Problem set from Y11 3U finals

You know who

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List of problems

1. For $P(x) = (4x + \frac{1}{k})^n$, the following is true:
   The coefficient of x^4 is 8 times that of x^3 and the coefficient of x^2 is 24 times that of x.
   Find n and k.

   (3)

2. For $0 \leq x < \pi$, evaluate x so that $\sin 4x = \sin 2x$.

   (2)

Soln. for problem 1

Understand that $P(x)$ may be expanded as:

$$\sum_{i=0}^{n} \binom{n}{i} (4x)^{n-i} \times k^{-n}$$

And that $\binom{n}{i} = \binom{n}{n-i}$ by the symmetric distribution of Pascal's triangle.
Thus taking the binomial coefficients of x^4, x^3, x^2, x the following is obtained:

- For x^4:

$$\begin{aligned} C &= \binom{n}{4} \times 4^4 \times k^{4-n} \\ &= \frac{n!}{(4!)(n-4)!} \times 256 \times k^{4-n} \\ &= \frac{n(n-1)(n-2)(n-3)}{4!} \times 256 \times k^{4-n} \\ &= \frac{32(n)(n-1)(n-2)(n-3)}{3} \times k^{4-n} \end{aligned}$$

- For x^3:

$$\begin{aligned} C &= \binom{n}{3} \times 4^3 \times k^{3-n} \\ &= \frac{n!}{(3!)(n-3)!} \times 64 \times k^{3-n} \\ &= \frac{(n)(n-1)(n-2)}{3!} \times 64 \times k^{3-n} \\ &= \frac{32(n)(n-1)(n-2)}{3} \times k^{3-n} \end{aligned}$$

- For x^2:

$$\begin{aligned} C &= \binom{n}{2} \times 4^2 \times k^{2-n} \\ &= \frac{n!}{(2!)(n-2)!} \times 16 \times k^{2-n} \\ &= \frac{(n)(n-1)}{2!} \times 16 \times k^{2-n} \\ &= 8(n)(n-1) \times k^{2-n} \end{aligned}$$

- For x:

$$\begin{aligned} C &= \binom{n}{1} \times 4^1 \times k^{1-n} \\ &= 4nk^{1-n} \end{aligned}$$