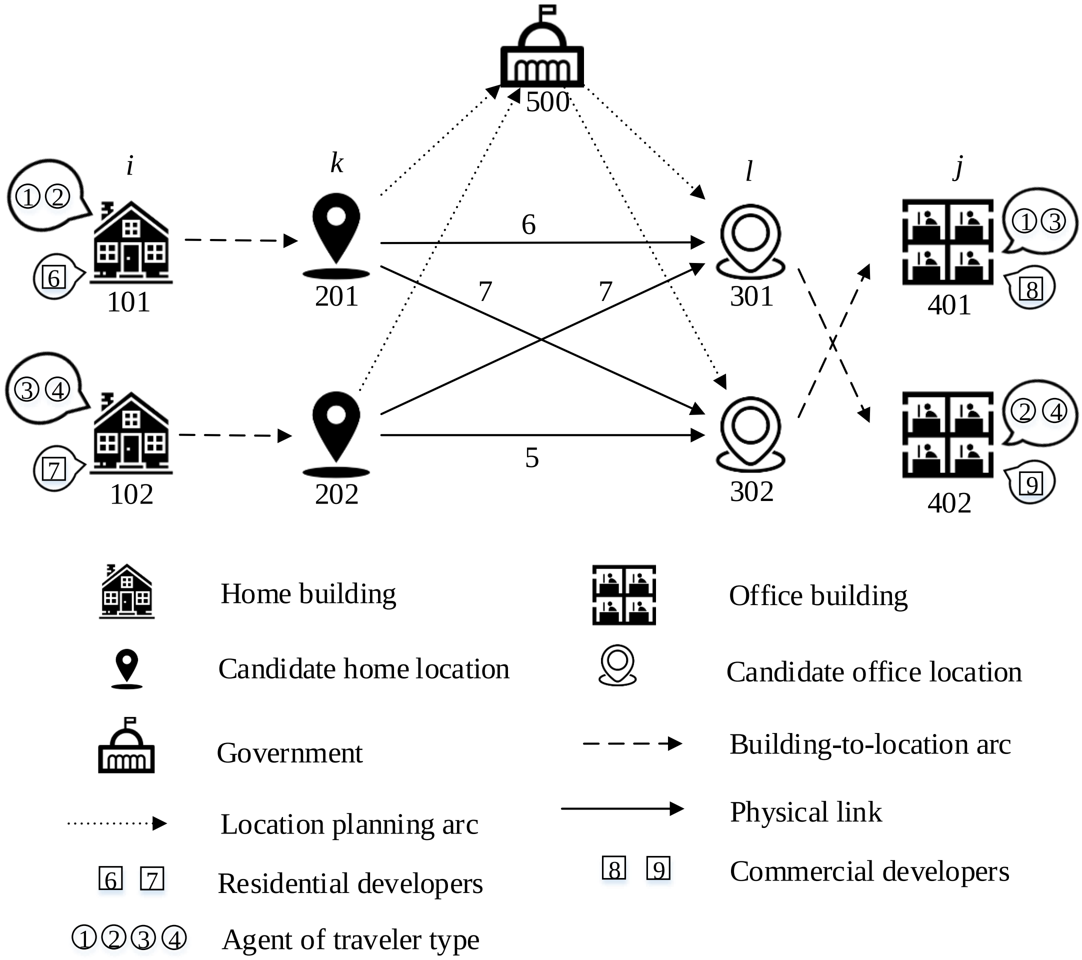

Fig. 2. Optimal solution of the illustrative example without road capacity

Table 1

The optimal choice of travelers of the illustrative example without road capacity.

Agent	Path of agent	Agent	Path of agent
1	101→201→302→401	5	101→201→500
2	101→201→301→402	6	102→202→500
3	102→202→302→401	7	500→302→401
4	102→202→301→402	8	500→301→402

The optimal layout is shown in Fig. . Table 1 shows the optimal path of each agent. Building 101, 102, 301, 302 are assigned location 202, 201, 402, 401 respectively. The total cost is 30.

