## Problem set from Y11 3U finals

You know who

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## List of problems

1. For  $P(x) = (4x + \frac{1}{k})^n$ , the following is true:

The coefficient of  $x^4$  is 8 times that of  $x^3$  and the coefficient of  $x^2$  is 24 times that of x.

Find n and k.

(3)

2. For  $0 \le x < \pi$ , evaluate x so that  $\sin 4x = \sin 2x$ .

(2)

## Soln. for problem 1

Understand that P(x) may be expanded as:

$$\sum_{i=0}^{n} \binom{n}{i} (4x)^{n-i} \times k^{-n}$$

And that  $\binom{n}{i} = \binom{n}{n-i}$  by the symmetric distribution of Pascal's triangle. Thus taking the binomial coefficients of  $x^4, x^3, x^2, x$  the following is obtained:

• For  $x^4$ :

$$C = \binom{n}{4} \times 4^4 \times k^{4-n}$$

$$= \frac{n!}{(4!)(n-4)!} \times 256 \times k^{4-n}$$

$$= \frac{n(n-1)(n-2)(n-3)}{4!} \times 256 \times k^{4-n}$$

$$= \frac{32(n)(n-1)(n-2)(n-3)}{3} \times k^{4-n}$$

• For  $x^3$ :

$$C = \binom{n}{3} \times 4^{3} \times k^{3-n}$$

$$= \frac{n!}{(3!)(n-3)!} \times 64 \times k^{3-n}$$

$$= \frac{(n)(n-1)(n-2)}{3!} \times 64 \times k^{3-n}$$

$$= \frac{32(n)(n-1)(n-2)}{3!} \times k^{3-n}$$

• For  $x^2$ :

$$C = \binom{n}{2} \times 4^{2} \times k^{2-n}$$

$$= \frac{n!}{(2!)(n-2)!} \times 16 \times k^{2-n}$$

$$= \frac{(n)(n-1)}{2!} \times 16 \times k^{2-n}$$

$$= 8(n)(n-1) \times k^{2-n}$$

• For x:

$$C = \binom{n}{1} \times 4^1 \times k^{1-n}$$
$$= 4nk^{1-n}$$