- **Considering road capacity**
  - **Input**

In order to show the layout affected by traffic capacity of physical link, we regard one traveler as one agent, the rest of the input data is consistent with the previous example. The value on physical link is the transportation cost of unit flow and capacity of physical link. At the same time, we reset the value of parameter in the model. In this example, each agent of traveler represents one traveler.

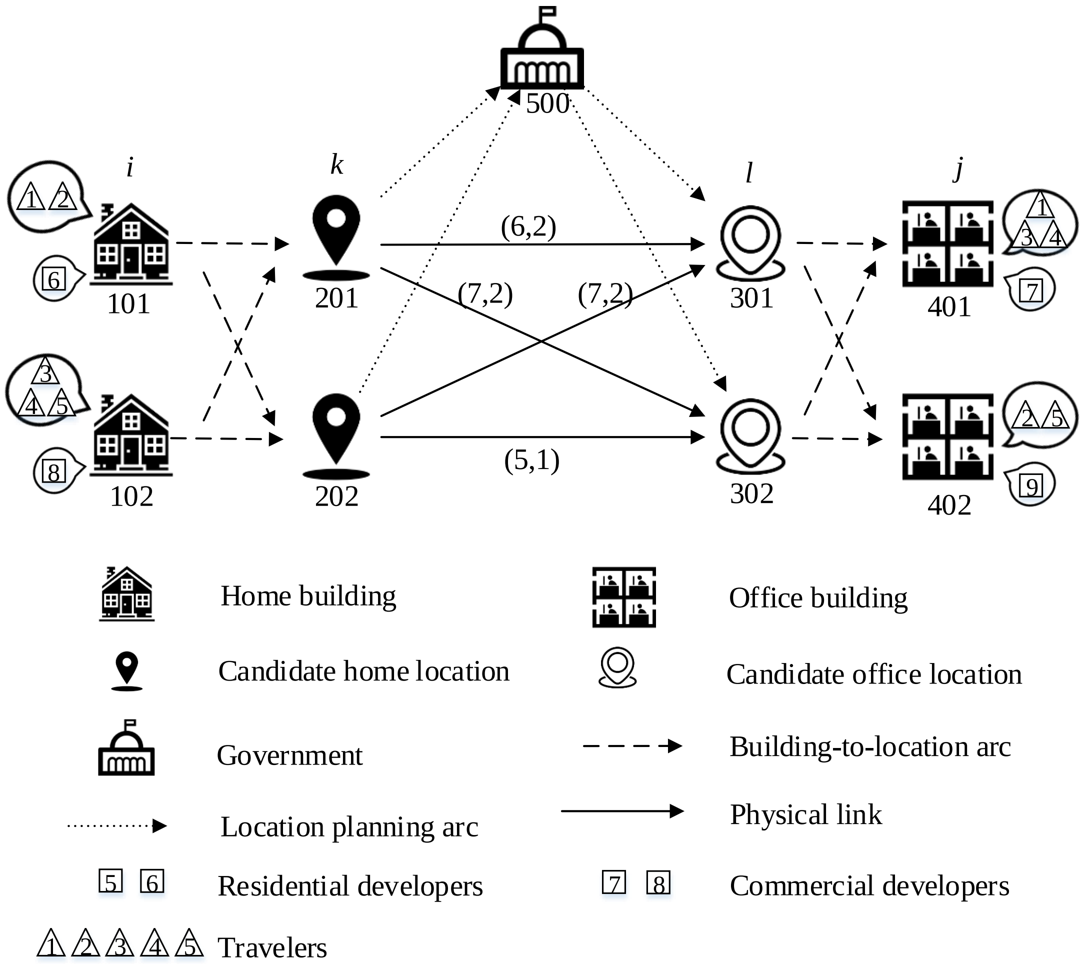

Fig. 3. Illustrative example of reformulation QAP with road capacity

  - **Output**

The optimal layout is shown in Fig. 4. Table 2 shows the optimal path of each agent. Due to the capacity of physical link between location 202 and location 302 is equal to 1. It can't undertake traveler 3 and 4. They must choose another path to get to destination. Therefore, the optimal solution is that building 101, 102, 301, 302 are assigned location 201, 202, 401, 402 respectively. The total cost becomes from 30 to 32.

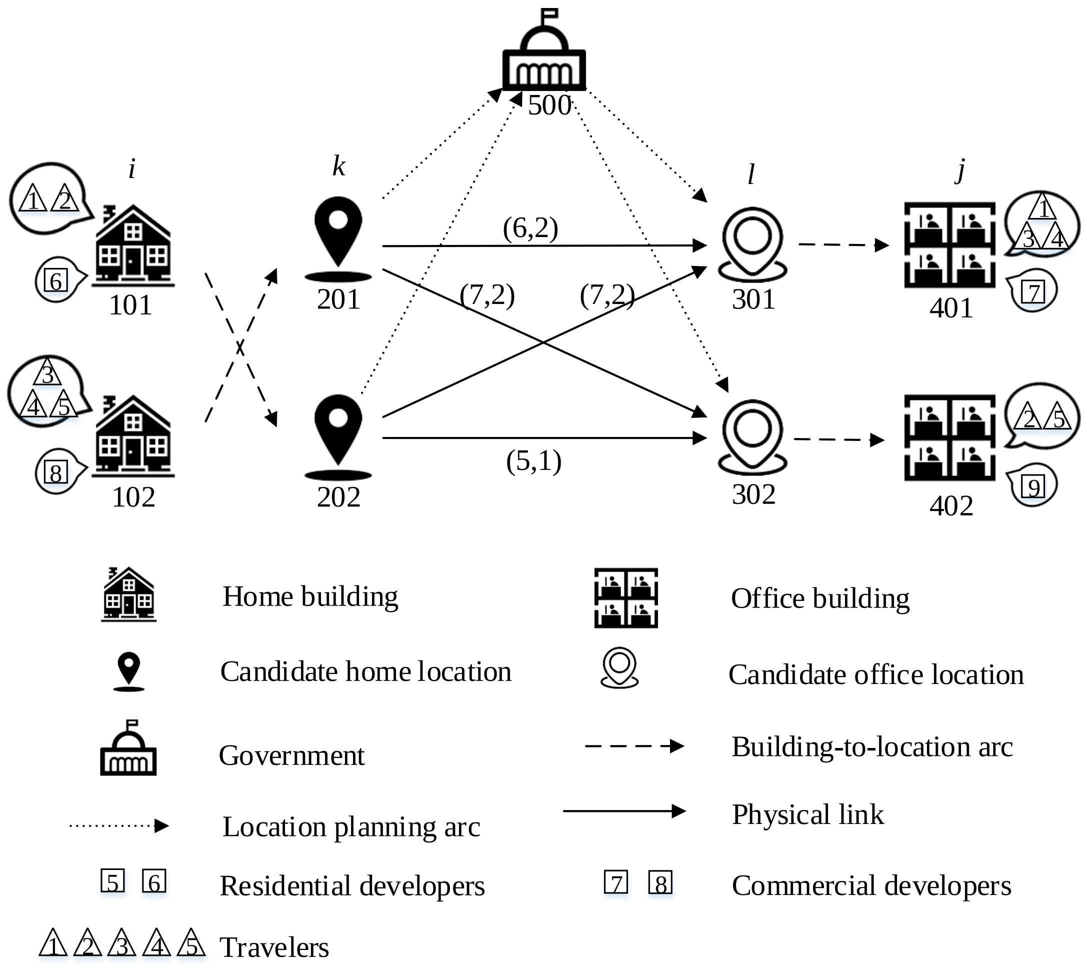

Fig. 4. Optimal solution of the illustrative example with road capacity

Table 2

The optimal choice of travelers of the illustrative example with road capacity.

Agent	Path of agent	Agent	Path of agent
1	101→202→301→401	6	101→202→500
2	101→202→302→402	7	102→201→500
3	102→201→301→401	8	500→301→401
4	102→201→301→401	9	500→302→402
5	102→201→302→402		